Distribution in Developing Markets

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Abstract

We model a dynamic game between a producer (principal) and a distributor (agent). The agent is liquidity constrained so she cannot pay upfront for the goods to be sold. Furthermore, the proceeds from the sale are private information. Neither party can commit to enforceable contracts. We set up the problem and characterize the optimal self-enforcing agreement between the players. Implementation is remarkably simple; the principal provides the agent with an initial endowment of the good and charges a fixed price for all additional units. The model is motivated by how to distribute higher priced items (cook stoves, solar lights etc.) in rural Africa.

1 Introduction

In distributing products to the end users, producers or wholesalers commonly rely on small retailers. This is particularly true in developing economies where reaching the rural population requires shopkeepers or traveling salespeople to transport the goods for sale. In addition to the logistical complications of reaching a widespread population with poor infrastructure the lack of efficient credit markets and poor enforcement of contracts hinder the retail sector.

This paper characterizes the optimal self-enforcing agreements in these environments. They have a remarkably simple implementation. The principal provides an initial endowment of the good to the agent and permits the agent to buy additional units at a fixed price. The initial transfer helps partially overcome the agent's liquidity constraint. The agent is free to walk away but has incentive to develop a long-run relationship with the principal in order to grow the retail business

We first consider a setting in which there is no uncertainty in the outcome of sales and hence no private information on the agent's side. In this case, the sole inefficiency is that the initial quantity sold is below the efficient level and it takes some time for the agent to grow the business to the efficient level. The agent's liquidity constraints require that the principal provide the agent with some form of credit but, since the agent can walk away at any point, any initial credit translates into a minimal amount of surplus that must be transferred to the agent. In determining the optimal amount of the endowment, the principal trades off the inefficiency of operating below capacity versus having to share more of the pie with the agent.

The dynamics of the relationship in this case have two distinct phases: *building up* and *cashing in*. In the building up phase, the agent reinvests his profits to grow his business by buying more and more product from the principal until reaching the efficient level. At that point, the agent cashes in on the relationship and from that point forward consumes profit in excess of inventory costs.

When final demand and hence the agent's cash flows are risky, the initial transfer and fixed price structure remains optimal during the building up stage. However, there is an additional inefficiency in the market. Although the agent grows the business *on average*, the growth rate is tied to the realization of the cash flows. Following good realizations, the agent will grow faster but if he has some bad luck he will be forced to shrink his business. A sufficiently long enough string of negative realizations can lead to the termination of the relationship.

The dynamics of the relationship now have three phases. As before there is the initial building up phase although now growth is stochastic, after high realizations the quantity allocated to the agent will increase more rapidly than after low ones. Once the agent reaches full capacity, the insurance phase begins. In this region, the agent does not consume nor does the quantity grow. Rather, the agent purchases insurance from the principal in the form of continuation value. This insurance protects against inefficient investment following a string of poor realizations. The quantity of insurance that the agent purchases depends both on the relative impatience of the agent and on the distribution of risky cash flows. If the agent is not more impatient than the principal then there is no cost of delaying consumption for the agent and accelerating it for the principal. Hence, in this case the agent will buy as much insurance as the participation constraint of the principal allows. The more impatient the agent relative to the principal, the more costly is the insurance. In such cases, the optimal agreements have less (and potentially zero) insurance. Once the agent has accumulated the optimal amount of insurance, the cashing in phase (and therefore consumption) begins.

Unlike in the case with no uncertainty, the fact that the agent starts consuming does not imply a steady state. The principal must constantly provide additional incentives for the agent to report truthfully. A sequence of negative shocks will force the agent to first stop consuming and eventually operate below the efficient level. In the long run the only absorbing steady state is when after a long sequence of poor realizations the relationship is terminated.

Development Agencies

This research was motivated by discussions with David Levine who wanted to expand the use of more efficient cook stoves and study the problems of adoption by the final consumers (Levine and Cotterman, 2012). Indeed, trade between the final consumers and the small retailers in our model share similar problems than the trades between the retailers and producers/wholesalers.

Development agencies many times try to introduce new products –efficient stoves, solar lights, water filters, mosquito nets, etc – When introducing a novel product as those listed above the development agencies believe they are great. Unfortunately these are typically experience goods and the final consumers are uncertain about their usefulness until they try them. The liquidity problem is exacerbated by this uncertainty. The small retailers thinking it will be hard to get the final consumers to try this product in the first place might be less willing to invest into carrying these new goods.

In some cases development agencies have simply freely distributed the goods. This is a costly way to introduce a product to the market. In addition, the expectation that they might receive the product for free severely distorts the development of a natural market for the goods (Oster, 1995). In addition when the good is freely distributed it will typically will not be used by those that value it the most and hence if one looks at use a few months after the delivery the results have in many instances been disappointing (Ashraf et al., 2010).

What this paper suggests is that it would be best for the NGOs to give the free goods to small shopkeepers for the them to sell at a profit and if indeed there is demand for the product, allow them to buy more units to sell. Note that since the initial units are given for free to the small retailers they have nothing to lose from trying to sell them. They can also offer them on credit to their customers since they did not have to take any loan to buy these units in the first place. If indeed the sale of these units is a success, then the market will take-off without much need for additional subsidies. If, on the other hand, the product is a poor match for the local conditions, the feedback from the small retailers will reveal it quickly providing more opportunities to tailor the product to the local needs.

1.1 Field Work

We plan to test our theory in the field. We are currently in the design phase of our study which we plan to conduct with BRAC-Uganda (a large NGO).

Overview

Community Health Promoters (CHPs) distribute a variety of development-promoting products to Uganda's poor while providing income to the CHPs. We would like to understand barriers to using CHPs in distributing higher-priced goods such as solar lights. If successful, distributing such products can improve health and education while also increasing CHPs incomes.

If BRAC offered solar lights we hypothesize that liquidity constraints and uncertainty about demand would inhibit CHPs from purchasing lights and trying to resell them. Providing financing to consumers and distributors should help overcome liquidity constraints. At the same time, if consumers are uncertain about how well the light works for them then they will turn down financing. If CHPs anticipate low demand, or even are uncertain about the level of demand, they will also turn down solar lights – even if offered with financing.

Lending the solar light to both distributors and consumers permits them to experience its benefits and overcome uncertainty. The goal of this study is to document growth impediments and distinguish between these two hypotheses in part by testing several mechanisms for overcoming them. We hope to solidify a partnership with BRAC in this research agenda.

Initial interviews

Before starting the pilot we would like to interview a few dozen CHPs to get a sense of the impediments to growth in sales of current products and the perceptions about potential demand for higher-priced items, in particular solar lights. We expect that these interviews as well as with feedback from BRAC will lead to various modifications of the proposal that we have outlined below.

Pilot Study

Product: After some consideration we think solar lights are an ideal product for the study. They would be a new product for the CHPs and there is flexibility in the price range. Lights are not particularly heavy nor bulky so they should not pose great logistical challenges. Lastly, lights clearly have large benefits but they will serve as an experience good for the end users. That is, they need to use and experience it in order to realize its advantages.

Initial Study: Our overall goal is to tease out what are the impediments for the sale of new higher priced items. We wish to test if the mechanisms we propose will overcome these difficulties and furthermore, do so in a way that provides the right incentives for the distributor (CHP in this case) not to default or run away with the goods.

Phase I: Make the solar lights available in seven regional centers. In each center, we would invite CHPs to attend an informational session explaining and demonstrating the benefits of the new product. We will then offer the CHPs a regional center the opportunity to sell the new product in one of three different methods. Each regional center will offer one of the three methods.

1. In three centers we would offer the solar light at a wholesale price of P_1.

2. In two centers we would offer the solar light at a wholesale price of P_2. We would also offer credit to purchase up to ten solar lights, with additional trade credit as the first purchases are repaid.

3. In two centers we would offer CHPs several solar lights at no charge, so they can test demand in their own community. They can then purchase additional lights at the wholesale price P_3.

Note: We may either set $P_1 = P_2 = P_3$, or charge $P_2, P_3 > P_1$ in attempt to recover the financing costs.

Phase II: After each offer has been in place at the regional center for two months, we will add a loan to the CHPs in group 1 who were originally just offered the lights without a loan.

We would also survey CHPs to determine their eagerness to sell these items and what constraints, such as their liquidity constraints, consumer liquidity constraints, uncertain demand, and so forth reduce their demand for inventory. These constraints may be driven by their own situation or arise from the preferences and constraints of their consumers. After two months we will re-survey CHPs to determine if experience has changed their views on the market solar lights, particularly for CHPs provided an initial loan (group 2) or inventory (group 3).

1.2 Related Theoretical Literature

Our paper is closely related to a large literature on optimal contracting and also to the literature on self-enforcing or relational contracts.

Within the contracting literature the closest papers are Clementi and Hopenhayn (2006); DeMarzo and Fishman (2007b); DeMarzo and Sannikov (2006); DeMarzo et al. (Forthcoming); Quadrini (2004); Abuquerque and Hopenhayn (2004); Li (October 2009). They all look at the problem of an entrepreneur that needs funding for a new venture. Although we do not allow for commitment in our model, one feature of the optimal contract that is present in many of these papers and shows up in our setup as well, is the idea that the agent starts consuming only after a sufficiently long stream of positive outcomes. This results from a combination of liquidity constraints and risk neutrality. At the beginning it is best to increase the agent's continuation value and have him reinvest all his cash in the venture, only when the liquidity constraint stops binding is it worth to start consuming.

Starting with DeMarzo and Fishman (2007a) a number of papers in this literature have looked at implementations where the agents get a loan and a credit line. When the cash flows are low the agent draws on the credit line to be able to meet the coupon payments. When the cash flows are high the agent pays back the credit line. If the credit line is repaid and cash is left the agent consumes. If, on the other hand, the agent has reached the limit of the credit line and still cannot meet the coupon payments then the firm is liquidated. Although the dynamics are similar, our implementation is quite different. One could possibly define our arrangement in terms closer to a credit line but thinking of it a direct transfer is a more direct way to recognize that any initial transfer to the agent provides at least that value to the agent. Even if mathematically equivalent, the framing might have an effect in the field. From an experimental point of view it will be interesting to test if the framing of the implementation has an important effect in terms of uptake.[1]

Within the relational contracts literature the closest paper is Thomas and Worrall (1994). Their model is very similar to our benchmark model without informational asymmetries (see Section 3). As we do, they show that in this case, the agent's continuation value must increase at a rate given by the agent's discounting.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 solves for the optimal self enforcing agreements in a benchmark case in which there are no informational asymmetries. In Section 4 we characterize the optimal agreements with informational asymmetries. Lastly, in Section 5 we consider what happens when we require the agreements to be renegotiation proof.

2 Model

There is a principal and an agent that can interact repeatedly over time $t = 0, 1, ...\infty$. The principal has the capacity to deliver to the agent up to $\bar{K}$ goods per period at a marginal cost of γ. The goods can then be sold on the local market. The principal does not have the technology to access this market and must contract with an agent to make the sale to the end consumer. The proceeds from the sale of the goods are risky and privately observed by the agent. We denote the per unit proceeds or cashflow in period t by p_t. Where p_t is independently and identically distributed over time with distribution F, support in $[p_{\min}, p_{\max}]$ and mean $\bar{p}$.

The agent has no capital and enjoys limited liability. Both the principal and the agent can walk way at any time they choose. If the relationship is terminated by either party the agent gets V_{out} and the principal Π_{out}.

At the beginning of each period the principal gives the agent some amount k_t of goods for the agent to sell. The agent then sells the goods and realizes a cashflow of $Y_t \equiv p_t k_t$. The agent then reports $\hat{p}_t k_t$ to the principal and makes a transfer T_t to the principal and consumes the rest. We then move on to the next period.

Both the principal and the agent are risk neutral and care about the expected present value of their per period payoffs. They discount the future at δ_P and δ_A respectively and we

[1] When the initial transfer is cast as a loan rather than as a gift we speculate that the agents will be less willing to carry this new product.

assume $\delta_P \geq \delta_A$. We use Π and U to denote these values:

$$\Pi = E\left[\sum_0^{\tau} \delta_P \left(T_t - \gamma k_t\right) + \delta_P^{\tau} \Pi_{out}\right]$$
$$U = E\left[\sum_0^{\tau} \delta_A \left(p_t k_t - T_t\right) + \delta_A^{\tau} V_{out}\right]$$

Where τ denotes the (random) time at which the relationship is terminated. We make the following assumption on primitives, which guarantees the set of equilibria is non-trivial.

Assumption 2.1. $\delta_A \bar{p} - \delta_P \gamma > 0$

The history of the game observed by the principal until time the start of period t is: $h_t^P \equiv \{k_i, \hat{p}_i, T_i\}_{i=0}^{t-1}$. When choosing an action in period t, the history of the game for the agent is $h_t^A \equiv \{h_t^P, k_t, p_0, ..., p_t\}$. A pure strategy for the principal is a sequence of functions $\left\{\sigma_t^P\right\}_{t=0}^{\tau}$ which determine the quantity of goods k_t to give to the agent (or to terminate the relationship) as a function of h_t^P. A pure strategy for the agent is a sequence of functions $\left\{\sigma_t^A\right\}_{t=0}^{\tau}$ which for each period determine the agents report $\hat{p}_t$, transfer T_t and decision whether terminate the relationship as a function of h_t^A. Mixed strategies are defined in the convential way and denoted by Σ^P and Σ^A.[2]

There is no external enforcement of contracts and hence the relationship will be government by self-enforcing agreements. An agreement is said to be self-enforcing if the strategy pair that implements it $\left\{\Sigma^P, \Sigma^A\right\}$ is a PBE of the game. We focus on the set of pareto efficient PBE, which can be parameterized by the expected continuation utility of the agent (Abreu et al., 1990), denoted by v.[3]

3 Benchmark Case: Deterministic Cash flows

We will start by focusing on the simpler case in which there is no uncertainty. For this we assume $p_{\min} = p_{\max} = \bar{p}$. The key feature of this environment is not the lack of uncertainty, but rather the lack private information on the agent's side that it implies. This exercise will help to isolate the two key frictions within the model as well as provide intuition for the more general setting.

Recalling that we use v to denote the continuation utility of the agent, an agreement consists of three functions $K(v), T(v), V(v)$ where the principal's maximization problem can

[2]e.g., Σ_t^A is a distributions over pure-strategies as a function of the h_t^A.

[3]In Section 5, we use a stronger notion by requiring the equilibrium to be renegotiation proof.

be stated recursively as:

$$\Pi(v) = \max_{K(\cdot),V(\cdot),T(\cdot)} [T(v) - \gamma K(v) + \delta_P \Pi(V(v))]$$

subject to

$$T(v) \in [0, \bar{p}K(v)] \tag{1}$$

$$K(v) \in [0, \bar{K}] \tag{2}$$

$$\delta_A V(v) - T(v) \geq V_{out} \equiv 0 \tag{3}$$

$$\bar{p}K(v) - T(v) + \delta_A V(v) \geq v \tag{4}$$

For ease of exposition, we have normalized the agents outside option to zero. The liquidity constraint (1) implies that neither agent nor principal has access to a bank. The only mechansim by which the consumption good is created is through selling to consumers. Also, it is worth noting that when we solve for the optimal agreement we will assume there is no hidden savings. In Section 5, we argue that it is robust to introducing a private storage technology for the agent. A key insight from our analysis is the simplicity with which the optimal agreement can be implemented.

Proposition 3.1. *The optimal agreement can be chacterized by a pair $(k_0^*, q^*) \in \mathbb{R}^2$. The principal provides the agent with a fixed initial endowment of the good, k_0^*, and sets a fixed price, q^*, at which the agent can purchase all future units of the good*

The proof goes as follows. First, the promise keeping constraint (4) must bind for all v such that $K(v) > 0$ otherwise the principal could reduce K and save the marginal cost. Next, consider continuation values $v < \bar{V} \equiv \bar{p}\bar{K}$, since $\delta_A V(v) - T(v) \geq 0$, the promise keeping constraint (4) implies that $K(v) < \bar{K}$, i.e., the capacity constraint (2) is slack. For all such v, it must be that $\Pi'(v) \geq \frac{-\gamma}{\bar{p}}$, since the principal can always give a little bit more capital to the agent. If $\Pi'(V(v)) \geq -\frac{\gamma}{\bar{p}}$ (recall that $\frac{-\gamma}{\bar{p}} > \frac{-\delta_A}{\delta_P}$), then the liquidity constraint (1) must bind, otherwise the principal can do better by increasing T by ϵ and and V by $\frac{\epsilon}{\delta_A}$. Thus,

$$T(v) = \bar{p}K(v), \ \forall V(v) < \bar{V} \tag{5}$$

The promise keeping constraint then implies that the agents continuation value must grow at a rate proportional to his discount factor:

$$V(v) = \frac{1}{\delta_A} v, \ \forall V(v) < \bar{V} \tag{6}$$

Finally, the participation constraint (3) must bind since otherwise the agreement could be improved upon by increasing both $K(v)$ and $T(v)$. Therefore,

$$\delta_A V(v) = T(v), \; \forall V(v) < \bar{V} \tag{7}$$

Solving (5)-(7), we get that

$$K(v) = \frac{v}{\bar{p}}, \; T(v) = v, \; V(v) = \frac{1}{\delta_A} v, \quad \forall V(v) < \bar{V} \tag{8}$$

Notice that when the agent transfers an amount $T(v) = v$ to the principal, he obtains a continuation value of $\frac{1}{\delta_A} v$, which means that in the next period he will receive a quantity of $K(\frac{v}{\delta_A}) = \frac{v}{\delta_A \bar{p}}$. Thus, the agent is effectively paying a price of $q^* = \delta_A \bar{p}$ for each unit of capital received.

This continues until the agent reaches a continuation value of $V(v) = \bar{V}$ (or $v = \delta_A \bar{V}$), at which point, in all subsequent periods, investment is fully efficient and the capacity constraint becomes binding ($K(\bar{V}) = \bar{K}$). The principal can no longer incent the agent by promising to grow the agents future capital stock and thus the agent begins to consume. For $v \in \left[\delta_A \bar{V}, (1 + \delta_A) \bar{V}\right]$, continuation value is constant (equal to $\bar{V}$) and the agent consumes the necessary amount so that promise keeping holds. That is,

$$V(v) = \bar{V}, \;\; T(v) = (1 + \delta_A) \bar{V} - v, \; K(v) = \bar{K}, \;\; \forall \; v \in \left[\delta_A \bar{V}, (1 + \delta_A) \bar{V}\right] \tag{9}$$

For $v > (1 + \delta_A) \bar{V}$, L binds at the lower boundary (i.e., $T(v) = 0$) and the principal begins promising the agent even more continuation value. Alternatively, if we relax the lower bound constraint on transfers and assume that the principal has access to a bank, then (9) continues to hold for $v > (1 + \delta_A) \bar{V}$.

The optimal agreement involves two phases. The first is a *building up* phase in which the agent builds up capital at a rate proportional to her discount factor. Although the agent does not consume during this period, her continuation value increases in this region as consumption nears. The second phase begins when the investment reaches its efficient level. The agent *cashes in* her built up continuation value and begins to consume. These two distinct phases arise in part due to the agent's linear preferences. With strictly concave utility, the optimal agreement would have similar features, but the distinction between the two regions would be less dramatic as the agent would consume prior to reaching the efficient investment level. We maintain linear preferences so as to preserve the ease with which the

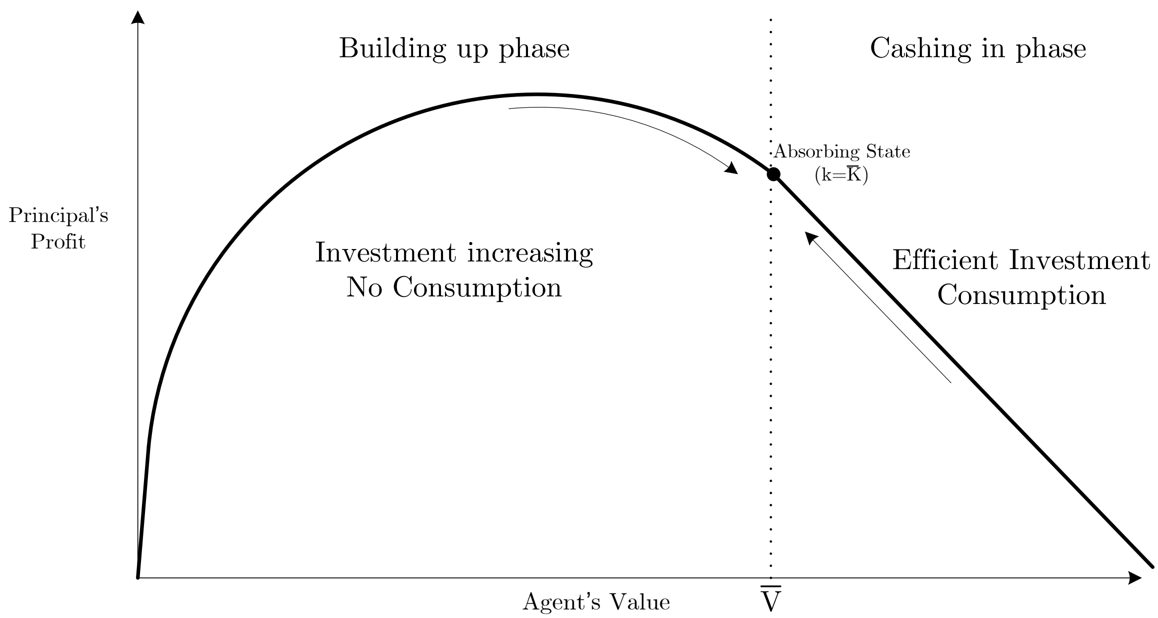

Figure 1: Illustration of the two phases in the optimal agreement for the benchmark case with deterministic cash flows.

agreement can be implemented.

4 Analysis of the General Model

The set up is the same as in the benchmark case, the only adjustment needed in the notation is that the agreement is now also contingent on the agents report. We assume (and later verify) that the optimal agreement can be implemented in pure strategies and restrict attention to K, V, T deterministic. In this set up, an agreement is a triple $K(v), T(\hat{p}, v), V(\hat{p}, v),$ where the agent begins the period with some promised continuation utility v and the timing of the stage game is as follows.

- $t = 0$: Principal chooses some amount of $K(v)$
- $t = 0.1$: Agent sells $K(v)$, obtains $pK(v)$ and reports $\hat{p}$ to the principal.[4]
- $t = 0.2$: Principal requires agent to make a transfer $T(\hat{p}, v)$ in exchange for a promised utility of $V(\hat{p}, v)$ which the agent will realize in the next period and so discounts at δ_A
- $t = 1$: Repeat with the new state variable $V(\hat{p}, v)$

In order for the agreement to be feasible, it must satisfy constraints similar to the benchmark case with the addition of an incentive compatibility constraint. We restate them in

[4]The "reports" are purely a technical/pedagogical construct. Only the transfers are crucial.

the following definition.

Definition 4.1. *We say that an agreement, (K, V, T), is feasible if it satisfies:*

$$T(p, v) \leq pK(v) \tag{L}$$

$$K(v) \in [0, \bar{K}] \tag{C}$$

$$\delta_A V(p, v) - T(p, v) \geq \delta_A V(\hat{p}, v) - T(\hat{p}, v) \tag{IC}$$

$$\delta_A V(p, v) - T(p, v) \geq V_{out} \tag{PC}$$

$$\mathbb{E}[pK(v) - T(p, v) + \delta_A V(p, v)] \geq v \tag{PK}$$

where all inequalities must hold for all $p, \hat{p} \in [p_{\min}, p_{\max}], v \geq V_{out}$. Notice that we have relaxed slightly the liquidity constraint by allowing the principal to access financing. This has little effect on the form of the optimal agreement but will simplify the exposition.

We say that a constraint binds at v if the constraint holds with equality for all p (and $\hat{p}$ where appropriate). Let Φ denote the set of all feasible agreements and let ϕ denote an arbitrary element of Φ. The principal's maximization problem is then

$$\Pi(v) = \max_{\phi \in \Phi} E_p \left[T(p, v) - \gamma K(v) + \delta \Pi_P(V(p, v)) \right] \tag{10}$$

Before characterizing the optimal agreement, we establish two useful, though nearly trivial, results.

Lemma 4.2 (Existence)**.** *An optimal feasible agreement exists.*

Proof. The feasible set of self-enforcing agreements is non-empty and closed. □

We use Π^{SB} to refer to the solution to (10) and Φ^{SB} to denote the set of feasible agreements (T, K, V), such that Π achieves its maximum.

Lemma 4.3 (Monotonicity)**.** *In the second best self-enforcing agreement, Π^{SB} is monotonically decreasing.*

Proof. $\Gamma(v') \subseteq \Gamma(v)$ for all $v \leq v'$ □

Corollary 4.4 (Differentiability)**.** *Π^{SB} is differentiable almost everywhere.*

The next lemma characterizes when the the liquidity constraint (L) binds in relation to the marginal cost of providing the agent with continuation value.

Lemma 4.5. *In any optimal agreement, $T(p,v) = pK(v)$ if and only if $\Pi'(V(v,p)) \geq -\frac{\delta_A}{\delta_P}$.*

Proof. First, we argue that if $\Pi'(V(v,p)) > \frac{-\delta_A}{\delta_P}$ then $L(V(v,p))$ must bind. If $L(V(v,p))$ were not binding then we can increase $T(p,v)$ by some small $\varepsilon > 0$ and $V(v,p)$ by $\frac{\varepsilon}{\delta_A}$. This would not violate any constraint and would lead to an improvement for the principal since $\varepsilon + \frac{\Pi'(V(v,p)) \times \frac{\varepsilon}{\delta_A}}{\delta_P} \geq 0$. The converse follows the same logic in reverse; the principal would benefit by reducing $T(p,v)$ and $V(p,v)$. □

Definition 4.6. *Let $V^L \equiv \inf \left\{ v : \Pi'(v) \geq \frac{-\delta_A}{\delta_P} \right\}$*

As in the benchmark case, for all $v > V^L$, it is optimal for the agent to consume an amount such that his continuation value is V^L (provided (L) does not bind at zero). Some additional properties of the optimal agreement in the benchmark case also apply here. In particular,

Lemma 4.7. *For all $V(v,p) \in [0, \bar{V})$, (L) binds (thus $\bar{V} \leq V^L$) and the (expected) continuation value grows at a rate equal to $\frac{1}{\delta_A}$. Further, (C) is slack if $v < \bar{V}$ and (PK) binds for all v such that $K(v) > 0$.*

We now turn to addressing the two remaining constraints: (IC) and (PC).

Lemma 4.8. *Incentive compatibility constraint must bind for all v.*

Proof. If it was slack for some (v, p, p') than it would be violated for (v, p', p). □

This implies that, for any feasible agreement, $I(v,p) \equiv \delta_A V(v,p) - T(v,p)$ is independent of p. The intuition is clear, the benefit to the agent of making a transfer must not depend on the size of the transfer made. Note that the participation constraint (PC) is equivalent to $I(v,p) \geq 0$. Recall that the participation constraint was binding for all $V(v) \leq \bar{V}$ in the benchmark case. Whether (PC) binds here depends on the severity of the asymmetric information problem.

Lemma 4.9. *If $p_{\min} > \gamma$ then (PC) binds for all (p,v) such that $V(p,v) < \bar{V}$.*

The argument is the same as above. Since (C) is slack for all such V, if (PC) was also slack, the principal could simply increase K by ε and increase T by (at least) $p_{\min}\varepsilon$ and be better off. For $p_{\min} < \gamma$, the same argument no longer applies. The principal must provide additional incentives in order to recover higher marginal costs.

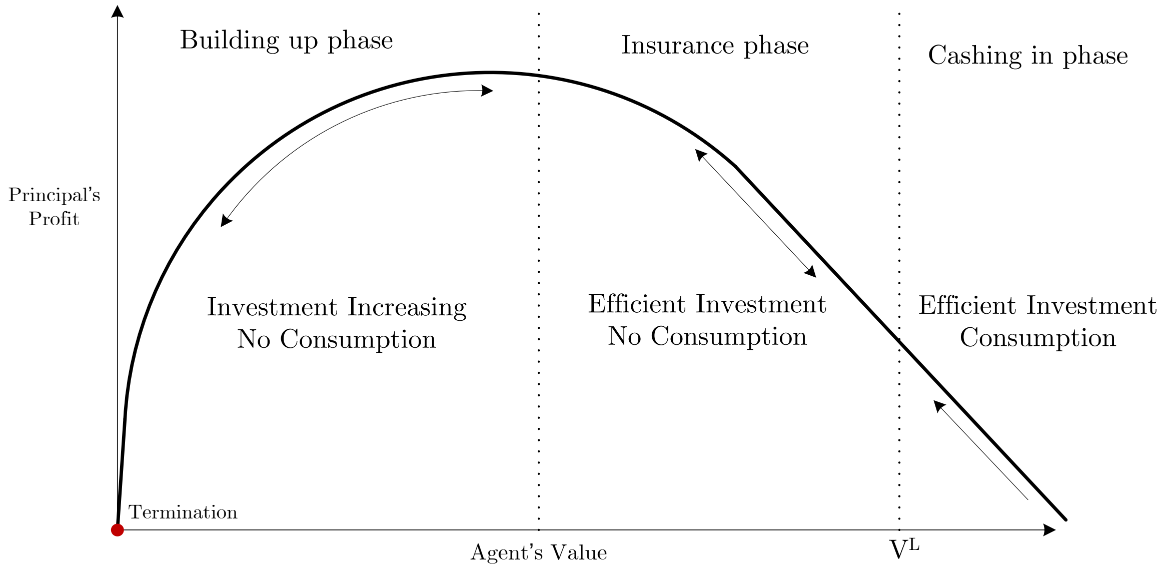

Figure 2: Illustration of the optimal self-enforcing agreement with risky cash flows.

4.1 Characterization of Optimal Self-Enforcing Agreements

Before stating a formal result, let us describe the intuition for the optimal agreement. The dynamics of the relationship now involves three phases. As before there is the initial building up phase although now growth is stochastic, after high realizations of p the quantity allocated to the agent will increase more rapidly than after low ones. Once the agent reaches full capacity, the *insurance* phase begins. In this region, the agent does not consume nor does the quantity grow. Rather, the agent purchases "insurance" from the principal in the form of continuation value. This insurance protects against inefficient investment following a string of poor realizations. The quantity of insurance that the agent purchases depends both on the relative impatience of the agent and on the distribution of risky cash flows.

If the agent is no more impatient than the principal ($\delta_A = \delta_P$) then there is no relative cost of delaying consumption for the agent. In this case the agent will buy as much insurance as the participation constraint of the principal will allow. The more impatient the agent relative to the principal, the more costly is the insurance and the shorter is the insurance phase. Once the agent has accumulated the optimal amount of insurance, the cashing in phase begins and the agent starts consuming.

Definition 4.10. *For an arbitrary non-negative function $I(v)$, let*

$$\hat{K}(v) \equiv \frac{v}{\bar{p}} - I(v) \qquad K^*(v) \equiv \min\{\hat{K}(v), \bar{K}\}$$

$$\hat{V}(p,v) \equiv \frac{1}{\delta_A}(pK^*(v) + I(v)) \qquad V^*(p,v) \equiv \min\{\hat{V}(p,v), V^L\}$$

$$\hat{T}(p,v) \equiv pK^*(v) - I(v) \qquad T^*(p,v) \equiv \delta_A V^*(p,v) - I(v)$$

The optimal agreement is summarized by the following proposition.

Proposition 4.11. *There exists a non-negative $I^*(v)$ such that $(K^*, V^*, T^*) \in \Phi^{SB}$.*

Proof. Follows from Lemmas 4.5-4.9. □

When the adverse selection problem is not too severe (i.e., $p_{\min} > \gamma$), a complete closed form characterization is possible.

Corollary 4.12. *If $p_{\min} \geq \gamma$ then $I^*(v) = \max\{0, v - \bar{V}\}$.*

5 Renegotiation Proof Agreements

In order to sustain the optimal self-enforcing agreement, the principal must "punish" the agent following a string of bad realizations. Such behavior is not robust to renegotiation. In this section, we characterize optimal renegotiation proof self-enforcing agreements and demonstrate that they obtain similar features to those already described.

Definition 5.1. *An agreement is* renegotiation proof *(RNP) if and only if for all histories, the principal's continuation profit (now denoted by $\tilde{\Pi}(v)$) lies on the pareto frontier. Let $\tilde{\Phi}^{SB}$ denote the set of optimal self-enforcing renegotiation proof agreements.*

Lemma 5.2. *In any renegotiation proof agreement, there exists some $k_0 > 0$ such that $K(v) \geq k_0$ for all (v, p).*

This lower bound on K means that the agent can always play the strategy "report $p_{\min}$" and obtain a payoff of at least

$$V_0 \equiv k_0 \times \max\left\{\frac{1}{1-\delta_A}(\bar{p} - p_{\min}), \frac{\bar{p}}{\delta_A}\right\} \tag{11}$$

In order to characterize optimal RNP agreements, the following definition will be useful

Definition 5.3. *Define the following functions:*

$$K'(v) \equiv k_0 + \frac{1-k_0}{\bar{p}(1-V_0)}v \qquad K^{**}(v) \equiv \min\{K'(v), 1\}$$

$$V'(p,v) \equiv \frac{(p-\bar{p})}{\delta_A}k^*(v) + \frac{v}{\delta_A} \qquad V^{**}(p,v) \equiv \max\{V'(p,v), V_L\}$$

$$T'(p,v) \equiv pK^*(v) - (V^*(p,v) - V'(p,v)) \qquad T^{**}(p,v) \equiv \max\{T'(p,v), 0\}$$

Theorem 5.4. *There exists a unique* k_0 *such that* $(T^{**}, K^{**}, V^{**}) \in \tilde{\Phi}^{SB}$.

Proof. To be completed. □

While the form of the optimal arrangement remains similar under the renegotiation-proof requirement, renegotiation-proofness decreases the principals profit as the severity with which he can punish the agent is limited. As a result, the principal is more likely to to "opt-out," especially as the severity of the information asymmetry grows.

Proposition 5.5. *If* $p_{\min} < \gamma$, $\tilde{\Pi}^{SB} \leq 0$ *for all* v *and the optimal renegotiation proof self-enforcing arrangement involves no distribution.*

Proof. To be completed. □

This result illustrates the importance of investing in monitoring technologies that would provide the principal with some information regarding outcomes. Without such a technology, the ability to sustain RNP self-enforcing arrangements is limited.

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