Market-making with Search and Information Frictions

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Abstract

We develop a dynamic model of trading through market-makers that incorporates two canonical sources of illiquidity: trading (or search) frictions and asymmetric information. We show that conventional predictions, derived from models that study either of these frictions in isolation, do not necessarily hold when both frictions are present. These results have important implications for technological innovations and regulatory initiatives that aim to reduce trading and/or information frictions. A key result is that reducing trading frictions slows down the rate at which market-makers learn about asset quality, amplifying the effects of adverse selection and ultimately leading to more illiquidity, as measured by, e.g., bid-ask spreads.

Keywords: Adverse selection, trading frictions, bid-ask spreads, liquidity, learning.

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1 Introduction

Financial markets have undergone significant changes in recent decades as a result of both technological innovations and regulatory initiatives. Many of these changes have targeted two of the fundamental sources of illiquidity in financial markets: trading frictions and information frictions. For example, technological innovations such as the introduction of the personal computer, the development of electronic trading platforms, and the advent of faster bandwidth have offered investors the opportunity to trade more quickly with a wider set of dealers.\(^1\) In addition, a number of regulatory initiatives have advocated a shift away from opaque over-the-counter (OTC) markets; some of these policies have promoted greater access to pre-trade price information, while others have required the collection and dissemination of post-trade prices and quantities.\(^2\) As these changes continue—and, in some cases, accelerate—a natural question arises: how does market liquidity respond to changes in the level of trading and/or information frictions?

The goal of this paper is to provide some answers to this question. To do so, we develop a model that incorporates both trading (or search) frictions and asymmetric information into a single, unified framework. We characterize equilibrium prices, trading decisions, and the corresponding evolution of beliefs, and then perform comparative statics to understand how the underlying frictions in the model determine market liquidity. We focus much of our attention on one particular measure of market liquidity, the bid-ask spread, though we also discuss implications for other measures, such as trading volume and price impact.

The effect of trading frictions and information frictions on bid-ask spreads have been studied extensively in separate literatures, which provide fairly stark predictions about the consequences of reducing either friction in isolation. In particular, the literature that formalizes OTC markets using search and matching models of trade, such as Duffie et al. (2005), offers a simple answer to the question posed above: if investors can contact dealers more easily, then competition among dealers will increase and bid-ask spreads will fall. Likewise, the literature that rationalizes bid-ask spreads as a consequence of asymmetric information, such as Glosten and Milgrom (1985), offers an equally straightforward prediction: if

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\(^1\)The transition of financial markets from dealer-based platforms to electronic platforms, and the ever-increasing execution speeds, have been well documented. Appendix A of Pagnotta and Philippon (2015) offers an excellent summary.

\(^2\)In the U.S., for example, the Dodd-Frank Wall Street Reform and Consumer Protection Act has called for the introduction of Swap Execution Facilities in the market for interest rate swaps; according to this legislation, an investor’s request to trade must be circulated to at least three dealers for price quotes before the trade can be executed. Similar regulatory requirements for pre-trade price transparency have been implemented in European markets as a consequence of MiFID II. Requirements for post-trade transparency are equally as prevalent. A notable example is legislation introduced by the Financial Industry Regulatory Authority (FINRA), which collects post-trade price information in markets for asset-backed securities and corporate bonds through the Trade Reporting and Compliance Engine (TRACE).
dealers have access to better information about the payoffs of the asset being traded, adverse selection
will be less severe and bid-ask spreads again should fall.

Our main result is that these conventional predictions are not necessarily true when both frictions are
present. Instead, reducing one of these frictions can make the other more severe. We illustrate this result,
primarily, by showing that reducing trading frictions can slow down the process by which dealers learn
about the quality of the assets being traded, which exacerbates the effects of asymmetric information and
can ultimately lead to wider bid-ask spreads.

To understand the intuition, it’s helpful to describe a few key features of the model. There are two
types of agents—traders and dealers—who trade a homogeneous asset that is either high or low qual-
ity. Traders know the quality of the asset but dealers do not; these are the information frictions. In each
period, traders are matched with a stochastic number of dealers. In particular, they might be matched
with one or more dealers, but they might not be matched with any dealers, in which case they can’t trade;
these are the trading frictions. Conditional on being matched, the dealer(s) offer bid and ask prices, and
traders decide whether or not to buy or sell based on their reservation value for the asset, along with the
realization of contemporaneous (aggregate and idiosyncratic) preference shocks. A trader’s reservation
value can be decomposed into two pieces: one that depends on the fundamental value of the asset and
another that depends on the expected gains from trading the asset in the future. Dealers observe aggre-
gate trading volume, which depends on the true quality of the asset and the (uncorrelated, unobserved)
aggregate preference shock. Hence, volume is a noisy signal of asset quality, and dealers update their
beliefs accordingly.

The first key piece of intuition is that dealers learn quickly when investors’ behavior—which is sum-
marized by their reservation value—is very different in the two states of the world, whereas they learn
more slowly when investors behave similarly regardless of asset quality. The second key result is that
reducing trading frictions implies that investors’ reservation values depend more heavily on the payoffs
associated with trading the asset in the future, and less on the fundamental value of the asset. Having
frequent opportunities to trade, with potentially many dealers, implies that traders can expect to take
advantage of favorable prices when their preference shocks dictate a desire to buy or sell. Alternatively,
when traders meet dealers infrequently, expected gains from trade shrink and reservation values depend
more heavily on the fundamental value.3 As a result, reducing trading frictions makes investors’ reserva-
tion values more similar across asset qualities, so that the (endogenously generated) signals that dealers

3In the limit, when traders live in autarky, the fundamental value is the only component of the reservation value.
observe are less informative. The last piece of the puzzle relates the speed of learning to bid-ask spreads: when learning slows down, dealers set wider bid-ask spreads to compensate for being more uncertain about the quality of the asset.

At a broader level, we think our analysis is potentially important for several reasons. First, it provides a single, unified framework that incorporates several ingredients that have been identified as crucial factors in financial markets: trading frictions, which have been studied extensively in search-based models of OTC markets; adverse selection, which lies at the heart of information-based models of market microstructure and the bid-ask spread; and learning, which is the focal point of dynamic models of information revelation. Second, our framework identifies key tradeoffs that can potentially help to interpret existing empirical evidence that has found ambiguous effects of reducing trading frictions in various financial markets. In the same spirit, our model could be useful for anticipating the effects of future changes to OTC market structures, as a number of current regulatory proposals are aimed at either eliminating trading frictions or reducing information asymmetries in financial markets. Lastly, since various measures of market liquidity do not respond symmetrically to changes in the severity of trading or information frictions, our analysis could potentially offer a guide for disentangling the effects of different types of shocks.

The rest of the paper is organized as follows. After reviewing the related literature below, we introduce the model in Section 2, characterize optimal behavior, and define an equilibrium. In Section 3, we consider a special case of the model that admits an analytical solution, and use this special case to illustrate the key results. Then, in Section 4, we consider an alternative specification, assign parameter values that are roughly consistent with either those in the existing literature or with the data, and explore additional properties of the model numerically. Section 5 concludes.

1.1 Related Literature

This paper is related to several strands of the literature. First, it is closely related to the large body of work that uses search frictions to model decentralized trading. Duffie et al. (2005), Lagos and Rocheteau (2009), and Hugonnier et al. (2014) focus on implications for bid-ask spreads under full information. Well-known examples of studies that found ambiguous effects from reducing trading or information frictions include Gemmill (1996), Madhavan et al. (2005), and Hendershott and Moulton (2011). In fact, Lagos and Rocheteau (2009) also find that bid-ask spreads can widen when trading frictions ease, though the mechanism is different. In particular, they document this property in an environment with no information frictions and investors who can hold arbitrary portfolios, whereas we establish our results in an environment with information frictions and investors who cannot.
(1993), Spulber (1996), and, more recently, Lester et al. (2015) analyze pricing under asymmetric information about preferences, i.e., about the traders’ private values of holding the asset. In our paper, the traders possess private information about their preferences and about a common value component of the asset, which leads to adverse selection. Moreover, since the common component is market-wide—that is, since all assets have the same common component—there is a role for learning over time by the uninformed market-makers.

This combination of adverse selection, learning, and decentralized trade in our model is also present in papers such as Wolinsky (1990), Blouin and Serrano (2001), Duffie and Manso (2007), Duffie et al. (2009), Golosov et al. (2014) and Lauermann and Wolinsky (2016). The key difference between these papers and our own is the source of learning: in these papers, agents learn only from their own trading experiences, while in our paper, learning occurs from observing market-wide outcomes, which we feel is a realistic feature of many financial markets.

In analyzing the effects of reducing trading frictions, we also make contact with the literature that studies the effects of high frequency trading, such as Biais et al. (2015), Pagnotta and Philippon (2015), Menkveld and Zoican (2017), and Du and Zhu (2017). A key distinction between our work and these papers—in addition to the many different modeling assumptions—is the crucial role that is assigned to the dealers’ learning process in our framework.

Finally, our analysis also contributes to several strands of a large literature that focuses on the effects of asymmetric information in settings without trading and/or search frictions. One strand of this literature focuses on the effects of asymmetric information on the bid-ask spread, such as the seminal contributions of Glosten and Milgrom (1985), Copeland and Galai (1983) and Kyle (1985). Our focus on the informational content of endogenous market signals is shared by the stand of this literature that studies information aggregation in rational expectations equilibrium (REE) models, pioneered by Grossman and Stiglitz (1980) and Hellwig (1980). In contrast to these earlier papers, however, our analysis highlights novel interactions between asymmetric information and search frictions, and shows how these interactions can lead to surprising and counter-intuitive implications for liquidity and prices.

who can only hold zero or one unit of the asset. See also Afonso (2011), who shows that reducing trading frictions can have counter-intuitive effects because of congestion externalities.

Another, more recent, example is Bethune et al. (2016).

This latter feature distinguishes our work from papers that study adverse selection stemming from private information about the idiosyncratic quality of an asset; a non-exhaustive list of papers in this tradition includes Camargo and Lester (2014), Guerrieri and Shimer (2014), Kaya and Kim (2015), Fuchs and Skrzypacz (2015), Chiu and Koeppel (2016), Choi (2016), and Kim (2017). In these papers, information revealed from a particular trade is asset-specific and therefore, is typically not useful in future trades.

A related literature also studies learning and information diffusion in network settings; see, e.g., Babus and Kondor (2016).
2 Model

2.1 Environment

Agents, Assets, and Preferences. Time is discrete and indexed by $t$. There are two types of risk neutral, infinitely-lived agents that we call “traders” and “dealers.” There is a single asset of quality $j \in \{l, h\}$. Traders can hold either zero or one unit of the asset, while dealers’ positions are unrestricted, i.e., dealers can take on arbitrarily long or short positions.

At the beginning of each period, the asset matures with probability $1 - \delta$, in which case the game ends. A trader who owns a unit of the asset receives a payoff $c_j$ when the asset matures, with $c_l < c_h$. A trader who owns a unit of the asset receives a flow payoff $\omega_t + \varepsilon_{i,t}$ if the asset does not mature, which we interpret as a liquidity shock. The aggregate portion of the shock, $\omega_t$, is an i.i.d. draw each period from a distribution $F(\cdot)$. The idiosyncratic portion of the liquidity shock, $\varepsilon_{i,t}$, is an i.i.d. draw for each trader in each period from a distribution $G(\cdot)$. We assume that the mean of both shocks is zero, and we normalize the payoffs to a trader without an asset to zero.

Finally, we assume that there is a large mass of dealers. Dealers receive a payoff $v_j$ when the asset matures, with $v_h > v_l$, but they do not receive any flow payoff from the asset before it matures. Given our assumption that dealers can take unrestricted positions, it follows that the payoff to a dealer from buying or selling a unit of the asset of quality $j \in \{l, h\}$ is $v_j$ and $-v_j$, respectively.

Trading and Frictions. There are two key frictions in the model. The first is an information friction: traders know the quality of the asset, $j \in \{l, h\}$, while the dealers do not. The dealers know the ex-ante probability that the asset is of quality $h$, which we denote by $\mu_0$.

The second key friction in the model is a search friction: in every period, each trader meets with a stochastic number of dealers. In particular, let $q_i$ denote the probability that a trader meets $i \in \{0, 1, \ldots\}$ dealers. It will be convenient to let

$$
\pi = 1 - q_0
$$

denote the probability that a trader meets with at least one dealer. Moreover, as we describe below, any meeting with $i \geq 2$ dealers will have the same outcome. Hence, let

$$
\alpha_m = \frac{q_1}{\pi} \quad \text{and} \quad \alpha_c = \frac{1 - q_0 - q_1}{\pi}
$$

denote the probabilities that a trader meets one dealer (a “monopolist” meeting) or more than one dealer (a “competitive” meeting), respectively, conditional on meeting at least one dealer.
After meetings occur, each dealer quotes a bid and an ask price, i.e. prices at which she’s willing to buy and sell a unit of the asset, respectively. Importantly, we assume that the number of dealers that a trader meets is common knowledge when dealers choose prices.\textsuperscript{10} We denote the prices quoted by the dealer when she is a monopolist by \((B^m_t, A^m_t)\), and the prices quoted by competing dealers by \((B^c_t, A^c_t)\).

\textbf{Information and Learning.} We assume that dealers observe the aggregate volume of trade at the end of each trading round. As we describe in detail below, this will turn out to be a noisy signal about asset quality, which will used by all the dealers to update their beliefs over time. This assumption will play a crucial role in making our analysis tractable. In particular, as we will show, it implies that (i) all dealers have identical beliefs at the beginning of each period and (ii) the actions of an individual trader and/or dealer will not alter the evolution of future beliefs. In what follows, we let \(\mu_t\) denote the beliefs of (all) dealers at the beginning of trading at time \(t\) that the asset is of quality \(h\).

\subsection{Traders’ Optimal Behavior}

Let \(W^q_{j,t}\) denote the expected discounted value of a trader who owns \(q \in \{0, 1\}\) unit of the asset at the beginning of period \(t\) when the asset is of quality \(j \in \{l, h\}\). Then, for an investor who does not own the asset, we have

\[
W^0_{j,t} = \delta E_{\omega, \varepsilon} \left[ \pi \sum_{k=c,m} \alpha_k \max \{-A^k_t + \omega_t + \varepsilon_{i,t} + W^1_{j,t+1}, W^0_{j,t+1}\} + (1 - \pi)W^0_{j,t+1} \right]. \tag{1}
\]

Note that the expectation is taken over \(\omega_t\) and \(\varepsilon_{i,t}\), which are drawn from \(F(\omega)\) and \(G(\varepsilon)\), respectively. All objects inside the brackets—including the current ask prices \(A^k_t\) and future payoffs \(W^q_{j,t+1}\)—can be calculated using the information available to a trader at time \(t\), which would include the true quality of the asset, along with the current beliefs of dealers. We describe in detail below how the trader uses this information to formulate beliefs.

In words, the first expression in equation (1) represents the expected payoff if the asset does not mature and the trader meets at least one dealer, whereupon he may either purchase a unit of the asset at price \(A^k_t\) or reject the offer and continue searching in period \(t + 1\). The second expression represents the expected payoff if the asset does not mature but the trader fails to meet a dealer. Recall that a trader with \(q = 0\) assets receives zero payoff if the asset matures, which occurs with probability \(1 - \delta\).

\textsuperscript{10}This is in contrast to the literature on price dispersion that follows, e.g., Burdett and Judd (1983).
Similar logic can be used to derive the expected payoff of a trader who owns one unit of the asset,

\[ W_{j,t}^1 = (1 - \delta)c_j + \delta \mathbb{E}_{\omega, \epsilon} \left[ \pi \sum_{k = c, m} \alpha_k \max \{ \omega_t + \epsilon_{i,t} + W_{j,t+1}^1, B_t^k + W_{j,t+1}^0 \} + (1 - \pi) \left( \omega_t + \epsilon_{i,t} + W_{j,t+1}^1 \right) \right]. \]  

(2)

Note that, when the asset matures, a trader who owns one unit receives a payoff \( c_j \).

We conjecture, and later confirm, that an individual trader’s decision to accept or reject an offer has no effect on dealers’ beliefs, and hence no effect on the path of future prices. An immediate consequence is that traders’ decisions to buy or sell follow simple cutoff rules: given asset quality \( j \in \{ l, h \} \), a trader who does not own the asset will buy in a meeting of type \( k \in \{ m, c \} \) if \( \epsilon_{i,t} \geq \epsilon_{j,t}^k \), while a trader who owns the asset will sell if \( \epsilon_{i,t} \leq \epsilon_{j,t}^k \), where these cutoffs satisfy

\[-A_t^k + \omega_t + \epsilon_{j,t}^k + W_{j,t+1}^1 = W_{j,t+1}^0 \]
\[\omega_t + \epsilon_{j,t}^k + W_{j,t+1}^1 = B_t^k + W_{j,t+1}^0.\]

Let us denote the reservation value of an investor at time \( t \) given asset quality \( j \in \{ l, h \} \) by

\[ R_{j,t} \equiv W_{j,t}^1 - W_{j,t}^0. \]

Then, the optimal behavior of traders is succinctly summarized by the cutoffs

\[ \epsilon_{j,t}^k = B_t^k - \omega_t - R_{j,t+1}, \]
\[ \epsilon_{j,t}^k = A_t^k - \omega_t - R_{j,t+1}. \]

(3)

(4)

defined for \( k \in \{ m, c \} \), along with the reservation value

\[ R_{j,t} = (1 - \delta)c_j + \delta \mathbb{E}_{\omega} \left\{ \pi \sum_{k = c, m} \alpha_k \left[ B_t G(\epsilon_{j,t}^k) + \int_{\epsilon_{i,t}^k}^{R_{j,t+1}} \omega_t + \epsilon_{i,t} + R_{j,t+1} \ dG(\epsilon_{i,t}) + A_t \left[ 1 - G(\epsilon_{j,t}^k) \right] \right] + (1 - \pi)R_{j,t+1} \right\}, \]

(5)

which is obtained by subtracting (1) from (2) and using the cutoff rules described above. Again, the expectation operator in (5) is taken over the aggregate shock, \( \omega_t \), as well as the prices and future payoffs, which we describe below.

**Demographics.** Given the trading rules described above, we can now describe the evolution of the distribution of asset holdings across traders over time. To do so, let \( N_t^0 \) denote the measure of traders
who have asset holdings \( q \in \{0, 1\} \) at time \( t \). When the asset is of quality \( j \in \{l, h\} \), then, we have

\[
N_{j,t+1}^1 = N_{t}^1 \left[ 1 - \pi + \pi \left( 1 - \sum_{k=c,m} \alpha_k G(\xi_{j,t}^k) \right) \right] + N_{t}^0 \pi \left[ 1 - \sum_{k=c,m} \alpha_k G(\tau_{j,t}^k) \right]
\]

\[
N_{j,t+1}^0 = N_{t}^1 \pi \sum_{k=c,m} \alpha_k G(\xi_{j,t}^k) + N_{t}^0 \left[ 1 - \pi + \pi \sum_{k=c,m} \alpha_k G(\tau_{j,t}^k) \right].
\]

Naturally, the measure of investors that own an asset in period \( t + 1 \) is equal to the measures of investors that owned an asset in period \( t \) and did not sell, plus the measure of investors that did not own an asset but chose to buy. The intuition behind the law of motion for the measure of investors that don’t own an asset follows the same logic. As we describe below, dealers will know the distribution of asset holdings, i.e. \((N_t^0, N_t^1)\) at the beginning of each period, but they will not be able to perfectly infer \( j \in \{l, h\} \).

### 2.3 Dealers’ Optimal Behavior

**Monopolist Pricing.** We first consider the optimal price offered in a meeting between a trader and a single dealer. When formulating this offer, the dealer takes as given the trader’s optimal behavior derived above. We will show that, under our assumptions, the dealer’s pricing problem is static: neither the price that she sets nor the trader’s response affects payoffs in future periods (e.g., through beliefs). We treat this as a conjecture, for now, and verify it later.

Under this conjecture, a monopolist dealer’s optimal prices \((A_t^m, B_t^m)\) solve

\[
\max_{A_t^m, B_t^m} \mathbb{E}_{j,\omega} \left[ N_{t}^0 \left[ 1 - G(\xi_{j,t}) \right] (A - \nu_j) + N_{t}^1 G(\xi_{j,t}) (\nu_j - B) \right],
\]

where the expectations operator is taken over the quality \( j \in \{l, h\} \) of the asset—using the dealer’s current beliefs, \( \mu_t \)—as well as the aggregate liquidity shock, \( \omega_t \), and future reservation values, \( R_{j,t+1} \), that determine the thresholds \( \xi_{j,t} \) and \( \tau_{j,t} \). Again, we postpone the derivation of how these latter expectations are formed until Section 2.4, below.

The optimal prices can be summarized by the two first-order conditions:

\[
0 = \mathbb{E}_{j,\omega} \left[ 1 - G(\xi_{j,t}) - g(\tau_{j,t}) (A_t^m - \nu_j) \right] \tag{6}
\]

\[
0 = \mathbb{E}_{j,\omega} \left[ -G(\xi_{j,t}) + g(\xi_{j,t}) (\nu_j - B_t^m) \right]. \tag{7}
\]

Re-arranging equation (6), we can write the optimal ask price as

\[
A_t^m = \mu_t \nu_h + (1 - \mu_t) \nu_l + \frac{1 - \mathbb{E}_{j,\omega} \left[ G(\tau_{j,t}) \right]}{\mathbb{E}_{j,\omega} \left[ G(\xi_{j,t}) \right]} + \mu_t (1 - \mu_t)(\nu_h - \nu_l) \frac{\mathbb{E}_{\omega} \left[ g(\xi_{h,1}) - g(\tau_{j,1}) \right]}{\mathbb{E}_{j,\omega} \left[ g(\tau_{j,1}) \right]}.
\]
This equation expresses the ask price as the sum of the expected fundamental value of the asset and two additional components. The first component derives from the dealer’s *market power* and is the inverse of the semi-elasticity of expected demand, akin to the standard markup in a monopolist’s optimal price. The second component is a premium that dealers charge to compensate for the presence of *asymmetric information*. Note that this second component can be re-written as

\[
\mu_t(1 - \mu_t)(v_h - v_l) \frac{\mathbb{E}_j,\omega [g(\tau_{m,j,t}^m) - g(\tau_{m,j,t}^m)]}{\mathbb{E}_j,\omega [g(\tau_{m,j,t}^m)]} = \text{Cov} \left( \frac{g(\tau_{m,j,t}^m)}{\mathbb{E}_j,\omega [g(\tau_{m,j,t}^m)]}, v_j \right).
\]

Hence, the asymmetric information component is essentially an adjustment that accounts for the relationship between the density of marginal buyers and the dealer’s valuation of the asset; it implies that dealers will adjust their asking price upward if the density of marginal buyers is relatively large when the asset is of high quality.\(^{11}\) Also note that this component disappears if there is no uncertainty over the quality of the asset, i.e., if \(\mu_t = 0, \mu_t = 1,\) or \(v_h = v_l.\)

Similar logic reveals that the bid price is equal to the expected fundamental value of the asset, adjusted downwards by the two components discussed above:

\[
B_t^m = \mu_tv_h + (1 - \mu_t)v_l - \frac{\mathbb{E}_j,\omega [G(\xi_{j,t}^m)]}{\mathbb{E}_j,\omega [g(\xi_{j,t}^m)]} - \mu_t(1 - \mu_t)(v_h - v_l) \frac{\mathbb{E}_\omega [g(\xi_{h,t}^m) - g(\xi_{h,t}^m)]}{\mathbb{E}_j,\omega [g(\xi_{j,t}^m)]}.
\]

**Competitive Pricing.** Next, we solve for equilibrium prices when a trader meets with two dealers. This situation corresponds almost exactly to the pricing problem in the canonical setting of Glosten and Milgrom (1985), where equilibrium bid and ask prices are set so that expected (static) profits are equal to zero. In other words, when two (or more) dealers compete, the bid price \(B_t^c\) is equal to the expected value of the asset conditional on a trader selling at that price, while the ask price \(A_t^c\) is equal to the expected value of the asset conditional on a trader buying at that price. Formally, this zero profit condition can be written

\[
0 = A_t^c - \frac{\mathbb{E}_j,\omega \left[v_j \left( 1 - G(\xi_{j,t}^c) \right) \right]}{\mathbb{E}_j,\omega \left[ (1 - G(\xi_{j,t}^c)) \right]}; \quad (10)
\]

\[
0 = B_t^c - \frac{\mathbb{E}_j,\omega \left[v_j G(\xi_{j,t}^c) \right]}{\mathbb{E}_j,\omega \left[ G(\xi_{j,t}^c) \right]}; \quad (11)
\]

\(^{11}\)Note that this asymmetric information component can be positive or negative.
Re-arranging yields

\[ A^c_t = \mu_t v_h + (1 - \mu_t) v_l + \mu_t (1 - \mu_t) (v_h - v_l) \frac{E_{j,\omega} [G(\bar{\epsilon}_{i, t}^c) - G(\bar{\epsilon}_{h, t}^c)]}{E_{j,\omega} [1 - G(\bar{\epsilon}_{j, t}^c)]} \]  \hspace{1cm} (12)

\[ B^c_t = \mu_t v_h + (1 - \mu_t) v_l - \mu_t (1 - \mu_t) (v_h - v_l) \frac{E_{j,\omega} [G(\bar{\epsilon}_{j, t}^c) - G(\bar{\epsilon}_{h, t}^c)]}{E_{j,\omega} [G(\bar{\epsilon}_{j, t}^c)]}. \]  \hspace{1cm} (13)

Again, it is worth noting that

\[ \mu_t (1 - \mu_t) (v_h - v_l) \frac{E_{j,\omega} [G(\bar{\epsilon}_{i, t}^c) - G(\bar{\epsilon}_{h, t}^c)]}{E_{j,\omega} [1 - G(\bar{\epsilon}_{j, t}^c)]} = \text{Cov} \left( \frac{1 - G(\bar{\epsilon}_{j, t}^c)}{E_{j,\omega} [1 - G(\bar{\epsilon}_{j, t}^c)]}, v_j \right) \]

\[ \mu_t (1 - \mu_t) (v_h - v_l) \frac{E_{j,\omega} [G(\bar{\epsilon}_{j, t}^c) - G(\bar{\epsilon}_{h, t}^c)]}{E_{j,\omega} [G(\bar{\epsilon}_{j, t}^c)]} = \text{Cov} \left( \frac{G(\bar{\epsilon}_{j, t}^c)}{E_{j,\omega} [G(\bar{\epsilon}_{j, t}^c)]}, v_j \right). \]

These expressions show that, under competition, bid and ask prices are equal to the expected value of the asset to the dealer, adjusted for adverse selection. This adjustment depends on the covariance between the probability of trade and the value of the asset. For example, the ask (bid) price is higher (lower) than the expected value since traders are more (less) likely to buy (sell) when the state is high. This creates a positive bid-ask spread, exactly as in Glosten and Milgrom (1985).

Comparing equations (8)-(9) and (12)-(13) reveals that prices under competition are similar in structure to monopoly prices, with two key differences. First, as one might expect, competitive prices do not contain the markup component found in monopoly prices. Second, the adjustment for asymmetric information in (8)-(9) depends on the mass of traders at the appropriate thresholds in the two states (i.e., the pdf), while the corresponding adjustment in (12)-(13) depends on the difference between the probability of trade in the two states (i.e., the cdf). Intuitively, this occurs because the monopolist’s optimal price is a function of the expected profit from the marginal trader, whereas competitive pricing is pinned down by the requirement that dealers earn zero profits on average.

### 2.4 Learning

We now explain how dealers update their beliefs about the quality of the asset, and how investors form expectations about dealers’ beliefs—and hence the prices they offer—in future periods.

As noted above, we assume that dealers learn by observing aggregate trading activity in each period. Notice immediately that this is equivalent to observing the thresholds \((\bar{\epsilon}_{i, t}^m, \bar{\epsilon}_{i, t}^c, \bar{\epsilon}_{j, t}^m, \bar{\epsilon}_{j, t}^c)\). Moreover, since dealers know which prices have been offered in equilibrium, each of these thresholds ultimately contains the same information. Consider, for example, \(\bar{\epsilon}_{j, t}^m\), which depends on the ask price \(A^m_t\), which dealers
know, along with the reservation value of the trader $R_{t+1}$ and the aggregate shock $\omega_t$, both of which the dealers do not observe. The reservation value clearly depends on the quality of the asset, while the aggregate liquidity shock is orthogonal to quality. Hence, the volume of asset purchases in monopoly meetings—which dealers can perfectly infer from the total volume of asset purchases given $\alpha_c$ and $\alpha_m$—is a noisy signal about asset quality, and the informational content can be summarized by:

$$S_t = R_{t+1} + \omega_t,$$

where $R_{t+1} = R_{j,t+1}$ when the true state of the world is $j \in \{l, h\}$.

Let us conjecture, for now, that investors’ reservation values depend only on dealers’ beliefs, along with the true state $j$. Then, given current beliefs $\mu_t$ and the observed signal $S_t$, a dealer’s updated belief $\mu_{t+1}$ depends on the likelihood of observing that signal when the asset’s quality is $h$ relative to that when it is $l$. To arrive at this likelihood, we first calculate the value of the aggregate shock, $\omega_t$, that is consistent with the observed signal $S_t$. Formally, define

$$\omega^*_{t,t} = S_t - R_{t,t+1}(\mu_{t+1}).$$

Using (14), $\omega^*_{t,t}$ is the value of $\omega_t$ consistent with the signal $S_t$ if the dealer conjectures that the asset is of quality $t \in \{l, h\}$ and future beliefs are $\mu_{t+1}$. Naturally, if $t$ is equated to the true asset quality, then $\omega^*_{t,t} = \omega_t$, i.e., the dealer’s conjecture corresponds to the true value of the aggregate liquidity shock.

Now, one might be concerned that the reservation values $R_{l,t+1}$ and $R_{h,t+1}$ in (15) are calculated under different information sets, but this is not the case since, by construction, both $\omega^*_{l,t}$ and $\omega^*_{h,t}$ are consistent with the signal $S_t$.

The law of motion for dealers’ beliefs is thus a function $\mu_{t+1}(\mu_t, S_t)$ that solves the fixed point problem

$$\mu_{t+1} = \frac{\mu_t}{\mu_t + (1 - \mu_t) \frac{f(S_t)}{f(\omega^*_{l,t})}} = \frac{\mu_t}{\mu_t + \frac{f(S_t - R_{l,t+1}(\mu_{t+1}))}{f(S_t - R_{h,t+1}(\mu_{t+1}))}}.$$

(16)

Now, even though dealers’ future beliefs cannot depend directly on the true quality of the asset (since they do not observe it), traders (who know the true quality) can certainly use this information to formulate expectations about dealers’ beliefs. In particular, it will be helpful to define the function $\tilde{\mu}_{j,t+1}(\mu_t, \omega_t)$ as the solution to the fixed point problem

$$\mu_{t+1} = \frac{\mu_t}{\mu_t + (1 - \mu_t) \frac{f(\omega_t + R_{j,t+1}(\mu_{t+1}) - R_{l,t+1}(\mu_{t+1}))}{f(\omega_t + R_{j,t+1}(\mu_{t+1}) - R_{h,t+1}(\mu_{t+1}))}}.$$

(17)

In words, when dealers have current beliefs $\mu_t$, the true quality of the asset is $j \in \{l, h\}$, and an aggregate liquidity shock $\omega_t$ is realized, investors can calculate that dealers’ beliefs in period $t + 1$ will be $\tilde{\mu}_{j,t+1}(\mu_t, \omega_t)$. 
This recursive law of motion validates our earlier conjectures about the formation of beliefs in equilibrium. First, since future beliefs (and therefore, future prices) only depend on current beliefs and realizations of the realization of the aggregate liquidity shock, it follows that traders’ reservation values depend only on beliefs and the true quality of the asset. Second, since future beliefs are independent of the actions of any one dealer or trader, both can formulate optimal behavior—prices for dealers and buy, sell, or don’t trade for traders—without affecting future beliefs.

2.5 Definition of Equilibrium

We now define a Markov equilibrium, where the strategies of all agents are functions of (at most) current dealer beliefs, \( \mu_t \), and realizations of the aggregate liquidity shock, \( \omega_t \). Such an equilibrium can be represented recursively as a collection of functions \( \{ \xi_j^k, \tau_j^k, R_j, A_j^k, B_j^k, \mu_j^+, \tilde{\mu}_j^+, N^{h+}_j, N^{l+}_j \} \) for \( j \in \{l, h\} \) and \( k \in \{m, c\} \) such that

1. Taking as given the way dealers set prices and update beliefs, investors’ decisions to buy or sell are determined by:

\[
\begin{align*}
\xi_j^k(\mu, \omega) &= B_j^k(\mu) - \omega - R_j \left( \tilde{\mu}_j^+(\mu, \omega) \right) \\
\tau_j^k(\mu, \omega) &= A_j^k(\mu) - \omega - R_j \left( \mu_j^+ \right) \\
R_j(\mu) &= (1 - \delta) c_j + \delta(1 - \pi) \int_{\omega} R_j \left[ \tilde{\mu}_j^+(\mu, \omega) \right] dF(\omega) + \delta \pi \int_{\omega} \left\{ \sum_{k \in \{m, c\}} \alpha_j^k B_j^k(\mu) G \left( \xi_j^k(\mu, \omega) \right) \right. \\
& \quad \left. + \int_{\xi_j^k(\mu, \omega)} \left[ \omega + \varepsilon + R_j \left( \tilde{\mu}_j^+(\mu, \omega) \right) \right] dG (\varepsilon) + A_j^k(\mu) \left[ 1 - G (\tau_j^k(\mu, \omega)) \right] \right\} dF(\omega).
\end{align*}
\]

2. Given investors’ behavior and expectations about future beliefs, prices are consistent with optimal behavior and, in the competitive case, zero profits. That is, \( A^m = A^m(\mu) \) and \( B^m = B^m(\mu) \) satisfy:

\[
\begin{align*}
0 &= \sum_{j \in \{l, h\}} \mu_j \int_{\omega} \left[ 1 - G (\tau_j^m(\mu, \omega)) - g (\tilde{\tau}_j^m(\mu, \omega)) (A^m - \nu_j) \right] dF(\omega) \quad \text{(21)} \\
0 &= \sum_{j \in \{l, h\}} \mu_j \int_{\omega} \left[ -G (\xi_j^m(\mu, \omega)) + g (\tilde{\xi}_j^m(\mu, \omega)) (\nu_j - B^m) \right] dF(\omega) \quad \text{(22)}
\end{align*}
\]

where \( \mu_h \equiv \mu \) and \( \mu_l \equiv 1 - \mu \), while \( A^c \equiv A^c(\mu) \) and \( B^c \equiv B^c(\mu) \) satisfy:

\[
\begin{align*}
0 &= A^c \left\{ \sum_{j \in \{l, h\}} \mu_j \nu_j \int \left[ 1 - G \left( \tau_j^c(\mu, \omega) \right) \right] dF(\omega) \right\} \quad \text{(23)} \\
0 &= B^c \left\{ \sum_{j \in \{l, h\}} \mu_j \nu_j \int \left[ G \left( \xi_j^c(\mu, \omega) \right) \right] dF(\omega) \right\} \quad \text{(24)}
\end{align*}
\]
3. Given a signal $S$, dealers’ beliefs evolve according to $\mu^+(\mu, S)$, which is a solution to:

$$\mu^+ = \frac{\mu}{\mu + (1 - \mu) \frac{f(S - R_l(\mu^+))}{f(S - R_h(\mu^+))}}.$$  

(25)

Given the true asset quality $j \in \{l, h\}$ and aggregate shock $\omega$, investors’ expectations of dealers’ beliefs evolve according to $\tilde{\mu}^+_j(\mu, \omega)$, which is a solution to

$$\mu^+ = \frac{\mu}{\mu + (1 - \mu) \frac{f(\omega + R_j(\mu^+) - R_l(\mu^+))}{f(\omega + R_j(\mu^+) - R_h(\mu^+))}}.$$  

(26)

Moreover, investors’ expectations are consistent with the evolution of dealers’ beliefs, so that

$$\tilde{\mu}^+_j(\mu, \omega) = \mu^+ \left(\mu, R_j \left(\tilde{\mu}^+_j(\mu, \omega) + \omega\right)\right) \text{ for } j \in \{l, h\}.$$  

(27)

4. Given true asset quality $j \in \{l, h\}$, beliefs $\mu$, and an aggregate shock $\omega$, the population evolves according to:

$$N^1_{j^+}(\mu, \omega) = N^1_j \left[1 - \pi + \pi \left(1 - \sum_{k \in \{m, c\}} G \left(\xi^k_j(\mu, \omega)\right)\right)\right] + N^0_\pi \left(1 - \sum_{k \in \{m, c\}} G \left(\tau^k_j(\mu, \omega)\right)\right)$$  

\[ \text{and} \]

$$N^0_{j^+}(\mu, \omega) = N^1_\pi \sum_{k \in \{m, c\}} G \left(\xi^k_j(\mu, \omega)\right) + N^0_\pi \left[1 - \pi + \pi \sum_{k \in \{m, c\}} G \left(\tau^k_j(\mu, \omega)\right)\right].$$  

(28)

(29)

Note that the laws of motion for $N^1_j$ and $N^0_j$ depend only on the thresholds $\{\xi^k_j, \tau^k_j\}$, for $j \in \{l, h\}$ and $k \in \{m, c\}$. Hence, dealers can always infer the distribution of assets across traders, even though they can’t directly observe asset quality.\(^\text{12}\)

3 Frictions, Learning, and Prices: A Tractable Case

In this section, we explore how the two key frictions in the model—namely, the information and search frictions—affect traders’ reservation values, the evolution of dealers’ beliefs, and, ultimately, equilibrium bid and ask prices. We show that, in isolation, each of these frictions has the expected effect: holding beliefs fixed, a reduction in search frictions causes bid-ask spreads to narrow; and holding search frictions constant, increasing uncertainty over the quality of the asset causes bid-ask spreads to widen.

However, the interaction between these two frictions generates novel predictions. First, we establish that reducing search frictions slows down learning. Intuitively, when investors have the opportunity to

\(^\text{12}\)Intuitively, by construction, $\omega^*_l$ and $\omega^*_h$ rationalize the aggregate trading volume that dealers observe, and hence the implied thresholds. As a result, the evolution of $N^0_1$ and $N^1_1$ when the asset quality is $l$ and the aggregate shock is $\omega^*_l$ are identical to the evolution of these variables when the asset quality is $h$ and the aggregate shock is $\omega^*_h$. 

trade more frequently, their behavior in the two states of the world is more similar, which implies that
the endogenous signal in the model (aggregate volume) is less informative. Second, since slower learning
implies more uncertainty—and more uncertainty implies wider spreads—we show that a reduction in
search frictions can ultimately lead to an increase in the bid-ask spread.

We are able to establish these results analytically by imposing several parametric assumptions. These
assumptions are not terribly special, per se, above and beyond the fact that they offer a certain amount of
tractability.13 In the next section, we establish that the key mechanisms derived here are preserved under
alternative assumptions.

3.1 Parametric Assumptions

We make three key parametric assumptions, described below.

Assumption 1 (Uniform Shocks). The aggregate liquidity shock, \( \omega \), is uniformly distributed over the interval
\([-m, m]\) for some \( 0 < m < \infty \), and the idiosyncratic liquidity shock, \( \varepsilon \), is uniformly distributed over the interval
\([-e, e]\) for some \( 0 < e < \infty \).

As we will show below, the assumption that \( \omega \) is uniformly distributed greatly simplifies the dealers’
learning process, while the assumption that \( \varepsilon \) is uniformly distributed simplifies the dealers’ pricing
problem.

Assumption 2 (Interior Thresholds). The bounds on the distributions of liquidity shocks are sufficiently large:

\[
m \geq \frac{1}{2} (v_h - v_l) \max \left\{ 1, \frac{\delta(1 - \pi/2)}{1 - \delta(1 - \pi/2)} \right\} \quad \text{and} \quad e \geq \sqrt[3]{\frac{3}{2}} (v_h - v_l).
\]

This second assumption ensures that, for all prices that are offered in equilibrium and all realizations
of \( \omega \), the thresholds \( \tau_{t,j}^k, \xi_{t,j}^k \) lie in the interior of \([-e, e]\) for \( j \in \{l, h\} \) and \( k \in \{m, c\} \), i.e., that some traders
always buy/sell in equilibrium.

Assumption 3 (Equal Valuations). On average, dealers and traders have the same valuation for an asset, i.e.,

\[
v_j = c_j \text{ for } j \in \{l, h\}.
\]

This last assumption keeps our analysis in line with models often used in finance (such as Glosten and
Milgrom, 1985), and also simplifies the analysis.

---

13 As (18)–(27) reveal, one can see that there is a fairly complicated fixed point problem at the heart of the equilibrium: the
law of motion for dealers’ beliefs is a convolution of both exogenous parameters (\( \omega \)) and endogenous variables (\( R_j \)), which
themselves depend on future prices and beliefs. This makes it difficult to derive analytical results for arbitrary distributions of
liquidity shocks.
3.2 Learning

The assumption that \( \omega \) is uniformly distributed greatly simplifies the dealers’ learning process. To see why, note from (16) that the updating process depends on current beliefs, \( \mu \), and the likelihood ratio

\[
\frac{f(S - R_l(\mu^+))}{f(S - R_h(\mu^+))}.
\]

When \( \omega \) is uniformly distributed, \( f(\omega) = \frac{1}{2m} \) for all \( \omega \in [-m, m] \) and \( f(\omega) = 0 \) for all \( \omega \not\in [-m, m] \). Hence, either the signal that dealers observe is uninformative or it is fully revealing about the state \( j \in \{l, h\} \).

Formally, let \( \Sigma_j(\mu) \) denote the set of signals (i.e., the values of aggregate trading volume) that are only feasible when the asset is of quality \( j \), given current beliefs \( \mu \), and let \( \Sigma_b \) denote the set of signals that are feasible in both states, \( l \) and \( h \), so that

\[
\mu^+(\mu, S) = \begin{cases} 
0 & \text{if } S \in \Sigma_l(\mu) \\
\mu & \text{if } S \in \Sigma_b(\mu) \\
1 & \text{if } S \in \Sigma_h(\mu). 
\end{cases}
\]

We conjecture, and later confirm, that

\[
\begin{align*}
\Sigma_l(\mu) &= [-m + R_l(0), -m + R_h(\mu)) \quad (30) \\
\Sigma_b(\mu) &= [-m + R_h(\mu), m + R_l(\mu)] \quad (31) \\
\Sigma_h(\mu) &= (m + R_l(\mu), m + R_h(1)) . \quad (32)
\end{align*}
\]

In words, suppose the true asset quality is \( j = h \). If the signal does not reveal the true asset quality, then \( \mu^+ = \mu \). Moreover, we will show below that reservation values are increasing in \( \mu \), so that \( R_h(\mu) \leq R_h(1) \). Therefore, under the candidate equilibrium, the minimum realization for \( S = \omega + R_j \) when \( j = h \) is \(-m + R_h(\mu)\); any \( S < -m + R_h(\mu) \) is only feasible if \( j = l \). Similar reasoning can be used to explain (31)–(32). Note that

\[
\Sigma_b(\mu) \neq \emptyset \iff R_h(\mu) - R_l(\mu) < 2m.
\]

Assumption 2 ensures that valuations always satisfy this condition.

Let \( p(\mu) \) denote the probability that the signal \( S = \omega + R(\mu) \in \Sigma_l \cup \Sigma_h \), i.e., the probability that the quality of the asset is fully revealed to the dealers. When \( \omega \) is uniformly distributed over the support \([-m, m]\), we have

\[
p(\mu) = \frac{R_h(\mu) - R_l(\mu)}{2m}.
\]

Since the expected number of periods before the quality is revealed is the inverse of \( p(\mu) \), the following insight follows immediately.
Remark 1. The expected speed of learning depends positively on \( R_h(\mu) - R_l(\mu) \).

Intuitively, learning occurs quickly when investors behave very differently when the asset is of high or low quality—that is, when \( R_h(\mu) - R_l(\mu) \) is relatively large. When investors’ behavior is less dependent on asset quality, and \( R_h(\mu) - R_l(\mu) \) is relatively small, it is more difficult for dealers to extract information from trading volume, and learning occurs more slowly.

3.3 Prices

We now derive equilibrium bid and ask prices in matches when a trader meets a single, monopolist dealer, and in matches when a trader meets competing dealers. Two aspects of our model make it possible to derive relatively simple pricing equations. First, the extreme learning process described above, which followed from the uniform distribution of \( \omega \), implies a straightforward relationship between current prices and future beliefs: beliefs are stationary until the state of the world is known with certainty. Second, the demand and supply functions that the dealers face are linear, given the uniform distribution over \( \epsilon \).

To start, it is helpful to define the expected continuation value of a trader when the asset quality is \( j \in \{l, h\} \) and current beliefs are \( \mu \):

\[
\tau_j(\mu) = \mathbb{E}_\omega \left[ R_j(\mu^+) \right] = (1 - p(\mu)) R_j(\mu) + p(\mu) R_j(1[j = h])
\]

(33)

Given this notation, it is straightforward to establish that the optimal price that a dealer offers when she is a monopolist is given by

\[
B^m(\mu) = \frac{\mathbb{E}_j \tau_j(\mu) + \mathbb{E}_j v_j - e}{2}
\]

\[
A^m(\mu) = \frac{\mathbb{E}_j \tau_j(\mu) + \mathbb{E}_j v_j + e}{2}
\]

In words, the bid and ask prices are simply the average of the expected value of the dealer and the trader, adjusted by a markup term \( \frac{e}{2} \). Importantly, the density of marginal buyers and marginal sellers is the same in both states of the world, so that the covariance between, e.g., \( g(\tau_j) \) or \( g(\xi_j) \) and \( j \in \{l, h\} \) is zero. From the optimal pricing equations, (8) and (9), this implies that the adverse selection term disappears when the trader meets with a single dealer. Moreover, in this case, the bid-ask spread is equal to \( e \) for all values of beliefs.

When a trader meets with two dealers, the bid and ask price consistent with zero profits are given by

\[
B^c(\mu) = \frac{\mathbb{E}_j \tau_j + \mathbb{E}_j v_j - e}{2} + \frac{1}{2} \sqrt{(e + \mathbb{E}_j (v_j - \tau_j))^2 - 4\text{Cov}(\tau_j, v_j)}
\]

\[
A^c(\mu) = \frac{\mathbb{E}_j \tau_j + \mathbb{E}_j v_j + e}{2} - \frac{1}{2} \sqrt{(e - \mathbb{E}_j (v_j - \tau_j))^2 - 4\text{Cov}(\tau_j, v_j)}
\]
In the Appendix we show that, under Assumptions 1–3,

\[ E_j r_j (\mu) = E_j v_j. \]  

(34)

Using this property, we can simplify the bid and ask prices as follows:

\[ B^m = E_j v_j - \frac{e}{2} \]  

(35)

\[ A^m = E_j v_j + \frac{e}{2} \]  

(36)

\[ B^c = E_j v_j - \frac{e}{2} + \sqrt{\left(\frac{e}{2}\right)^2 - Cov (r_j, v_j)} \]  

(37)

\[ A^c = E_j v_j + \frac{e}{2} - \sqrt{\left(\frac{e}{2}\right)^2 - Cov (r_j, v_j)}. \]  

(38)

Equations (35)–(38) illustrate that the model with uniformly distributed shocks and two types of meetings (with one or two dealers) is tractable enough to admit analytical solutions, and yet rich enough to capture the key economic mechanisms at work. On the one hand, the markup term in \( B^m \) and \( A^m \) implies that the dealers capture some rents that are unrelated to asymmetric information, as in, e.g., Duffie et al. (2005). On the other hand, the adverse selection term in \( B^c \) and \( A^c \) captures the portion of the bid-ask spread that is attributed to adverse selection, as in, e.g., Glosten and Milgrom (1985). Since \( E_j r_j = E_j v_j \), the term \( Cov (r_j, v_j) \) is maximized at \( \mu = \frac{1}{2} \), which implies that this portion of the bid-ask spread is also maximized at \( \mu = \frac{1}{2} \), i.e., when the information asymmetry between traders and dealers is maximal.

3.4 Reservation Values

Using the optimal bid and ask prices derived above, we show in the Appendix that the reservation value of an investor, given current beliefs \( \mu \in (0, 1) \) and asset quality \( j \), can be written as

\[ R_j (\mu) = (1 - \delta)v_j + \delta r_j (\mu) + \delta \pi \sum_{k = m, c} \alpha_k \Omega^k_j (\mu), \]  

(39)

where \( r_j (\mu) \) is defined in (33) and \( \Omega^k_j (\mu) \) is what we call the net option value of holding a quality \( j \) asset in a type \( k \) meeting, i.e., the option value of selling the asset net of the option value of buying the asset.\(^{14}\)

Under Assumptions 1–3, one can show that

\[ \Omega^k_j (\mu) = \frac{B^k - A^k + 2e}{2e} \left( \frac{A^k + B^k}{2} - r_j (\mu) \right). \]  

(40)

Intuitively, \( \Omega^k_j (\mu) \) is the expected surplus an investor earns from future trading opportunities in type \( k \in \{m, c\} \) meetings, given that the asset is of quality \( j \in \{l, h\} \). The first term on the left-hand side of (40) is

\(^{14}\)This net option value derives from the fact that acquiring a unit of the asset offers the investor the option value of selling it a later date but—given our assumption that investors can only hold one unit of the asset at a time—acquiring a unit of the asset also implies that the forfeiture of the option value of buying a unit of the asset at a later date, too.
the ex ante probability (before \( \epsilon \) and \( \omega \) are realized) that the investor will optimally choose to trade in a type \( k \) meeting, given prices \( A^k \) and \( B^k \). The second term,

\[
\frac{A^k + B^k}{2} - r_j(\mu) = \frac{B^k - r_j(\mu) - (r_j(\mu) - A^k)}{2},
\]

is the expected difference between the surplus the investor will earn from selling the asset at a later date and the surplus he could have earned from buying an asset at a later date.

Since the bid and ask prices are independent of asset quality, the net option value is decreasing in the expected continuation value \( r_j \). One can show that \( r_h(\mu) > r_l(\mu) \), which implies that the net option value is larger when the asset is of quality 1. We highlight this property in the remark below, as it will play a key role in the ensuing results.

**Remark 2.** The net option value is decreasing in \( v_j \), so that \( \Omega^k_1(\mu) \geq \Omega^k_h(\mu) \) for any \( \mu \in (0, 1) \), \( k \in \{m, c\} \).

### 3.5 Equilibrium Characterization

To characterize the equilibrium, we can use (39)–(40) to write

\[
R_h(\mu) - R_l(\mu) = (1 - \delta) (v_h - v_l) + \delta (r_h(\mu) - r_l(\mu)) + \delta \pi \sum_{k=c,m} \alpha_k [\Omega^k_h(\mu) - \Omega^k_l(\mu)]
\]

\[
= (1 - \delta) (v_h - v_l) + \delta (r_h(\mu) - r_l(\mu)) - \delta \pi \sum_{k=c,m} \alpha_k \frac{B^k - A^k + 2e}{2e} (r_h(\mu) - r_l(\mu)).
\]

Then, using (35)–(38), along with

\[
p(\mu) = \frac{R_h(\mu) - R_l(\mu)}{2m},
\]

we establish in the Appendix that (41) can be written as a single equation in one unknown, \( p \), given beliefs \( \mu \) and parameters \( \Xi \equiv (\delta, \pi, \alpha_c, v_h, v_l) \). In particular, let \( p^*(\mu) \) denote the solution to \( Z(p, \mu; \Xi) = 0 \), where

\[
Z(p, \mu; \Xi) = -2mp + (1 - \delta) (v_h - v_l) + \delta (1 - \pi) (p (v_h - v_l) + 2mp (1 - p))
\]

\[
- \frac{\delta \pi \alpha_c}{2} \sqrt{1 - \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) [2m (1 - p) p + p (v_h - v_l)] (p (v_h - v_l) + 2mp (1 - p))}. \tag{42}
\]

**Proposition 1.** Under assumptions 1–3, there exists a unique \( p^*(\mu) \) such that \( Z(p^*(\mu), \mu; \Xi) = 0 \).

Importantly, solving for the speed of learning is sufficient for a full characterization of the model: one can use \( p^*(\mu) \) to construct the reservation values, \( \{R_j(\mu)\}_{j \in \{l, h\}} \), along with equilibrium prices, \( \{A^k, B^k\}_{k \in \{m, c\}} \). In the next section, we exploit several of the results derived above, along with the characterization afforded by equation (42), to understand how changes to the parameters affect equilibrium outcomes.

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\( ^{15} \)A key result is that \( E_j r_j = E_j v_j \). This implies that \( R_j(1 | j = h) = v_j \), i.e., that reservation values under full information are equal to the value of owning the asset and not trading it.
3.6 Comparative Statics

In this section, we explore how bid-ask spreads change in response to changes in the underlying economic environment, with a focus on understanding the interaction between search frictions, asymmetric information, and learning. We proceed in two steps. We start by examining how reservation values, the speed of learning, and the bid-ask spread depend on the dealers’ beliefs, \( \mu \). The results we derive are informative for our next step, where we explore the effects of changing the degree of search frictions in our model, either by changing the frequency of trading opportunities (\( \pi \)) or the fraction of trading opportunities that are competitive (\( \alpha_c \)).

Comparative Statics with Respect to Beliefs

Since \( A^m - B^m = e \) for all \( \mu \), the effect of beliefs on the bid-ask spread operate exclusively through the prices in competitive meetings. In the Appendix, we establish the following results.

**Lemma 1.** The bid-ask spread in competitive meetings, \( A^c - B^c \), the difference in reservation values, \( R_h - R_l \), and the probability that the true asset quality is revealed, \( p \), are all hump-shaped in \( \mu \) with a maximum at \( \mu = 1/2 \).

Intuitively, the bid-ask spread is largest when uncertainty is maximal, i.e., when \( \mu = 1/2 \). When the bid-ask spread is widest, the probability of trade is smallest, as investors are more likely to draw idiosyncratic liquidity shocks that lie in the “inaction region.” This causes the difference in the net option values of trading, \( \Omega_h - \Omega_l \), to decline; a decrease in the probability of trade causes a disproportionate decline in the option value to sell when the asset quality is \( l \) and the option to buy when the asset quality is \( h \), making \( \Omega_h - \Omega_l \) less negative. As a result, the difference in the reservation values, \( R_h - R_l \), widens. As investors’ behavior in the two states of the world becomes more distinguishable, the probability that the true asset quality is revealed, \( p \), increases and learning occurs, on average, more quickly.

The Effect of Search frictions

Now consider a decrease in the severity of search frictions. In what follows, we will focus on the effect of an increase in \( \pi \), though we show in the Appendix that similar results obtain for an increase in \( \alpha_c \). Our first result, summarized below, implies that an increase in \( \pi \) unambiguously slows down the learning process.

**Proposition 2.** For any \( \mu \in (0, 1) \), \( \frac{\partial p^*(\mu)}{\partial \pi} < 0 \).

In the discussion above, more extreme values of \( \mu \) caused bid-ask spreads to narrow and increased the probability of trade, causing \( R_h - R_l \)—and thus \( p \)—to fall. The intuition behind the result in Proposition
2 is similar: an increase in \( \pi \) causes a direct increase in the probability of trade, as opposed to operating through the bid-ask spread. As a result, the difference in net option values increases (i.e., becomes more negative), and \( R_h - R_l \propto p \) falls. In words, in the presence of asymmetric information, an increase in the probability of trade again makes the option value of selling (buying) relatively high when the asset is quality \( l \) \((h)\). As a result, the behavior of investors in the two states of the world becomes more similar and it takes longer, on average, to learn the state of the world.

The effect on bid-ask spreads, however, is more nuanced as there are two, opposing effects. The first is what we call the static effect: holding beliefs constant, an increase in \( \pi \) causes the difference in reservation values \( R_h - R_l \) to narrow, which implies a decrease in the covariance of a trader’s expected valuations, \( r_j \), and a dealer valuations, \( v_j \). Since average bid-ask spreads are given by

\[
\sum_k \alpha_k (A^k - B^k) = e - \alpha c \sqrt{e^2 - 4 \text{Cov}(r_j, v_j)},
\]

a decline in this covariance leads to a lower bid-ask spread (holding beliefs fixed). Intuitively, since the increase in \( \pi \) makes investors behave more similarly in the two states of the world—i.e., the likelihood of an investor buying or selling at a given price becomes more even for \( j \in \{l, h\} \)—the problem of adverse selection is diminished and spreads fall.

However, even though an increase in \( \pi \) causes bid-ask spreads to fall for a given level of beliefs, in equilibrium beliefs are changing over time. This leads us to the second effect of increasing the frequency of trade, which we call the dynamic effect: since an increase in \( \pi \) implies that dealers will remain uncertain about the true asset quality for longer (Lemma 2), and bid-ask spreads are larger when dealers are more uncertain (Lemma 1), more frequent meetings can ultimately lead to larger bid-ask spreads in the future.

To formalize this trade-off, let \( \sigma_{t,j} \) denote the expected value of the average bid-ask spread in period \( t \) when the asset is of quality \( j \in \{l, h\} \). Plugging in equilibrium bid and ask prices yields:

\[
\sigma_{t,j} = \mathbb{E}_{\omega_t} \left[ \sum_k \alpha_k (A^k_t - B^k_t) j \right] \\
= \left( 1 - (1 - p(\mu))^t \right) \left[ e - \alpha c e \right] + (1 - p(\mu))^t \left( e - e \alpha c \sqrt{1 - \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) (r_h(\mu) - r_l(\mu))} \right)
\]

Differentiating with respect to \( \pi \), we have

\[
\frac{\partial}{\partial \pi} \sigma_{t,j} = -t (1 - p(\mu))^t \frac{\partial p}{\partial \pi} \alpha c e \left[ 1 - \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) (r_h(\mu) - r_l(\mu)) \right] \\
+ (1 - p(\mu))^t \alpha c e \frac{4 \mu (1 - \mu) (v_h - v_l) \frac{\partial}{\partial \pi} (r_h(\mu) - r_l(\mu))}{2 \sqrt{1 - \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) (r_h(\mu) - r_l(\mu))}}
\]
The first term in the expression above is positive while the second term is negative. However, as $t$ converges to infinity, the first term becomes large relative to the second term. Hence, $\frac{\partial}{\partial \pi} \sigma_{t,j} > 0$ for values of $t$ that are sufficiently large. The following proposition summarizes this result.

**Proposition 3.** There exists a $\tau < \infty$ such that $\frac{\partial \sigma_{t,j}}{\partial \pi} > 0$ for all $t \geq \tau$.

### 4 General Model

In this section, we relax Assumptions 1–3 and confirm the results above numerically. Most importantly, we consider more general distributions for the aggregate and idiosyncratic liquidity shocks. This makes the analysis considerably more complicated, as the interaction between the optimal bid and ask prices, reservation values, and future prices and beliefs introduce a number of additional affects that were absent from our analysis with uniform distributions. While these complications make analytical results harder to derive, it remains fairly straightforward to solve the model computationally. We describe the simple, iterative algorithm here:

1. Given a grid of values for $\mu$ and $\omega$, we start with an initial guess for the reservation values, $R_j(\mu), j = h, l$.

2. Given reservation values, we can determine traders’ expectations of dealer beliefs $\mu^+$ for each $\omega$ by solving (17).

3. Given the updating equations, we compute prices for any beliefs $\mu$.

4. The law of motion for dealer beliefs and the pricing formulae can then be combined to yield an updated guess for the reservation value functions $R_j(\mu)$, using the expression in (5).

5. We repeat this iterative process until convergence.

Using this algorithm, we solve the model and show that the main insights from Section 3 continue to hold, despite the presence of the additional feedback effects mentioned above.

#### 4.1 Full Information Benchmark

As a benchmark, we first analyze the full information case, i.e., where dealers know the quality of the asset (but not the traders’ liquidity shocks, which are still privately observed). In this case, reservation values and prices are constant across time, and it is fairly straightforward to conduct comparative statics.
on the matching frequency, \( \pi \). We show that, under mild restrictions on the distributions of shocks, the standard relationships between trading frequency and bid-ask spreads emerge.

**Condition 1.** Let \( \tilde{G} \) be the distribution of \( \omega + \epsilon \). \( \tilde{G} \) is symmetric around 0 and \( \frac{\tilde{G}}{g} \) is increasing and convex.

This condition is satisfied by a number of commonly used distributions, including the normal, logistic and the Pareto distributions. It implies that the full information markup charged by monopolist dealers increases with volume at an increasing rate. This assumption is sufficient to ensure that full information spreads shrink as trading opportunities become more frequent.

**Proposition 4.** When asset quality is common knowledge, and Condition 1 holds, the spread charged by monopolist dealers, \( A_m^j - B_m^j \), is decreasing in \( \pi \) for \( j \in \{l, h\} \).

To see the intuition, we focus on the case with \( v_j > c_j \), i.e. when dealers value the asset more than the average obtained by the traders upon exit. This implies that, on average, traders are more likely to be sellers than buyers. As a result, the option to sell is worth more than the option to buy, i.e. the net option value is positive, \( \Omega_j > 0 \). Indeed, in the proof of Proposition 4, we show that

\[
\frac{dR_j}{d\pi} = \Omega_j + \pi \frac{d\Omega_j}{d\pi} > 0,
\]

so that more frequent trading opportunities raise reservation values. Moreover, under Condition 1, we also show that the inverse semi-elasticity of the investors that are buying the asset is greater than the inverse semi-elasticity of the investors that are selling the asset. As a result, the increase in \( \pi \) causes the bid price to increase more than the ask price, and the spread falls. Since this analogous to the relationship between the frequency of trading opportunities and spreads in the canonical Duffie-Garleanu-Pedersen framework, we refer to it as the “DGP effect.”

### 4.2 Asymmetric Information and Non-Uniform Distributions

Let us now return to the model with asymmetric information, where asset quality is privately observed by the traders. As noted above, this model has to be solved numerically. For the sake of illustration, we impose assumptions on the distributions of liquidity shocks and the values of the model’s key parameters. While all of our results depend, in some sense, on the parametric assumptions that are imposed, it is important to note that there is nothing special about these particular assumptions; that is, the results reported below obtain in a large range of the parameter space.

To start, we assume that both aggregate and idiosyncratic liquidity shocks are drawn from a mean-zero normal distribution, i.e. \( \omega \sim N(0, \sigma_\omega^2) \) and \( \epsilon \sim N(0, \sigma_\epsilon^2) \). We set the dealer valuations in the two
states to $v_h = 1.45$ and $v_l = 1$, and the corresponding valuations for traders to $c_h = 1.35$ and $c_l = 0$. We set the probability of staying in the market after each trading round, $\delta$, equal to 0.5 and the variances of the liquidity shocks, $\sigma^2_\omega$ and $\sigma^2_\varepsilon$, equal to 1. Finally, since the monopoly prices under non-uniform distributions contain both the markup component and the adverse selection component, we set $\alpha_m = 1$.

Figure 1 plots key equilibrium objects for two different values of $\pi$, namely 0.25 and 0.75. The top left panel plots the traders’ reservation values in the high and low state, while the top right figure plots the difference between the two. Importantly, this figure confirms that $R_h(\mu) - R_l(\mu)$ decreases with $\pi$ for all $\mu$. The key force behind this result is the same as in the special case: more frequent trading opportunities increases the weight of the net option value $\Omega_j(\mu')$ in the reservation value $R_j(\mu)$. Since the difference $\Omega_h(\mu') - \Omega_l(\mu') < 0$, this has the effect of bringing the reservation values closer to each other.

Figure 1: Effect of $\pi$ on prices and spreads.

The lower two panels plot prices and spreads. Naturally, both the bid and the ask prices are increasing functions of $\mu$, the probability that the asset quality is $h$. Spreads, however, are non-monotonic in $\mu$: they are small when there is no uncertainty—as the asymmetric information component of the bid and the ask is shut down—and large when there is more uncertainty. Moreover, note that the bottom right figure also
reveals that increasing $\pi$ lowers the spread for all $\mu$, i.e., that more frequent trading opportunities reduces spreads for any fixed set of beliefs.

Intuitively, increasing $\pi$ reduces $R_h - R_l$, and hence causes traders to act more alike in the two states of the world. As a result, dealers’ optimal prices require a smaller adjustment for asymmetric information. To see why, recall from equations (8) and (9) that the asymmetric information component of the dealers’ optimal prices depend on the density of traders around certain key thresholds. When the difference in reservation values narrows, so too does the difference between these thresholds, and the asymmetric information component shrinks. To put it differently, when trading decisions are driven to a greater extent by the option to rebalance in response to future liquidity shocks, there is less adverse selection from the dealers’ perspective, leading to tighter spreads.

However, this effect on spreads is essentially a static one since, by construction, it holds beliefs fixed. The decline in the difference between reservation values also slows down learning, exactly as in the special case. As a result, dealers remain relatively more uncertain about the true asset quality. This force makes the asymmetric information component of prices larger, putting upward pressure on spreads. Which force dominates— i.e. whether spreads widen or narrow as $\pi$ increases—depends on parameters.

To illustrate these two effects, in Figure 2 we plot the evolution of spreads over time. In this example, we assume that the true quality of the asset is $h$, and the dealers’ initial beliefs are $\mu_0 = 0.5$. The two panels plot (the average values of) dealer beliefs and spreads over time for two different values of $\pi$.\footnote{Recall that the realized path of dealer beliefs – and therefore, prices – depends on the realized sequence of $\omega_t$. The graph plots the average computed over 10000 such sample paths.} In both cases, beliefs drift upwards: since the true state is $h$, they both eventually converge to 1. However, the pace is clearly slower when $\pi$ is higher; again, more frequent trading opportunities leads to slower
learning. Spreads are tighter initially in the high $\pi$ case, but eventually end up wider. This is because, initially, beliefs are very similar in both cases (since they both start at 0.5 by assumption), so the static effect dominates and spreads are narrower under high $\pi$. Over time, however, the differential pace of learning kicks in, keeping uncertainty high and spreads wide in the high $\pi$ scenario, relative to the low $\pi$ case.

5 Conclusion

In the previous sections, we have laid out a unified framework for analyzing asset markets characterized by both informational and search frictions. Our results uncover novel interactions between these two fundamental imperfections, overturning conventional wisdom based on studying them in isolation. They help reconcile a puzzling feature of many OTC markets in recent years – even as technological and policy-induced changes have reduced trading frictions and increased transparency, measures of liquidity (e.g. bid-ask spreads) have largely remained unchanged, or even widened. Our analysis suggests that this may not be particularly surprising in markets where both types of frictions are present.

There are many directions for future research. A key challenge in many financial markets is to quantify the severity of different types of frictions. This is particularly relevant for regulators, who can then devise appropriate policies. In ongoing work, we use the theoretical framework developed in this paper to inform an empirical strategy that can disentangle information and search frictions from observable transaction data. Another interesting direction would be add other sources of market illiquidity (e.g. inventory costs).
References


**Appendix**

**A Proofs**

**A.1 Proof of Proposition 1**

*Proof.* For ease of notation, we suppress depends of $R_j, r_j, p$ to $\mu$. From (41), we have

\[
R_h - R_l = (1 - \delta) (v_h - v_l) + \delta (r_h - r_l) - \delta \pi \sum_k \alpha_k \frac{B^k - \Lambda^k + 2e}{2e} (r_h - r_l)
\]

\[
= (1 - \delta) (v_h - v_l) + \delta (r_h - r_l) - \delta \pi \alpha_c \frac{r_h - r_l}{2}
\]

\[
- \delta \pi \alpha_m \frac{\sqrt{e^2 - 4 \text{Cov}_j (r_j, v_j)} + e}{2e} (r_h - r_l)
\]

Page 29
Note that \( r_j = (1 - p) R_j + pv_j \). Therefore,

\[
\text{Cov} (r_j, v_j) = (1 - p) \text{Cov} (R_j, v_j) + p \text{Var} (v_j) = (1 - p) \text{Cov} (R_j, v_j) + p(1 - \mu) (v_h - v_t)^2
\]

Moreover,

\[
\mathbb{E}_j R_j = \mathbb{E}_j v_1 \to \mu R_h + (1 - \mu) R_l = \mu v_h + (1 - \mu) v_t
\]

\[
R_l = \frac{\mathbb{E}_j v_j - \mu R_h}{1 - \mu} \to R_h - R_l = \frac{R_h - \mathbb{E}_j v_j}{1 - \mu}
\]

which then it implies that

\[
\text{Cov} (R_j, v_j) = \mu R_h v_h + (1 - \mu) \left[ v_t - \frac{\mu}{1 - \mu} (R_h - v_h) \right] v_t - (\mathbb{E} v_j)^2
\]

\[
= \mu R_h (v_h - v_t) + v_t \mathbb{E} v_j - (\mathbb{E} v_j)^2
\]

\[
= \mu (v_h - v_t) (R_h - \mathbb{E} v_j)
\]

\[
= \mu (1 - \mu) (v_h - v_t) (R_h - R_l)
\]

Finally, we realize that \( r_h - r_t = (1 - p) (R_h - R_t) + p (v_h - v_t) \) and that from Bayesian updating, \( p = \frac{R_h - R_l}{2mp} \). The above expressions allow us to write (43) as an equation in \( p \) given by

\[
2mp = (1 - \delta) (v_h - v_t) + \delta [(1 - p) 2mp + (v_h - v_t) p]
\]

\[- \delta \pi \alpha_c [(1 - p) 2mp + (v_h - v_t) p]
\]

\[- \delta \pi \alpha_m \sqrt{\frac{e^2 - 4(1 - \mu) (v_h - v_t) [2mp (1 - p) + p (v_h - v_t)] + e}{2e}} [(1 - p) 2mp + (v_h - v_t) p]
\]

Since \( \alpha_c + \alpha_m = 1 \), we can write the above as

\[
0 = -2mp + (1 - \delta) (v_h - v_t) + \delta \left( 1 - \frac{\pi}{2} \right) [(1 - p) 2mp + (v_h - v_t) p]
\]

\[- \frac{\delta \pi \alpha_m}{2} \sqrt{\frac{1 - \frac{4}{e^2} \mu (1 - \mu) (v_h - v_t) ((1 - p) 2mp + (v_h - v_t) p) [(1 - p) 2mp + (v_h - v_t) p]}{[(1 - p) 2mp + (v_h - v_t) p]}}
\]

which is the same as (42). We refer to the right hand side of the above as \( Z (p, \mu; \Xi) \). We show that \( Z (p, \mu; \Xi) \) has a unique solution. To see this note that

\[
Z (0, \mu; \Xi) = (1 - \delta) (v_h - v_t) > 0
\]

\[
Z \left( \frac{v_h - v_t}{2m}, \mu; \Xi \right) = - (v_h - v_t) + (1 - \delta) (v_h - v_t) + \delta \left( 1 - \frac{\pi}{2} \right) (v_h - v_t)
\]

\[- \delta \pi \alpha_m \sqrt{\frac{1 - \frac{4}{e^2} \mu (1 - \mu) (v_h - v_t)^2}{1 + \alpha_m \sqrt{1 - \frac{4}{e^2} \mu (1 - \mu) (v_h - v_t)^2}}} < 0
\]
This implies that there exists a \( p^* \in \left[ 0, \frac{v_h - v_l}{2m} \right] \) that solves our equation. In addition, if we define \( g(p) = p(2m(1-p) + v_h - v_l) \), then

\[
Z_p = -2m + \delta \left( 1 - \frac{\pi}{2} \right) g'(p) \\
- \frac{\delta \pi \alpha_c}{2} \sqrt{1 - \frac{4}{e^2} \mu (1 - \mu) (v_h - v_l) g(p) g'(p)} \\
+ \frac{\delta \pi \alpha_c}{4} g(p) \left[ \frac{4}{e^2} \mu (1 - \mu) (v_h - v_l) g'(p) \right] \\
= -2m + \delta \left( 1 - \frac{\pi}{2} \right) g'(p) \\
- \frac{\delta \pi \alpha_c}{4} g(p) \left[ 2 - \frac{12}{e^2} \mu (1 - \mu) (v_h - v_l) g(p) \right] \\
4 \sqrt{1 - \frac{4}{e^2} \mu (1 - \mu) (v_h - v_l) g(p)}
\]

Note that

\[
g'(p) = 2m (1 - 2p) + v_h - v_l
\]

by the Assumption (2)

\[
(v_h - v_l + 2m) \delta \left( 1 - \frac{\pi}{2} \right) - 2m < 0 \rightarrow g'(0) \delta \left( 1 - \frac{\pi}{2} \right) - 2m < 0
\]

\[
\forall p \in \left[ 0, \frac{v_h - v_l}{2m} \right] \delta \left( 1 - \frac{\pi}{2} \right) g'(p) - 2m < 0
\]

where the second inequality follows since \( g'(p) \) is decreasing in \( p \). In addition,

\[
\frac{12\mu (1 - \mu)}{e^2} (v_h - v_l) g(p) \leq \frac{3}{e^2} (v_h - v_l)^2 < 2
\]

Finally, \( \forall p \in \left[ 0, \frac{v_h - v_l}{2m} \right] \), \( g'(p) > 0 \) given (2). This implies that the last expression in (44) is negative and therefore, \( Z_{p_m} = \sum_{p=0}^{\left[ 0, \frac{v_h - v_l}{2m} \right]} g(p) \). Hence, \( Z(p^*_m, \mu; \Xi) = 0 \) has a unique solution in \( \left[ 0, \frac{v_h - v_l}{2m} \right] \). Note that since \( R_j(\mu) \)'s are increasing in \( \mu \), \( R_h(\mu) \leq v_h \) and \( R_l(\mu) \geq v_l \). This implies that \( R_h(\mu) - R_l(\mu) \leq v_h - v_l \) which then implies that \( p \leq \frac{v_h - v_l}{2m} \).

### A.2 Proof of Lemma 1

**Proof.** Recall the equation that describes \( \mu \):

\[
0 = -2mp + (1 - \delta) (v_h - v_l) + \delta \left( 1 - \frac{\pi}{2} \right) [(1 - p) 2mp + (v_h - v_l) p] \\
- \frac{\delta \pi \alpha_m}{2} \sqrt{1 - \frac{4}{e^2} \mu (1 - \mu) (v_h - v_l) [(1 - p) 2mp + (v_h - v_l) p] [(1 - p) 2mp + (v_h - v_l) p]}
\]

31
The above function, \( Z(p, \mu; \Xi) \), depends on \( \mu \) only through \( \mu (1 - \mu) \). Moreover, \( Z(p, \mu; \Xi) \) is increasing in \( \mu (1 - \mu) \). Therefore, \( Z_{\mu} > 0 \) when \( \mu < 1/2 \) and \( Z_{\mu} < 0 \) when \( \mu > 1/2 \). Note that we have

\[
Z_p dp^* + Z_\mu d\mu = 0 \rightarrow \frac{dp^*}{d\mu} = -\frac{Z_p}{Z_\mu}
\]

From proof of proposition 1 we know that \( Z_p < 0 \). Therefore, \( \frac{dp^*}{d\mu} > 0 \) when \( \mu < 1/2 \) and vice versa.

Additionally, bid-ask spreads are given by

\[
A^c - B^c = e - \sqrt{e^2 - 4\mu (1 - \mu) (v_h - v_l) [(1 - p^*) 2mp^* + (v_h - v_l) p^*]}
\]

The above is increasing in \( \mu (1 - \mu) \) and \( p^* \). Since both of these are hump-shaped in \( \mu \) with maximum at \( \mu = 1/2 \), so is the bid-ask spread.

Finally, recall that

\[
R_h - R_l = 2mp^*
\]

and hence \( R_h - R_l \) is also hump-shaped in \( \mu \) and maxed at \( \mu = 1/2 \). This concludes the proof.

A.3 Proof of Proposition 2

Proof. We have

\[
Z_\pi = -\delta \left[ (1 - p) 2mp + (v_h - v_l) \right]
\]

\[
-\frac{\delta}{2} \left[ 1 - \frac{4}{e^2} \mu (1 - \mu) (v_h - v_l) [(1 - p) 2mp + (v_h - v_l) p] [(1 - p) 2mp + (v_h - v_l) p] \right]
\]

and therefore, \( Z_\pi < 0 \). Hence,

\[
\frac{dp^*}{d\pi} = -\frac{Z_p}{Z_\pi} < 0.
\]

A.4 Proof of Proposition 3

Proof. From the text we have

\[
\frac{\partial}{\partial \pi} \sigma_{t,j} = -\alpha_c (1 - p^*) t^{-1} \frac{dp^*}{d\pi} \left( 1 - \sqrt{1 - \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) g(p^*)} \right)
\]

\[
+ \alpha_c (1 - p^*) \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) g'(p^*) \frac{dp^*}{d\pi} \right] \left[ 1 - \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) g(p^*) \right]
\]

\[
= \alpha_c (1 - p^*) \frac{dp^*}{d\pi} \left[ \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) g'(p^*) \right] \left[ 1 - \frac{4}{e^2} (v_h - v_l) \mu (1 - \mu) g(p^*) \right]
\]

In the above, \( \frac{dp^*}{d\pi} < 0 \) and for large enough \( t \) the expression in the brackets is negative. Hence, when \( t \) is large enough \( \frac{\partial}{\partial \pi} \sigma_{t,j} > 0 \).
A.5 Proof of Proposition 4

**Proof.** Note that the full information model is described by the following relationships

\[
\delta R = \delta c + \pi \left[ \int_{B-R}^{A} \varepsilon dG(\varepsilon) + (B-R)G(B-R) + (A-R)(1-G(A-R)) \right]
\]

\[
\begin{align*}
A &= v + \frac{1-G(A-R)}{g(A-R)} \\
B &= v - \frac{G(B-R)}{g(B-R)}
\end{align*}
\]

where

\[
\xi_a(A-R) = \frac{1-G(A-R)}{g(A-R)} \quad \text{and} \quad \xi_b(B-R) = \frac{G(B-R)}{g(B-R)}
\]

are the semi-elasticities of demand (supply) with respect to prices and are therefore equal to the markups charged by the dealer.

Note. We have

\[
\xi''_a(\varepsilon) = \frac{d^2}{d\varepsilon^2} \frac{1-G(\varepsilon)}{g(\varepsilon)} = \frac{d}{d\varepsilon} \left( 1 - \frac{(1-G(\varepsilon))g'(\varepsilon)}{g(\varepsilon)^2} \right) = -\frac{d}{d\varepsilon} \left( \frac{(1-G(\varepsilon))g'(\varepsilon)}{g(\varepsilon)^2} \right) = \frac{g'}{g} \left( 1 - \frac{(1-G(\varepsilon))g''(\varepsilon)}{g' \cdot g^2} \right) + \frac{2(1-G(\varepsilon))g'(\varepsilon)^2}{g^3}
\]

Note that

\[
G(-\varepsilon) = 1 - G(\varepsilon) \\
g(-\varepsilon) = g(\varepsilon) \\
-g'(-\varepsilon) = g'(\varepsilon) \\
g''(-\varepsilon) = g''(\varepsilon)
\]

Thus

\[
\xi''_a(\varepsilon) = -\frac{g'(-\varepsilon)}{g(-\varepsilon)} - \frac{G(-\varepsilon)g''(-\varepsilon)}{g(-\varepsilon)^2} + 2 \frac{G(-\varepsilon)(g'(-\varepsilon))^2}{g(-\varepsilon)^3} = \xi''_b(-\varepsilon) > 0
\]

We show the claim first by assuming that \( v > c \) - the case where \( v < c \) follows a similar argument is thus omitted. The following Lemma proves useful in showing the main result:

**Lemma 1.** Suppose Condition 1 holds and that \( v > c \), then we have

\[
R > c, A - R > R - B.
\]
Proof. Suppose that $R \leq c$, then

$$\int_{B-R}^{A-R} \varepsilon dG(\varepsilon) + (B - R) G(B - R) + (A - R) (1 - G(A - R)) \leq 0$$

We can write the above as function $\Omega(A - R, B - R)$. Note that

$$\Omega(x, -x) = 0$$
$$\Omega_y(x, y) = G(y) > 0$$
$$\Omega_x(x, y) = 1 - G(x) > 0$$

This implies that $\Omega(A - R, B - R) \leq 0$ must lead to $A - R \leq R - B$, i.e., there is more buyers than sellers!

Hence by Condition 1

$$\frac{1 - G(A - R)}{g(A - R)} \geq \frac{1 - G(R - B)}{g(R - B)} = \frac{G(B - R)}{g(B - R)}$$

Thus

$$A + B = 2v + \frac{1 - G(A - R)}{g(A - R)} - \frac{G(B - R)}{g(B - R)} \geq 2v$$

Since

$$A - R \leq R - B \rightarrow A + B \leq 2R \rightarrow 2v \leq 2R \leq 2c$$

which is a contradiction. Thus the claim follows.

The above lemma implies that when there is gains from trade, the trader’s capture some of these gains and that there are more sellers than buyers.

To prove the main claim, we first note that

$$\frac{dA}{dR} = -\xi'_a(A - R)$$
$$\frac{dA}{dR} = \frac{1}{1 - \xi'_a(A - R)} \in [0, 1]$$
$$\frac{dB}{dR} = \frac{\xi'_b(B - R)}{1 + \xi'_b(B - R)} \in [0, 1]$$

Thus

$$\frac{d(A - B)}{dR} = -\xi'_a(A - R) \frac{1}{1 - \xi'_a(A - R)} - \xi'_b(B - R) \frac{1}{1 + \xi'_b(B - R)}$$

where

$$-\xi'_a(\varepsilon) = 1 + \frac{(1 - G(\varepsilon)) g'(\varepsilon)}{g(\varepsilon)^2}$$
$$\xi'_b(\varepsilon) = 1 - \frac{G(\varepsilon) g'(\varepsilon)}{g(\varepsilon)^2}$$
$$\xi'_b(-\varepsilon) = 1 - \frac{G(-\varepsilon) g'(-\varepsilon)}{g(-\varepsilon)^2} = 1 + \frac{(1 - G(\varepsilon)) g'(\varepsilon)}{g(\varepsilon)^2}$$
$$\xi'_b(-\varepsilon) = -\xi'_a(\varepsilon)$$
From the previous lemma we have \( A - R > R - B \) and therefore, from assumption 1, we must have

\[-\xi_a' (A - R) < -\xi_a' (R - B) = \xi_b' (B - R)\]

We, therefore, have

\[\frac{d (A - B)}{dR} = \frac{-\xi_a' (A - R)}{1 - \xi_a' (A - R)} - \frac{\xi_b' (B - R)}{1 + \xi_b' (B - R)}\]

\[= \frac{-\xi_a' (A - R) - \xi_b' (B - R)}{(1 - \xi_a' (A - R)) (1 + \xi_b' (B - R))} < 0\]

It is thus left to show that \( \frac{dR}{d\pi} > 0 \). We have

\[\delta dR = \pi [\Omega_x (dA - dR) + \Omega_y (dB - dR)] + d\pi \Omega (A - R, B - R)\]

\[= \pi [1 - G (A - R)] (dA - dR) + G (B - R) (dB - dR)] + d\pi \Omega (A - R, B - R)\]

\[= \pi \left[ - (1 - G (A - R)) \frac{dR}{1 - \xi_a'} - G (B - R) \frac{dR}{1 + \xi_b'} \right] + d\pi \Omega (A - R, B - R)\]

\[= \frac{\Omega (A - R, B - R)}{\delta + \pi \left[ \frac{1 - G (A - R)}{1 - \xi_a'} + \frac{G (B - R)}{1 + \xi_b'} \right]} > 0\]

This concludes the proof.

\[\Box\]

**B Other Derivations**

**B.1 Valuations of traders and dealers are equal in expectation**

Here we establish that (34) which states that in expectation - given dealers’ information - valuation of dealers and traders are equal.

We first show this when \( \mu = 0 < 1 \), i.e., when dealers are fully informed about the \( j \). Note that when \( \mu = 1 \),

\[B^c (1) = \frac{r_h (1) + v_h - e}{2} + \frac{1}{2} \sqrt{(e + v_h - r_h (1))^2} = v_h\]

\[A^c (1) = \frac{r_h (1) + v_h + e}{2} - \frac{1}{2} \sqrt{(e + v_h - r_h (1))^2} = v_h\]

\[B^b (1) = \frac{r_h (1) + v_h - e}{2}\]

\[A^b (1) = \frac{r_h (1) + v_h + e}{2}\]

where the above holds because with full information, \( \text{Cov}_j (r_j, v_j) = 0 \).

Note that \( r_h (1) = R_h (1) \) and value functions can be written as

\[r_h (1) = (1 - \delta) v_h + \delta r_h (1) + \delta \pi \sum_{k=c,m} \alpha_k \frac{(B^k - A^k + 2e)}{2e} \left( \frac{A^k + B^k}{2} - r_h (1) \right)\]

\[(1 - \delta) r_h (1) = (1 - \delta) v_h + \delta \pi \alpha_c (v_h - r_h (1)) + \delta \pi \alpha_m \frac{1}{2} v_h - r_h (1)\]
Obviously the unique solution of the above equation is \( r_h (1) = v_h \). Similarly, we can show that \( r_l (0) = v_l \).

This implies that
\[
 r_j (\mu) = p (\mu) R_j (\mu) + (1 - p (\mu)) v_j
\]

As a result
\[
 E_j r_j (\mu) = p (\mu) E_j R_j (\mu) + (1 - p (\mu)) E_j v_j
\]

Suppose to the contrary that \( E_j r_j > E_j v_j \). Given the above expression, this implies that \( E_j R_j (\mu) > E_j v_j \). Note that the Bellman equations for traders net-option values are given by
\[
 R_j (\mu) = (1 - \delta) v_j + \delta r_j (\mu)
\]
\[
 + \delta \pi \sum_{k=c, m} \alpha_k \left( B_k - A^k + 2e \right) \left( A^k + B_k - r_j (\mu) \right)
\]

Using the formulas for prices provided in the text, we have
\[
 A^c (\mu) = A^m (\mu) - \Psi (\mu)
\]
\[
 B^c (\mu) = B^m (\mu) + \Phi (\mu)
\]

with
\[
 \Phi (\mu) = \frac{1}{2} \sqrt{\left( E_j (v_j - r_j) + e \right)^2 - 4 Cov_j (r_j, v_j)}
\]
\[
 \Psi (\mu) = \frac{1}{2} \sqrt{\left( E_j (r_j - v_j) + e \right)^2 - 4 Cov_j (r_j, v_j)}
\]

Note that when \( E_j r_j > E_j v_j \), then \( \Psi (\mu) > \Phi (\mu) \)
\[
 r_j (\mu) = (1 - p (\mu)) R_j (\mu) + p (\mu) v_j
\]
\[
 E_j r_j = (1 - p) E_j R_j + p E_j v_j
\]

Using the above, we can write the Bellman equations as
\[
 R_j (\mu) = (1 - \delta) v_j + \delta r_j (\mu)
\]
\[
 + \delta \pi \sum_{k=m, c} \frac{1}{2} \alpha_k \frac{v_j - r_j (\mu)}{2}
\]
\[
 + \delta \pi \alpha_c \frac{\Phi (\mu) + \Psi (\mu) + e}{2} \left( v_j - r_j (\mu) + \Phi (\mu) - \Psi (\mu) \right)
\]

Taking expectation with respect to \( j \) in the above equation, we have
\[
 E_j R_j = (1 - \delta) E_j v_j + \delta E_j r_j
\]
\[
 + \delta \pi \sum_{k=m, c} \frac{1}{4} \alpha_k E_j v_j - E_j r_j
\]
\[
 + \delta \pi \alpha_c \frac{\Phi (\mu) + \Psi (\mu) + e}{2} \frac{E_j v_j - E_j r_j + \Phi (\mu) - \Psi (\mu)}{2}
\]
Since $\Phi(\mu) < \Psi(\mu)$ and $E_j v_j < E_j r_j$, the last two terms in the right hand side of the above expression are negative. Hence,

$$E_j R_j < (1 - \delta) E_j v_j + \delta E_j r_j = (1 - \delta + \delta (1 - p(\mu))) E_j v_j + \delta p(\mu) E_j R_j$$

Therefore,

$$(1 - \delta p(\mu)) E_j R_j < (1 - \delta p(\mu)) E_j v_j$$

The above inequality is in contradiction to the initial assumption where $E_j r_j > E_j v_j$. This is the required contradiction that proves our initial assumption wrong. Similarly, we can show that $E_j r_j < E_j v_j$ cannot hold. Hence, $E_j r_j = E_j v_j$.

**B.2 Pricing under Monopoly and Competition**

**B.2.1 Monopoly Pricing**

Suppose that for all realizations of $\omega$, $B^k - R_j(\mu) - \omega \in [-e, e]$. This means that if updated beliefs are given by $\mu'$, then the probability of a sale by a trader is given by

$$\frac{B^k - \omega - R_j(\mu') + e}{2e}$$

This implies that for a realization of $\omega$, the probability of sale by a trader is given by

$$\frac{B^k - \omega - r_j(\mu) + e}{2e}$$

where $r_j(\mu) = (1 - p(\mu)) R_j(\mu) + p(\mu) R_j(1[j = h])$. Therefore profits of a dealer who buys is given by

$$\sum_j \mu_j \int_{-m}^{m} \frac{B - \omega - r_j + e}{2e} (v_j - B) \frac{d\omega}{2w}$$

The derivative of the above objective with respect to $B$ is given by

$$\frac{E_j v_j - B}{2e} - E_j \frac{B - r_j + e}{2e} = 0 \rightarrow B^m = \frac{E_j v_j + E_j r_j - e}{2}$$

Similarly, if we assume that $R_j(\mu) + \omega - A^k \in [-e, e]$, then the profits for a selling monopolist dealer is given by

$$\sum_j \mu_j \int \frac{e - A + \omega + r_j}{2e} (A - v_j) \frac{d\omega}{2w}$$

The first order condition with respect to $A$ is given by

$$-\frac{A - E_j v_j}{2e} + E_j \frac{e - A + r_j}{2e} = 0 \rightarrow A^m = \frac{E_j v_j + E_j r_j + e}{2}$$
B.2.2 Pricing under Competition

In equilibrium and under competition, profits of a buying trader must be zero and we must have that

\[-B^2 + (E_j r_j + E_j v_j - e) B - (E_j v_j (r_j - e)) = 0\]

The above equation has two solutions. By Bertrand competition, equilibrium must be the higher solution. To see this, suppose that \(B_1 < B_2\) are the roots of this equation. Suppose, further, that equilibrium bid is \(B_2\). Then one of the dealers can deviate to \(B_2 - \epsilon\) for a small but positive value of \(\epsilon\) and achieve positive profits. The reason is that if \(B > B_2\) profits must be negative since ultimately they are negative and the equation has only two roots. Therefore, just below \(B_2\), they must be positive. Hence, this is a profitable deviation. Therefore, equilibrium bid must be given by

\[
B^e = \frac{E_j r_j + E_j v_j - e + \sqrt{(E_j r_j + E_j v_j - e)^2 - 4E_j v_j (r_j - e)}}{2}
\]

The discriminant in the above can be written as

\[
(E_j v_j)^2 + (E_j r_j)^2 - 2eE_j r_j + e^2 + 2E_j v_j E_j r_j - 2eE_j v_j + 4eE_j v_j - 4E_j v_j r_j =
\]

\[
E_j v_j^2 + E_j r_j^2 - 2eE_j r_j + e^2 + 2E_j v_j E_j r_j - 2eE_j v_j - 4E_j v_j r_j + 4E_j v_j E_j r_j - 4E_j v_j =
\]

\[
E_j v_j^2 + E_j r_j^2 - 2eE_j r_j + e^2 - 2E_j v_j E_j r_j + 2eE_j v_j - 4\text{Cov}_j (v_j, r_j) =
\]

\[
(E_j v_j - E_j r_j + e)^2 - 4\text{Cov}_j (v_j, r_j)
\]

Therefore, we can write the above as

\[
B^e = \frac{E_j (v_j + r_j) - e + \sqrt{(E_j (v_j - r_j) + e)^2 - 4\text{Cov}_j (v_j, r_j)}}{2}
\]

For the above to be a valid solution, we must have that

\[e + E_j (v_j - r_j) \geq 2\sqrt{\text{Cov}_j (v_j, r_j)}\]

In other words, \(e\) must be large enough. We will verify that these conditions are indeed satisfied under Assumptions 1-3.

As for the ask, we have that profits are given by

\[
\sum_j \mu_j \int \frac{e - A + \omega + r_j}{2e} (A - v_j) \frac{d\omega}{2w}
\]
Therefore, the zero-profit condition must be given by

$$-A^2 + (E_jr_j + e + E_jv_j)A - E_jv_j(e + r_j) = 0$$

Then, a similar Bertrand type argument as above implies that the equilibrium ask price is the lower of the two roots of the above. Thus, we have

$$A^c = \frac{E(r_j + v_j) + e - \sqrt{(E(r_j + v_j) + e)^2 - 4Ev_j(e + r_j)}}{2}$$

$$= \frac{E(r_j + v_j) + e - \sqrt{(Er_j)^2 + (Ev_j)^2 + e^2 + 2eEv_j + 2eEr_j + 2Er_jEv_j - 4Ev_j(e + r_j)}}{2}$$

$$= \frac{E(r_j + v_j) + e - \sqrt{(Er_j)^2 + (Ev_j)^2 + e^2 - 2eEv_j + 2eEr_j - 2Er_jEv_j - 4Cov(v_j, r_j)}}{2}$$

For this to exist, we must have that

$$e + E_j(r_j - v_j) \geq 2\sqrt{Cov(v_j, r_j)}$$