An Analysis of Shareholder Agreements∗

Gilles Chemla, Michel Habib, and Alexander Ljungqvist†
UBC and CEPR, LBS, and NYU

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Abstract

We provide an explanation for the main financial clauses in standard shareholder agreements, including joint ventures and venture capital contracts. These clauses are put and call options, drag-along and tag-along rights, pre-emption rights, and initial public offering clauses.

We view these clauses as a response to a double moral hazard problem, whereby the value of the joint enterprise depends on non-contractible investments made by both partners, and whereby each partner may engage in a value decreasing transfer. We allow for uncertainty in that there is a probability that a buyer with a higher valuation than the continuation value appears, and a positive probability that one partner has to sell his shares.

Contract clauses i) minimize transfers from the joint enterprise to one or both partners or the trade buyer while saving costs associated with dispute resolution, and ii) elicit efficient levels of specific investments by allocating state-contingent residual claims. (JEL:G34).

Keywords: Shareholder Agreements; Joint Ventures; Put Options; Call Options; Drag-Along Rights; Tag-Along Rights; Pre-Emption Rights; IPO Clauses..

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†Correspondence to: Gilles Chemla, Faculty of Commerce, University of British Columbia, Vancouver BC V6T 1Z2, Canada. Fax: 604 822 4695, tel: 604 822 8490, e-mail: gilles.chemla@commerce.ubc.ca. Chemla gratefully acknowledges financial support from SSHRC and the Bureau of Asset Management. Part of this paper was written while Habib was visiting HEC Lausanne and ESA Beyrouth. The hospitality of these two institutions is gratefully acknowledged.
1 Introduction

Shareholder agreements are aimed to tailor the respective duties and rights of shareholders whenever the duties and rights reached by law are not believed to be the most appropriate. Shareholder agreements govern mostly privately held companies. They generally identify the shareholders, describe their duties and rights, describe a process to resolve disputes, and for their most part describe restrictions on the transfer of shares as well as the allocation of various types of options. The clauses on the restricted transfer of shares and the allocation of options that form the bulk of shareholder agreements indicate that the shareholders, who are often actively involved in the operations of the company, are believed to have a unique expertise in operating the company and in creating value (Bernstein, 1988, Freedman, 1994, Martel, 1991, Racette, 1998, Richardson, 2000, Stedman and Jones, 1990).

Shareholder agreements generally grant the parties to the contracts the option to put their stakes to their partners in the contracts or to call the stakes of their partners; pre-emption rights that confer precedence to the parties in buying their partners' stakes in case the partners should wish to exit the joint enterprise; drag-along rights that allow the parties to force their partners to join them in selling their stakes to a trade buyer in case of a trade sale; demand rights that allow the parties to force their partners to agree to taking the joint enterprise public in an IPO; and tag-along (or piggy-back) rights that allow the parties to demand of a trade buyer buying their partners' stakes the same.

1In contrast, public companies typically have large memberships where shareholders are free to buy and sell shares on a readily available market. Public companies exhibit a separation between ownership and control, and the role of the owners is mostly restricted to voting on some important decisions. In addition, the stock exchange in which public companies are listed generally requires companies to abide by “Continuing Obligations” that are aimed at protecting shareholders.
treatment as received by their partners.

In fact, a cursory look at standard shareholder agreements, joint venture contracts (Campbell and Reuer, 2000, Herzfeld and Wilson, 1996, Linklaters et al, 1990, Reiter and Shishler, 1999, Scott, 1999), and venture capital contracts (Bartlett, 199., Stedman and Jones, 1990) that govern the three forms of joint enterprise that are closely-held corporations with multiple shareholders, joint ventures, and venture-capital backed new ventures reveals that these clauses are common to these three types of contracts\(^2\). This is not surprising since a joint venture is nothing but a shareholder agreement between two partner firms that hold equity stakes in it, while a venture capital contract is typically a joint venture between the founder and a venture capitalist. The fact that put and call options, pre-emption rights, and drag-along, demand and tag-along rights are common to all three types of contracts suggests that the problems which these arrangements are intended to address are common to the three forms of joint enterprise.

Two main problems appear to arise in joint enterprises. One is the double moral hazard that results from the fact that no party is the unique residual claimant to the value created by the investment he makes in the enterprise (Holmstrom, 1982, Grossman and Hart, 1986, Hart and Moore, 1988, Bhattacharyya and Lafontaine, 1995). The other is the transfer a party can effect from the enterprise to himself. Such transfers are attractive, for their cost is shared with the other parties to the joint enterprise whereas their benefit is received by the transferring party alone. Value-decreasing transfers must be minimized and, while the attainments of the first-best value of the enterprise is

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\(^2\)These clauses appear to be strikingly similar in different countries and legal systems.
precluded by the presence of the problem of double moral hazard, the stakes and clauses are chosen in order to induce the parties to make the investments that maximize the value of the enterprise subject to the problem of double moral hazard. We show that the clauses found in shareholder agreements allow the firm to save on costs of dispute resolution, that emerge in our model as distortions on ex ante investments. Our purpose in this paper is to show how the clauses presented above serve to minimize the costs of transfers and renegotiation.

Joint ventures and venture capital have received much attention. Thus, to take but a few examples, Allen and Phillips (2000), Darrough and Stoughton (1989), McConnell and Nantell (1985), Mohanram and Nanda (1998), Pisano (1989), Oxley (1997), and Rey and Tirole (1998) study various aspects of joint ventures. Similarly, Admati and Pfleiderer (1994), Aghion, Bolton and Tirole (2000), Bergemann and Hege (1998), Berglöf (1994), Cornelli and Yoshia (1997), Gompers (1995), Kaplan and Strömberg (1999), Repullo and Suarez (1998), and Smith (2001) study various aspects of venture capital contracts. Standard shareholder agreements have received much less attention, although the distribution of equity ownership is widely believed to be important in creating value (Jensen and Meckling (1976); Shleifer and Vishny (1986), Burkart, Gromb and Panunzi (1997), Morck et al (1990); etc...). Although most of the analysis of equity-holdings has been developed through a contract theoretic approach, the finance profession has so far not concentrated much on the analysis of existing clauses in shareholder agreements, nor on the optimal design of such clauses. We are not aware of any previous work on the clauses we examine, with the exception of the call option that is embedded in convertible preferred stock\(^3\).

\(^3\)In this paper, we do not attempt to address the optimality of these contracts (see Aghion, Bolton and Tirole, 2001, on the optimality of one restricted type of contract in venture capital.
We proceed as follows. We present the clauses found in shareholder agreements, joint venture contracts, and venture capital contracts in Section 2. We provide the initial setting in Section 3. We address the problem of transfers in Section 4. We address that of bargaining in the case of a trade sale or an IPO in Section 5 and in case of a buyout by one partner in Section 6. We conclude in Section 7. Proofs of most propositions are contained in the Appendix.

2 A Brief Overview of Shareholder Agreements

The most common articles and groups of articles in standard shareholder agreements are the following (Bernstein, 1988, Freedman, 1994, Martel, 1991, Racette, 1998, Richardson, 2000, Stedman and Jones, 1990):

- Definitions and interpretation of terms such as shareholders, board of directors, and fair market value.

- Termination of prior agreements between some or all parties regarding the organization and affairs of the company.

- Warranties and covenants specifying that all shares are free and clear of all claims.

- Provision of control, e.g. designation of the rights and duties of the shareholders in the management of the company, and requirement of prior unanimous consent for major decisions such as the declaration of any dividend and the issuance or sale of shares.

- Operation and financing contracts). Instead, we consider these contracts as given and we consider their effect on investments and transfers.
• Restrictions on transfers of shares: The shareholders commit not to sell, pledge, or charge their shares except with the prior written consent of all other shareholders.

• Survivorship arrangements: Upon the death of any shareholder, the personal representatives of the deceased, “the Vendor”, shall sell of of the shares to the corporation of the Vendor to the Corporation, typically at a price specified in the article on valuation. Life insurance policies will be issued on the other partners to ensure that this article can be enforced.

• Right of first refusal: A partner wishing to sell his stake at a given price to a buyer having made an offer must offer his shares at this price to the other partners. If the other partners have refused this offer, the seller will be allowed to sell to another buyer, and/or

• Pre-emption rights, which oblige a partner wishing to sell its stake in the firm/joint venture to offer its shares to the other partner. Pre-emption rights can take several forms. To the extreme, selling the shares to an outside investor is actually prohibited. Alternatively, they can simply end up offer the shares to the other partner if it comes with an offer equal to an outside investor.

• Put: A partner is granted put options on the shares held by the other partners. The exercise price may be either unspecified, in particular it may be based on a valuation previously agreed upon by the partners, or reached at by an impartial expert, or it may be specified at a premium over the true value. In this latter case, the put option may be allocated only if the partner is proven to have failed to abide by his duties.
• Call: Allocation of call options similarly to the put options, except that the strike price may be either reached at by the partners or an impartial expert, or chosen at a discount (Roberts and Mahesh).

• Valuation: The valuation can be either based upon a previous valuation agreed upon by the partners, or based upon a valuation from an impartial external expert.

• Drag-along rights. In case a partner sells its stake to an outside investor, these allow the investor to buy out the other partner’s stake, at the same price and on the same terms as the first partner. Drag-along rights amount to offering a call option to the outside investor on the remaining partner’s shares when the first partner sells its stake to the investor.

• Tag-along (or piggy-back) rights. In case a partner sells its stake to an outside investor, these allow the other partner to oblige the outside investor to buy the second partner’s stake at the same price and on the same terms as it is buying the first partner’s stake. They can be thought of as a put option attached to the first partner’s stake.

• Initial public offering (IPO) clauses, whereby the two partners agree in advance the circumstances in which they will take the firm/joint venture public, usually make sure that the firm will be taken public once a pre-specified level of profit is achieved or when the firm has a specific need for outside finance, and that if a partner wants to take the firm or the joint venture public by selling some of its stake, the other partner also has to sell its stake.

• Non competition: Each partner agrees not to compete with the company.
• Dispute resolution and arbitration: The partners agree to follow a specified procedure to resolve disputes, including the appointment of an arbitrator.

• General contract and sale provisions

3 The initial setting

3.1 The model

Consider an enterprise jointly undertaken by two parties, $a$ and $b$. Each party must make a non-contractible investment towards the success of the enterprise and must be granted a stake in the enterprise in order to be induced to make such investment. Let $i_a$ denote the investment made by party $a$ and $\gamma_a$ denote $a$’s stake in the enterprise. Let $i_b$ and $\gamma_b = 1 - \gamma_a$ denote the corresponding values for party $b$. Assume both investments have unit marginal cost.

Should the enterprise remain the joint property of the two parties, the enterprise would have value $V_{ab}(i_ia_i, s_w)$, where $s_w$ denotes the state of the world. The multiplicative formulation $i_ia_i$ is intended to capture two widespread characteristics of joint enterprises: i) every party’s investment is essential and ii) the parties’ investments are expected to be equally important.

In addition to remaining joint, the enterprise can be acquired in its entirety by one or the other party or by a trade buyer. Specifically, with probability $p_{ts}$, a trade buyer appears who has a valuation higher than the value of the existing joint enterprise if he can buy the shares of at least one partner. With probability $p_a$ (resp. $p_b$), partner $b$ (resp $a$) has to sell its stake to partner $a$ (resp. $b$). Following standard shareholder agreements, this may be due to the fact that i) one partner deceases, or ii) one partner defaults on its obligations.
and proves to be unable to engage in an appropriate investment level, or iii) the outcome of investment requires a further ex post investment by one partner. The corresponding values of the enterprise are denoted \( V_{ts}(i_a i_b, s_w) \), \( V_a(i_a i_b, s_w) \), and \( V_b(i_a i_b, s_w) \), respectively. The value of the enterprise thus depends on the investments made by the parties and on the realized state of the world.

For simplicity, we assume that these outcomes are mutually exclusive. In other words, we restrict our analysis to four states of the world, \( s_{ab}, s_{ts}, s_a, \) and \( s_b \). These are defined by the nature of the highest-valued use of the enterprise.

For example, the state \( s_{ab} \) is such that:

\[
V_{ab}(i_a i_b, s_{ab}) = \max \left\{ V_{ab}(i_a i_b, s_{ab}), V_a(i_a i_b, s_{ab}), V_b(i_a i_b, s_{ab}), V_{ts}(i_a i_b, s_{ab}) \right\}
\]

State \( s_w, w \in W \equiv \{ab, a, b, ts\} \), has probability \( p_w \). It is not contractible. \( V_w \) is increasing and concave.

The investments \( i_a \) and \( i_b \) are made at time 0, prior to the realization of the state at time 1. At time 2, following the realization of the state, we allow the parties to the enterprise to modify their stakes, and then to engage in transferring value from the enterprise. Any renegotiation or modification of the stakes is assumed to be efficient, e.g. to follow an asymmetric Nash bargaining solution, so that the organizational form with the highest value will emerge in equilibrium. We make the important assumption that transfers are value-decreasing. There are therefore no transfers in the states \( s_a, s_b, \) and \( s_{ts} \) when the enterprise in those states has but a single owner, who bears the entire cost of any value-decreasing transfer.

In contrast, transfers may occur in state \( s_{ab} \) and in state \( s_{ts} \) when the trade buyer buys the shares of one partner, for the enterprise has two owners in those states. These may be the founding parties \( a \) and \( b \), or the trade buyer \( tb \) who
has bought one or the other party’s stake, but not both. We denote $t_a$ the value transferred by party $a$, with $t_b$ and $t_{tb}$ similarly defined. We assume that transfer $t_a$ (resp. $t_b, t_{ts}$) has a unit cost to the value $V_{ab}(i_a i_b, s_{ab})$ (resp $V_{ab}(i_a i_b, s_{ab}), V_{ts}(i_a i_b, s_{ts})$) and generates a non verifiable benefit $B_a(t_a)$ to $a$ (resp. $B_b(t_b)$ to $b$, $B_{ts}(t_{ts})$ to $tb$). That transfers are value-decreasing implies, in the case where the enterprise has remained the joint property of the founding parties $a$ and $b$:

$$B_a(t_a) < t_a, \quad B_a(t_b) < t_b$$

We assume that $B_w()$ is increasing and concave with a derivative in zero lower than 1. Hence, a party may engage in a transfer because the cost of the transfer is shared with the other party, whereas the benefit of the transfer is received exclusively by the party engaging in the transfer.

$$B_a(t_a) > \gamma_a t_a, \quad B_b(t_b) > (1 - \gamma_a)t_b \tag{1}$$

Similar inequalities are assumed to hold in the cases where the trade buyer buys out one or the other party. This simple specification implies that

$$\frac{\max_{\gamma_a} B_a(t_a) + B_b(t_b) - t_a - t_b}{s.t. t_a \in \arg \max B_a(t_a) - \gamma_a t_a, \quad t_b \in \arg \max B_b(t_b) - (1 - \gamma_a)t_b}$$

is attained when $t_a = B_a'^{-1}(\gamma_a), t_b = B_b'^{-1}(1 - \gamma_a)$. We call $\gamma_a^R$ the unique solution.

### 3.2 Results without the clauses

We now turn to the determination of the initial stakes $\gamma_a$ and $\gamma_b$. It is clear that the first-best investments are precluded by the joint ownership of the enterprise,
which makes it impossible for each partner to be the unique beneficiary to the investment he makes (Holmström, 1982, Grossman and Hart (1986), Hart and Moore (1988)).

The equal importance of the investments made by the two parties, as represented by the multiplicative formulation $i_a i_b$, suggests that the parties’ stakes should be identical: $\gamma_a = \gamma_b = \frac{1}{2}$. We show that this is indeed the case:

**Proposition 1** The problem:

$$\max_{\gamma_a, \gamma_b} \sum_{w \in W} p_w V_w (i_a i_b, s_w) - i_a - i_b$$

where:

$$i_a = \arg \max_{\hat{i}_a} \sum_{w \in W} p_w \gamma_a V_w (\hat{i}_a i_b, s_w) - \hat{i}_a$$

and:

$$i_b = \arg \max_{\hat{i}_b} \sum_{w \in W} p_w \gamma_b V_w (i_a \hat{i}_b, s_w) - \hat{i}_b$$

has solution $\gamma_a = \gamma_b = \frac{1}{2}$.

Note that the solution $\gamma_a = \gamma_b = \frac{1}{2}$ does not depend on the characteristics of the states $s_w$, $w \in W$. This suggests that the value of the enterprise cannot be increased by allowing the stakes $\gamma_a$ and $\gamma_b = 1 - \gamma_a$ to depend on the realized state $s_w$, that is $\gamma_a (s_w)$ and $\gamma_b (s_w) = 1 - \gamma_a (s_w)$. We show this in Proposition 2.

**Proposition 2** The value of the enterprise $\sum_{w \in W} p_w V_w (i_a i_b, s_w) - i_a - i_b$ cannot be increased by allowing the parties’ stakes to depend on the realized state.

Proposition 2 is important for it implies that the equal division of the value of the enterprise mandated by Proposition 1 must be maintained in all realized
states. Bargaining between the parties that would alter the parties’ payoffs in state $s_w$ from the desired $\frac{V_{w(i_a, s_w)}}{2}$ for each party is therefore to be precluded. Note that the results of Propositions 1 and 2 pertain when introducing the transfers, since the minimization of the costs from the transfers can be made separately from the maximization of the value created by the investment, and because both problems solve for $\gamma_a = \frac{1}{2}$.

Yet, the possibility of bargaining upon the realization of the state exists in the states $s_{ab}$ and $s_{ts}$, for the value-decreasing transfers in those states presents the parties with the opportunity to bargain over the value created by reduced transfers\(^4\). We denote $\beta_a$ party $a$’s bargaining power and $\beta_b = 1 - \beta_a$ that of party $b$. We show in Sections 5 and 6 that contract clauses can be used to save on bargaining costs in the states $s_{ab}$ and $s_{ts}$ for the purpose of maintaining the equality of the parties’ stakes and payoffs.

4 Transfers in state $s_{ab}$

4.1 Puts/calls with a strike price depending on a future valuation.

In this section, we point out that puts and calls with a strike price that is the value of the firm after the state is revealed, but before transfers take place, minimize the costs of value-destroying transfers.

Consider put options. First consider the case where only partner $a$ can engage in value destroying transfers, i.e. $B_b(.) = 0$. Then, renegotiation would make sure that $b$ transfers his shares to avoid transfers, and $a$ has all shares but one, so that $a$ only engages in no transfer. In addition, it appears that

\(^4\)There may be other scopes for renegotiation. For instance, if we assume that the trade buyer needs to buy both partners’ stakes to realize the value $V_{ts}$, partners may be tempted to form a coalition to obtain a higher bargaining power with the trade buyer (Chemla (2000)).
a put option that grants \( b \) the right to sell its stake to \( a \) at a strike price 
\[ (1 - \gamma_a) V_{ab}(i_a i_b, s_{ab}) \] makes sure that \( a \) does not transfer value from the venture either. Indeed, a transfer by partner \( a \) would then trigger the exercise of the put option by \( b \) and would make \( a \) bear the full cost of his value-destroying transfer.

**Lemma 3** In the joint enterprise state, when only \( a \) can engage in a transfer, granting \( b \) a put option with a strike price 
\[ (1 - \gamma_a) V_{ab}(i_a i_b, s_{ab}) \] fixed at the beginning of period 2 makes sure that \( a \) does not transfer value from the venture. Put options then deter any transfer while saving the need for a costly renegotiation.

The expectation that \( b \) will exercise its put option in case \( a \) engages in a value-decreasing transfer serves to deter \( a \) from engaging in such a transfer. Indeed, a value-decreasing transfer on \( a \)'s part would result in \( b \) exercising its put, making \( a \) bear the full cost of the value destroyed. The put option therefore negates any benefit to value-decreasing transfers and ensures that these do not take place. Note that in this case \( a \) does not have a put option, so that \( a \) will have to keep its shares if it engages in a transfer.

When both \( a \) and \( b \) can engage in a transfer, the situation is more complex. If both partners are given a put option with exercise price \( P_a \) and \( P_b \), it appears that although giving partner \( a \) a put option reduces partner \( b \)'s incentive to transfer value, it also increases partner \( a \)'s incentive to transfer value, as \( a \) can now transfer value from the venture and then sell its shares at the strike price if the strike price is higher than the shares of the value of the venture after the transfer. It is also clear that the strike prices that make sure that \( a \) does not transfer value cannot guarantee that \( b \) does not transfer value. It appears that the best that put options can achieve is to maximize the surplus created in the
joint venture state by minimizing the cost of the transfers. This boils down to
the same program as the renegotiation program, i.e. to allocate ownership so
that the second best outcome that would be reached through renegotiation can
be achieved without renegotiation. If, say, $\gamma_a > \gamma_a^R$, giving a a put option at a
strike price $(\gamma_a - \gamma_a^R)V_{ab}(i_a i_b, s_{wb})$ on a fraction $\gamma_a - \gamma_a^R$ of its shares would elicit
the same second-best outcome as renegotiation, while saving on renegotiation
cost.

**Proposition 4** When $\gamma_a > \gamma_a^R$ (resp. $\gamma_a < \gamma_a^R$), granting partner a a put
option on a fraction $\gamma_a - \gamma_a^R$ (resp. b a put option on a fraction $\gamma_a^R - \gamma_a$) at a
strike price $(\gamma_a - \gamma_a^R)V_{ab}(i_a i_b, s_{wb})$ (resp. $(\gamma_a^R - \gamma_a)V_{ab}(i_a i_b, s_{ab}))$ achieves the
same second best transfer levels as renegotiation while saving the renegotiation
cost.

The effect of call options can be analyzed similarly. If only partner a has a
call option with a strike price lower than $V_{ab}(i_a i_b, s_{ab})$ on an appropriate fraction
of the shares owned by the other partner, it may be tempted to buy out these
shares.

**4.2 Call at a discount, and put at a premium**

In this section, we argue that granting each party the option to sell his stake to
the other party at a premium to the value of the enterprise, or to buy the other
party’s stake at a discount when that other party can be shown in court, with
probability $\pi$, $0 < \pi \leq 1$, to have engaged in a transfer can serve to preclude
such transfers.

That put and call options are used following transfers is suggested by the
following extract from Linklaters, Paines, and Nightingale (p. 60, 1990):

14
Typically the right to compel transfer will arise when one of the parties is in breach of the shareholders' agreement. The non-defaulting party is given the option of buying out the defaulter or of disposing of his own interest to the defaulter ...

That the strike price of these options is at a premium or a discount for the purpose of penalizing the party that has engaged in a transfer is suggested, for the case of call options which have strike price at a discount to firm value, by the following paragraph from Asselin (1998):

As for the forced sale of a defaulting shareholder's shares, the agreement should also normally specify that the shares will be sold at a much lower price than their value so as to encourage shareholders to abide by their commitments.

Recall that $\pi$, $0 < \pi \leq 1$, denotes the probability of showing in court that a transfer has taken place. We now show:

**Proposition 5** Transfers from the enterprise to one or the other party can be precluded by granting the party that has not engaged in a transfer the option to buy the stake of the party that has engaged in a transfer at a discount, or to sell his stake to that same party at a premium.

The expression for $X_C$ in (13) can be interpreted as follows. The first term, $\gamma_a V_{ab}(i_a, i_b, t_a, 0, s_{ab})$, represents the value of party $a$’s stake, which is being bought by party $b$. The second term represents the discount. It is intended to deny party $a$ any benefit to having transferred the amount $t_a$. It equals the net benefit to party $a$ from engaging in the transfer, and is weighted by the reciprocal of the probability of having the option exercised by party $b$, for it is only when the option is exercised that party $a$ bears the cost of the discount.
The expression for \( X_P \) in (14) can be similarly interpreted. The first term represents the value of party \( b \)'s stake, which party \( a \) acquires. The second is identical to and plays the same role as the second term in the expression for \( X_C \). It is added rather than subtracted because \( X_P \) is a payment made by party \( a \), rather than a payment received as was \( X_C \). The third term, \( \gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) - \gamma V_a (i_a i_b, t_a, 0, s_{ab}) \), is new. It represents the decrease in the value of party \( b \)'s stake that results from having the enterprise be owned by party \( a \) alone in state \( s_{ab} \), when it should jointly be owned by parties \( a \) and \( b \). Since this loss is borne by party \( a \), it serves to deter him from engaging in the transfer, and can therefore be subtracted from the premium he has to pay party \( b \).

As all the determinants of the strike prices are known at the outset, the strike prices of the options can be fixed at time 0, at the time of writing the contract. Alternatively, the valuation of the enterprise may be left to time 2, and the computation of the strike prices as functions of that value be specified at time 0.\(^5\) For example, in the case of the strike price \( X_C \) of the call option, the value of the enterprise under present circumstances \( V_{ab} (i_a i_b, t_a, 0, s_{ab}) \) can be estimated at time 2, and the strike price computed as that part of the value that belongs to party \( a \), minus a discount that equals a multiple of party \( a \)'s estimated net gain from engaging in the transfer. This net gain equals the value of \( a \)'s stake in the firm, plus an estimate of the transfer made by \( a \), minus an estimate of what \( a \)'s stake would have been had he not engaged in the transfer. It is a variation on the concept of 'expectations damages.' The net gain is assessed at some multiple of its value, in a manner reminiscent of the concept of 'triple

\(^5\)In this alternative case, the valuation may be done according to some formula (e.g. market-to-book) or it may be left to an independent valuator (Racette, 1998).
damages.

In the case of the strike price $X_P$ of the put option, the value of the enterprise under the sole ownership of party $a$, $V_a(i_a, t_a, 0, s_{ab})$, must also be estimated. That contracts recognize the need for different valuations of what may appear to be the same underlying asset is suggested by the following extract from Cole and Sokol (1999):

*The stockholders agreement may also include provision for the purchase of the stock ..., with provision for different valuations applicable to the purchase, depending on the circumstances.*

It is clear is that the computation of the strike prices relies, in total or in part, on estimates made at time 0. These estimates need not be exact, but they must be unbiased. Such unbiasedness can reasonably be assumed to hold regarding the two founding parties $a$ and $b$, for they are unlikely to undertake a joint enterprise without learning a fair amount about each other. But what about a new party $t_b$, who may buy out one or the other founding party? Suppose the new party is in all respects identical to party $a$, except in that he can engage in a transfer $t_{tb} > t_a$, with:

$$\gamma V_{ab}(i_a, t_{tb}, 0, s_{ab}) + t_{tb} > \gamma V_{ab}(i_a, t_a, 0, s_{ab}) + t_a$$  \hspace{1cm} (2)

and:

$$\gamma V_{ab}(i_a, t_{tb}, t_b, s_{ab}) + t_{tb} > \gamma V_{ab}(i_a, t_a, t_b, s_{ab}) + t_a$$  \hspace{1cm} (3)

Then party $t_b$ may buy out party $a$ for the sole purpose of engaging in transfer. The undesirability of such incoming parties is expressed by the following extract from Racette (1998):

*The objectives of shareholders agreements are ... [to] prevent the sale or...*
transfer of [a shareholder’s] interest to persons deemed undesirable by the remaining shareholders.

We note that the put and call options that party \( b \) holds may not suffice to deter party \( tb \) from engaging in a transfer for these were directed at founding party \( a \) rather than incoming party \( tb \). Indeed, consider the expected exercise of the call option of strike price \( X_C \) by party \( b \). Party \( tb \) has expected payoff:

\[
(1 - \pi) [\gamma V_{ab}(i_a i_b, t_{tb}, 0, s_{ab}) + t_{tb}] + \pi [X_C + t_{tb}]
\]

\[
= (1 - \pi) [\gamma V_{ab}(i_a i_b, t_{tb}, 0, s_{ab}) + t_{tb} - [\gamma V_{ab}(i_a i_b, t_a, 0, s_{ab}) + t_a]]
\]

\[
+ \pi [t_{tb} - t_a] + \gamma V_{ab}(i_a i_b, s_{ab})
\]

\[
> \gamma V_{ab}(i_a i_b, s_{ab}) \quad (4)
\]

where the equality is true by using (13) and the inequality is true by \( t_{tb} > t_a \) and (2). Thus, the expectation that party \( b \) will take party \( tb \) to court fails to deter the latter party from engaging in the transfer \( t_{tb} \).\(^6\)

It is possible that the put option succeeds where the call option has failed and deters party \( tb \) from engaging in the transfer \( t_{tb} \), thereby deterring him from buying party \( a \)’s stake in the first place. However, suppose that party \( b \) is an ‘indispensable’ party. This means that the involvement of party \( b \) is critical to the success of the enterprise.\(^7\) It may imply that party \( b \) is prohibited, at least for some initial period of time, from selling his stake in the enterprise. Evidence

---

\(^6\)A sufficient condition for party \( b \) to take party \( tb \) to court is that:

\[
(1 - \pi) [\gamma V_{ab}(i_a i_b, t_a, 0, s_{ab}) - \gamma V_{ab}(i_a i_b, t_{tb}, 0, s_{ab})]
\]

\[
= \pi [V_b(i_a i_b, t_a, 0, s_{ab}) - V_b(i_a i_b, t_{tb}, 0, s_{ab})]
\]

\[
< \gamma V_{ab}(i_a i_b, t_{tb}, 0, s_{ab}) - \gamma V_{ab}(i_a i_b, t_{tb}, t_{tb}, s_{ab})
\]

The condition has an intuitive interpretation. It states that the additional decrease in the value of party \( b \)’s stake that results from the replacement of party \( a \) by party \( tb \) is smaller if party \( b \) takes party \( tb \) to court than if the former follows the latter in engaging in a transfer.

\(^7\)Formally, \( p_a = p_{ts} = 0 \).
of such prohibition is provided by Linklaters, Paines, and Nightingale (p. 59, 1990):

Restrictions on voluntary transfers will reflect the respective roles played by the parties to the shareholders’ agreement. If one party possesses critical skills and facilities, the withdrawal of that party in the early years of the joint venture would effectively mean dissolution and an absolute prohibition may be appropriate.

In such case, party b will be denied a put option and another arrangement must be devised to deter party tb from acquiring party a’s stake when such an acquisition is motivated solely by party tb’s greater ability to extract value from the enterprise. Pre-emption rights may serve to do so. These are (Linklaters, Paines, and Nightingale, pp. 50-51, 1990):

... articles which, except in the case of a transfer to certain close relatives of a shareholder, required the shareholder to give notice of transfer to the company and then to offer the shares to the other members pro rata. If a member was willing to purchase the shares at their fair value, the shares could not be transferred to a non-member.

Note that by mandating that the departing party’s stake be sold to the remaining party at fair value, pre-emption rights preclude any bidding for the departing party’s stake between the remaining party and a third party. This is desirable in our case. Should there be any such bidding, the third party tb would be in a position to offer a higher price for the stake of the departing party a than would the remaining party b, for tb’s offer would in effect be subsidized by b through the transfer t_{tb} whose cost would in part be borne by b. Indeed,
the value of party a’s stake is $\gamma V_b(i_a i_b, s_{ab})$ to party b, whereas it is:

$$(1 - \pi) \left[ \gamma V_{ab}(i_a i_b, t_{ab}, 0, s_{ab}) + t_{tb} \right] + \pi \left[ X_C + t_{tb} \right]$$

$> \gamma V_{ab}(i_a i_b, s_{ab})$

$> \gamma V_b(i_a i_b, s_{ab})$

to party $t_b$, where the first inequality was established in (4) and the second inequality is true by the definition of state $s_{ab}$. Of course, both founding parties will wish to avoid a sale of party a’s stake to party $t_b$, for the assumption that transfers are value-decreasing implies:

$$V_b(i_a i_b, s_{ab}) > V_b(i_a i_b, t_{ab}, 0, s_{ab}) + t_{tb}$$

Bargaining will therefore ensue, in which party b will offer to increase party a’s payoff in order to induce him not to sell his stake to party $t_b$. But, as noted in Section 3, such bargaining will distort the parties’ investments and is therefore to be precluded. We show that pre-emption rights serve to do so, under the sufficient condition that:

$$V_{ab}(i_a i_b, t_a, 0, s_{ab}) + t_a < V_b(i_a i_b, s_{ab}) \quad (5)$$

The condition can be interpreted to mean that a greater loss in enterprise value is sustained from the transfer $t_a$ than from the inappropriate use of the enterprise in state $s_{ab}$. It is an intuitive condition for it implies that a party would rather own the entire enterprise than share it with a party that engages in transfers.

**Proposition 6** Pre-emption rights serve to preclude bargaining that would be intended to induce a founding party not to sell his stake to a third party who is
in a position to transfer more value from the enterprise than are the founding parties.

5 Sale to a trade buyer in state $s_{ts}$

We now turn to state $s_{ts}$ in which the enterprise should be sold in its entirety to a trade buyer $tb$. As noted in Section 3, bargaining between the founding parties over the value created by such sale would alter the parties’ payoffs and distort their investments. It is therefore to be precluded. Drag-along rights serve to do so, for they entitle one party to force the other party to join the first party in selling his stock to a third party, on the same terms and conditions as the first party. Indeed, we can show:

Proposition 7 Drag-along rights preclude bargaining between the founding parties regarding the sale of the enterprise to a trade buyer.

We note that this interpretation of the role of drag-along rights is confirmed by the following extract from Tannenbaum (1999), in the case of a firm whose ownership is divided between majority and minority investors:

... if a buyer desires to purchase all ownership interests in a business, the minority’s refusal may result in a blocked sale or a reduced price. Majority owners should consider requiring minority owners to participate in the sale in those circumstances through the so-called “drag-along” provisions. These clauses require the minority to sell their shares in the business, on the same terms and conditions as the majority’s sale ...

Drag-along rights clearly conflict with pre-emption rights. In the former case, a party that wishes to sell his stake to a third party can force a reluctant party to sell to the third party. In the latter case, the reluctant party can
preclude the party that wishes to sell from selling to the third party by buying the selling party’s stake at fair value. Note, however, that pre-emption rights will be found where one party is indispensable and $p_{ts} = 0$, whereas drag-along rights will be found where a trade sale is possible and $p_{ts} > 0$. There should therefore be no conflict, for these different clauses should be used under different conditions.

Closely related to drag-along rights are demand rights. These entitle one party to force the other party to agree to taking the enterprise public in an IPO. They can be viewed as the counterpart to drag-along rights in a state in which the value of the enterprise can be increased by taking it public, and bargaining over that increase in value is to be precluded.\footnote{Formally, one would introduce a state $s_{ipo}$ which has probability $p_{ipo}$, and is such that: $V_{ipo}(i_a i_b, s_{ipo}) = \max \left[ V_{ab}(i_a i_b, s_{ipo}), V_a(i_a i_b, s_{ipo}), V_b(i_a i_b, s_{ipo}), V_{ts}(i_a i_b, s_{ipo}), V_{ipo}(i_a i_b, s_{ipo}) \right]$}

We thus have:

**Corollary 8** Demand rights preclude bargaining between the founding parties regarding the IPO of the enterprise.

Tag-along rights are in a sense a complement to drag-along rights. When one founding party, say party $a$, has arranged the sale of his stake in the enterprise to a trade buyer $tb$, tag-along rights allow the remaining founding party $b$ to require the trade buyer $tb$ to purchase party $b$’s stake on the same terms and conditions as party $a$’s stake.

Tag-along rights may be necessary when party $a$ and the trade buyer $tb$ are in a position to collude to divide the increase in value that results from the sale of part of the enterprise to the trade buyer between the two of them, and deny party $b$ any part of that value, or decrease party $b$’s part of that value below that
mandated by his stake $\gamma$ in the enterprise. Foreseeing such collusion, party $b$ will alter his investment $i_b$, thereby decreasing the value of the enterprise. Tag-along rights serve to preclude such an outcome, by ensuring that both founding parties sell their stakes to the trade buyer on the same terms.

We formalize the preceding reasoning as follows. Consider the case where:

$$V_{ts}(i_a i_b, s_{ts}) = V_{ab}(i_a i_b, s_{ts}) + S_{tb}(i_a i_b, s_{ts})$$

The term $S_{tb}(i_a i_b, s_{ts})$ denotes synergy gains that are received exclusively by the trade buyer $tb$ by virtue of him having acquired a stake in the enterprise in state $s_{ts}$.\(^9\) That the synergies gains are received exclusively by the trade buyer $tb$ provide him and one founding party, say party $a$, with the opportunity to collude to increase their payoffs at the expense of the other founding party, party $b$.

Let the trade buyer $tb$ acquire party $a$’s stake at a price:

$$P = \gamma V_{ab}(i_a i_b, s_{ts}) + \frac{3}{4} S_{tb}(i_a i_b, s_{ts})$$

$$> \gamma V_{ab}(i_a i_b, s_{ts}) + \gamma S_{tb}(i_a i_b, s_{ts})$$

$$= \gamma V_{ts}(i_a i_b, s_{ts})$$

where $\gamma = \frac{1}{2}$ is recalled. Both the trade buyer $tb$ and party $a$ are made better by such an acquisition than if the trade buyer were to acquire the entire enterprise at a price $V_{ts}(i_a i_b, s_{ts})$, with each founding party having payoff $\gamma V_{ts}(i_a i_b, s_{ts})$ in proportion to his stake $\gamma$. Party $a$ has payoff $P > \gamma V_{ts}(i_a i_b, s_{ts})$, and the

---

\(^9\)This condition is made to simplify the exposition and is stronger than necessary. It is sufficient that part of the increase in value is not reflected in the value of the enterprise after the sale of party $a$’s stake to the trade buyer.
trade buyer $tb$ has payoff:
\[
\gamma V_{ab}(i_a i_b, s_{ts}) + S(i_a i_b, s_{ts}) - P = \frac{1}{4} S_{tb}(i_a i_b, s_{ts}) > 0
\]

The trade buyer and party $a$’s gains are at the expense of party $b$, whose payoff is:
\[
\gamma V_{ab}(i_a i_b, s_{ts}) < \gamma V_{ts}(i_a i_b, s_{ts})
\]

Clearly, party $b$’s investment $i_b$ will be altered as a result. This can be avoided by requiring the trade buyer to buy both founding parties’ stakes on the same terms, thereby making both founding parties payoffs equal to the desired $\gamma V_{ts}(i_a i_b, s_{ts})$. The purpose of tag-along rights is to impose this requirement on the trade buyer. We have thus shown:

**Proposition 9** Tag-along rights ensure that the gains from a trade sale are shared by the founding partners in proportion to their stakes.

We note that this interpretation of the role of tag-along rights is confirmed by the following extract from Clinton and Richardson (2000), in the case of a firm whose ownership is divided between management shareholders and investors:10

*While co-sale agreements do not prevent management shareholders from selling their shares at a profit, they do force them to share the benefit of their sale with the investors by allowing the investors to include their shares in the sale.*

---

10 Tag-along rights are also referred to as co-sale agreements.
6 Buyouts in states $s_a$ and $s_b$

Note that states $s_a$ and $s_b$ can boil down to state $s_{ab}$ with an optimal allocation of shares being $\{0, 1\}$ and $\{1, 0\}$. In this section we point two further roles for put and call options. One is to reduce bargaining costs. The other one is to ensure an optimal level of specific investment. We now consider the states $s_a$ and $s_b$, in which the enterprise is to be acquired in its entirety by one or the other founding partner.

Initially consider state $s_b$. Party $b$ must buy out party $a$, for such buyout makes possible an increase in the value of the enterprise from $V_{ab}(i_a, i_b, s_b)$ to $V_b(i_a, i_b, s_b)$. In this case too, bargaining between the parties over the value thus created is to be precluded. This can be achieved by granting party $a$ the option to put his stock to party $b$ at fair value, where the value to be estimated is that of the enterprise under the sole ownership of party $b$.\footnote{Recall from Section 4 and from the reference to Cole and Sokol (1999) therein that the parties are able to make ‘provision for different valuations applicable to the purchase, depending on the circumstances.’} The strike price is then $X_{P,a \rightarrow b} \equiv \gamma V_b(i_a, i_b, s_b)$, it is clearly in party $a$’s interest to exercise his put option as his payoff from doing so is:

$$X_{P,a \rightarrow b} = \gamma V_b(i_a, i_b, s_b)$$

$$> \gamma V_{ab}(i_a, i_b, s_b)$$

and party $b$ has payoff:

$$V_b(i_a, i_b, s_b) - X_{P,a \rightarrow b}$$

$$= V_b(i_a, i_b, s_b) - \gamma V_b(i_a, i_b, s_b)$$

$$= \gamma V_b(i_a, i_b, s_b)$$
Each party therefore has the payoff \( \gamma V_b(i_a i_b, s_b) \) that induces him to make the investment that maximizes the value of the enterprise, subject to the constraints impose by the problem of double moral hazard.

Now consider state \( s_a \). A similar reasoning establishes that party \( b \) should be granted the option to put his stock to party \( a \) at fair value, where the value to be estimated is that of the enterprise under the sole ownership of party \( a \).

Is there a danger of either party wishing to use his put option as a threat to increase his payoff in the states in which he is expected to honor the other party’s option? Consider party \( b \) in state \( s_b \). What if he were to threaten to exercise his put option for the purpose of bargaining with party \( a \) over the value \( V_b(i_a i_b, s_b) - V_a(i_a i_b, s_b) \) that would be destroyed if party \( b \) were to carry out his threat? His payoff would then be:

\[
\gamma V_a(i_a i_b, s_b) + \beta_b [V_b(i_a i_b, s_b) - V_a(i_a i_b, s_b)]
\]

where \( \gamma V_a(i_a i_b, s_b) \) is party \( b \)’s payoff from exercising his put option in state \( s_b \), as it equals his share of the fair value of the enterprise under the sole ownership of party \( a \). The total payoff \( \gamma V_a(i_a i_b, s_b) + \beta_b [V_b(i_a i_b, s_b) - V_a(i_a i_b, s_b)] \) would be lower than the payoff \( \gamma V_b(i_a i_b, s_b) \) obtained from honoring party \( a \)’s put option if \( \beta_b < \gamma \). Thus, if \( \beta_b < \gamma \), party \( b \)’s threat to exercise his put option in state \( s_b \) is not credible.

The preceding has assumed that party \( a \) would not exercise his put option in state \( s_b \) if party \( b \) were to threaten doing so. This is indeed the case, as party \( a \)’s payoff would be:

\[
\gamma V_a(i_a i_b, s_b) + \beta_a [V_b(i_a i_b, s_b) - V_a(i_a i_b, s_b)] > \gamma V_b(i_a i_b, s_b)
\]
if he were to enter into bargaining with party $b$. The inequality is true as $\beta_b < \gamma$, $\gamma = \frac{1}{2}$, and $\beta_a = 1 - \beta_b$ combine to imply $\beta_a > \gamma$.

Now consider party $a$ in state $s_a$. If he were to threaten to exercise his option and had precedence over party $b$ in doing so, party $a$ would have payoff:

$$\gamma V_b (i_a i_b, s_a) + \beta_a [V_a (i_a i_b, s_a) - V_b (i_a i_b, s_a)] > \gamma V_a (i_a i_b, s_a)$$

Clearly, party $a$’s threat to exercise his put option in state $s_a$ is credible. The resulting bargaining can be precluded by granting party $b$ precedence over party $a$ in exercising his put option. Party $b$ will therefore exercise his put option in state $s_a$, as desired. As shown above, he will not exercise or threaten to exercise his put option in state $s_b$, despite having precedence over party $a$, for party $b$’s payoff from doing so is lower than his payoff from honoring party $a$’s put option.

The analysis of the case $\beta_b > \gamma$ is similar. As expected, precedence in exercising the put option should now be granted to party $a$.

Evidence that contract clauses specify an order of precedence between the parties is provided by Linklaters, Paines, and Nightingale (1990, p. 61):\(^{12}\)

... the appropriate party will have the right within a specified period of time to purchase all of the other’s shares. Failure to exercise that right will entitle the other to purchase the former’s shares.

We have thus shown:

**Proposition 10** Put options combined with the granting of precedence to the party with lower bargaining power in renegotiation preclude bargaining between

\(^{12}\)Admittedly, the extract applies to call options rather than put options, but this is because it is part of an example.
Can call options achieve the same result? Interestingly, under the present formulation, the answer must be in the negative. This is because an option for one party to buy the other party’s stake at fair value, where the value to be estimated is that of the enterprise under the sole ownership of the buying party, is never strictly in the money for the buying party. The buying party will therefore prefer not to exercise the option and engage in bargaining when his bargaining power is larger than his stake in the enterprise.

To see this, assume that $\beta_b < \gamma$ and consider state $s_a$. If party $a$ were to exercise his option to buy out party $b$ at fair value, party $a$’s payoff would be $\gamma V_a (i_a i_b, s_a)$, as desired. However, suppose that party $a$ were not to exercise his call option, and were instead to bargain with party $b$ over the increase in value $V_a (i_a i_b, s_a) - V_{ab} (i_a i_b, s_a)$ that can be achieved by having party $a$ buy party $b$’s stake. Party $a$’s payoff would then be:

$$\gamma V_{ab} (i_a i_b, s_a) + \beta_a [V_a (i_a i_b, s_a) - V_{ab} (i_a i_b, s_a)] > \gamma V_a (i_a i_b, s_a)$$

where the inequality is true by the fact that $\beta_b < \gamma$ implies $\beta_a > \gamma$. Party $a$’s reluctance to exercise his call option in state $s_a$ extends to the case where party $b$ threatens to exercise his call option in that state, for party $a$’s payoff is then:

$$\gamma V_b (i_a i_b, s_a) + \beta_a [V_a (i_a i_b, s_a) - V_b (i_a i_b, s_a)] > \gamma V_a (i_a i_b, s_a)$$

The preceding result holds regardless of the manner in which precedence is assigned, if at all. In the case where $\beta_b > \gamma$, it is party $b$ who will be reluctant to exercise his call option in state $s_b$. Call options therefore fail to preclude
bargaining under the present formulation of the value of the enterprise.\footnote{It is of course conceivable that call options preclude bargaining under a different formulation of the value of the enterprise, but we were unable to identify such a formulation.}

**Proposition 11** Assume that the value of the joint enterprise in state \(s_a\) is also an increasing and concave function of a specific investment \(j_a\) (resp. \(j_b\)). Call options of the appropriate strike prices combined with the appropriate order of precedence serve to elicit the efficient investments \(j_a^*\) and \(j_b^*\) from partners \(a\) and \(b\), respectively.


7 Conclusion

This paper explains the prevalent use of six prevalent financial clauses in shareholder agreements, including joint ventures and venture capital contracts. The rationale developed is based on a double moral hazard problem, so that the clauses affect the partners’ incentives to engage in non-contractible investment and value-destroying transfer. A natural question is then how ownership of the firm or the venture jointly owned by two or more partners would be affected when considering physical investment. In particular, it may be the case that one partner invests a lot of physical capital, while the other partner’s investment is mostly non-contractible. In that case, the partner who makes mostly contractible, physical investment can be compensated for this investment with a very simple contract, which specifies its payoff as a function of physical investment. In particular, securities with low sensitivity to payoffs, like debt, can be written to ensure that the partner which makes physical investment gets an appropriate rate of return on its investment. On the other hand, non-contractible
investment from the partner who makes non-contractible investment is subject to opportunism from the other partner. Reasonable levels of non-contractible investment can then be reached by providing the partner with highly payoff-sensitive securities like equity, call options, and preemption rights.

In addition, shareholder agreements and joint venture contracts also contain clauses other than those governing the transfer of shares, but that should be of interest to financial managers and academics. These clauses, that are not necessarily common to joint venture contracts and more general shareholder agreement contracts, include the liability of each partner, the establishment of different classes of shares, clauses protecting the minority partner (e.g. major decisions require the agreement of both partners), provisions to break deadlocks in 50:50 split of control, the establishment and the workings of the board of directors and staffing, the place of incorporation, the name and role of each partner and guarantor (e.g. if one partner has set up a new subsidiary with few assets that owns the shares in the joint venture, a guarantee from the parent firm may be required), restrictions on the partners (e.g. non-competition, non-solicitation, and confidentiality requirements), intellectual property rights, and the country law governing the firm or the joint venture in international agreements (Campbell and Reuer (2000)).
8 Appendix

8.1 Proof of Proposition 1

The investments $i_a$ and $i_b$ are the solutions to the first-order conditions:

\[ i_b \sum_{w \in W} p_w \gamma_a V_{w,1} (i_a i_b, s_w) - 1 = 0 \]  

(6)

and:

\[ i_a \sum_{w \in W} p_w (1 - \gamma_a) V_{w,1} (i_a i_b, s_w) - 1 = 0 \]  

(7)

where $V_{w,1} (i_a i_b, s_w) = \frac{\partial V_w(i_a i_b, s_w)}{\partial (i_a i_b)}$. Combining equations (6) and (7), we have:

\[ \sum_{w \in W} p_w V_{w,1} (i_a i_b, s_w) = \frac{1}{(1 - \gamma_a)i_a} = \frac{1}{\gamma_a i_b} \]  

(8)

This implies:

\[ \frac{\partial i_a}{\partial \gamma_a} = \frac{i_b}{(1 - \gamma_a)^2} + \frac{\gamma_a}{1 - \gamma_a} \frac{\partial i_b}{\partial \gamma_a} \]  

(9)

The stake $\gamma_a$ is the solution to the first order condition:

\[ \left[ \sum_{w \in W} p_w V_{w,1} (i_a i_b, s_w) \right] \frac{\partial (i_a i_b)}{\partial \gamma_a} = \frac{\partial i_a}{\partial \gamma_a} + \frac{\partial i_b}{\partial \gamma_a}. \]  

(10)

Using (6) and (7), (10) becomes:

\[ (1 - \gamma_a)^2 \frac{\partial i_a}{\partial \gamma_a} + \gamma_a^2 \frac{\partial i_b}{\partial \gamma_a} = 0 \]

Substituting (9) into the preceding equation, we obtain:

\[ \frac{\partial i_b}{\partial \gamma_a} = - \frac{i_b}{\gamma_a} \]  

(11)

and, using (8) again:

\[ \frac{\partial i_a}{\partial \gamma_a} = \frac{i_a}{1 - \gamma_a} \]  

(12)

\[ ^{14}\text{Recall that } \gamma_b = 1 - \gamma_a. \]
In addition, (10) can be rewritten

\[
\frac{1}{(1-\gamma_a)i_a} \left[ -\frac{i_a i_b}{\gamma_a} + \frac{i_a i_b}{1-\gamma_a} \right] = \frac{i_a}{1-\gamma_a} - \frac{i_b}{\gamma_a}
\]

which implies \((2\gamma_a - 1)i_a = 2\gamma_a - 1\). Since \(i_a = 1\) cannot be an optimum, this implies that \(\gamma_a = \frac{1}{2}\).

Alternatively, it is sufficient to note that the strict concavity of the objective function ensures that a local maximum is the global maximum and to complete the proof by showing that \(\gamma_a = \frac{1}{2}\) is a local maximum of the objective function.

At \(\gamma_a = \frac{1}{2}\), \(i_a = i_b \equiv i\) by equation (8) and the objective function has value:

\[
\sum_{w \in W} p_w V_w (i^2, s_w) - 2i
\]

Now increase \(\gamma_a\) to \(\gamma'_a = \frac{1}{2} + \delta\). Using equations (12) and (11), \(i_a\) and \(i_b\) now equal:

\[
i'_a = i + \frac{i}{1 - \frac{1}{2} \delta} = i [1 + 2\delta]
\]

and:

\[
i'_b = i - \frac{i}{\frac{1}{2} \delta} = i [1 - 2\delta]
\]

respectively. The objective function now has value:

\[
\sum_{w \in W} p_w V_w (i'_a i'_b, s_w) - i'_a - i'_b
\]

\[
= \sum_{w \in W} p_w V_w \left(i^2 \left[1 - 4\delta^2\right], s_w\right) - 2i
\]

\[
< \sum_{w \in W} p_w V_w \left(i^2, s_w\right) - 2i \]
8.2 Proof of Proposition 2

We shall show that the solutions $\gamma_a(s_w)$ and $\gamma_b(s_w) = 1 - \gamma_a(s_w)$ are such that

$$\gamma_a(s_w) = \gamma_b(s_w) = \frac{1}{2} \quad \forall s_w, w \in W.$$ 

The investments $i_a$ and $i_b$ are the solutions to:

$$i_b \sum_{w \in W} p_w \gamma_a(s_w) V_{w,1}(i_a i_b, s_w) - 1 = 0$$

and:

$$i_a \sum_{w \in W} p_w (1 - \gamma_a(s_w)) V_{w,1}(i_a i_b, s_w) - 1 = 0$$

Consider party $a$‘s stake in state $s_{ab}$, $\gamma_a(s_{ab})$. It is the solution to:

$$I_b \left( \sum_{w \in W} p_w (1 - \gamma_a(s_w)) V_{w,1}(i_a i_b, s_w) \right) \frac{\partial i_a}{\partial \gamma_a(s_{ab})} + I_a \left( \sum_{w \in W} p_w \gamma_a(s_w) V_{w,1}(i_a i_b, s_w) \right) \frac{\partial i_b}{\partial \gamma_a(s_{ab})} = 0$$

... 

Using the Implicit Function Theorem to derive $\frac{\partial i_a}{\partial \gamma_a(s_{ab})}$ and $\frac{\partial i_b}{\partial \gamma_a(s_{ab})}$ and
substituting into the preceding equation, we obtain:\textsuperscript{15}

\[
\sum_{w \in W} p_w (1 - \gamma_a (s_w)) V_{w,1} (i_a, s_w) = \sum_{w \in W} p_w \gamma_a (s_w) V_{w,1} (i_a, s_w)
\]

The same equation is obtained when considering the stakes \(\gamma_a (s_a), \gamma_a (s_b),\) and \(\gamma_a (s_{ab}).\) That the first-order condition is identical for the stakes in the different states suggests that these stakes are equal. Setting \(\gamma_a (s_w) = \gamma_a \forall s_w\) we can solve for \(\gamma_a = \frac{1}{2} = \gamma_b.\)

8.3 Proof of Proposition 4

Consider the problem faced by a party, say party \(a,\) that is contemplating transferring the amount \(t_a\) from the enterprise. The party expects to be required have to sell his stake at a price \(X_C\) to party \(b\) with probability \(\pi,\) or to have to buy party \(b\)'s stake at price \(X_P\) with that same probability. For party \(a\) to refrain from engaging in such transfer when party \(b\) is expected to do so, the prices \(X_C\) and \(X_P\) must be such that:

\textsuperscript{15}We have:

\[
\frac{\partial i_a}{\partial \gamma_a (s_{ab})} = \frac{1}{\Delta} \left[ -i_a^2 p_{ab} V_{ab,1} (i_a, s_{ab}) \sum_{w \in W} p_w V_{w,11} (i_a, s_w) \right. \\
- \left. i_a p_{ab} V_{ab,1} (i_a, s_{ab}) \sum_{w \in W} p_w \gamma_a (s_w) V_{w,1} (i_a, s_w) \right]
\]

and:

\[
\frac{\partial i_b}{\partial \gamma_a (s_{ab})} = \frac{1}{\Delta} \left[ i_a^2 p_{ab} V_{ab,1} (i_a, s_{ab}) \sum_{w \in W} p_w V_{w,11} (i_a, s_w) \right. \\
+ \left. i_b p_{ab} V_{ab,1} (i_b, s_{ab}) \sum_{w \in W} p_w (1 - \gamma_a (s_w)) V_{w,1} (i_b, s_w) \right]
\]

where:

\[
\Delta = i_a^2 \sum_{w \in W} p_w \gamma_a (s_w) V_{w,11} (i_a, s_w) \sum_{w \in W} p_w (1 - \gamma_a (s_w)) V_{w,1} (i_a, s_w)
\]

\[
- \left[ \sum_{w \in W} p_w (1 - \gamma_a (s_w)) V_{w,1} (i_a, s_w) + i_a \sum_{w \in W} p_w (1 - \gamma_a (s_w)) V_{w,11} (i_a, s_w) \right]
\]

\[
\times \left[ \sum_{w \in W} p_w \gamma_a (s_w) V_{w,1} (i_a, s_w) + i_a \sum_{w \in W} p_w \gamma_a (s_w) V_{w,11} (i_a, s_w) \right]
\]

and \(V_{w,11} (i_a, s_w) = \frac{\partial V_{w,1} (i_a, s_w)}{\partial (i_a, s_w)}.\)
\[
(1 - \pi) [\gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + t_a] + \pi [X_C + t_a] \\
\leq \gamma V_{ab} (i_a i_b, s_{ab}) \\
\Leftrightarrow X_C \leq \gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) - \frac{1}{\pi} \left[ \begin{array}{c} \gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + t_a \\ -\gamma V_{ab} (i_a i_b, s_{ab}) \end{array} \right] \\
\equiv X_{C,1} \quad (13)
\]

and:

\[
(1 - \pi) [\gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + t_a] \\
+ \pi [V_b (i_a i_b, t_a, 0, s_{ab}) + t_a - X_P] \\
\leq \gamma V_{ab} (i_a i_b, s_{ab}) \\
\Leftrightarrow X_P \geq \gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + \frac{1}{\pi} \left[ \begin{array}{c} \gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + t_a \\ -\gamma V_{ab} (i_a i_b, s_{ab}) \end{array} \right] \\
- [\gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) - \gamma V_a (i_a i_b, t_a, 0, s_{ab})] \\
\equiv X_{P,1} \quad (14)
\]

where \(\gamma \equiv \frac{1}{2}\). The preceding assumes that party \(b\) will take party \(a\) to court rather than remain in the joint enterprise and follow party \(a\) in engaging in a transfer. For this to be the case, the strike price \(X_C\) must be such that:

\[
(1 - \pi) \gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + \pi [V_b (i_a i_b, t_a, 0, s_{ab}) - X_C] \\
\geq \gamma V_{ab} (i_a i_b, t_a, t_b, s_{ab}) + t_b \\
\Leftrightarrow X_C \leq \gamma V_b (i_a i_b, t_a, 0, s_{ab}) - \frac{1}{\pi} \left[ \begin{array}{c} \gamma V_{ab} (i_a i_b, t_a, t_b, s_{ab}) + t_b \\ -\gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) \end{array} \right] \\
- [\gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) - \gamma V_b (i_a i_b, t_a, 0, s_{ab})] \\
\equiv X_{C,2} \quad (15)
\]
The corresponding condition for the strike price $X_P$ is:

$$(1 - \pi) \gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + \pi X_P \geq \gamma V_{ab} (i_a i_b, t_a, t_b, s_{ab}) + t_b$$

$$\Leftrightarrow X_P \geq \gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + \frac{1}{\pi} \left[ \frac{\gamma V_{ab} (i_a i_b, t_a, t_b, s_{ab}) + t_b}{\gamma V_{ab} (i_a i_b, t_a, 0, s_{ab})} \right]$$

$$\equiv X_{P,2}$$

(16)

We assume that $X_{C,1} \leq X_{C,2}$ and $X_{P,1} \geq X_{P,2}$. We set $X_C \equiv X_{C,1}$ and $X_P \equiv X_{P,1}$.

The strike prices $Y_C$ and $Y_P$ that are intended to preclude party $b$ from transferring the amount $t_b$ are similarly defined.

### 8.4 Proof of Proposition 5

The fair value of party $a$'s stake equals either $\gamma V_{ab} (i_a i_b, s_{ab})$ (its value under present circumstances) or $\gamma V_b (i_a i_b, s_{ab})$ (its value under the sole ownership of party $b$). In either case, it is no more than party $a$'s payoff under present circumstances. The expectation that party $b$ would exercise his pre-emption rights in case party $a$ should wish to sell his stake is therefore sufficient to deter $a$ from doing so. It remains to show that party $b$ will exercise his pre-emption rights if and when given notice of party $a$’s desire to sell his stake. This is clearly the case when the fair value of party $a$’s stake is appraised at $\gamma V_b (i_a i_b, s_{ab})$.

---

16 A necessary condition for this to be the case is that:

$$\gamma V_{ab} (i_a i_b, t_a, 0, s_{ab}) + t_a \geq \gamma V_{ab} (i_a i_b, t_a, t_b, s_{ab}) + t_b$$

This in effect assumes that a transfer is more valuable to a party when that party alone engages in transfers.

17 Note that the solution $X_C = 0$ or $X_P = \infty$ would serve the desired purpose, but such extreme values would provide very strong incentives for party $b$ falsely to claim that party $a$ has engaged in a transfer.
When the fair value of party \(a\)’s stake is appraised at \(\gamma V_{ab}(i_a i_b, s_{ab})\), party \(b\) will compare his payoff when he exercises his pre-emption rights:

\[
V_b(i_a i_b, s_{ab}) - \gamma V_{ab}(i_a i_b, s_{ab})
\]

to his payoff when he does not, and therefore expects party \(a\) to sell his stake to party \(tb\), who will engage in the transfer \(t_{tb}\) despite party \(b\)’s expected exercise of his call option. This second payoff is:

\[
(1 - \pi) \gamma V_{ab}(i_a i_b, t_{tb}, 0, s_{ab}) + \pi [V_b(i_a i_b, t_{tb}, 0, s_{ab}) - X_C] = (1 - \pi) \gamma V_{ab}(i_a i_b, t_{tb}, 0, s_{ab}) + \pi V_b(i_a i_b, t_{tb}, 0, s_{ab}) - t_a \\
< (1 - \pi) V_{ab}(i_a i_b, t_a, 0, s_{ab}) + \pi V_b(i_a i_b, t_{tb}, 0, s_{ab}) + t_a \\
\leq V_{ab}(i_a i_b, t_a, 0, s_{ab}) + \pi V_b(i_a i_b, t_{tb}, 0, s_{ab})
\]

where the first inequality is true by the fact that \(t_{tb} > t_a\) and transfers are value-decreasing, the second by the fact that \(t_{tb} > t_a\) and the definition of state \(s_{ab}\), and the third by assumption (5).

\[\Box\]

### 8.5 Proof of Proposition 6

It suffices to show that at least one party will wish to exercise his drag-along rights. In case the drag-along rights are exercised, each party has payoff \(\gamma V_{ts}(i_a i_b, s_{ts})\), for the requirement that both parties’ stakes be sold and that they be sold on the same terms ensures that the value of the enterprise is divided between the
parties in proportion to their stakes. In contrast, in case bargaining occurs, the
parties’ payoffs are, from Section 3:

\[ \beta_a V_{ts}(i_a i_b, s_{ts}) + (\gamma - \beta_a) V_{ab}(i_a i_b, s_{ts}) \]

and:

\[ (1 - \beta_a) V_{ts}(i_a i_b, s_{ts}) + (\beta_a - \gamma) V_{ab}(i_a i_b, s_{ts}) \]

Clearly, unless \( \beta_a = \gamma \), one party will be made worse off by bargaining. This
party will therefore find it in his interest to exercise his drag-along rights.■
9 References


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