Asset Securitization and Optimal Asset Structure of the Firm

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First Version: January 5, 2001
This Version: October 22, 2001

*I am grateful to Alexander Batchvarov, Gregory Duffee, Jesse Fried, Dwight Jaffee, Dirk Jenter, Hayne Leland, Francois Robinet, Dan Rogers, Klaus Toft, Nancy Wallace, and to participants at the EFMA 2001 Meeting in Lugano and FMA 2001 Meeting in Toronto for provided information, constructive suggestions, and insightful discussions. Correspondence Information: Jure Skarabot, UC Berkeley, Haas School of Business, 545 Student Services Bldg. # 1900, Berkeley, CA 94720, tel: (510) 642 8543, fax: (510) 643 1420, e-mail: skarabot@haas.berkeley.edu, WWW: http://haas.berkeley.edu/~skarabot

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Abstract

The process of asset securitization is a new and innovative financing method used for funding and risk management purposes. Evolved over the last few decades, securitization represents a substantial and established part of US and global capital markets. In addition to its importance as a financial and asset restructuring tool, securitization originated various streams of academic research. Different models have been developed in order to analyze the theoretical aspects of the process. But some important questions have remained unexplained. The established models don't address why the firm should securitize its assets. They don't investigate if securitization can be a viable alternative for the firm when it chooses its optimal corporate structure. We develop a model of the multi-asset firm which provides an answer to these questions. The model analyzes various corporate structures and validates asset securitization as one of the value maximizing options which can accomplish the optimal incorporation of the assets in the firm. In the framework of the model, the multi-asset firm can optimally choose between aggregating all the assets in one firm, securitizing a part of them through a securitization vehicle, or spinning them off into single-asset firms. For each structure, the debtholders and the equityholders choose the firm's optimal leverage ratio. The optimal asset and capital structure have an effect on the value of tax and non-debt bankruptcy claims. That phenomena leads to a potential increase in the overall value of the firm. For a specific set of the assets, securitization can be a preferred choice, because it avoids the costs associated with the standard bankruptcy procedure for the firm. The model provides a set of testable implications related to the optimal corporate and financial structure of the multi-asset firm and to the subsequent role of securitization.

Key words: asset securitization, structured finance, optimal asset structure, credit risk.

JEL Classification: G21, G32, G34.

1 Introduction

Firms own multiple assets and the nature of the assets varies across the firms. For a specific multi-asset firm, the assets can be similar to each other, indicating that the firm is focused and it has minimized the potential conflicts which are caused by holding widely diverse assets. Alternatively, the assets can be different from each other, suggesting that the firm is diversified and it has captured the coinsurance benefit of various assets. It is also common that some of the assets are separated from the original firm and incorporated in a new entity. Those assets are not always put into a smaller firm with the same structure as the original firm. For example, they can be transferred into an entity which looks more like a trust and where the role of the equityholders is minimized to holding only the cash flow rights. The trust securitizes the assets by issuing its own non-recourse claims and it establishes a substantially cheaper bankruptcy restructuring procedure. Essentially, firms face the task to find their optimal corporate structure and scope.

To examine this issue we develop a valuation model for the multi-asset firm. Our model addresses the following fundamental question, does the corporate structure of the firm matter? More specifically, if the firm owns multiple assets, is it better that the assets are jointly incorporated in the multi-asset firm or that they are separated among several independent single-asset entities? If the assets are separated, should they be owned by the firms which have similar structure as the multi-asset firm, or should these single-asset entities use more restricted governance structure and accordingly issue securities with substantially lower risk characteristics? And finally, can we predict the firm's optimal choice of incorporation depending on the parameters which characterize the assets?

We take the concept of a standard Leland's (1994) capital structure model and we extend it to a multi-asset framework. In such setting, the firm issues debt and equity securities and it optimizes its capital structure through the balance between the tax credit of debt and the bankruptcy costs associated with the potential default. The optimal capital structure generates the highest overall value of the firm. The default boundary is endogenously chosen by the equityholders to maximize the value of their claims. Alternatively, we allow the firm to structure itself into similar stand-alone firms, where each of them owns a single asset. And in order to capture the effect of asset securitization, we introduce the option of a stand-alone trust – securitization vehicle[1] – which can own a transferred asset and issues asset-backed securities[2]. Securitization vehicle issues two classes of asset-backed securities, the debt-like class and the residual equity-like class. In comparison to the stand-alone firms, the equityholders in the securitization vehicle waive their control rights and they are entitled only to the residual cash flows. In particular, the equityholders are not allowed to infuse additional capital into the trust after the value of the assets hits the predetermined lower boundary. If the value of the assets drops, the trust rapidly prepays its claims. For that reason, the trust never enters into a standard bankruptcy procedure. The transactions costs related to the prepayment are significantly lower than the bankruptcy costs for the firm.

The analytic clarity of continuous-time framework and the closed-form solutions for the value of contingent claims allow us to compare the values of these structures for a continuum set of parameters. We choose the volatility of each assets and the correlation between them as the main variables and we keep the rest of the firm's parameters fixed. We allow the firm to choose its capital structure in order to maximize the total value of the firm. The overall value is equal to the sum of the total values of the original firm and its spun-off entities. We find that for the specific regions in the set of parameters, a certain organizational structure yields the highest overall value of the firm. There also exists a set of parameters where it is optimal for the original firm to transfer a part of its assets into a securitization vehicle. That result indicates that asset securitization can be the optimal choice for the value maximizing firm.

[1] Securitization vehicle, also called special purpose vehicle, is an entity which acquires ownership of the transferred assets and issues asset-backed securities. This entity is established only for the purpose of specific securitization and it is legally different and independent from the original owner of the assets. The securitization vehicle has a different governance structure than the originating firm. In particular, its specific structure restricts any chance of a standard bankruptcy procedure.

[2] For the purpose of the model, we assume that each of the assets in the multi-asset firm can be potentially securitized. We analyze the more realistic case – where only a specific kind of assets can be used in securitization – in our future work.

We can ask several questions. What is the intuition behind these results? What is the reason that the overall values of potential structures are different from each other? The general explanation is simple. In the model, the firm can elect one of the described asset structures. When the firm chooses one structure over another, the debtholders and the equityholders costlessly adjust firm's optimal leverage ratio. The optimal asset and capital structure has the effect on the value of tax and non-debt bankruptcy claims. As a consequence of this optimization process, the value of tax credits can be increased and the value of bankruptcy costs can be decreased. This change leads to a potential increase in the overall value of the firm. Whichever structure generates the highest total value, that one is the optimal choice for the value maximizing firm. The fact that the value can be transferred from the tax and non-debt bankruptcy claimants to the equity and debt holders through the properly chosen structure of the firm, explains why the organizational structure of the firm matters[3]. This concept is similar to the explanation for the value transfer in the presence of secured debt, (see Scott (1977), Bebchuk and Fried (1996)), or through the process of securitization, (see Lupica (2000)).

After we have justified the existence of the optimal asset structure in the context of our model, we want to understand the intuition behind each of the specific results. We change the volatility and correlation parameters and the first question which we want to address is when and why the firm should keep the assets together and when and why they should be separated in stand-alone entities.

We assume that the firm owns multiple imperfectly correlated assets and we vary the volatilities and the correlation. Our model indicates that the assets should be kept in the firm if the volatilities are relatively similar to each other and they should be separated if the volatilities are further apart. More specifically, if the relative difference between the volatilities is inside the interval around value zero, the joint incorporation of the assets in a multi-asset firm generates the structure with the highest value. Otherwise the firm should divide the assets into stand-alone single-asset entities. The explanation for this result is clear. When the imperfectly correlated assets are kept in the firm, they

[3]In the real world, such value transfers are restricted by the enforcement of the tax code and the protection of all bankruptcy claimants by the power of the bankruptcy courts.

provide a coinsurance in the sense that the overall value of the assets is less volatile. For the firm which can adjust its optimal leverage, a decrease in the assets' volatility is reflected in an increase in the total value of the firm, (see Leland (1994)). At the same time, there are advantages associated with the separation of the assets into stand-alone entities. When the volatilities of the assets differ from each other, each of the single-asset entities can adjust its optimal leverage more precisely than the multi-asset firm. This advantage increases the overall value of single-asset firms relatively to the overall value of the multi-asset firm, as the volatilities of the assets diverge further apart, (see John and John (1991), Flannery, Houston and Venkataraman (1993), John (1993)).

In addition to the interactions between the assets with different volatility levels, there is a value increasing effect of the correlation between the assets on the overall value of the multi-asset firm, (see Lewellen (1971)). A decrease in the correlation of the assets increases the coinsurance effect, therefore it should be reflected in an increase of the overall value of the firm. Our model confirms this intuition. We can observe that if we keep the volatilities fixed and we lower the correlation of the assets, then the overall value of the firm increases. The region where the joint incorporation of the assets is optimal becomes wider.

As a second question, our model addresses the issue of asset securitization. After the firm decides to separate its assets into non-recourse single-asset entities, we want to know when it is optimal to choose a standard type firm or an entity with a trust-like structure. Our model of the trust-like structures captures the fundamental characteristics of securitization vehicles. In the same way as the standard firm, the securitization vehicle issues two classes of securities. One class are the debt-like notes and the other is the residual equity-like class. It can choose its leverage to maximize the total value of the entity for a given set of the parameters. We model the equityholders as the owners of residual cash flow rights with no control rights. This role of equityholders is offset by the assumption that the securitization vehicle never enters into the costly bankruptcy procedure. When the value of the assets hits exogenous lower level, the securitization vehicle declares an early amortization period, it liquidates the assets and prepays the issued claims. Therefore we assume, that the restricted role of equity allows for an inexpensive early prepayment instead of the costly bankruptcy procedure in the case that the value of the assets deteriorates below some predetermined level. The model captures the advantage of the trust structure over the standard firm. This advantage is reflected in a greater protection for the trust claimholders against the costly standard bankruptcy procedure, (see Frost (1997), Hansmann and Kraakman (2000)). And finally – for the purpose of the model – we assume that the securitization vehicle chooses a corporation status for the tax purposes. Therefore, the securitization vehicle pays taxes, but the taxes are offset by the tax credits of debt. We keep the same structure of tax credits as in the case of the original company. Here we don't focus on specific tax advantages of asset securitization, (for a more detailed analysis, see Sullivan (1998)).

The results of our model show that for the high and extremely low levels of volatility, a stand-alone asset should be owned by the standard firm, where for the middle to lower range of volatility, the securitization vehicle is the optimal choice. The explanation for this outcome is straightforward. Transaction costs associated with the prepayment process for the securitization vehicle are substantially lower than the standard bankruptcy costs for the firm[4]. On the other side, tax credit is higher for the standard firm than for the securitization vehicle[5]. These two offsetting facts allow for a change in the overall value of the firm. From the tax benefit perspective, the standard firm is the optimal structure. But the variation in the difference between the bankruptcy costs for the standard firm and the prepayment costs for the securitization vehicle drives the optimality of securitization. Therefore, for the high levels of volatility, the standard firm is optimal because the difference between the bankruptcy and the prepayment costs is relatively low comparing to the tax benefit advantage. As the volatility decreases, securitization is optimal because the growth in the difference between the bankruptcy and the pre-

[4]In the real world, debtholders in the securitization vehicle always receive almost full payment. Fixed prepayment level for the value of the assets guarantees the principal payment. The optimal default boundary for the equityholders is lower than the boundary chosen by the debtholders. At the level chosen by the debtholders, equityholders would prefer the keep the firm solvent by adding the additional capital for coupon payments into the firm. The trust-like structure of the securitization vehicle prohibits such behaviour. In the model, we capture that phenomena by a substantially lower prepayment costs factor for the securitization vehicle than the bankruptcy costs factor for the standard firm.

[5]The tax benefit of leverage is lower when a fixed bankruptcy point used. This observation was already made by Leland (1994).

payment costs is faster than the growth in the tax advantage of the standard firm. As the volatility decreases towards the zero level, the tax benefit advantage of the standard firm is still present, but the bankruptcy costs advantage of the securitization vehicle becomes insignificant. This last phenomena is driven by the following fact. Close to the zero level of volatility, the standard firm and the securitization vehicle are nearly risk-free. The bankruptcy and the prepayment costs are near zero, hence the difference is also near zero.

We agree that our model cannot explain all the aspects of asset securitization. The volatility of the assets and the correlation between them are not the only factors which are relevant for the securitization. In fact, main three requirements for securitization are the following: assets generate a steady cash flow in the future, assets require a minimal managerial involvement and control, assets are observable and verifiable. We leave the development of a more extensive model which would capture the above characteristics for our future research.

And finally, our securitization model can be viewed as a model of the firm issuing secured debt. That might imply that the firm could achieve the same value increasing effect with the use of secured debt instead of asset securitization. In fact, asset-backed securities have much in common with secured debt (e.g., equipment trust certificates and mortgage bonds) and securitization is in many respect equivalent to secured financing. But there are some key differences between two methods. The holders of asset-backed securities are in a superior position in the case of the originator's bankruptcy. Securitization effectively removes the assets from the originator for bankruptcy purposes. The transfer is held as a "true sale" and the securitization trust has an ownership interest in the assets[6]. Historically, holders of asset-backed securities have resisted legal challenges from other claimants and they have retained the exclusive ownership rights to the securitized assets. Secured creditors cannot avoid the bankruptcy procedure. Even if they ultimately get the full amount of their claims repayed, secured creditors needs to wait

[6]If the transfer is held not to be a sale for bankruptcy purposes, the trust would then be a creditor of the originator and have a security interest in the assets. In such case, the originator's bankruptcy would result in a stay of all actions by creditors to obtain property of the originator, (see Schwarcz (1993)).

and expend costs until the bankruptcy procedure is finished.

Although the model doesn't directly distinguish between the two forms of financing, we implicitly relay on that main advantage of securitization over the secured debt. We assume that the use of secured debt by a multi-asset firm would still keep the bankruptcy costs high and our model captures the advantage of securitization in a form of a large difference between the bankruptcy costs of standard firms versus the prepayment costs associated with the securitization structure. Only the true separation of the assets and a trust-like structure of the securitization vehicle substantially decreases the bankruptcy costs and therefore increases the overall value of the multi-asset entity.

2 Related Literature

Our approach follows a part of the corporate finance literature, which is concentrated on the valuation of contingent claims and it analyzes the default in terms of issuers' incentive or ability to meet their obligations. This area of research – structural models of credit risk – is well established, starting with the pioneering work of Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978), Leland (1994), and Leland and Toft (1996). As an important extension of traditional models, we focus on the valuation problem with multiple underlying assets[7]. Regardless of a variety of different structural models, the appropriate multi-asset model has not yet been developed. One of the reasons is that the valuation of multi-asset contingent claims normally doesn't generate closed-form solution. We approach to this problem in the following way. In order to obtain closed-form solutions, we apply the method which is similar to the approximate valuation of arithmetic Asian options using their geometric mean[8]. The modification of the underlying multi-asset process leads to analytic-form solutions for the value of the claims. We believe that this novel approach is not only suitable for our project, but it

[7] The topic is similar to the pricing of multi-assets options. The multi-asset option research started with the work of Magrabe (1978), Stulz (1982), and Rubinstein (1991, 1994).

[8] See Kemma and Vorst (1990), Turnbull and Wakeman (1991), and Vorst (1992) for analytic approximations of Asian options that produce closed-form expressions.

can be applied in a large set of valuation models which are based on multiple underlying uncertainties.

From a conceptual perspective, our model is similar to the study by Flannery, Houston and Venkataraman (1993). In that study the firm wants to invest in risky projects. To implement these projects, the initial owners must issue external debt and equity. They also must decide between forms of corporate organization, choosing separate or joint incorporation of the projects. The model is based on a trade-off between the tax advantage of debt and the asset substitution problem. The authors determine whether multiple projects should be operated within a single firm, or in separate legal subsidiaries of a holding firm. Joint incorporation is more valuable when project returns are less positively correlated and when the projects' risks are more similar to one another. Separate incorporation is preferable when project risks are very different or their return correlations are very high. Although this study does not consider any alternative organizational structure – like asset securitization – the results are consistent with our findings. On the other side, the model is built in a very restrictive setting. It is a one period model, it doesn't allow for any bankruptcy outcome and it is not capable to derive the optimal capital structure. The numerical methods must be applied to derive the results and the model can cover only a few discrete examples from the parameter set. In that view, our model successfully eliminates the shortfalls from the Flannery, Houston and Venkataraman (1993) study.

Several authors analyzed the asset structure of the firm in the association with the organizational form that allows the separation between the firm's assets. Greenbaum and Thakor (1987) was one of the first studies which addressed the relevance of asset structure in connection with the process of securitization. They analyzed the effect of the adverse selection on asset structure of financial institutions. They showed that if the financial institution posses private information about their assets which are not available to investors, then the institution is better off if it sell and securitize better quality assets and it keeps worse quality assets on its books and finance them with deposits. Their study showed that not only the asset structure matters, but also it indicated the suitability of securitization in transforming the asset structure.

There is a strong similarity between the goal of our model and the objective of project finance research. The project finance studies, like Shah and Thakor (1987), John and John (1991) and Chemmanur and John (1996) analyzed the interactions between the project financing and the optimal incorporation with a similar aim as our model. Shah and Thakor (1987) developed a model of optimal capital structure based on corporate taxes and asymmetric information. Their model predicts that in order to minimize signaling costs, projects which are riskier than the parent firm should be separated from the parent firm. Separation is conducted using the limited recourse project financing arrangements. Separated projects should be financed so that they have higher leverage ratios than the original firm. According to their model, the economic motivation for project financing is driven by the attempt to minimize the costs of asymmetric information. John and John (1991) developed a model of project financing based on a balance between debt-related tax shields and agency costs of underinvestment decisions caused by the outstanding debt[9]. They conclude that when the projects are far apart in the growth characteristics of their technology, the value of gains from project financing should be high. In addition, less debt should be allocated to the financed projects when managerial discretion in investment is high. Chemmanur and John (1996) constructed a model of project financing based on symmetric information. They focus on a firm with multiple projects and analyze the relationships among its organizational, capital and ownership structures. Differences in managerial ability across projects, the benefits of control, and the probability of loss of control through a takeover or bankruptcy are driving factors in their model. The authors derive implications for spin-off and project financing decisions. They conclude that management's ability to govern different projects and the control benefits available to them from the projects, play important roles in deciding which of the projects should be kept in the original firm and which should be spun off into a separate firm or funded through a limited recourse project financing structure. If management's relative abilities across the projects are very different, then projects will be spun off into separate firms. When the relative abilities are similar but the control benefits are significantly different, then project financing should be used. They discuss optimal debt ratios and show that, in the case of project financing ar-

[9] John (1993) developed a similar theoretical analysis which justified the optimality of spin-offs.

rangements, the projects with smaller control benefits than the parent firm should have higher debt ratios.

As we can see, these studies used a different approach than our model. They assumed some form of asymmetric information or private benefits of control framework. Their setting can provide a qualitative justification of different organizational forms but it can't measure the differences in the overall value between the different structures of the firm. Our model not only provides an efficient valuation method, but it also explicitly models different structures for a continuum range parameters in a unified variance-covariance framework.

There exists a literature which focuses specifically on the valuation of asset-backed securities. In most cases, it utilizes a different structure than our model. The framework is usually aligned with the following assumptions. For a given class of asset-backed securities, the pool of underlying assets – which serves as a collateral for the issued obligations – is formed. Once the performance of each asset – in most cases the assets are financial securities generating regular income – is simulated, the generated cash flows are aggregated, supporting the coupon and principal payments of asset-backed securities. The default and prepayment rates of the underlying pool are reflected in the value of issued tranches. We mention two representative studies. Childs, Ott and Riddiough (1996) developed a model for pricing commercial mortgage-backed securities. They used the structural model for the value of each mortgage in the pool. Based on the correlation structure, they aggregated generated cash flows which support a class of CMBS securities. They priced those securities using Monte Carlo techniques. Duffie and Gârleanu (2001) addressed the valuation of collateralized debt obligations. They used the reduced-form model of credit risk to represent each the underlying debt obligations. Specific assumptions about the default intensity processes for the single obligations allowed them to express the default intensity of a pool in a similar analytic form. That result lead to the estimation of the cash flows from the collateral pool. The authors developed an efficient pricing simulation of CDO tranches under different priority schemes.

It is true that a more precise formulation for the pool of underlying assets – like in the above studies – leads to an accurate pricing technique for asset-backed securities. We want to stress that our model primarily focuses on of the explanation why the firm should securitize its assets. None of the pricing models is able to answer this question. Because we model the firm with all of its assets included, our results could potentially be useful for the enterprise wide risk management.

From a related perspective, there exists a well established security design literature which analyzes the optimal design of asset-backed securities[10]. Models developed in these studies assume the presence of asymmetric information with some form of Akerlof's (1970) "lemons problem". These models seek to explain the gains achieved by bundling assets and then further tranching them before they are sold into the capital markets. Our goal for the future research is to analyze the optimal design of asset-backed securities from a perspective of credit risk and frictions related to the potential default. In that way, we could built a model for the issuing process in a symmetric information framework.

We can conclude this section with the observation that our contribution to the literature is similar to the contribution of Leland's (1994) study which resolved – in a continuous-time framework – the puzzles related to the optimal capital structure[11], (see Myers (1984)). Our model uses similar continuous-time methods in order to resolve questions related to the optimal asset structure. Deriving the rules for the optimal asset structure of the firm also gives an answer to the fundamental question why the firm should securitize its assets.

3 The Model

Consider the firm which owns two different assets. The two-asset structure is not

[10] See Boot and Thakor (1993), Glaeser and Kallal (1997), Riddiough (1997), DeMarzo and Duffie (1999) and DeMarzo (1999).

[11] It is true that in a pure Modigliani-Miller world, the issue of optimal asset structure and securitization comes under the power of the indifference theorems. Using the standard assumptions about market imperfections, we are able to pierce Modigliani-Miller structure and we can determine the optimal capital and asset structure of the firm in a multi-asset setting.

restrictive. Our model is general enough to cover the cases with an arbitrary number of assets. Results don't depend on the number of different assets in the way that the extension to a more general multi-asset case would be prevented. We assume that the uncertainty in the model is presented by the random process for the value of the assets $V = (V_1, V_2)$. Process V follows 2-dimensional Ito's process in the following sense. Given a probability space $(\Omega, \mathcal{F}, P)$ we assume that each process V_i solves a stochastic differential equation of the form

$$dV_i = \mu_i(V_i, t)dt + \sigma_i(V_i, t)dw_i, \tag{1}$$

where each w_i is a standard one-dimensional Brownian motion and motions w_i and w_j are correlated

$$E(dw_i dw_j) = \rho_{ij}(t)dt. \tag{2}$$

We assume that Brownian motions w_i are linearly independent, therefore correlation matrix $\Sigma = [\rho_{ij}]_{i,j=1}^2$ has full rank. Coefficients $\mu_i : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ and $\sigma_i : (0, \infty) \times [0, \infty) \to \mathbb{R}$ satisfy conditions for the existence and uniqness of solution (1) for each initial condition $V(0)$, (see Karatzas and Shreve (1991), (1998)).

We assume that the process V for the value of the assets is exogenous, therefore it is unaffected by the organizational structure of the firm or by the characteristics of the actual owner or manager of the assets. That means the process V is unchanged regardless if all of the assets stay in the multi-asset firm, or if some of them are separated into the single-asset firm or transferred to the securitization vehicle.

We also assume that there exists a riskless bond $B(t)$ with a constant riskless rate r which satisfies the following

$$dB = rBdt. \tag{3}$$

In order to stay comparable with the standard contingent claim valuation literature, we assume that each V_i follows a geometric Brownian motion

$$dV_i = \mu_i V_i dt + \sigma_i V_i dw_i \tag{4}$$

and the correlation coefficient between the Brownian motions w_i and w_j is constant

$$E(dw_i dw_j) = \rho_{ij} dt. \tag{5}$$

Therefore from (1), (4) and (3) we get by Ito's lemma

$$B(t) = B(0)e^{rt}, \tag{6}$$

$$V_i(t) = V_i(0)e^{\left(\mu_i - \frac{1}{2}\sigma_i^2\right)t + \sigma_i w_i}. \tag{7}$$

We can use the existence of an equivalent martingale measure Q under which the value of each asset is determined by

$$V_i(t) = V_i(0)e^{\left(r - \frac{1}{2}\sigma^2\right)t + \sigma w_i^Q}, \tag{8}$$

where each w_i^Q is a standard Brownian motion under measure Q and the correlation matrix $\Sigma = [\rho_{ij}]_{i,j=1}^2$ stays the same as in (5).

The main part of our model is the valuation of securities. Consider a contingent claim $F(V,t)$ on the firm's assets. This contingent claim pays a payout $C(V,t)$ which is financed by the issue of additional equity. Using multi-dimensional Ito's lemma and standard replication techniques, it can be derived that $F(V,t)$ satisfies the 2-dimensional version of Black-Scholes equation[12]

$$\frac{\partial F}{\partial t} + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\rho_{ij}\sigma_i\sigma_j V_i V_j \frac{\partial^2 F}{\partial V_i \partial V_i} + \sum_{i=1}^{2} r V_i \frac{\partial F}{\partial V_i} - rF + C(V,t) = 0 \tag{10}$$

where $V \in \mathcal{B} \subset \mathbb{R}_{++}^2$ and $0 \leq t \leq T$ with the appropriate boundary conditions for V at the boundary surface $\partial\mathcal{B}$ and the endpoints $t = 0$, $t = T$.

[12] More generally, if net payout (dividend yield) generated by the asset V_i – and not financed by further equity issuance – is denoted $P_i(V_i,t)$, then claim F satisfies the following equation

$$\frac{\partial F}{\partial t} + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\rho_{ij}\sigma_i\sigma_j V_i V_j \frac{\partial^2 F}{\partial V_i \partial V_i} + \sum_{i=1}^{2}\left(rV_i - P_i(V_i,t)\right)\frac{\partial F}{\partial V_i} - rF + C(V,t) = 0. \tag{9}$$

If we imagine that the above claim $F(V,t)$ represents a debt-like security with a coupon C and principal payment P at the maturity then the boundary conditions must reflect the main features of the debt. That means that at the maturity $t = T$ or at the default boundary $\partial\mathcal{B}$, debtholders receive the whole principal payment, if the firm is solvent and if the value of the assets is less than the principal, then they take over the firm and therefore receive the total value of the assets less some bankruptcy cost.

In order to compare the values of contingent claims between different structural models, we first analyze the case with all the assets kept in the firm. This structure is an extension of the original single-asset valuation by Leland (1994). Our analysis uses the same basic valuation concept. The value of tax benefits is added and value of bankruptcy cost is subtracted from the value of the assets and the derived amount equals the total value of the firm, (see Leland (1994)). On the other side, the total value of the firm is equal to the value of debt and equity claims issued. In our model we allow for one class of debt and a single equity class. The second case is the firm which securitize a part of its assets. The original multi-asset firm keeps one asset and transfers the second assets to the securitization vehicle. Securitization vehicle issues two classes of securities. The third case is a complete separation of assets into two independent firms, where each of the firms owns one of the assets and issues debt and equity securities which are structured in the same way across the firms.

3.1 Case I: Multi-asset firm

Consider a debt claim $D(V,C,t)$ that continuously pays a coupon C, as long as the firm is solvent and it gives the right to debtholdes to capture the firm's assets in case it declares the bankruptcy. Coupon payments are financed by selling additional equity. Following Leland (1994), we assume that debt promises a perpetual coupon payment so that the debt claim has no explicit time dependence. Valuation equation (10) can be rewritten as

$$\mathcal{L}F(V,t) \stackrel{\text{def}}{=} \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\rho_{ij}\sigma_i\sigma_j V_i V_j \frac{\partial^2 D}{\partial V_i \partial V_i} + \sum_{i=1}^{2} r V_i \frac{\partial D}{\partial V_i} - rD + C = 0, \tag{11}$$

where V is in the solvency region $\mathcal{B} \subset \mathbb{R}^2_{++}$, $\rho_{11} = \rho_{22} = 1$ and $\rho_{12} = \rho_{21} = \rho$.

The above equation must be equipped with the proper boundary conditions. Naturally, if the value of assets V grows large, then the firm is far away from bankruptcy and the value of debt $D(V_1, V_2)$ should be close to the value of risk-free perpetual debt

$$V_1 + V_2 \to \infty \;\Rightarrow\; D(V_1, V_2) \to C/r. \tag{12}$$

If the firm enters bankruptcy, that means V is at the boundary $\partial\mathcal{B}$ of the solvency region $\mathcal{B}$, then

$$(V_1, V_2) \in \partial\mathcal{B} \;\Rightarrow\; D(V_1, V_2) = (1-\alpha)\left(V_1 + V_2\right), \tag{13}$$

with $0 < \alpha \leq 1$. Here we assumed that if the bankruptcy occurs, a fraction α of the total value of the assets will be lost due to the bankruptcy costs. Debtholders receive the remaining part of the assets and equityholders receive nothing. In addition, the boundary of the solvency region $\partial\mathcal{B}$ is endogenized as in Leland (1994). The problem (11) becomes a free-boundary problem with additional smooth-pasting condition at the default boundary.

There exists a problem which is preventing us from simply rewriting Leland's (1994) model to a multi-asset setting. The value of the firm's assets is represented by the sum of two geometric Brownian motions. Because the sum of lognormal processes is not a lognormal process, that represents a serious problem in case that we are searching for some variation of closed-form solutions for equation (11). To make the analysis tractable and to stay aligned with similar one-dimensional models, we make a simplifying assumption for the value of the firm's assets.

The problem is similar to the valuation of arithmetic Asian options where the payoff from the option is a function of the sum of the prices of underlying securities. Again, security prices are lognormally distributed. Because there is no recognizable closed-form probability density function for the sum of lognormal random variables, a variety of approximation techniques has been developed to value arithmetic Asian options. Our approach follows the research method which uses analytic approximations of Asian options that produce closed-form solutions. In particular, we substitute the sum

$$\text{multi-asset value} = V_1 + V_2, \tag{14}$$

with the corresponding multiplier of the geometric mean

$$\text{multi-asset value} \approx 2\sqrt{V_1 V_2}. \tag{15}$$

In the same way as in the case of geometric Asian option, which are easy to price in Black-Scholes framework with known closed-form solutions, (see Kemma and Vorst (1990), Turnbull and Wakeman (1991), and Vorst (1992)), the substitution (15) allows us to derive closed-form solutions for equation (11). Because the geometric average is always lower than the corresponding arithmetic average (equality holds only in the case when all the elements are the same), the substitution (15) lowers the total value of the firm. To estimate the exact difference, we assume – without loss of generality – that $V_1 \geq V_2$ and $kV_1 \leq V_2$ for some fixed constant k , $0 < k < 1$. Then the difference between the sum and the corresponding geometric mean can be estimated from the above as

$$(V_1 + V_2) - 2\sqrt{V_1 V_2} = \left(\sqrt{V_1} - \sqrt{V_2}\right)^2 \leq V_1 \left(1 - \sqrt{k}\right)^2. \tag{16}$$

We minimize this approximation error by focusing our analysis on the example where the assets V_1 and V_2 are approximately similar in the size, that means k is close to one[13].

Once the total value of the assets is approximate by $2\sqrt{V_1 V_2}$, the boundary conditions (12) and (13) are rewritten as

$$2\sqrt{V_1 V_2} \to \infty \ \Rightarrow \ D(V_1, V_2) \to C/r \tag{17}$$

[13] Alternatively, we could improve the estimate by decreasing the bankruptcy boundary, (see Vorst (1992)), or by modifying the drift and volatility of the processes, (see Levy (1992)).

and

$$(V_1, V_2) \in \partial\mathcal{B} \;\Rightarrow\; D(V_1, V_2) = (1-\alpha)\left(2\sqrt{V_1 V_2}\right). \tag{18}$$

As a candidate for solution of equation (11) with boundary conditions (17) and (18) we propose

$$D(V_1, V_2) = \frac{C}{r} - \frac{y}{(V_1 V_2)^x} \tag{19}$$

with suitable constants y and x which we need to determine at some later point[14]. From (11) and (19) we get

$$\mathcal{L}D(V_1, V_2) =$$

$$= \frac{-y}{(V_1 V_2)^x}\left[\frac{1}{2}\left(\sigma_1^2 x(x-1) + 2\rho\sigma_1\sigma_2 x^2 + \sigma_2^2 x(x-1)\right) - rx - rx - r\right] =$$

$$= \frac{-y}{(V_1 V_2)^x}\left[\frac{1}{2}\left(\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2\right)x^2 - \left(2r - \frac{1}{2}\left(\sigma_1^2 + \sigma_1^2\right)\right)x - r\right]. \tag{20}$$

In order to satisfy

$$\mathcal{L}D(V_1, V_2) = 0 \tag{21}$$

for all $(V_1, V_2) \in \mathcal{B}$, the following equation must hold

$$\frac{1}{2}(\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2)x^2 - \left(2r - \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2\right)\right)x - r = 0. \tag{22}$$

The above equation (22) has two solutions for x, but only the positive one satisfies the required boundary condition (17). Therefore, we determine x as x_m where

$$x_m = \frac{\left(2r - \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2\right)\right) + \sqrt{\left(2r - \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2\right)\right)^2 + 2(\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2)r}}{\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2}. \tag{23}$$

Using the bankruptcy boundary condition (18), we can determine the constant y. Here we assume – and later we give the proof using smooth-pasting condition for the

[14]The geometric mean approximation for the value of the assets affects the valuation of contingent claims. The exact estimation of the difference is beyond the scope of this article.

value of equity – that the solvency region $\mathcal{B}$ is defined as

$$\mathcal{B} = \left\{ (V_1, V_2); V_1 > 0, V_2 > 0, 2\sqrt{V_1 V_2} > V_B \right\} \tag{24}$$

for some positive constant V_B which needs to be determined later. Therefore, at the point when the bankruptcy is declared, the following is true for values of the assets (V_1, V_2)

$$2\sqrt{V_1 V_2} = V_B. \tag{25}$$

From (18), (19) and (25) we can conclude that

$$y = \left(\frac{C}{r} - (1-\alpha)V_B \right) \left(\frac{V_B}{2} \right)^{2x_m}. \tag{26}$$

Therefore the value of the debt $D(V_1, V_2)$ is given as

$$D(V_1, V_2) = \frac{C}{r} - \left(\frac{C}{r} - (1-\alpha)V_B \right) \left(\frac{V_B}{2\sqrt{V_1 V_2}} \right)^{2x_m}. \tag{27}$$

Equation (27) represents a natural extension of Leland's (1994) debt valuation to a multi-asset setting. It can be rewritten as

$$D(V_1, V_2) = (1 - p_B) \left(\frac{C}{r} \right) + p_B \left((1-\alpha) V_B \right) \tag{28}$$

where

$$p_B = \left(\frac{V_B}{2\sqrt{V_1 V_2}} \right)^{2x_m}$$

has the interpretation of the present value of \$1 contingent on future bankruptcy, (see Leland (1994)).

In order to continue with the valuation of other contingent claims, we must determine the value of bankruptcy costs $BC(V_1, V_2)$ and tax benefits $TB(V_1, V_2)$.

The value of bankruptcy costs $BC(V_1, V_2)$ is modelled as a security which satisfies equation (11), pays no coupon and has value equal to the bankruptcy costs αV_B at

$(V_1, V_2) \in \partial\mathcal{B}$. Factor α is the same as in (13). Here again we approximate the sum of the values for the assets by their corresponding geometric mean. Therefore, $BC(V_1, V_2)$ satisfies boundary conditions

$$2\sqrt{V_1 V_2} \to \infty \;\Rightarrow\; BC(V_1, V_2) \to 0 \tag{29}$$

and

$$(V_1, V_2) \in \partial\mathcal{B} \;\Rightarrow\; BC(V_1, V_2) = \alpha V_B. \tag{30}$$

Using the same steps as in the valuation of debt notes, we find that the value of bankruptcy costs is given as

$$BC(V_1, V_2) = \alpha V_B \left(\frac{V_B}{2\sqrt{V_1 V_2}} \right)^{2x_m}. \tag{31}$$

Value of tax benefits $TB(V_1, V_2)$ can be determined in the same way as the value of bankruptcy costs $BC(V_1, V_2)$. Tax benefits can be modelled as a security which satisfies equation (11), pays a coupon equal to the tax credit value of interest payments τC, where $0 < \tau < 1$, as long as the firm is solvent and pays nothing at the bankruptcy boundary. Therefore, $TB(V_1, V_2)$ satisfies boundary conditions

$$2\sqrt{V_1 V_2} \to \infty \;\Rightarrow\; TB(V_1, V_2) \to \frac{\tau C}{r} \tag{32}$$

and

$$(V_1, V_2) \in \partial\mathcal{B} \;\Rightarrow\; TB(V_1, V_2) = 0. \tag{33}$$

We find that the value of tax benefits is given as

$$TB(V_1, V_2) = \frac{\tau C}{r} - \left(\frac{\tau C}{r} \right) \left(\frac{V_B}{2\sqrt{V_1 V_2}} \right)^{2x_m}. \tag{34}$$

The total value of the firm $v(V_1, V_2)$ with two assets is equal to the value of the assets plus the value of tax credits minus the value of bankruptcy costs

$$v(V_1, V_2) = 2\sqrt{V_1 V_2} + TB(V_1, V_2) - BC(V_1, V_2) =$$

$$= 2\sqrt{V_1 V_2} + \frac{\tau C}{r} - \left(\frac{\tau C}{r} + \alpha V_B\right)\left(\frac{V_B}{2\sqrt{V_1 V_2}}\right)^{2x_m}. \qquad (35)$$

The value of equity is equal to the total value of the firm minus the value of debt

$$E(V_1, V_2) = v(V_1, V_2) - D(V_1, V_2) =$$

$$= 2\sqrt{V_1 V_2} - \frac{(1-\tau)C}{r} + \left(\frac{(1-\tau)C}{r} - V_B\right)\left(\frac{V_B}{2\sqrt{V_1 V_2}}\right)^{2x_m}. \qquad (36)$$

Our next goal is to prove that the bankruptcy boundary – endogenously determined by the equityholders – is specified as

$$\partial\mathcal{B} = \left\{(V_1, V_2); V_1 > 0, V_2 > 0, 2\sqrt{V_1 V_2} = V_B\right\} \qquad (37)$$

for some constant V_B. As a part of that proof we determine the specific value for the constant V_B.

In order to justify bankruptcy boundary (37), we follow Leland's (1994) procedure. If the firm is not constrained by debt covenants, bankruptcy will occur when the firm cannot meet the coupon payments by issuing additional equity, therefore when equity value is zero. The above valuation is designed in the way that when $(V_1, V_2) \in \partial\mathcal{B}$ the value of equity is zero. The requirement for equityholders to maximize equity value by choosing the optimal bankruptcy boundary is given by smooth-pasting condition at $(V_1, V_2) \in \partial\mathcal{B}$.

Similar free-boundary optimization method in a multi-asset setting was used by Hu and Øksendal (1998), using optimal stopping theory, (see Øksendal (1989), (1998)). As discussed by Øksendal (1989), to use smooth-pasting approach, we need to check that the gradient of $E(V_1, V_2)$ is equal to the gradient of payoff function at the boundary

$\partial\mathcal{B}$[15]. Equityholders receive nothing when the firm goes bankrupt, therefore the smooth-pasting condition is given as

$$\nabla E(V_1, V_2)|_{(V_1,V_2)\in\partial\mathcal{B}} = 0. \tag{38}$$

The condition (38) can be rewritten as two separate equations

$$\frac{1}{V_1}\left(\frac{V_B}{2} - x_m\left(\frac{(1-\tau)C}{r} - V_B\right)\right) = 0, \tag{39}$$

$$\frac{1}{V_2}\left(\frac{V_B}{2} - x_m\left(\frac{(1-\tau)C}{r} - V_B\right)\right) = 0. \tag{40}$$

From (39) and (40) we can conclude that (38) holds if

$$V_B = \frac{(1-\tau)Cx_m}{r(x_m + 1/2)}. \tag{41}$$

Therefore we determined that

$$\partial\mathcal{B} = \left\{(V_1, V_2); V_1 > 0, V_2 > 0, 2\sqrt{V_1V_2} = \frac{(1-\tau)Cx_m}{r(x_m+1/2)}\right\} \tag{42}$$

is the optimal stopping boundary for the equityholders to declare bankruptcy.

Substituting expression for V_B into equations (27), (36), (35) we derive the value of debt, equity, and the total value of the firm.

Values of debt, equity, and the total value of the firm are given as

$$D(V_1, V_2) = \frac{C}{r}\left[1 - \left(\frac{C}{2\sqrt{V_1V_2}}\right)^{2x_m} k\right], \tag{43}$$

$$E(V_1, V_2) = 2\sqrt{V_1V_2} - \frac{(1-\tau)C}{r}\left[1 - \left(\frac{C}{2\sqrt{V_1V_2}}\right)^{2x_m} \frac{m}{2}\right], \tag{44}$$

[15]This choice of bankruptcy condition also maximizes the value of equity at any level of V, $dE/dV_B = 0$. The equivalence of these two conditions suggests that the boundary $\partial\mathcal{B}$ is incentive compatible for the equityholders, (see Merton (1973) and Leland (1994)).

$$v(V_1, V_2) = 2\sqrt{V_1 V_2} + \frac{\tau C}{r}\left[1 - \left(\frac{C}{2\sqrt{V_1 V_2}}\right)^{2x_m} h\right], \tag{45}$$

where

$$k = \left[(x_m + 1/2) - (1-\alpha)(1-\tau)x_m\right] m,$$
$$h = \left[(x_m + 1/2) + \frac{\alpha(1-\tau)x_m}{\tau}\right] m,$$
$$m = \frac{1}{(x_m + 1/2)}\left(\frac{(1-\tau)x_m}{r(x_m + 1/2)}\right)^{2x_m}.$$

Now we can determine the interest R paid by risky debt as a coupon C divided by the value of debt $D(V_1, V_2)$

$$R = \frac{C}{D(V_1, V_2)} = r\left[1 - \left(\frac{C}{2\sqrt{V_1 V_2}}\right)^{2x_m} k\right]^{-1} \tag{46}$$

and the yield spread s

$$s = R - r = r\left[\left(1 - \left(\frac{C}{2\sqrt{V_1 V_2}}\right)^{2x_m} k\right)^{-1} - 1\right]. \tag{47}$$

We can determine the optimal leverage by choosing the coupon C which maximizes the total value of the firm $v(V_1, V_2)$, given the current value of the assets (V_1, V_2). Differentiating equation (45) with respect to C and setting the derivative to zero determines the optimal coupon. Therefore, optimal leverage is given by the level of coupon

$$C^* = 2\sqrt{V_1 V_2}\left(\frac{1}{h(1+2x_m)}\right)^{1/2x}. \tag{48}$$

We can substitute the optimal coupon C^* into equations (43), (45), (47) we derive the optimal values of debt $D^*(V_1, V_2)$, the total value of the firm $v^*(V_1, V_2)$, the spread s^*, and the optimal bankruptcy point $V^{b\,*}$

$$D^*(V_1, V_2) = \frac{2\sqrt{V_1 V_2}}{r}\left(\frac{1}{h(1+2x_m)}\right)^{1/2x_m}\left(1 - \frac{k}{h(1+2x_m)}\right), \tag{49}$$

$$v^*(V_1, V_2) = 2\sqrt{V_1 V_2}\left(1 + \frac{\tau}{r}\left(\frac{1}{h(1+2x_m)}\right)^{1/2x_m}\left(\frac{2x_m}{1+2x_m}\right)\right), \tag{50}$$

$$s^* = r\left(\frac{k}{h(1+2x_m)-k}\right), \tag{51}$$

$$V^{b\,*} = \frac{2\sqrt{V_1 V_2}(1-\tau)x_m}{r(1+1/2)}\left(\frac{1}{h(1+2x_m)}\right)^{1/2x_m}. \tag{52}$$

3.2 Case II: Single-asset firm

3.2.1 Single-asset originating firm

Now we perform the same analysis as in the previous section, focusing on the case when the firm decides to spin-off or securitize a part of its assets. The analysis should allow us to compare different asset structures of the firm and decide which of them is favorable depending on the set of parameters.

Without loss of generality we assume that the firm decides to spin-off the asset V_2 and to keep the asset V_1. The value of the assets which now belongs to the original firm, follows a one-dimensional geometric Brownian motion

$$dV_1 = \mu_1 V_1 dt + \sigma_1 V_1 dw_1. \tag{53}$$

There is no recourse between the originator and securitization vehicle, (see Schwarcz (1993) and Frankel (1991)). Therefore each contingent claim $F_o(V_1, t)$ with payout $C_o(V_1, t)$ satisfies a one-dimensional version of Black-Scholes equation

$$\mathcal{L}^1 F_o(V_1, t) \stackrel{\text{def}}{=} \frac{\partial F_o}{\partial t} + \frac{1}{2}\sigma_1^2 V_1^2 \frac{\partial^2 F_o}{\partial V_1^2} + rV_1 \frac{\partial F_o}{\partial V_1} - rF_o + C_o(V_1, t) = 0 \tag{54}$$

where $V_1 \geq V_B^o$ and $0 \leq t \leq T$ with the appropriate boundary conditions for $F_o(V_1, t)$ at the boundary point $V_1 = V_B^o$ and $t = 0$, $t = T$.

As in section 3.1, we assume that contingent claims have no explicit time dependence, therefore we can rewrite equation (54) as

$$\frac{1}{2}\sigma_1^2 V_1^2 \frac{\partial^2 F_o}{\partial V_1^2} + rV_1 \frac{\partial F_o}{\partial V_1} - rF_o + C_o(V_1) = 0 \tag{55}$$

where $V_1 \geq V_B^o$.

The valuation is the same as in Leland's (1994) study and we can just rewrite the results. Bankruptcy costs factor α and the tax rate τ stays the same as in the case of the multi-asset firm. Coupon payment C_o is different. Equation (55) has a standard form solution. Each contingent claim depends on its payoff and boundary conditions. Define the exponent x_o as

$$x_o = \frac{r - \frac{1}{2}\sigma_1^2 + \sqrt{\left(r - \frac{1}{2}\sigma_1^2\right)^2 + 2\sigma_1^2 r}}{\sigma_1^2}. \tag{56}$$

Value of the debt $D_o(V_1)$ is equal to

$$D_o(V_1) = \frac{C_o}{r} + \left((1-\alpha)V_B^o - \frac{C_o}{r}\right)\left(\frac{V_B^o}{V_1}\right)^{x_o}. \tag{57}$$

Value of the equity $E_o(V_1)$ is equal to

$$E_o(V_1) = V_1 - \frac{(1-\tau)C_o}{r} + \left(\frac{(1-\tau)C_o}{r} - V_B^o\right)\left(\frac{V_B^o}{V_1}\right)^{x_o}. \tag{58}$$

The total value of the firm $v_o(V_1)$ is equal to the value of assets V_1 plus the value of tax benefits $TB_o(V_1)$ minus the value of bankruptcy costs $BC_o(V_1)$

$$v_o(V_1) = V_1 + TB_o(V_1) - BC_o(V_1) = D_o(V_1) + E_o(V_1) =$$

$$= V_1 + \left(\frac{\tau C_o}{r}\right)\left[1 - \left(\frac{V_B^o}{V_1}\right)^{x_o}\right] - \alpha V_B^o \left(\frac{V_B^o}{V_1}\right)^{x_o} \tag{59}$$

Applying the smooth-pasting condition, the endogenous bankruptcy point V_B^o is derived as

$$V_B^o = \frac{(1-\tau)C_o x_o}{r(x_o+1)}. \tag{60}$$

Substituting the value for the bankruptcy point (60) into (59), (57), (58) we get

$$D_o(V_1) = \frac{C_o}{r}\left[1 - \left(\frac{C_o}{V_1}\right)^{x_o} k_o\right], \tag{61}$$

$$E_o(V_1) = V_1 - \frac{(1-\tau)C_o}{r}\left[1 - \left(\frac{C_o}{V_1}\right)^{x_o} m_o\right], \tag{62}$$

$$v_o(V_1) = V_1 + \frac{\tau C_o}{r}\left[1 - \left(\frac{C_o}{V_1}\right)^{x_o} h_o\right] \tag{63}$$

where

$$k_o = \left[(x_o+1) - (1-\alpha)(1-\tau)x_o\right] m_o,$$

$$h_o = \left[(x_o+1) + \frac{\alpha(1-\tau)x_o}{\tau}\right] m_o,$$

$$m_o = \left(\frac{(1-\tau)x_o}{r(x_o+1)}\right)^{x_o} \frac{1}{(x_o+1)}.$$

We can determine the interest R_o paid by the single-asset firm for its debt

$$R_o = \frac{C_o}{D_o(V_1)} = r\left[1 - \left(\frac{C_o}{V_1}\right)^{x_o} k_o\right]^{-1} \tag{64}$$

and the relevant yield spread s_o

$$s_o = R_o - r = r\left[\left(1 - \left(\frac{C_o}{V_1}\right)^{x_o} k_o\right)^{-1} - 1\right]. \tag{65}$$

We can also analyze the optimal leverage of the single-asset firm. The originator can choose the coupon C_o which maximizes the total value $v_o(V_1)$, given the current value of the assets V_1. Differentiating equation (63) with respect to C_o, setting the derivative to zero yields the optimal coupon. Therefore, the optimal leverage is given by the level of coupon

$$C_o^* = V_1 \left(\frac{1}{h_o(1+x_o)} \right)^{1/x_o}. \tag{66}$$

We can substitute the optimal coupon C_o^* into equations (61), (63), and (65) we derive the optimal values of debt $D_o^*(V_1)$, total value of the firm $v_o^*(V_1)$, and spread s_o^*

$$D_o^*(V_1) = \frac{V_1}{r} \left(\frac{1}{h_o(1+x_o)} \right)^{1/x_o} \left(1 - \frac{k_o}{h_o(1+x_o)} \right), \tag{67}$$

$$v_o^*(V_1) = V_1 \left(1 + \frac{\tau}{r} \left(\frac{1}{h(1+x_o)} \right)^{1/x_o} \left(\frac{x_o}{1+x_o} \right) \right), \tag{68}$$

$$s_o^* = r \left(\frac{k_o}{h_o(1+x_o) - k_o} \right). \tag{69}$$

3.2.2 Securitization vehicle with single class of debt notes

Now we can model the securitization vehicle and estimate the value of its asset-backed notes. The valuation methods are the same as above, but the value of asset-backed notes is determined by the legal and structural requirements related to the structure of securitization vehicles.

We assume that the securitization vehicle pays taxes and we keep the structure of tax credits the same as in the case of the original company[16]. Holders of equity-like claims have only cash flow rights. As one of the consequences, securitization vehicles are designed in the way that the chance of voluntary bankruptcy is eliminated. There is a remote chance of involuntary bankruptcy, (see Schwarcz (1993), The Committee on Bankruptcy and Corporate Reorganization (1995, 2000), Hill (1996)). On the other side, securitization vehicles structures normally include early amortization triggers. Certain events trigger the amortization of debt notes on a pass through basis. These triggers are established to protect the debtholders against further deterioration of the assets, (see

[16]In most cases, securitization vehicles manage to avoid taxation, but that is not always the case, (see Schwarcz (1993), Kravitt (1997), Frankel (1991)) A "pay-through" structure, where the vehicle issues debt securities, will normally not qualify as a non-taxable entity.

Bhattacharya and Fabozzi (1996), Merrill Lynch (2000)). For the modelling purposes, we substitute the bankruptcy procedure with the early prepayment process. And finally, in order to prevent potential moral hazard problems associated with the governance, we assume that the equityholders in the securitization vehicle are a different entity from the equityholders in the original firm.

Now we model the securitization vehicle with a single class of perpetual debt-like notes. The originator transfers a part of its assets into a securitization vehicle. The value of the assets which now belong to the securitization vehicle, follows a one-dimensional geometric Brownian motion

$$dV_2 = \mu_2 V_2 dt + \sigma_2 V_2 dw_2. \tag{70}$$

The value of the notes issued by the securitization vehicle depend only on securitized assets V_2, therefore each asset-backed contingent claim $F_s(V_2)$ with payout $C_s(V_2)$ satisfies one-dimensional version of Black-Scholes equation

$$\mathcal{L}^2 F_s(V_1, t) \stackrel{\text{def}}{=} \frac{1}{2}\sigma_2^2 V_2^2 \frac{\partial^2 F_s}{\partial V_2^2} + rV_2 \frac{\partial F_s}{\partial V_2} - rF_s + C_s(V_2) = 0 \tag{71}$$

where $V_2 \geq V_B^s$ and with the appropriate boundary conditions for $F_s(V_2)$ at the boundary point $V_2 = V_B^s$.

Securitization vehicle issues one class of debt-like notes $D_s(V_2)$ and one class of residual equity-like notes $E_s(V_2)$. Similar to the above valuation we can determine that the value of the debt $D_s(V_2)$ is equal to

$$D_s(V_2) = \frac{C_s}{r} + \left((1 - \alpha_s)V_B^s - \frac{C_s}{r}\right)\left(\frac{V_B^s}{V_2}\right)^{x_s}, \tag{72}$$

where

$$x_s = \frac{r - \frac{1}{2}\sigma_2^2 + \sqrt{\left(r - \frac{1}{2}\sigma_2^2\right)^2 + 2\sigma_2^2 r}}{\sigma_2^2}. \tag{73}$$

Here we assumed that if the prepayment occurs, a fraction α_s, $0 < \alpha_s \leq 1$, of the total value of the assets will be lost due to the transaction costs, debtholders receive the remaining part of the assets and equityholders receive nothing.

We assume that the prepayment trigger point is determined as a minimum net-worth covenant[17]. Therefore the fixed prepayment point V_B^s is given as

$$V_B^s = \frac{C_s}{r} \tag{74}$$

and we can rewrite (72) as

$$D_s(V_2) = \frac{C_s}{r}\left(1 - \alpha_s\left(\frac{C_s}{rV_2}\right)^{x_s}\right). \tag{75}$$

In a similar way as in section 3.1, we can determine the value of prepayment costs for the securitization vehicle

$$BC_s(V_2) = \alpha_s V_B^s \left(\frac{V_B^s}{V_2}\right)^{x_s} \tag{76}$$

and using (74) we can rewrite (76)

$$BC_s(V_2) = \alpha_s \frac{C_s}{r}\left(\frac{C_s}{rV_2}\right)^{x_s}. \tag{77}$$

We can determine the value of tax benefits for the securitization vehicle

$$TB_s(V_2) = \frac{\tau_s C_s}{r}\left(1 - \left(\frac{V_B^s}{V_2}\right)^{x_s}\right) \tag{78}$$

and using (74) we can rewrite (78)

$$TB_s(V_2) = \frac{\tau_s C_s}{r}\left(1 - \left(\frac{C_s}{rV_2}\right)^{x_s}\right). \tag{79}$$

[17] Alternatively, we could assume that the assets in securitization vehicle are generating cash-flow λV_2 which is paid out in form of coupon payment C_s and the remaining part is paid to equityholders. If the value of the assets decreases to the point that $\lambda V_2 = C_s$, the early prepayment is triggered. Conceptually, such structure leads to the same bankruptcy condition.

The total value of the securitization vehicle is equal to

$$v_s(V_2) = V_2 - BC_s(V_2) + TB_s(V_2) =$$

$$= V_2 - \alpha_s \frac{C_s}{r}\left(\frac{C_s}{rV_2}\right)^{x_s} + \frac{\tau_s C_s}{r}\left(1 - \left(\frac{C_s}{rV_2}\right)^{x_s}\right). \tag{80}$$

The value of equity-like residual claim $E_s(V_2)$ is equal to

$$E_s(V_2) = v_s(V_2) - D_s(V_2) = V_2 - \frac{C_s}{r} + \frac{\tau_s C_s}{r}\left(1 - \left(\frac{C_s}{rV_2}\right)^{x_s}\right). \tag{81}$$

We can determine the interest R_s paid by the securitization vehicle for its debt

$$R_s = \frac{C_s}{D_s(V_2)} = r\left(1 - \alpha_s\left(\frac{C_s}{rV_2}\right)^{x_s}\right)^{-1} \tag{82}$$

and the relevant yield spread s_s

$$s_s = R_s - r = r\left[\left(1 - \alpha_s\left(\frac{C_s}{rV_2}\right)^{x_s}\right)^{-1} - 1\right]. \tag{83}$$

Now we analyze the optimal leverage of the securitization vehicle. The securitization vehicle can choose the coupon C_s which maximizes the total value $v_s(V_2)$, given the current value of the assets V_2. Differentiating equation (80) with respect to C_s, setting the derivative to zero yields the optimal coupon. Therefore, the optimal leverage is given by the level of coupon

$$C_s^* = rV_2\left(\frac{\tau_s}{(\alpha_s + \tau_s)(1 + x_s)}\right)^{1/x_s}. \tag{84}$$

Again, we can substitute the optimal coupon C_s^* into equations (75), (80), (83) and we derive the optimal values of debt $D_s^*(V_2)$, total value of the firm $v_s^*(V_2)$, and spread s_s^*

$$D_s^*(V_2) = V_2 n_s^{1/x_s}(1 - \alpha_s n), \tag{85}$$

$$v_s^*(V_2) = V_2\left(1 - \alpha_s n_s^{1+1/x_s}\right), \tag{86}$$

$$s_s^* = \frac{r\alpha_s n_s}{1 - \alpha_s n_s} \tag{87}$$

where

$$n_s = \frac{\tau_s}{(\alpha_s + \tau_s)(1 + x_s)}. \tag{88}$$

4 Comparative analysis of different models

Our research continues with a comparative analysis of the models developed in the previous sections. There are two main goals which we want to achieve. First we want to focus on the multi-asset firm. Using the closed-form solutions for the total value of the multi-asset firm, the value of its contingent claims, and the optimal leverage, we can provide a detailed valuation of the firm. Changing the parameters, we can observe the sensitivity of the model and derive asymptotic behavior for the cases when the parameters approach the boundary values. Second, we want to compare the overall values of different organizational structures of the multi-asset firm from the perspective of the optimal asset structure. The firm can keep both assets in the multi-asset entity, it can securitize either of them through the securitization vehicle, or it can separate them into two non-related single-asset firms. We vary the volatility of each of the two assets and closed-form solutions provide an easy way to determine which of the asset structures generates the highest overall value.

To facilitate the comparison, we consider a base case example. There are two assets which are determined by the process $V = (V_1, V_2)$. Initial asset value is normalized[18] to $V_1 = V_2 = 100$. The observable parameters are the following. Risk-free rate $r = 0.02$, bankruptcy costs fraction $\alpha = 0.4$ for the firm, prepayment transaction costs fraction $\alpha_s = 0.02$ for the securitization vehicle, tax rate $\tau = 0.1$ for the firm and $\tau_s = 0.1$ for the

[18]It is not necessary to keep the size of each assets equal. We checked the valuation with the different sizes and the results didn't change much. Although in order not to undervalue the multi-asset firm, it is important that the difference between the size of the assets is not too large.

securitization vehicle, and correlation between the assets V_1 and V_2 is given as $\rho = 0.5$. The observable parameters are roughly consistent with the historical data. In particular, they follow parameters used in Leland (1994), and Leland (1998).

Values of debt (43), equity (44), and total value (45) for the multi-asset firm closely resembles single-asset expressions derived in Leland's (1994) paper. Therefore we can expect that most of the comparative static observations related to the multi-asset firm are the same as the observations found by Leland (1994). For example, Figure 1 shows the relationship between the total value of the firm $v(V_1, V_2)$ and the coupon C for varying levels of volatility of the assets. In a similar way, Figure 2 presents relationship between the debt value $D(V_1, V_2)$ and the coupon C. Each of these figures is conceptually similar to Leland's (1994) findings, implying that the multi-asset debt and the multi-asset total value of the firm should have analogous properties as the single-asset debt and the single-asset debt total value the firm.

But there are differences between the single-asset firm represented by Leland's model and the multi-asset firm represented by our extended model. For example, we focus on the endogenously defined bankruptcy point for each of the firms. In a single-asset setting, bankruptcy occurs when the value of assets reaches the following boundary, (see Leland (1994))

$$V_B^o = \frac{(1-\tau)C_o x_o}{r(x_o+1)}, \tag{89}$$

and in two-asset setting, same bankruptcy point is derived as the following value, (see (41))

$$V_B = \frac{(1-\tau)C x_m}{r(x_m+1/2)}. \tag{90}$$

Regardless of the fact that the exponents x_o for a single-asset firm and x_m for a two-asset firm are different, we hypothesize that multi-asset firms have relatively higher bankruptcy level. Leland (1994) observed that V_B^o decreases with an increase in the volatility of the firm's assets. In that spirit, we can make an observation that the multi-assets firm performs as the single-asset firms with lower volatility of its assets. We verified that V_B is higher than V_B^o as long as the volatility σ_2 of the asset held by the multi-asset firm (but not held by single-asset firm) is not substantially higher than volatility σ_1 of the asset held by single-asset firm[19].

Now we turn our attention to the multi-asset firm. Previously we derived expression for the optimal coupon C^* which maximizes the total value of the two-asset firm. Coupon C^* determines the optimal values of debt $D^*(V_1, V_2)$, total value of the firm $v^*(V_1, V_2)$, spread $s^*(V_1, V_2)$ over the risk-free rate, and leverage $D^*(V_1, V_2)/v^*(V_1, V_2)$. We want to understand how these values change as the volatility of each asset varies. Changing the volatility parameters σ_1 and σ_2 and keeping all the parameters fixed, captures the variation across the different assets. We generated charts for the total value of the firm $v^*(\sigma_1, \sigma_2)$, see Figure 3, the value of debt $D^*(\sigma_1, \sigma_2)$, see Figure 4, the spread $s^*(\sigma_1, \sigma_2)$, see Figure 5, and the leverage $D^*(\sigma_1, \sigma_2)/v^*(\sigma_1, \sigma_2)$, see Figure 6. We can observe that the total value of the firm stays the same for larger values of volatility and increases once volatility parameters of both assets become lower. The same observation holds for the value of debt. It seems that the combination of a low volatility asset with a higher volatility asset doesn't substantially increases the total value of the firm as compared to the case of two assets with high volatility. On the other side, in the region where the total value is flat, there is a corresponding increase in the spread as the volatility of either assets becomes higher. Finally, we can observe that the optimal leverage of the two-asset firm is between the relative bounds as observed in the real world.

The observation about the interactions between assets is a part of a large and well studied research area. Diversification value of imperfectly correlated projects has been discussed frequently in the literature. In order to better understand this issue, we present two charts, first for the value of the firm $v^*(\rho)$ and second for the value of debt $D^*(\rho)$, see Figure 7 and Figure 8. In both cases we vary the correlation coefficient ρ between the assets and we keep all other parameters fixed. For each value of ρ, we choose the optimal coupon C^* which maximizes the total value of the two-asset firm. From each of the figures we observe that the value of the firm and the value of debt increase as the correlation between the assets decreases.

We analyze the optimal leverage in the case of a single-asset originating firm and

[19] This observation holds if both firms choose optimal coupon C_o^*, C^*. In case of the same relative coupon levels C_o, C, then V_B is higher than V_B^o only when σ_2 is strictly lower than σ_1.

in the case of a single-asset securitization vehicle. Again, the main difference between two models is driven by the bankruptcy condition. Securities issued by the originating firm are determined by the endogenous bankruptcy point V_B^o, and securities issued by the securitization vehicle are determined as exogenous minimum net-worth covenant V_B^s. We focus on the total value, the value of debt, spread and leverage, as we did in the case of the multi-asset firm. Each of the claims is a function of volatility σ_1 and σ_2 and the coupon is chosen to maximize the total value of the specific firm in question. We generate charts only for the securitization vehicle. We present the graph for total value of the firm $v_s^*(\sigma_1, \sigma_2)$, see Figure 9, the value of debt $D_s^*(\sigma_1, \sigma_2)$, see Figure 10, the spread $s_s^*(\sigma_1, \sigma_2)$, see Figure 11, and the leverage $D_s^*(\sigma_1, \sigma_2)/v_s^*(\sigma_1, \sigma_2)$, see Figure 12. Because we assume that the securitization vehicle owns only the asset which value follows process V_2, none of the values depends on volatility σ_1. The spread $s_s^*(\sigma_1, \sigma_2)$ is at the risk-free level. Surprisingly, the leverage $D_s^*(\sigma_1, \sigma_2)/v_s^*(\sigma_1, \sigma_2)$ is unexpectedly low for a chosen set of parameters. We don't observe this phenomena in the real world securitizations, where the residual equity-like piece is normally at the level of few percent. Most likely explanation for this apparent inaccuracy of our model is the fact that we structured the securitization vehicle with only two classes of securities. Therefore we are not able to capture any credit enhancement effects of subordination which are generated by issuing several classes of debt-like securities. In our future work we extend the securitization vehicle model to multiple classes of asset-backed securities in order to generate the leverage results consistent with the real world observations, (see Skarabot (2001)) .

We can compare different organizational structures for the firm. We want to determine which structure provides the highest overall value. Multi-asset firm owns two assets and it can chose among five different options for the asset structure. It can keep both assets in the firm and the overall value is equal to the total value of the multi-asset firm $v_s^*(V_1, V_2)$. It can separates the assets into two firms, where each of the individual firms owns one asset and the overall value is equal to the sum of total values of the single-asset firms $v_o^*(V_1) + v_o^*(V_2)$. It can keep one of the assets in the original firm and transfer the other asset into a securitization vehicle. In this case the overall value is equal to the total value of the single-asset firm which owns the first asset plus the total value of the securitization vehicle which owns the second asset, that means $v_o^*(V_1) + v_s^*(V_2)$ or $v_s^*(V_1) + v_o^*(V_2)$. Finally, both of the assets can be transferred into separate securitization vehicles. In this last case the overall value is equal to the sum of the total values of the securitization vehicles $v_s^*(V_1) + v_s^*(V_2)$. In the same way as in the previous part, we vary volatility of each of the assets between the lower an the upper bound. We parametrize the bounds to determine a suitable volatility region[20]. At each volatility level (σ_1, σ_2) we determine the optimal coupon C^* which maximizes the overall value of the firm for each of the five organizational structures. Then we choose the structure with the highest value and we generate a "winner plot" for the optimal asset structure, see Figure 13. Each region in that chart represents a subset of volatility pairs for which the corresponding organizational structure deliver the highest overall value of the firm.

Based on the Figure 13 we can make several observations. As long as the volatility parameters of two assets are close to each other – indicating that the assets have similar characteristics – the multi-asset structure prevails. When the volatility parameters are further apart, other structures are optimal. If both volatility parameters are above some level, then the separate incorporation is optimal. As one of the volatility parameters gets lower, we see that the optimal choice is to securitize the asset with low volatility and keep the asset with high volatility in the original firm. This results is broadly aligned with the main concept of asset securitization, which states that securitized assets are the one which have relatively predictable stream of future cash flows, therefore lower volatility. As one of the volatility parameters approaches to zero, our model predicts a different optimal structure as we might expect from real world securitization scenarios. Instead of the outcome where securitization is the optimal structure for lowest levels of volatility, we see that there is a reversal in the optimal choice. The separation of the assets into two single-asset firms prevails as the structure generating the highest overall value.

We also performed some comparative statics, changing other parameters of the model. The "winner plots" stay are similar to each other, but there are some changes in the specific distribution of the optimal structures. The most important observation is related

[20]We avoid the potential singularity for the value if the volatility approaches to zero.

to the effect of the correlation on the optimal structure. We can observe that if the correlation ρ between the assets gets lower, the multi-asset firm range becomes wider, see Figure 14.

It is intuitively clear that the assets with lower volatility should be securitized and the assets with higher volatility should be kept in the standard firm. Equityholders in the standard firm have the option to optimally declare the default. The value of this option is reflected in the fact that the standard firm should – in principle – have higher total value. As the volatility increases, the value of the voluntary default option goes up and the relative advantage of the standard firm over the securitization vehicle widens. As the volatility decreases, the value of that option goes down and the bankruptcy costs advantage of the securitization vehicle becomes a leading factor in the valuation. Therefore the securitization vehicle is the optimal choice for the lower levels of volatility. As the volatility decreases to zero, the securitization vehicle and the standard firm become risk-free and there is a very low chance of bankruptcy in either case. Therefore the bankruptcy costs advantage of the securitization vehicle disappears. Because the standard firm has higher value than the securitization vehicle – not taking into the account the bankruptcy costs advantage – we get the result that at the extremely low levels of volatility the standard firm is the optimal choice.

More specifically, this phenomena is driven by the relative value of tax credit and bankruptcy costs. Prepayment costs for the securitization vehicle are lower than the bankruptcy costs for the standard firm. Tax credit is higher for the standard firm than for the securitization vehicle. Securitization is used when the difference between the bankruptcy and prepayment costs is higher than the tax differential. When the gap between the bankruptcy and prepayment costs closes, the tax credit advantage of the standard firm determines the optimal structure, see Figure 15.

Finally, we want to stress that our assumptions about the assets lead to necessary but not sufficient conditions for securitization. To illustrate this fact we focus on the assets which require some level of managerial involvement after they are transferred into the securitization trust. We assume that the managerial compensation is paid in the form of the dividend which depletes the assets in the trust. Under this assumption, the valuation equation for asset-backed claims (71) should be rewritten as

$$\frac{1}{2}\sigma_2^2 V_2^2 \frac{\partial^2 F_s}{\partial V_2^2} + (r-\delta)V_2 \frac{\partial F_s}{\partial V_2} - rF_s + C_s = 0, \tag{91}$$

where δ is a constant representing payment flow, (see footnote 12). Using (91), each contingent claim in securitization trust can be determined accordingly[21]. In this case, the prepayment advantage of the securitization trust is not high enough to offset the tax credit disadvantage, see Figure 16.

5 Conclusions

The main goal of our project is to provide an answer to the question why firms securitize their assets. In order to clarify this issue we developed a valuation models for the multi-asset firm, the single-asset firm and the securitization vehicle. Each of these business entities issues its own debt and equity securities. We also determined the optimal capital structures in the way that the total value of the specific firm is maximized. The models are derived in a continuous-time environment and all the claims have closed-form solutions which determine their values. In order to achieve the analytic clarity of the valuation model, we made some simplifying assumptions about the assets, the structure of the firms, and the securities issued. Once we derived valuation expression for each of the model, it is easy to find the optimal asset structure which maximizes the overall value of the firm. We shoved that for a specific region of variance-covariance parameters, asset securitization provides the optimal choice for the value maximizing firm.

[21]We assume that the original firm can avoid those costs because of the stronger role of the equity-holders.

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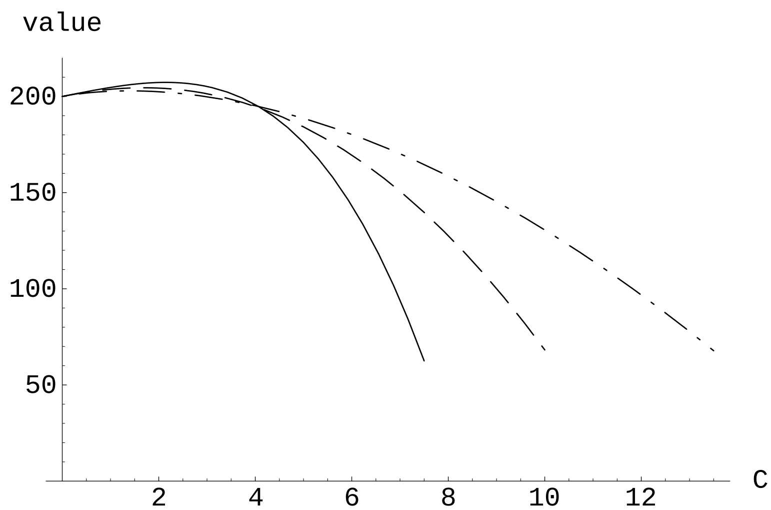

Figure 1: Multi-asset firm. Total value of the firm $v(V_1, V_2)$ as a function of C. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\tau = 0.1$, $\rho = 0.5$, $r = 0.02$; full line $\sigma_1 = \sigma_2 = 0.15$, dashed line $\sigma_1 = \sigma_2 = 0.2$, dot-dashed line $\sigma_1 = \sigma_2 = 0.25$.

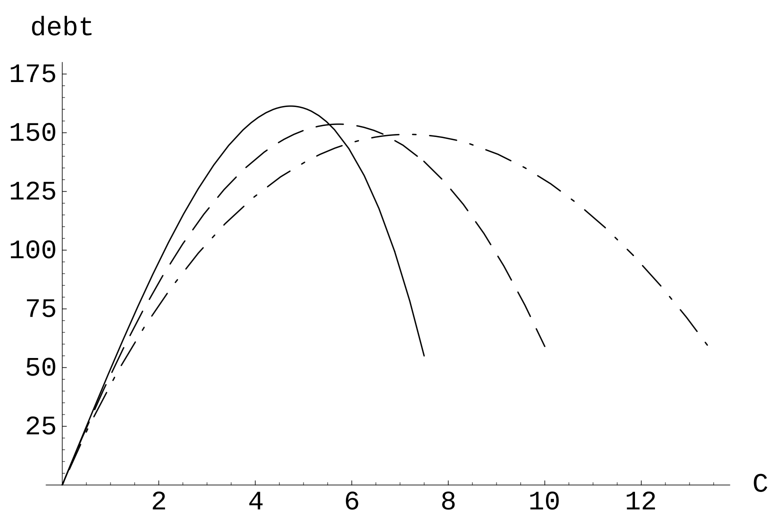

Figure 2: Multi-asset firm. Debt $D(V_1, V_2)$ as a function of C. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\tau = 0.1$, $\rho = 0.5$, $r = 0.02$; full line $\sigma_1 = \sigma_2 = 0.15$, dashed line $\sigma_1 = \sigma_2 = 0.2$, dot-dashed line $\sigma_1 = \sigma_2 = 0.25$.

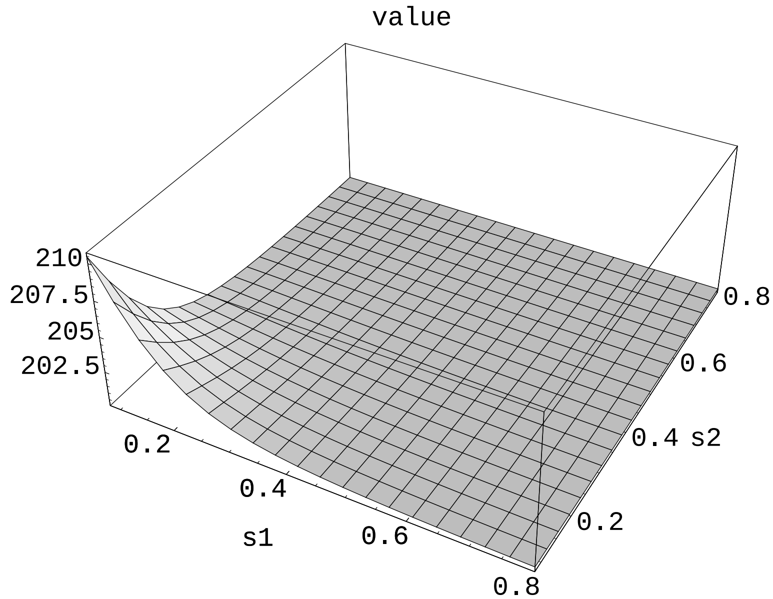

Figure 3: Multi-asset firm. Total value of the firm $v^*(V_1, V_2)$ as a function of volatility s_1, s_2. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\tau = 0.1$, $\rho = 0.5$, $r = 0.02$. Coupon C^* chosen to maximize the value of the firm.

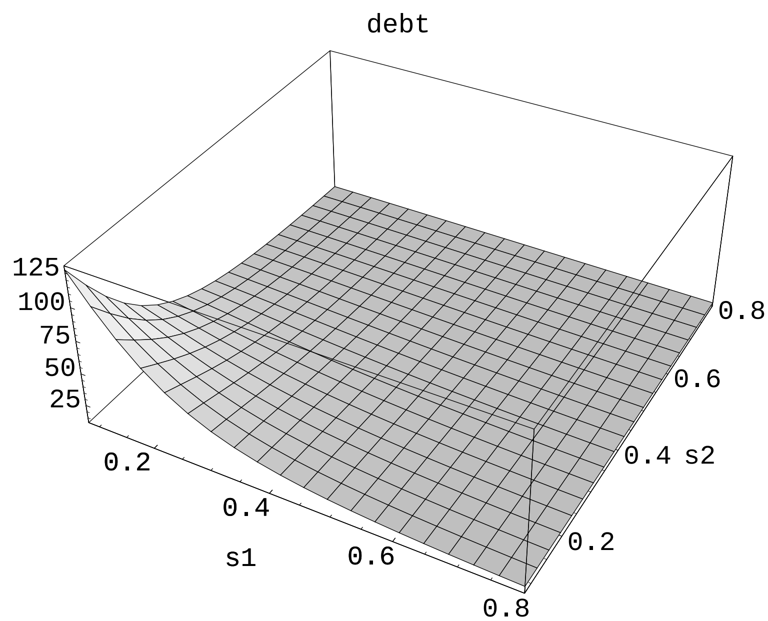

Figure 4: Multi-asset firm. Debt $D^*(V_1, V_2)$ as a function of volatility s_1, s_2. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\tau = 0.1$, $\rho = 0.5$, $r = 0.02$. Coupon C^* chosen to maximize the value of the firm.

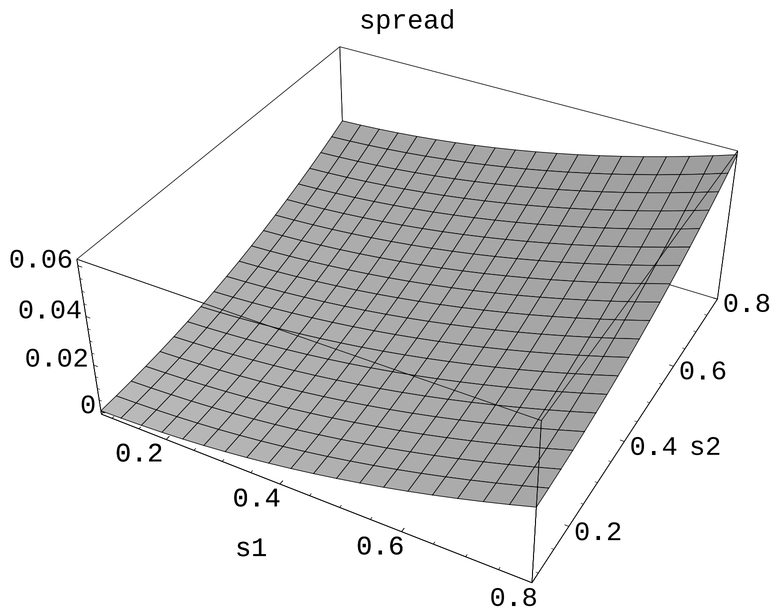

Figure 5: Multi-asset firm. Spread $s^*(V_1, V_2)$ as a function of volatility s_1, s_2. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\tau = 0.1$, $\rho = 0.5$, $r = 0.02$. Coupon C^* chosen to maximize the value of the firm.

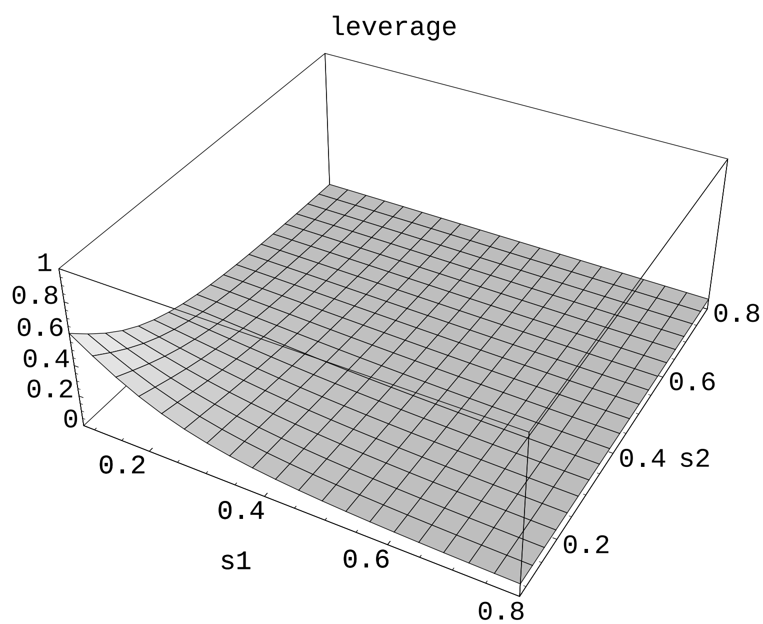

Figure 6: Multi-asset firm. Leverage $D^*(V_1, V_2)/v^*(V_1, V_2)$ as a function of volatility s_1, s_2. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\tau = 0.1$, $\rho = 0.5$, $r = 0.02$. Coupon C^* chosen to maximize the value of the firm.

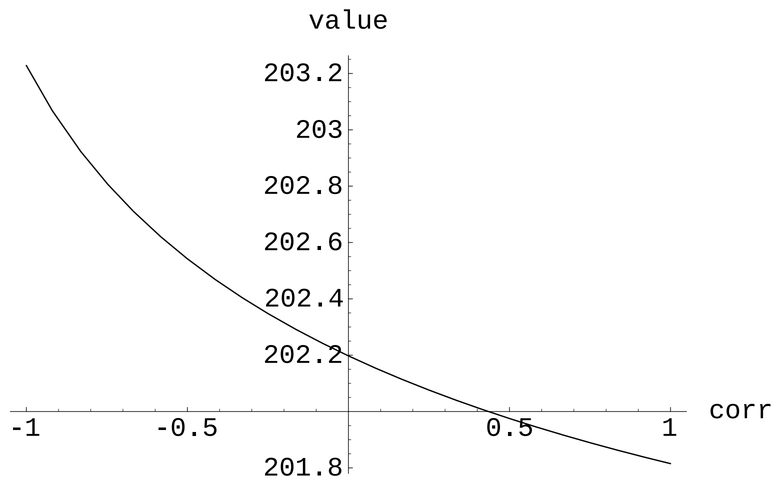

Figure 7: Multi-asset firm. Total value of the firm $v^*(V_1, V_2)$ as a function of ρ. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\tau = 0.1$, $\sigma_1 = 0.3$, $\sigma_2 = 0.3$, $r = 0.02$. Coupon C^* chosen to maximize the value of the firm.

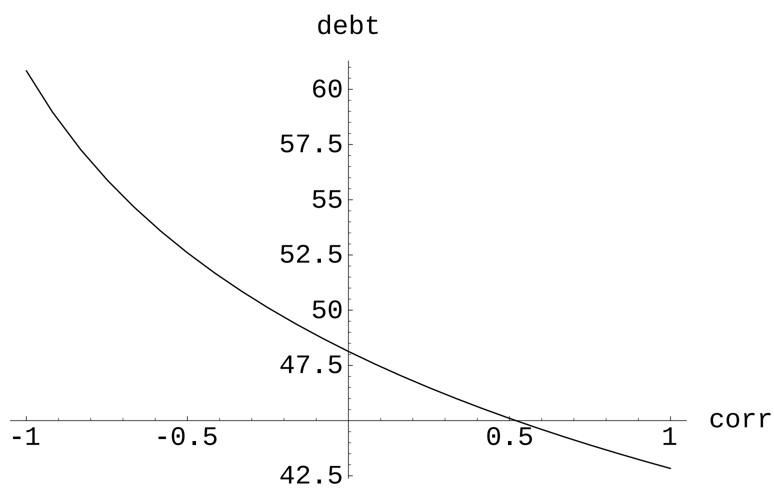

Figure 8: Multi-asset firm. Debt $D^*(V_1, V_2)$ as a function of ρ. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\tau = 0.1$, $\sigma_1 = 0.3$, $\sigma_2 = 0.3$, $r = 0.02$. Coupon C^* chosen to maximize the value of the firm.

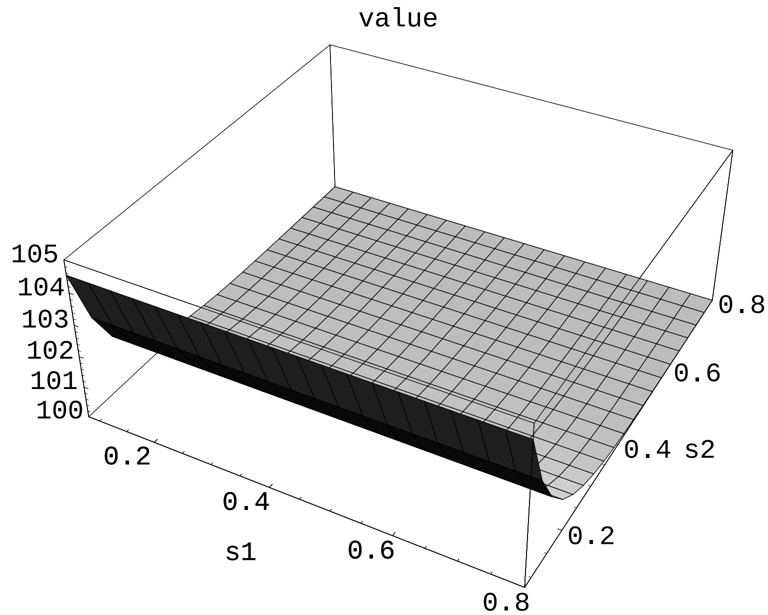

Figure 9: Securitization vehicle. Total value of the firm $v_s^*(V_2)$ as a function of volatility s_1, s_2. Parameters $V_2 = 100$, $\alpha_s = 0.02$, $\tau_s = 0.1$, $r = 0.02$. Coupon C_s^* chosen to maximize the value of the firm.

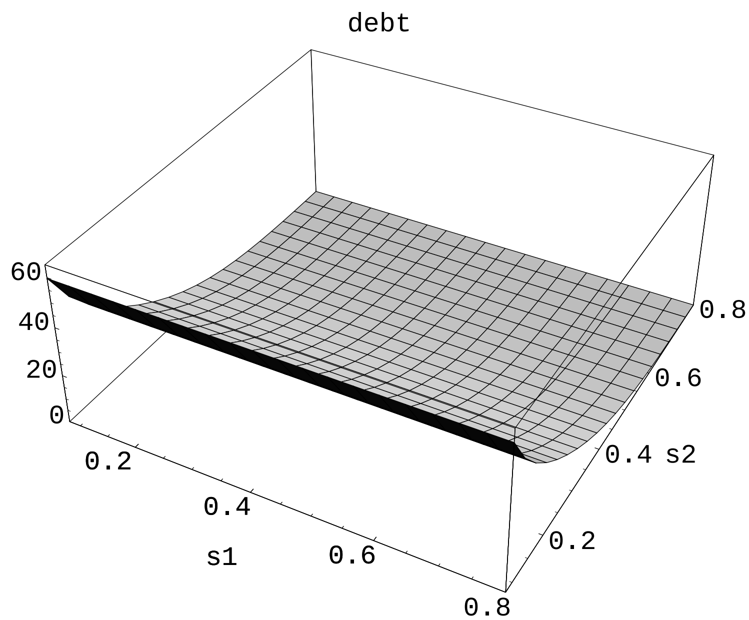

Figure 10: Securitization vehicle. Debt $D_s^*(V_2)$ as a function of volatility s_1, s_2. Parameters $V_2 = 100$, $\alpha_s = 0.02$, $\tau_s = 0.1$, $r = 0.02$. Coupon C_s^* chosen to maximize the value of the firm.

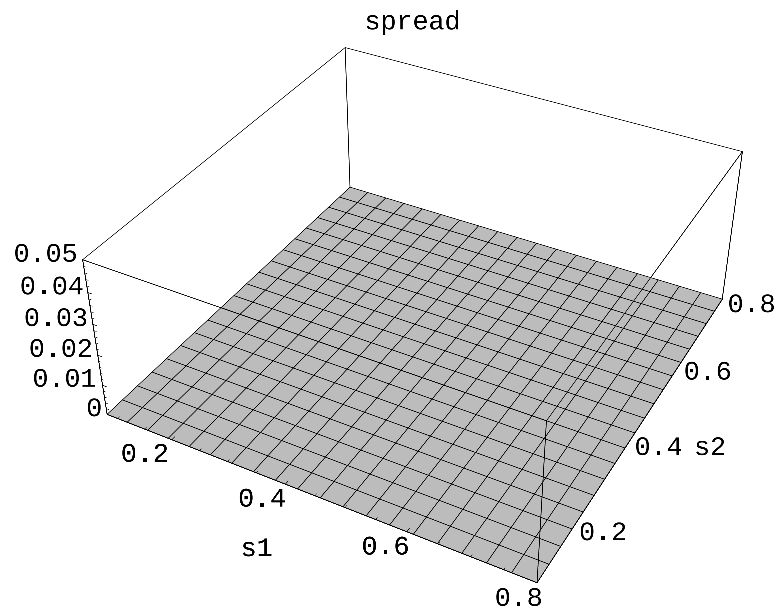

Figure 11: Securitization vehicle. Spread $s_s^*(V_2)$ as a function of volatility s_1, s_2. Parameters $V_2 = 100$, $\alpha_s = 0.02$, $\tau_s = 0.1$, $r = 0.02$. Coupon C_s^* chosen to maximize the value of the firm.

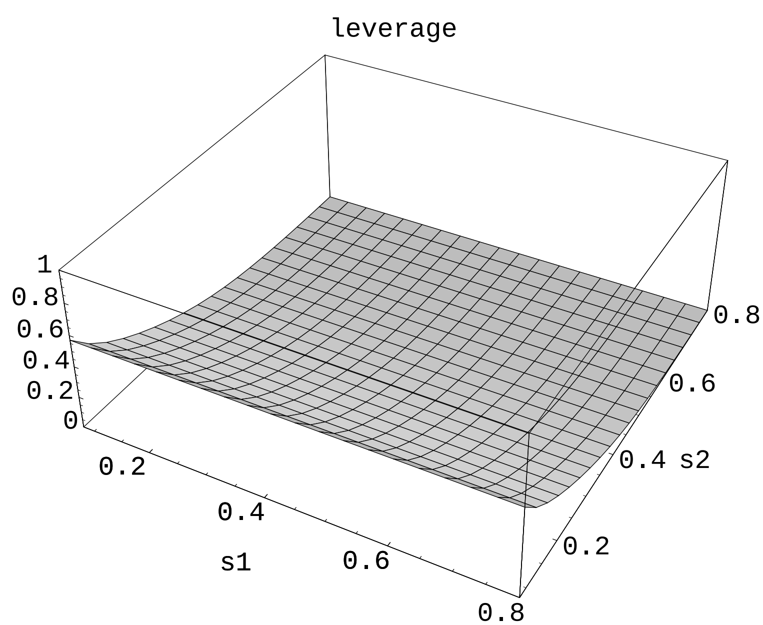

Figure 12: Securitization vehicle. Leverage $D_s^*(V_2)/v_s^*(V_2)$ as a function of volatility s_1, s_2. Parameters $V_2 = 100$, $\alpha_s = 0.02$, $\tau_s = 0.1$, $r = 0.02$. Coupon C_s^* chosen to maximize the value of the firm.

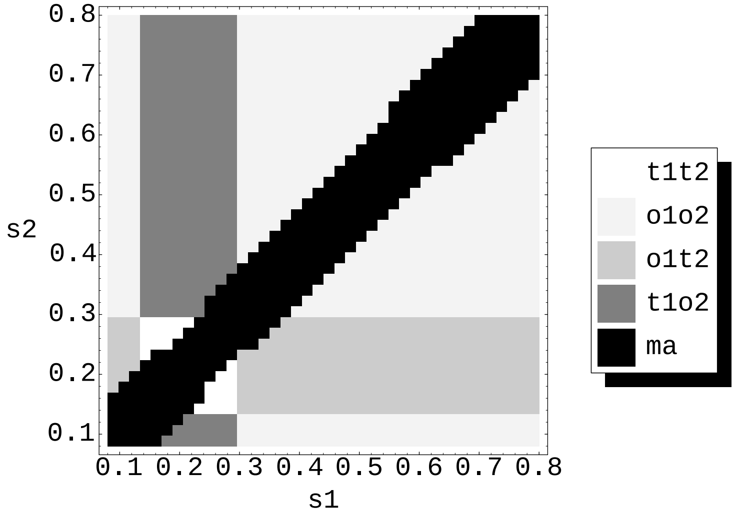

Figure 13: Optimal structure as a function of volatility. `s1` $= \sigma_1$, `s2` $= \sigma_2$, `t1t2` $= v_s^*(V_1) + v_s^*(V_2)$, `o1o2` $= v_o^*(V_1) + v_o^*(V_2)$, `o1t2` $= v_o^*(V_1) + v_s^*(V_2)$, `t1o2` $= v_s^*(V_1) + v_o^*(V_2)$, `ma` $= v_s^*(V_1, V_2)$. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\alpha_s = 0.02$, $\tau = 0.1$, $\tau_s = 0.1$, $\rho = 0.5$, $r = 0.02$.

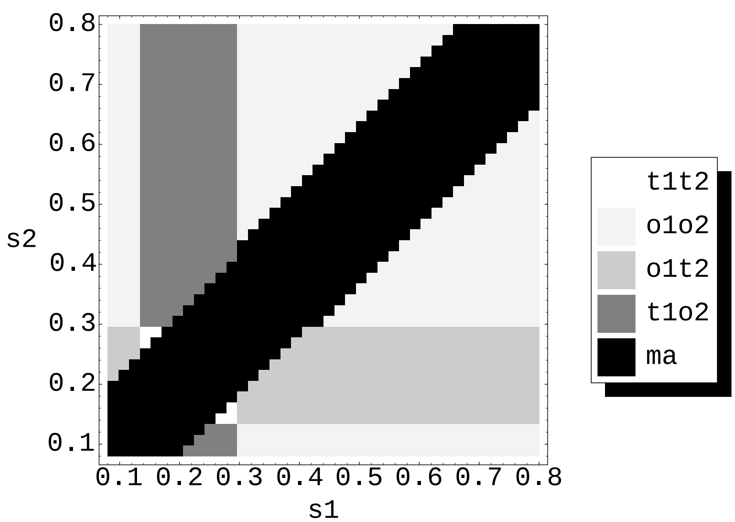

Figure 14: Optimal structure with ρ decreased. `s1` $= \sigma_1$, `s2` $= \sigma_2$, `t1t2` $= v_s^*(V_1) + v_s^*(V_2)$, `o1o2` $= v_o^*(V_1) + v_o^*(V_2)$, `o1t2` $= v_o^*(V_1) + v_s^*(V_2)$, `t1o2` $= v_s^*(V_1) + v_o^*(V_2)$, `ma` $= v_s^*(V_1, V_2)$. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\alpha_s = 0.02$, $\tau = 0.1$, $\tau_s = 0.1$, $\rho = 0$, $r = 0.02$.

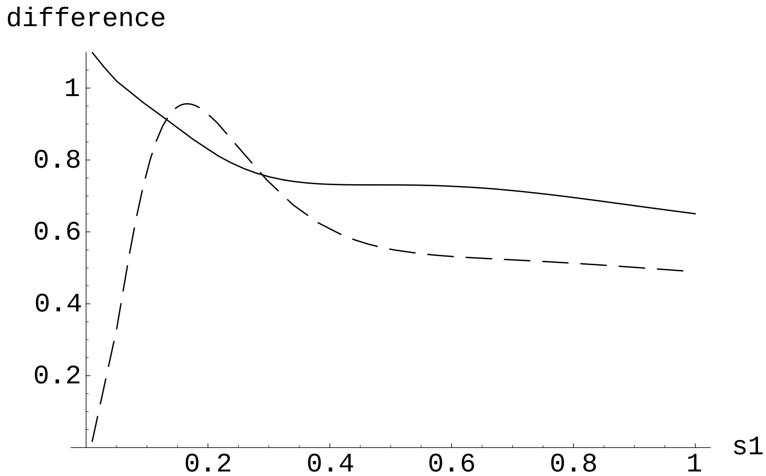

Figure 15: Standard firm versus securitization vehicle. Full line = tax benefit difference $TB_o(V_1) - TB_s(V_1)$ as a function of volatility s_1. Dashed line = bankruptcy/prepayment costs difference $BC_o(V_1) - BC_s(V_1)$ as a function of volatility s_1. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\alpha_s = 0.02$, $\tau = 0.1$, $\tau_s = 0.1$, $\rho = 0.5$, $r = 0.02$.

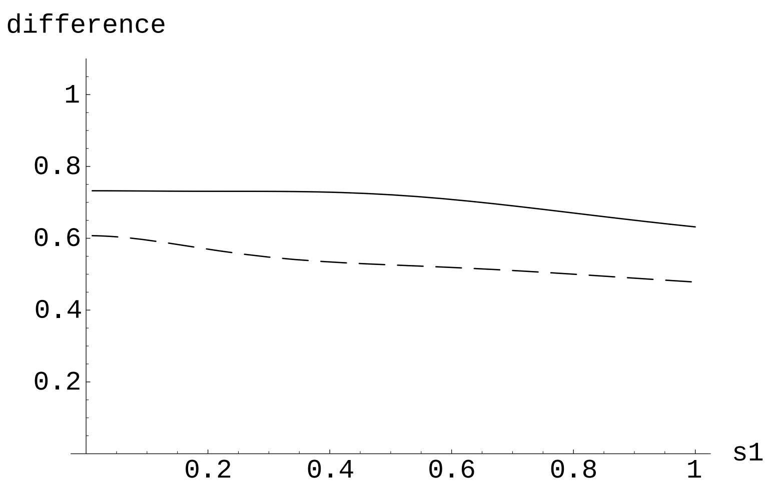

Figure 16: Standard firm versus securitization vehicle (with management fees). Full line = tax benefit difference $TB_o(V_1) - TB_s^\delta(V_1)$ as a function of volatility s_1. Dashed line = bankruptcy/prepayment costs difference $BC_o(V_1) - BC_s^\delta(V_1)$ as a function of volatility s_1. Parameters $V_1 = 100$, $V_2 = 100$, $\alpha = 0.4$, $\alpha_s = 0.02$, $\tau = 0.1$, $\tau_s = 0.1$, $\rho = 0.5$, $r = 0.02$, $\delta = 0.01$.