External Finance, Internal Capital Markets and the Real Economy*

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Abstract

We analyze the allocation of capital to investment projects in an equilibrium context when firms' ability to pledge cash flows is limited. We argue that an increase in the supply to the market can improve the allocation of capital because this additional supply drives down the required return to the point where project-owners can attract capital. We study two specific scenarios: external financing of investments, and conglomeration. External finance increases the supply of capital because external investors liquidate projects too often. With deconglomeration capital is supplied to the market rather than being distributed internally in the conglomerate. These benefits occur even though external finance and deconglomeration are always privately inefficient in our model. Also, when we allow for information acquisition, we find a stonger benefit of an increase in supply since better ex-post allocation provides incentives for firms to obtain information.

1 Introduction

External capital markets serve the useful role of allocating resources to the most productive projects. However, financial imperfections might hamper this allocative role. In a situation

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where the allocation of capital is constrained by market imperfections, actions that promote market activity might be socially beneficial, even when they are not privately optimal. In this paper, we study two such instances: the need for external finance and deconglomeration.

We analyze the allocation of capital to investment projects in an equilibrium context. This allocation is affected by an imperfection in the market for capital: project-owners can only pledge a fraction of the returns generated by the additional investment. The required return on capital might be higher than the return that project-owners can pledge. In this case, an increase in the supply of capital to the market can improve the allocation if this additional supply drives down the required return to the point where project-owners can attract capital.

The need for external finance is one way in which the supply of capital is increased. In the model, because there is limited pledgeability of cash-flows, external finance introduces 'excessive' liquidation of projects. If firms cannot pledge enough cash-flows to outside investors, these investors have a tendency to liquidate the part of the firm's assets. Excessive liquidation is coslty for firms because they no longer be able to continue long term investment projects. For a firm in isolation, the payoff of investment would clearly decrease with the amount of external financing. This reasoning is consistent with most of the corporate finance literature, which generally argues that external finance is costlier than internal finance due to agency or information problems (Jensen and Meckling, 1976, Myers and Majluf, 1984). However, as explained above, the equilibrium effect of this increase in supply is that, by driving down the required return on capital, it allows firms to attract more capital. Thus, even though external finance is costly for each firm in isolation, it can improve the allocation of capital.

The potential benefit of external finance relies on the externality it generates.[1] An entrepreneur who does not need external finance does not benefit from an external investor that forces him to liquidate. Entrepreneurs only seek external finance when they need it to set up their projects. Therefore, it is the *need* for external finance that is at the core of this positive externality.

[1] Our externality is a pecuniary externality (the effect of liquidation on the required rate of return). The reason why a pecuniary externality has real allocative effects here is because of the main inefficiency in the model (the capital market inefficiency). Because of limited pledgeability market prices (which are influenced by the inefficiency) deviate from social values, and thus there is room for price effects to have real effects on welfare.

This effect sheds light into two why countries that rely heavily on external finance (e.g. the US) are also countries with high productivity and growth. One possible explanation for this evidence is that external finance is beneficial because, without it, firms would not be able to make the necessary investments. La Porta, Lopez-de-Silanes, Shleifer and Vishny (1997) argue that better institutions, e.g. the legal system, mitigate agency costs and allows firms to raise more external finance. Demirguc-Kunt and Maksimovic (1998) show that firms in countries with active stock markets and developed legal systems grow faster, and Rajan and Zingales (1998a) show that this is specially true for firms in industrial sectors that are relatively more in need of external finance. These arguments do not imply that external finance is preferred to internal finance. Rather, they imply that external finance is beneficial when there are no other sources of finance. While undoubtedly this is an important role of external finance, our model suggest an alternative and complementary possibility. The allocation of capital improves when firm finance their initial investments with external finance.

Whether external finance is better depends on the degree of pledgeability. External finance leads to higher supply in the reallocation market. If pledgeability is not too low, this increase in supply will make it possible for firms to attract capital in the reallocation market. However, if pledgeability is too low, then not even the increase in the supply in the reallocation market is sufficient to allow firms to attract new capital. In this case, there is no benefit of reallocation and the economy only suffers the cost of excessive liquidation. Finally, for very high levels of pledgeability, firms are able to attract capital regardless of the supply of capital.

This insight generates predictions associated with the degree of pledgeability, and the need for external finance. The degree of pledgeability should be related to institutional and informational features of financial markets. One possible empirical proxy for it is the degree of investor protection in the economy (as in Shleifer and Wolfenzon, 2002). As explained above, external finance will have costs but not benefits when the reallocation market works poorly (low pledgeability). External finance can be beneficial when the reallocation market has the potential to work well (average pledgeability), and is equivalent to internal finance when pledgeability is very high. This suggests a positive interaction between the need for external finance and investor protection. A combination of high investor protection and external financing needs leads to more active reallocation markets, and thus to a more efficient allocation of capital. This is consistent with the empirical results in Wurgler (2000).

These results are related to the discussion about the benefits of liquidation. The 'liquidationist' view (De Long, 1990) argues that liquidation of existing firms is healthy for the economy since it releases factors of production that can be used more productively for new projects. This position goes back to Schumpeter's (1942) idea of 'Creative Destruction'. Contrary to this view, Caballero and Hammour (1996, 1998, 1999) argue that in an economy with frictions, liquidation might be excessive since the released factors will end up being unemployed. Our model is consistent with both views. For low levels of pledgeability, the reallocation market does not exist. In this case, the resources freed by liquidation are not redeployed to productive activities.[2] However, at higher levels of pledgeability, the reallocation market appears and some of the released resources find their way to productive activities. Thus, both views are special cases in our model: the degree of frictions (as measured by pledgeability) determines whether liquidation is beneficial.

We extend the model to allow for an endogenous set of investment opportunities by embedding our previous model in a framework where there is information acquisition about projects. The decision whether to acquire information depends on the depth of the reallocation market. In a deeper reallocation market, firms that turn out to be productive can easily raise additional capital and therefore their incentives to acquire information are stronger. In this model there are two benefits of excessive liquidation. The first is the improvement in the ex-post allocation discussed before. The second is the increase in incentives to acquire information. Thus the effect of external finance on the allocation of capital can be even stronger than we suggested above.

This effect points to another benefit of liquidation that, to the best of our knowledge, is not present in the current discussion about the its costs and benefits. Increases in liquidation can provide incentives for entrepreneurs to invest since they anticipate a higher probability of raising capital. In this respect, even if liquidation were excessive from a static point of view, it might still be optimal in a dynamic setting because of the positive incentives that deep markets provide to entrepreneurs. The idea that static inefficiencies can be viewed as providing incentives in a

[2] This is also similar to the arguments in Shleifer and Vishny (1992), who argue that specific assets which are sold because of liquidity constraints might not be redeployed to the buyers with the highest valuations because such buyers might not have enough funds to finance the acquisition.

dynamic setting also goes back to Schumpeter (1942):

> "A system ... that at *every* given point of time fully utilizes its possibilities to the best advantage may yet in the long run be inferior to a system that does so at *no* point of time, because the latter's failure to do so may be a condition for the level or speed of long-run performance" (emphases in the original, p. 83).

The model also predicts that measures of external finance will be positively correlated with the availability of information about firms. The model is thus consistent with evidence that measures of the quality of information about firms (such as accounting standards) are positively correlated with equity market capitalization and with the fraction of investments financed via the equity market (see Rajan and Zingales, 1998a and 1999). Of course, this correlation can be explained by the fact that information production is a necessary condition for external finance to exist. In countries in which there is more information about the corporate sector, investors would be more willing to finance firms. However, our model points to a different channel for this correlation: Firms which are financed with external funds will have higher incentives to produce information.

A second way in which supply of capital to the external market can be affected is through conglomeration. Recently, a number of studies have analyzed the costs and benefits of internal capital markets in conglomerates (see Stein, 2001, for a survey). This literature suggests that a potential benefit internal capital markets is its ability to allocate capital more efficiently than external investors (Stein, 1997).[3] The opposite view is that internal capital markets reduce efficiency in a conglomerate since they do not allocate capital efficiently but rather in a 'socialistic' way (Rajan, Servaes and Zingales, 2000; Scharfstein and Stein 2000).

All the studies in this literature analyze conglomerates in isolation. In this paper, we extend our previous analysis to study conglomerates in equilibrium and find a novel cost of internal capital markets: conglomerates, by reallocating capital among their divisions, reduce the depth of the external capital market and thus compromise the ability of the economy to reallocate capital *across* conglomerates.

[3] A related idea is explored in Gertner, Scharfstein and Stein (1994) who assume that conglomerates can do internal restructuring of assets while external capital markets cannot.

Again, the potentially harmful effect of conglomerates on the allocation of capital is an externality. Because we assume that conglomerates allocate capital efficiently, there are always private incentives to create conglomerates. The implication of our model is that such privately efficient arrangements can make the external market for capital thinner, and consequently worsen the allocation of capital. Recently, according to Khanna and Palepu (1999), governments of developing countries have been under pressure to take actions to dismantle their business groups. Our model suggest that this dismantling can be socially beneficial (for intermediate values of investor protection) but will never be done by business groups themselves.

Our results are also related to the ideas in Rajan and Zingales (1998b). They argue that even though relationship-based financial systems (such as those with a high proportion of business groups) can increase or preserve value in some circumstances (specially when contractibility in the economy is low), they contribute to misallocation of resources in the external capital markets.

We start in the next section (section 2) by analyzing the effect of external finance on the allocation of capital. In section 3 we analyze the effects of conglomeration. We embed our model in a framework with information acquisition in section 4. Section 5 presents our final remarks. All the proofs are relegated to the Appendix.

2 External finance and the allocation of capital

We consider a model with risk-neutral agents that do not discount the future. There are three periods in the model t_0, t_1, and t_2, and an aggregate amount of capital equal to $1 + K$ (with $K > 1$). There is a set J with measure 1 of agents ('entrepreneurs'), each with one project opportunity (projects are described below) and a different set of agents with no project opportunities ('investors'). We analyze two different distribution of the capital endowment in sections 2.3 and 2.4. In the first case, entrepreneurs have exactly the amount of capital needed to finance the initial investment of their projects. In the second case, entrepreneur's capital endowment is not sufficient to pay for the project set up cost and hence need to finance it from the external capital market.

2.1 Technologies

There are two types of technologies with payoffs in period t_2. In addition, capital can be stored and does not depreciate. We refer to the first technology as the 'general technology'.[4] All agents in the economy can invest in the general technology at date t_1 and investment pays off in the final period. Letting ω be the aggregate amount of capital invested in the general technology, we define $x(\omega)$ as the *per-unit* pay off.

Assumption 1 *The general technology satisfies:*

A. $\frac{\partial[\omega x(\omega)]}{\partial \omega} \geq 0$

B. $\frac{\partial^2[\omega x(\omega)]}{\partial \omega^2} \leq 0$

C. $x'(\omega) \leq 0$

D. $x(1+K) > 1$

Total payoff is increasing in the amount invested (first assumption) and exhibits decreasing returns to scale (second assumption). The third assumption implies that the per unit payoff is decreasing in the amount invested. We make the last assumption for simplicity. It implies that storage is always dominated by the general technology and consequently it is not used in equilibrium.

We refer to the second type of technologies as projects. Each project is of infinitesimal size. At date t_0 each project requires an investment of one unit of capital. At date t_1 projects' productivity is revealed to be high ('good' projects) or medium ('medium' projects) with probability p_H and $p_M \equiv 1 - p_H$, respectively. At this date, a project can be liquidated, can receive an additional unit of investment, or can simply be continued with no change until t_2. If the project is liquidated at time t_1 the entire unit of capital is recovered and can be switched to the general technology or to other projects. We describe how the reallocation market works below.

At date t_2 the projects' payoffs are realized. The good (medium) project generates cash of Y_H (Y_M) for each unit invested, with ($Y_H > Y_M$).

Assumption 2 $Y_M > x(0)$.

[4] This is the same as the "autarky sector" for capital in Caballero and Hammour (1998).

This assumption implies that the medium project is more productive than the general technology.

2.2 The reallocation market and limited pledgeability

Initially, we assume that entrepreneurs have exactly one unit of capital. The rest of the $1+K$ capital endowment is held by investors. In this case the initial investment at date t_0 does not need to be financed externally and there is no capital market at date t_0. We analyze the case of external finance of the initial investment in section 2.4 and describe the date t_0 capital market there. Since entrepreneurs have only one unit of capital, the additional capital invested at date t_1 must be financed by external investors. We refer to the capital market at date t_1 as the 'reallocation market' and explain how it works in this section.

At date t_1, after learning the productivity of the projects, agents decide whether to liquidate their investments. We let $l_i^j \in [0,1]$ be the probability that agent j liquidates its project when the state is $i = H, M$. The total supply of capital is then $K + \int \sum_{i=H,M} p_i l_i^j dj$. This capital is allocated to firms with projects that are not liquidated and to the general technology. To attract capital, firms have to pledge part of their future returns. We introduce an imperfection in this reallocation market. Firms cannot pledge to investors the entire cash flow they generate. In particular, firms can only pledge a fraction λ of the returns of the second unit invested.[5] We assume that this 'limited pledgeability' applies only to projects but does not apply to the general technology.

The limited pledgeability assumption can be justified by appealing to different contracting frameworks. For example, limited pledgeability is an implication of the framework in Hart and Moore (1994). In Hart and Moore (1994), the human capital needed to run investment projects is inalienable.[6] Managers (entrepreneurs) may use a threat to withdraw their human capital in order to renegotiate repayments. The outcome of the bargaining process is that investors and managers get to split the period's cash flow. This bargaining limits the fraction of the cash flows

[5] The assumption that a only a fraction of the returns of the second unit –and not of the first– can be pledged is made only for simplicity. The results of the model still go through if we assume that a fraction λ of the returns of both units can be pledged. We further discuss the role and implications of this hypothesis later (section ??).

[6] Under this interpretation, a 'project' requires inalienable human capital while the general technology requires only unspecialized capital.

that can be credibly pledged to investors.[7] Limited pledgeability is also an implication of the Holmstrom and Tirole (1997) model of moral hazard in project choice. In that model limited pledgeability arises from the fact that investors must leave a high enough fraction of the payoff to entrepreneurs in order to induce them to choose the project with low private benefits but high potential profitability.[8] Furthermore, limited pledgeability also arises in models in which insiders can expropriate investors. In Shleifer and Wolfenzon (2002), expropriation has costs that limit the optimal amount of expropriation that the insider undertakes. In this model, higher levels of investor protection (i.e., higher costs of expropriation) leads to lower expropriation and consequently higher pledgeability.

The reallocation market is modeled as follows. First, the projects that continue (i.e., those that were not liquidated) simultaneously announce the price they are willing to pay for the first unit of capital they received[9]. We let Q_i^j $(i = H, M)$ be the price that firm j announces when its productivity is i. Limited pledgeability implies that $Q_i^j \leq \lambda Y_i$. Next, agents with capital allocate it to either projects or the general technology. If there are more projects offering to pay the same price than capital available, each project gets one unit of capital with the same probability. An allocation in the capital market can be described by r_i^j, the probability that project j gets one unit of capital when its productivity is i and it has not been liquidated, and ω the amount allocated to the general technology. This last quantity can be written as $\omega = K + \int \sum_{i=H,M} p_i l_i^j dj - \int \sum_{i=H,M} p_i r_i^j dj$. Since $K > 1$, there is always enough capital to provide additional finance to all projects. This assumption implies that $\omega > 0$.

Definition (equilibrium in the reallocation market): An equilibrium satisfies

- Agents allocate their capital to maximize their return.
- No continuing firm can profitably change the announcement Q^j.

As we explain below, in the symmetric equilibrium all technologies that get capital offer the

[7] See Diamond and Rajan (2000, 2001) for a recent example of an application of the Hart and Moore (1994) framework.

[8] Under this interpretation, a 'project' requires entrepreneurial effort while the general technology does not.

[9] Projects have no use for a second additional unit of capital and thus offer to pay zero for that unit.

same return. We let R be this equilibrium interest rate (or gross rate of return) offered in the reallocation market.

2.3 Internal financing of the initial investment

We concentrate in the symmetric equilibria of the model. In these equilibria, all firms with good (medium) projects liquidate with the same probability l_H (l_M), offer the same return Q_H (Q_M) and receive one additional unit of capital with the same probability r_H (r_M).

Lemma 1 *In all equilibria, if $r_i > 0$ then $Q_i = x(\omega)$ for $i = H$, M.*

Since the general technology always receives capital, it pays $x(\omega)$. It is not an equilibrium for firms with projects of type i to offer $Q_i < x(\omega)$ and receive capital with probability $r_i > 0$ since this implies that agents investing in technology i are not maximizing their payoffs. It is also not possible to have in equilibrium $Q_i > x(\omega)$. Clearly a firm with a project of type i could deviate and offer Q' with $Q_i > Q' > x(\omega)$. This firm will pay less and also receive a unit of capital with probability 1 since the investor will first allocate their capital to investment opportunities offering the highest return. This lemma implies that all the technologies that receive capital in equilibrium offer the same return $R = x(\omega)$.

The liquidation decision is simple. If a firm liquidates, it gets the market return R and, if it continues, it gets no less than Y_i $(i = H, M)$ and possibly more if additional capital is allocated. But since $Y_M > x(0)$ (by assumption 2) and $x' < 0$,then $Y_M > R$. Consequently, it never pays to liquidate, that is, in all equilibria $l_j = 0$ and the supply of capital in the reallocation market is K.

We define $\omega^*(\lambda)$ as the ω that satisfies $\lambda Y_H = x(\omega)$ and $\omega^{**}(\lambda)$ as the ω that satisfies $\lambda Y_M = x(\omega)$. In what follows we typically write ω^* and ω^{**}, but it should be understood that these quantities depend on λ. We also define λ_1^{IF}, λ_2^{IF}, λ_3^{IF}, and λ_4^{IF} as follows:

$$\begin{aligned}
\lambda_1^{IF} Y_H &= x(K) \\
\lambda_2^{IF} Y_H &= x(K - p_H) \\
\lambda_3^{IF} Y_M &= x(K - p_H) \\
\lambda_4^{IF} Y_M &= x(K - 1)
\end{aligned}$$

Since $x' < 0$ and $Y_H > Y_M$, then $\lambda_1^{IF} < \lambda_2^{IF} < \lambda_3^{IF} < \lambda_4^{IF}$. The equilibrium in the reallocation market is given by:

Lemma 2 *In all equilibria, $l_H = l_M = 0$ and the supply K is allocated as follows:*

a. if $\lambda \leq \lambda_1^{IF}$ then $r_H = r_M = 0$, $\omega = K$,

b. if $\lambda_1^{IF} < \lambda \leq \lambda_2^{IF}$ then $r_H = \frac{K-\omega^*}{p_H} < 1$, $r_M = 0$, and $\omega = \omega^*$

c. if $\lambda_2^{IF} < \lambda \leq \lambda_3^{IF}$ then $r_H = 1$, $r_M = 0$, and $\omega = K - p_H$,

d. If $\lambda_3^{IF} < \lambda \leq \lambda_4^{IF}$ then $r_H = 1$, $r_M = \frac{K-p_H-\omega^{**}}{p_M} < 1$, and $\omega = \omega^{**}$

e. If $\lambda_4^{IF} < \lambda$ then $r_H = 1$, $r_M = 1$, and $\omega = K - 1$.

In case a) not even good firms are able to attract capital because pledgeability is very low. All the supply of capital is allocated to the general technology. The return offered in the market is $R = x(K) = \lambda_1^{IF} Y_H$ which is above what good firms can pledge, λY_H. As λ increases the ability of firms to raise capital improves. In case b), good projects are able to raise capital, although with probability less than 1. In this case, the equilibrium return offered in the market is $R = x(\omega^*) = \lambda Y_H$. Good firms benefit from getting funds, since one unit of capital generates Y_H but costs only λY_H. These firms would like to offer a slightly higher return to improve the probability of getting funds. However, limited pledgeability prevents them from doing so. In all the equilibria of case b), capital flows to the general technology up to the point at which the return of this technology is λY_H, the maximum that a good firm can offer. At this point, investors start allocating their capital to the good firms. As λ increases, good firms can offer a higher return and consequently the amount ω that needs to be allocated to the general technology decreases. As a result, more capital is allocated to the good firms and so the probability that they get funds is higher. In case c), pledgeability is so high that all the good firms receive one unit but it is not high enough for medium firms to receive capital. In case c) medium firms get capital with probability less than 1. Finally, in case e) pledgeability is so high that all good and medium firms get one additional unit of capital.

Note that the first best allocation of capital involves both good and medium firms getting one unit of capital. Not surprisingly, when there are no imperfections, the competitive equilibrium allocation is first best. However, with imperfections, the allocation deviates from first best. The lower the degree of pledgeability, the higher the wedge between the marginal product of capital and the return offered in the market and consequently the worse the allocation.

2.4 External financing of the initial investment

In this section we assume that the capital endowment is distributed in such a way that each entrepreneurs has $1 - Z$ of capital, and the remaining capital is held by investors. In this case, entrepreneurs need to raise external finance at date t_0. We show that external finance of the initial investment (henceforth only 'external finance') forces firms to liquidate their projects in situations when this is not privately optimal. However, this increase in liquidation, which is "excessive" from a private viewpoint, increases the supply of capital available for reallocation. This increase in the supply of capital can make the reallocation market deeper and more efficient. Even though external finance is always privately inefficient, it can improve the social allocation of capital. Whether external finance is better than internal finance depends on the relative benefits of reallocation, vis a vis optimal continuation of projects.

At date t_0 entrepreneurs borrow Z in exchange for a set of promised payments and a state-contingent allocation of the right to liquidate. To focus on imperfections brought about by limited pledgeability of payoffs, we assume that initial contracts can be made fully contingent on dates and states (realized productivity of projects). A contract offered by entrepreneur j is defined by a vector $\{q_H^j, D_{1H}^j, D_{2H}^j, q_M^j, D_{1M}^j, D_{2M}^j\}$ where q_i^j is the probability of liquidating the project for realized productivity of $i = H, M$, D_{1i}^j is the payment to the investor at date t_1 when productivity is i and the project is liquidated (since projects produce no cash-flow at date t_1, the payment D_{1i}^j can be positive only when there is liquidation), and D_{2i}^j is the payment to the investor at date t_2 when productivity is i.

Entrepreneurs approach investors and make them a take-it-or-leave-it offer. The assumption that entrepreneurs have all the bargaining power can be justified by the fact that there is excess supply of capital at date t_0. Initial investors accept the contract if it offers them a payoff of at least RZ at date t_2, where R is the equilibrium return they expect in the reallocation market. Entrepreneurs maximize their payoff subject to the participation constraint of the investor.

The reallocation market is similar to the one described for the internal finance case. The only difference is that, because entrepreneurs raise capital date t_0, they might enter date t_1 with claims against the reallocation payoff Y_i. The obligations to the initial investors are senior to those of date t_1 investors.[10] Therefore, the ability to raise new capital is hampered by these commitments since firms have less cash flow available to pledge to new investors in the reallocation market. In particular for entrepreneur j, $Q_i^j \leq \lambda Y_i - D_{2i}^j$ with $i = H, M$.

Again, we focus on the symmetric equilibria, that is, in equilibria where all entrepreneurs offer the same contract at date t_0, and offer the same state contingent price Q_i $(i = H, M)$ in the reallocation market. As before, we let r_i $(i = H, M)$ be the probability that a firm with productivity i gets one unit of additional capital in the reallocation market.

Lemma 1 applies to this case and therefore the equilibrium rate of return in the reallocation market is $R = x(\omega)$.

The date t_2 payoff of an entrepreneur j, is given by:

$$\sum_{i=H,M} p_i \left\{ q_i^j (1 - D_{1i}^j) R + (1 - q_i^j)[Y_i + r_i^j (Y_i - D_{2i}^j - Q_i^j)] \right\} \tag{1}$$

and the date t_2 payoff of the date t_0 investor who lends him the funds is

$$\sum_{i=H,M} p_i \left\{ q_i^j D_{1i}^j R + (1 - q_i) r_i^j D_{2i}^j \right\} \tag{2}$$

When productivity is i, the project is liquidated with probability q_i. In this case the investor receives D_{1i}^j and the entrepreneur receives $1 - D_{1i}^j$. They can invest these amounts in the

[10] The seniority of initial claims can be shown to be optimal.

reallocation market an earn a return of R. This explains the first term equations 1 and 2. If the project is not liquidated, the firm always generates cash flows of Y_i. In addition the firm gets funds reallocated with probability r_i, generates Y_i from that unit of capital and pays D_{2i} to the date t_0 investors and Q_i to the date t_1 investors. Note that D_{2i} is only paid when additional capital is allocated to the firm since the payoff from the first unit of capital is not pledgeable (by assumption). This explain the second term in equations 1 and 2.

We define λ_1^{EF}, and λ_2^{EF} as:

$$\begin{aligned} \lambda_1^{EF} Y_H &= x(K+Z) \\ \lambda_2^{EF} Y_H &= x(K+Z-p_H) \end{aligned}$$

Lemma 3 *Suppose* $p_M \geq Z$[11]. *In all equilibria,* $D_{1i} = 1$, $q_H = 0$ *and*

a. *If* $\lambda \leq \lambda_1^{EF}$ *then* $D_{2i} = 0$ *for* $i = H, M$, $q_M = \frac{Z}{p_M}$, $r_H = r_M = 0$, *and* $\omega = K + Z$.

b. *if* $\lambda_1^{EF} < \lambda \leq \lambda_2^{EF}$ *then* $D_{2i} = 0$ *for* $i = H, M$, $q_M = Z/p_M$, $r_H = \frac{K+Z-\omega^*}{p_H} \leq 1$, $r_M = 0$, *and* $\omega = \omega^*$

When a project is liquidated the coalition of the date t_0 investor and the entrepreneur receives one unit which can be invested in the market at a return of R. However, if the project is not liquidated, even when no additional capital is allocated, it generates $Y_i > R$. Therefore it is optimal to pay the maximum possible to the investor when liquidation occurs and decrease the probability of liquidation. This implies that $D_{1i} = 1$.

Also, since $p_M \geq Z$, it is possible to pay the date t_0 investor simply by liquidating the project in the medium state. Since liquidating the project in the medium state is better than liquidating it in the good state, there will be no liquidation in the good state ($q_H = 0$).

Firms prefer to pay date t_0 investors out of the date t_2 proceeds ($D_{2i} > 0$) and reduce liquidation. However, when pledgeability is low, price competition in the reallocation market makes it impossible to have an equilibrium with $D_{2i} > 0$. With low pledgeability, capital is

[11] The case in which $p_M < Z$ is similar. In case a, firms liquidate with probability 1 in the medium state and with some probability in the good state. In case c, as λ increases, firms first decrease q_H until $q_H = 0$ and only then start decreasing q_M. We do not discuss this case because the number of cases to consider increases.

rationed in the reallocation market (i.e., $r_i < 1$). Since the amount firms can pledge out of date t_2 proceeds is capped ($Q_i + D_{2i} \leq \lambda Y_i$), firms have incentives to reduce D_{2i} and increase Q_i to attract more capital. The only equilibrium is then one in which $D_{2i} = 0$. When pledgeability is sufficiently high ($\lambda > \lambda_2^{EF}$), good firms are not rationed anymore and thus have no incentives to reduce D_{2H} in order to offer a better deal in the reallocation market. In this case, equilibria with $D_{2i} > 0$ can be sustained.

Since for the range of λ considered in the lemma, the only way to pay the investor is out of date t_1 liquidation proceeds. Since liquidation is inefficient, the optimal contract minimizes liquidation. That is, $p_M q_M = Z$.

We now state the main result of this section.

Proposition 4 *Suppose that $Z \leq \min[p_M,\ p_H]$. There exists a treshold $\lambda_{\min}$, satisfying $\lambda_1^{EF} < \lambda_{\min} < \lambda_1^{IF}$, such that external finance dominates internal finance for all $\lambda_{\min} \leq \lambda \leq \lambda_2^{EF}$, while internal finance dominates for all $\lambda \leq \lambda_{\min}$.*

The main trade-off between internal and external finance is as follows. External finance increases liquidation of the medium firms at a rate which is proportional to the financing need Z. Since this increase in liquidation is inefficient, external finance introduces a cost. However, the increase in the supply of capital coming from higher liquidation can be beneficial because of limited pledgeability. The equilibria with limited pledgeability has an inefficient amount of capital reallocation to the good firms ($r_H < 1$). An increase in external finance (increase in Z), increases the supply of capital and decreases the interest rate. Thus, reallocation of capital to good firms is higher with external finance ($r_H^{EF} > r_H^{IF}$). This effect (which we call the *deep market effect*) introduces a social benefit of external finance.

Whether external finance is better depends on the degree of pledgeability. If pledgeability is too low ($\lambda \leq \lambda_{\min}$), good firms have a difficult time attracting funds irrespective of the increase in the supply of capital, and the deep market effect either never materializes or is too weak to overcome the effect of liquidation. For larger levels of pledgeability ($\lambda \geq \lambda_{\min}$), the deep market effect kicks in strongly, and can improve the allocation of capital.

3 Internal Capital Markets

Recently, a number of studies have analyzed the costs and benefits of internal capital markets in conglomerates (see Stein, 2001, for a survey). This literature suggests that a potential benefit internal capital markets is its ability to allocate capital more efficiently than external investors (Stein, 1997).[12] The opposite view is that internal capital markets reduce efficiency in a conglomerate since they do not allocate capital efficiently but rather in a 'socialistic' way (Rajan, Servaes and Zingales, 2000; Scharfstein and Stein 2000).

All the studies in this literature analyze conglomerates in isolation. In this section we extend our previous analysis to study conglomerates in equilibrium and find a novel cost of internal capital markets: conglomerates, by reallocating capital among their divisions, reduce the depth of the external capital market and thus compromise the ability of the economy to reallocate capital *across* conglomerates. A key point is that, due to limited pledgeability, a conglomerate might find it optimal to reallocate capital internally to mediocre divisions (and get the full return) rather than supply it to the market (and get a fraction of the marginal product of a good project). Good projects could in principle have other sources of capital in the market. However, the internal reallocation which is performed by conglomerates reduces the (external) supply of funds and raises the interest rate. The high interest rate prevents conglomerates with good projects from raising capital in the (external) market. In other words, conglomerates make the external capital market thinner.

In the case of stand alone firms, the supply of funds to the (external) market is higher (since there are no internal capital markets) and consequently the equilibrium interest rate is lower. In this case, it is possible that good projects can raise funds. Thus, it is possible that the overall allocation of capital is better when firms rely solely on the external capital market. In a sense, this is the reverse side of the coin of the external finance argument. External finance works by increasing the supply in the (external) market for reallocation of capital. Internal capital markets will have the opposite effect, and can thus be socially harmful.

We consider a simple extension of the model presented above. Our goal is to compare the

[12] A related idea is explored in Gertner, Scharfstein and Stein (1994) who assume that conglomerates can do internal restructuring of assets while external capital markets cannot.

equilibrium capital allocation in two situations. One situation is when firms are standalones (i.e., one project per firm), and all reallocation takes place via the external capital market. The other situation is when firms are 'conglomerates'. In the model, a conglomerate is a firm with multiple projects in which headquarters have control rights over the allocation of capital, and so it can reallocate capital perfectly among its divisions. In fact, any organization in which an individual has control rights over the allocation of funds (e.g., a business group) fits into our definition of 'conglomerate'. Our goal here is not to analyze the incentives for conglomeration. Rather, we take the number of projects in each firm as given and analyze the resulting capital allocation. To keep the model as simple as possible, we consider conglomerates with only two projects.

As in the benchmark model the state of each project is not known at date t_0. We consider only the case of internal finance of the initial investment. We also assume that projects can be either good, medium, or bad. The reallocation payoffs can be Y_H, Y_M, or 0 (with $Y_H > Y_M > 0$) with probabilities p_H, p_M, and $p_L \equiv 1 - p_H - p_M$, respectively.

The first best allocation of capital is trivial. It requires that the bad projects liquidate its investment, and one additional unit be invested in the good and medium projects at date t_1.[13]

Consider now the decentralized equilibrium of the model. We make the following two assumptions:

Assumption 3 $p_L > p_H$

and

Assumption 4 $p_H > p_L - p_L^2$

These assumptions are sufficient conditions to have a range of λ in which the standalone equilibrium dominates (see proposition below). We explain why they are needed below.

We define U_{SA} and U_C as the aggregate payoff in the standalone and the conglomerate equilibrium respectively.

[13] Of course, this is true only if there is enough capital, i.e., if $K + p_B \geq p_H + p_M$.

Proposition 5 *There exist values* $(\overline{\lambda}_1, \overline{\lambda}_2, \overline{\lambda}_3)$, *satisfying* $\overline{\lambda}_1 < \overline{\lambda}_2 < \overline{\lambda}_3$, *such that:*

a. for $\lambda < \overline{\lambda}_1$, $U_{SA} < U_C$

b. for $\overline{\lambda}_1 < \lambda < \overline{\lambda}_2$, $U_{SA} > U_C$, *and*

c. for $\overline{\lambda}_2 < \lambda < \overline{\lambda}_3$, $U_{SA} < U_C$

Since the probabilities p_H, p_M, and p_L are project specific and do not depend on whether the project is in a conglomerate, the number of good, medium and bad projects is the same in the conglomerate and in the standalone case. However in the conglomerate case, we need to keep track of six (ex-post) types of conglomerates which we refer by their projects: (H, H), (M, M), (L, L), (H, M), (H, L), (M, L).

In all the cases of Proposition 5, the supply of funds in the standalone case is higher than the supply of funds in the conglomerate case. The reason is that, in the stand alone case, all the capital in the bad projects is supplied to the market. In the conglomerate equilibrium, however, some of the bad projects are in conglomerates that reallocate capital internally rather than supply it to the market. Conglomerates (H, M), (H, L), (M, L) reallocate capital from their less productive projects to the more productive ones. Thus the capital that is in the bad projects in the (H, L), (M, L) conglomerates that is supplied to the external market in the standalone case is not supplied in the conglomerate case. Assumption 3 guarantees that p_L is sufficiently high to have a significant difference in supply between the standalone and the conglomerate case. Assumption 4 guarantees that the number of good firms is sufficiently high so that the increased supply can be used productively.

In case a, this difference in the supply of funds is not important since pledgeability is so low that no external reallocation can take place (firms find it impossible to raise funds). As a result no reallocation takes place in the standalone equilibrium. However, in the conglomerate equilibrium, some reallocation takes place in internal capital markets. This reallocation increases the aggregate payoff as capital is allocated to higher productivity projects. Consequently the conglomerate equilibrium does a better job in allocating capital.

In case b, pledgeability is average. In the range specified, the supply of funds in the standalone case reduces the interest rate sufficiently so that the good projects can attract some capital. However, there is still no external reallocation in the conglomerate equilibrium since the supply of capital is smaller and consequently the interest rate higher. Therefore in the standalone equilibrium most of the capital supplied finds its way to good projects. However, in the conglomerate equilibrium, there is no external reallocation to good projects. In addition, some of the internal reallocation, though privately optimal is not socially efficient. For example, conglomerate (M, L) does a inefficient reallocation since the unit it allocates to the medium project would have a higher productivity in a good project. Since the conglomerate gets the full return of allocating internally, whereas it gets only a fraction of the returns if it allocates via the external markets, it is privately optimal for this conglomerate to undertake this socially inefficient internal reallocation. In this case, the standalone equilibrium is associated with better allocation of capital.

Finally, in case c, pledgeability is so high that even with the low supply of funds in the conglomerate equilibrium there is significant external reallocation to good firms. Thus the conglomerate equilibrium is better at allocating capital since it reallocates well internally and in the external capital markets.[14]

As explained above one of the interpretations of λ is the degree of investor protection. This section shows that countries with poor investor protection benefit from having organizations in which headquarters makes allocation decisions. This is consistent with the evidence of business groups in developing countries, which typically have poor investor protection. According to Leff (1976) and, more recently Khanna and Palepu (1997), one of the functions of business groups is the reallocation of funds among member firms. Although we do not have similar measures for countries with good investor protection, the extent to which business groups dominate these economies seem to confirm our results. Claessens, Djankov, Fan, and Lang (2000) find that 68 percent of listed firms in 9 Asian economies belong to business groups. The listed companies of a single group in India, the Tata group, accounts for approximately 8% of the country's public companies (Khanna and Palepu, 1997). In addition, the large size of business groups suggest that their potential for reducing the depth of the external markets is not a second order effect.

[14] When the reallocation market is so efficient that all the good firms are getting one unit of capital, the internal reallocation of conglomerate (M, L) is not harmful. The reason is that the unit in the bad project cannot be better used than in a medium project as all the good projects are already receiving one unit.

This section also shows that conglomerates might be socially costly when the level of investor protection is intermediate. Although we did not discuss the conglomeration decision above, firms in this model have incentives to conglomerate. They benefit from internal capital markets and impose an externality to other firms by reducing the depth of the market. This model then explains why, according to Khanna and Palepu (1999), governments of developing countries have been under pressure to take actions to dismantle their business groups. This dismantling can be socially beneficial (for intermediate values of investor protection) but will never be done by business groups themselves.

Our results are also related to the ideas in Rajan and Zingales (1998b). They argue that even though relationship-based financial systems (such as those with a high proportion of business groups) can increase or preserve value in some circumstances (specially when contractibility in the economy is low), they may also contribute to misallocation of resources because such systems rely less on price signals from the external market to allocate capital. In the context of our model, conglomerates sometimes allocate capital to medium projects, even though there better projects elsewhere in the economy who are starving for additional capital. In addition, we explain why this behavior can be consistent with equilibrium in the market for capital. Our model suggests that there is a reinforcing mechanism at work. Because conglomerates allocate capital to medium projects, the external capital market is thin, and prices are such that capital does not flow to the best projects. Given this equilibrium scenario, it is indeed optimal for conglomerates to allocate capital inside the conglomerate to projects which are worse than the marginal project searching for funds in the market.

4 Equilibrium with endogenous information acquisition

In section 2 we showed that an increase in external financing needs can improve the allocation of capital. External finance introduces a typical private inefficiency (excessive liquidation), but also a benefit. Excessive liquidation increases the supply of capital, and the associated decrease in the interest rate allows capital to flow to good projects (the *deep market effect*).

In this section, we extend the model to allow for an endogenous set of investment opportunities. More specifically, we assume that at date t_1 agents have to pay a cost to find out the profitability of their projects. We focus again in the comparison between internal and external finance of the initial investment.

Information acquisition is an ex-ante (date t_1) investment choice that influences the outcome of the reallocation market. In turn, the outcome of the reallocation market influences firms' incentives to acquire information. In a deeper reallocation market, firms that turn out to be productive can easily raise additional capital and therefore their incentives to acquire information are stronger. This additional effect (which we call the *incentive effect*) increase the potential benefits of excessive liquidation and external finance.

4.1 The model

The model is very similar to the one we analyzed in section 2. The only substantial difference is that the profitability of a particular project will only be revealed at date t_1 (before the reallocation market opens), if a cost c is paid (per project). Otherwise, profitability is revealed at the final date (t_2), when cash-flows are produced.

We also make a few simplifying assumptions (none of them crucial), which make the model in this section slightly different than the one in section 2. First, we assume that the state of nature in which profitability is lower (state M) is such that the payoff of a project is lower than the payoff of the general technology. This means that if profitability is revealed to be low, the project should be optimally liquidated. For simplicity, we take the payoff in this case to be equal to zero.

If instead the project is revealed to be profitable, it will generate positive cash flows at date t_2. If no additional capital is invested at date t_1, the project generates a cash-flow equal to Y_1. If one additional unit of capital is invested at date t_1, the project generates an additional amount Y_2. For simplicity, we assume that the probability that a project is successful is equal to $\frac{1}{2}$. Again, all uncertainty is idiosyncratic so that exactly half the projects that continue turn out to be profitable.

We assume that the production technology exhibits decreasing returns to scale, that is $Y_1 > Y_2 > 0$. In addition, we assume that the productive technology is better than the general one, even when there is no capital invested at the general technology and no information acquisition:

Assumption 5 $\frac{Y_1}{2} > x(0)$

Notice that the payoff of an uninformed firm ($\frac{Y_1}{2}$) can be interpreted in a sense as an average, safe payoff (similar to Y_M in the previous models). All firms can achieve that simply by investing the first unit of capital at date t_0.

We make the following additional assumptions about the payoffs and the cost of acquiring information:

Assumption 6 $c > \frac{x(0)}{2}$

Assumption 7 $\frac{Y_2}{2} > c$

Assumption 8 $\frac{\partial[\omega x(\omega)]}{\partial \omega} |_{\omega=K} > c$

There are two potential benefits of acquiring information. First, one benefit is that a firm that finds out that it will not be profitable can liquidate optimally and invest its capital in the general technology. An additional benefit arises when there is reallocation among firms. A firm that learns that it will be profitable receives additional capital at date t_1 and generates additional cash flows of Y_2.The first two assumptions mean that while the first benefit alone is not enough to justify paying the cost of acquiring information (assumption 6), the second benefit makes information worthwhile. Assumption 7 guarantees that if there is reallocation among firms it is optimal to gather information. Finally, assumption 8 means that the marginal productivity of the general technology is large enough such that it does not pay to simply allocate all the additional capital at date t_1 to uninformed firms.

We assume that once information is acquired, it becomes available to everyone else in the economy. Thus, our model does not differentiate between information acquisition and information disclosure. Once information is acquired, it is automatically disclosed. We also assume that the cost c is measured in units of wealth, and it is a private cost entirely born by the firm.

At date t_1, there will then be firms that gathered information and found out that they are productive, firms that gathered information and find out they are unproductive and firms that do not gather information. To simplify exposition we refer to these types of firms as 'good', 'bad', and 'uninformed' firms, respectively. As before, there is an aggregate amount of capital equal to $1 + K$ (with $K > 1$) distributed among a set of agents (with measure higher than 1). There is a set J with measure 1 of agents ('entrepreneurs'), each with one project opportunity (projects are described below) and a different set of agents with no project opportunities ('investors').

We let i be the fraction of firms that acquire information. Half of these firms will turn out to be good and will demand additional funds, and half of these firms will turn out to be bad and supply funds to the market. We let l be the fraction of the firms that does not acquire information (uninformed firms) and that liquidate the investment at date t_1. These firms reallocate their investments. That is, at date t_1, there will be $K + \frac{1}{2}i + (1-i)l$ of funds available for reallocation.

As before, we let r be the fraction of firms that are known to be productive (good firms) that receive additional capital at date t_1 and we let s be the fraction of firms that do not gather information (uninformed firms) that gets one additional unit of capital. We can interpret r (s) as the probability that a good (uninformed) firm receives additional capital. The capital not allocated to firms is invested in the general technology. We let ω be the amount of capital going to the general technology.

The aggregate payoff as a function of the choice variables (i, l, r, s) will then be:

$$U = \frac{1}{2}i\left[Y_1 + rY_2\right] + (1-i)(1-l)\left[\frac{Y_1}{2} + s\frac{Y_2}{2}\right] + \omega x(\omega) - ic \tag{3}$$

where:

$$\omega = K + \frac{1}{2}i(1-r) + (1-i)[l - (1-l)s]$$

With the assumptions we make, we can easily show that the first best allocation of resources involves full information ($i = 1$), and full reallocation to good firms ($r = 1$). We do not provide a proof here, but the intuition is straightforward. First, at the first best it must be that $r^{FB} = 1$ since the productivity of capital reallocated to good firms, Y_2, is higher than that of the general technology, $x(\omega)$, ($Y_2 > x(0) > x(\omega)$) and than that of uninformed firms, $Y_2/2$. Thus, all good firms get an additional unit of funding.

The impact of an increase in i on the aggregate payoff is essentially given by $\frac{Y_2}{2} - c$, which is positive by assumption 6. The only situation when this is not true is when it is potentially optimal to reallocate funds to all uninformed firms as well. In this case the gain of increasing information is due to the fact that information makes reallocation more efficient, since if capital is reallocated to uninformed firms it will be wasted with probability $\frac{1}{2}$. Assumption 8 is the technical condition which guarantees that the social costs of this waste are larger enough such that $i^{FB} = 1$. Finally, since all firms are informed in the first best any value for s is equally good (s is essentially irrelevant since there are no uninformed firms).

4.2 Decentralized equilibria with internal and external finance

We now analyze whether this allocation is achieved in a decentralized equilibrium with limited pledgeability of cash flows. Limited pledgeability will affect the efficiency of reallocation of investments at date t_1, as we already showed in section 2. The new effect we show here is that, when the efficiency of the reallocation market is compromised, the incentives to gather information are affected. This is because firms might not be prepared to invest in information acquisition if they anticipate that they will not be able to attract additional capital in the reallocation market.

4.2.1 Equilibrium with internal finance

The sequence of events are as follows. At date t_0, entrepreneurs invest 1 in their projects. Next, they decide whether they want to gather information at a cost of c. At date t_1, half the entrepreneurs who gathered information learn that their firms will be productive at date t_2 (good firms) and the other half learns that their firms will not be productive (bad firms). Uninformed firms decide whether to liquidate their investments or continue. Firms with liquidated investments and investors allocate their funds to firms that continue and to the general technology. We start by analyzing the reallocation market, which works exactly like it did in section 2.[15]

Total supply of capital is fixed at $S = K + \frac{1}{2}i + (1 - i)l$ and the aggregate demand schedule is the sum of the individual demands generated by the general technology, the good firms and the uninformed firms that continue. The demand from the general technology is given by the

[15] Although we use a noncooperative game framework for the formal proofs, the equilibrium that we obtain, except for minor technical issues, is similar to what can be obtained using a demand and supply framework. We explain this latter framework here.

inverse of the function $x(\omega)$. The demand of the good firms is zero above a rate of λY_2 and is one below λY_2. Similarly, the demand from uninformed firms is zero at a rate above $\frac{\lambda Y_2}{2}$ and is 1 below that rate.

Figure 1 shows the aggregate demand and supply schedules under the assumption that $x(K) > \lambda Y_2$. We define ω^* by $x(\omega^*) = \lambda Y_2$. Similarly, we define, ω^{**} by $x(\omega^{**}) = \frac{\lambda Y_2}{2}$. We discuss three different cases depending on where the supply curve falls relative to the demand schedule.

I) No reallocation to either good or uninformed firms

This case, shown in panel A, arises when supply is relatively low (the condition is $S < \omega^*$) and as a result the interest rate is above λY_2. Good firms are not able to pledge sufficient funds to offer a comparable return so they cannot attract capital and the entire supply of capital is allocated to the general technology. The equilibrium interest rate is $R = x(S) > \lambda Y_2$.

II) Partial reallocation to good firms and no reallocation to uninformed firms

In this case, shown in Panel B, supply is in an intermediate range ($\omega^* \leq S < \omega^* + \frac{1}{2}i$). An amount ω^* is allocated to the general technology. The rest is allocated to the good firms. There is not enough capital to allocate one unit to every good firm. Rather, a fraction $r = \frac{S-\omega^*}{(1/2)i}$ of good firms receive one unit. In this case, the equilibrium interest rate is $R = \lambda Y_2$.[16]

III) Complete reallocation to good firms

This case, shown in Panel C, arises when supply is high ($S \geq \omega^* + \frac{1}{2}i$). All good firms receive one unit of capital. In this case the equilibrium interest rate is $R \leq \lambda Y_2$. As can be seen from

[16] This is the only case where the demand and supply analogy does not work well because it gives a different answer than the non-cooperative game. At $R = \lambda Y_2$ every good firm demands one unit of capital since the total payoff from reallocation is Y_2 and the firm pays only λY_2. Therefore, total demand is $\omega^* + \frac{1}{2}i$ and not the interval $[\omega^*, \omega^* + \frac{1}{2}i]$ as shown in figure 1. Clearly, with this framework there is no equilibrium.

However, in the non-cooperative game framework we use for the proofs, an equilibrium exists. Every good firm announces that it pays λY_2, an amount ω^* is allocated to the general technology and the rest is allocated to good firms. This is an optimal allocation for investors since the return to investing in the general technology and in the good firms is the same. Even though good firms benefit when they receive a unit of capital and they receive this unit with probability of less than one, they cannot raise the amount they offer due to limited pledgeability. Also, they do not decrease it since doing so implies getting no funds for sure (the general technology and the other good firms are offering λY_2). Therefore the equilibrium exists and is the one shown in the figure.

the figure, whether uninformed firms receive one unit of capital depends on how high supply is relative to $\omega^{**} + \frac{1}{2}i$.

Now, we can go back to date t_0 and solve for the equilibrium amount of information and liquidation. Firms have three options: they can invest in information, they can forego information gathering and liquidate at date t_1, or they can forego information gathering and continue. Figures 2 and 3 show the decisions that firms can take and their associated payoffs:

$$\begin{aligned} U_I &= \frac{1}{2}[Y_1 + r(Y_2 - R)] + \frac{1}{2}R - c \\ U_{NI}^l &= R \\ U_{NI}^c &= \frac{1}{2}[Y_1 + s(Y_2 - 2R)] \end{aligned}$$

The values of r, s and R are determined in equilibrium in the reallocation market.

Equilibrium with high pledgeability We start by considering the case in which the amount of capital that good firms can pledge is always higher than the return on the general technology, that is, $\lambda Y_2 \geq x(K)$.

Proposition 6 *If $\lambda Y_2 \geq x(K)$, there is a unique equilibrium. This equilibrium is equivalent to the first best allocation. All firms acquire information ($i_{IF}^* = 1$), and all good firms receive additional capital ($r_{IF}^* = 1$)*

The intuition for this result is as follows (all proofs are in the appendix). All good firms will attract one unit of capital in the reallocation market ($r_{IF}^* = 1$) since they can pledge a large fraction of the cash flow (regardless of the amount invested in the general technology, since we have $\lambda Y_2 > x(K)$). Therefore, the benefit of acquiring information is high. This explains why all the firms want to gather information.

Equilibrium with lower pledgeability We consider now the case in which $\lambda Y_2 < x(K)$. This inequality implies that when the amount of capital going to the general technology is low, good firms will be unable to attract additional capital. Here we will show that this failure of the reallocation market will in turn affect the incentives for information gathering. In this case, there is the possibility of multiple equilibria. We describe them in the following two propositions:

Proposition 7 *If $\lambda Y_2 < x(K)$, there always exists an equilibrium in which no firm acquires information ($i^*_{IF} = 0$), and all uninformed firms continue their projects ($l^*_{IF} = 0$). The expected aggregate payoff in such an equilibrium is $\frac{1}{2}Y_1$.*

The intuition for this result is as follows. Since in equilibrium no firm gathers information and no firm liquidates, the supply of capital in the reallocation market will be "low" (equal to K). This situation corresponds to case I of the reallocation market described above (figure 1 - panel A) with $S = K$. The equilibrium interest rate in the reallocation market will be higher ($R = x(K)$) than the amount good firms can pledge. As a result, a good firm will not be able to attract capital in the reallocation market. The benefits of getting information are significantly reduced and consequently no firm deviates from this equilibrium to acquire information.

In addition to the equilibrium described, there might be two additional equilibria. One of them, however, is not stable, so we will not discuss it here. The other one is described in the following proposition. Recall that we define ω^* by $x(\omega^*) = \lambda Y_2$.

Proposition 8 *Suppose $\lambda Y_2 < x(K)$. The additional equilibria with internal finance are given by:*

A) For $x(K + \frac{1}{2}) \leq \lambda Y_2$:

- *If $\omega^* \leq K + \frac{1}{2}\frac{Y_2 - 2c}{Y_2(1-\lambda)}$, there exists an equilibrium with $i^*_{IF} = 1$ and $r^*_{IF} = 1 - 2(\omega^* - K) < 1$. The aggregate payoff in this equilibrium is greater than the payoff in the equilibrium described in proposition 7*
- *Otherwise the unique equilibrium is the one described in proposition 7.*

B) For $\lambda Y_2 < x(K + \frac{1}{2})$:

- *The unique equilibrium is the one described in proposition 7.*

The intuition for this result is as follows. Half the firms that get information liquidate and supply their funds to the market. In equilibrium, all the firms get information and therefore the supply of capital in the reallocation market is large. This situation corresponds to case II of the reallocation market described above (figure 1 - panel B). The large supply of capital causes the equilibrium interest rate in the reallocation to be low enough such that good firms can compete with the general technology and attract capital. Since good firms get additional capital in the reallocation market, the benefit of getting information is high and no firm has incentives to deviate.

This equilibrium does not always exists. The reason is that information acquisition only pays off if a good firm can attract new capital. However, only a fraction of the good firms get additional capital in the reallocation market. If the probability of getting new capital is low, there might not be enough incentives for information acquisition and the equilibrium breaks down.

The reason for the (potential) existence of multiple equilibria is that there is an externality involved in information acquisition. When few firms gather information, the reallocation market is 'thin'. This reduces the incentives of other firms to gather information since good firms will be unlikely to get additional capital. Therefore, it is an equilibrium to have no firm acquiring information. Conversely, if many firms acquire information, the reallocation market is deeper, and the incentives to acquire information potentially higher. Thus, an equilibrium with all firms acquiring information is also possible.

This result is related to recent work on the determinants of entrepreneurship (Gromb and Scharfstein, 2001 and Landier, 2001). These papers have a similar mechanism to the one which generates these multiple equilibria. For example, in Gromb and Scharfstein (2001) an increase in 'entrepreneurship' increases the supply of failed entrepreneurs to the labor market and thus increases the probability that a 'project owner' in need of a manager can find good managers. This in turn gives incentives for further entrepreneurial activity. The most important difference (besides the obvious one that the focus is on redeployment of skilled labor and not capital) is that they do not analyze the equilibrium effect of external finance and liquidation of projects, as we do in the next section. Thus, the main mechanism we emphasize here is absent from their analysis.

4.2.2 Equilibrium with external finance

In this section, we analyze the equilibria with external finance. As in section 2, we model external finance by assuming that firms only have a fraction $(1-Z)$ of the unit of capital required for the initial investment. If investment is to be undertaken, firms must borrow the remaining fraction Z from investors. Everything else in the model is unchanged. In particular, we maintain the assumption that only a fraction λ of the payoff arising from reinvestment, Y_2, can be pledged to investors, and that the payoff Y_1 cannot be pledged at all.

Contracting at date t_0 The contracting framework is very similar to the one we analyzed in section 2. Firms borrow a fraction Z of the initial investment, in exchange for a set of promised payments and an implied allocation of control rights. Again, we assume initial contracts can be made fully contingent on dates and states. This implies that payments and the probability of liquidation can depend on whether or not information was gathered and on the productivity of the firm. We let D_1, D_{1B}, and D_{1G} denote the payment to the investor at date t_1 when the firm gathers no information, when the firm gathers information and finds that it is an unproductive firm (bad firm) and when the firm gathers information and finds it is a productive firm (good firms), respectively. We let D_{2G}^{ni} and D_{2G}^{i} denote the payments to the investor at date t_2, where the superscripts i and ni refer to whether or not the firm gathered information at date t_1 and the subscripts B and G refer to whether the firm is unproductive or productive, respectively. With this notation, the contract consists of a vector $\{B_0, q_1, q_{1G}, q_{1B}, D_1, D_{1G}, D_{1B}, D_{2G}^{ni}, D_{2G}^{i}\}$, where B_0 is the amount borrowed, and q_{1i} is the probability of liquidating a firm of type i (again, q_1 is the probability of liquidating an uninformed firm)[17]

The reallocation market is similar to the one described for the internal finance case. The only difference is that, because firms raise capital date t_0, they might enter date t_1 with claims against the reallocation payoff Y_2. Thus, good firms are only able to pledge $\lambda Y_2 - D_{2G}^{i}$ and uninformed firms that continue can offer at most $\frac{\lambda Y_2}{2} - D_{2G}^{ni}$.

We focus on a case where $Z \leq \frac{1}{2}$ (external financing needs are not very high). Also, we focus on the case where $\lambda Y_2 < x(K)$. As in the internal financing case, we can easily show that if

[17] Since firms produce no cash-flows at those states, we have $D_{2B}^{i} = D_{2B}^{ni} = 0$. We will not mention these variables in what follows.

$\lambda Y_2 \geq x(K)$ and $Z \leq \frac{1}{2}$, the unique equilibrium with external finance involves full information and full reallocation (the first best equilibrium, $i^*_{EF} = 1$ and $r^*_{EF} = 1$). Thus, when $\lambda Y_2 \geq x(K)$ and $Z \leq \frac{1}{2}$ there is no difference between the internal and the external finance case.

Since there is extra supply of capital at date t_0, competition among investors drive down their expected return to R. Therefore, the contract maximizes firms' payoffs subject to the initial investor receiving ZR at date t_2. We do not discuss optimal contracts in detail here, since they are similar to those explored in section 2. The optimal contract should have the following features. It should have as little liquidation of uninformed firms as possible (since such liquidation is privately inefficient), and it should have the lowest possible payment when the project is good (because this limits firm's ability of raising funds in the reallocation market). Thus, if $Z \leq \frac{1}{2}$, the optimal contract that implements information gathering has $B_0 = Z$, $D_{1B} = 2Z$, $q_{1B} = 1$, $D_{1G} = 0$, and $D^i_{2G} = 0$. The expected date t_2 payoff of the investor is ZR and the payoff of the firm:

$$U_I = \frac{1}{2}\left[Y_1 + r(Y_2 - R)\right] + \frac{1}{2}R - c - ZR$$

If a firm does not acquire information, the optimal contract has $B_0 = Z, D_1 = 1$ and $q_1 = Z$. The payoff for a firm which does not acquire information is then:

$$U_{NI} = (1 - q_1)\frac{Y_1}{2} = (1 - Z)Y_1$$

Investors liquidate firms that do not gather information with probability $q_1 = Z$, such that they breakeven from an ex-ante perspective.[18]Thus, we have $l^*_{EF} = Z$ with external finance.[19]

[18]If the firms decides to liquidate its project without acquiring information, the investor must get at least ZR. Thus, the payoff to the firm is $(1 - Z)R$ which is dominated by U_{NI}.

[19]We are implicitly using the fact that uniformed firms will be unable to raise additional capital. The reason for this is that at date t_1, uninformed firms compete for capital with good firms, and good firms can pledge more

External finance, deep markets, and incentives to acquire information We can now describe how external finance will change the equilibria described above. We proceed in a constructive way. We start from an hypothetical situation with zero information acquisition. This is always an equilibrium with internal finance. We then show that for a large enough financing requirement Z, this equilibrium will disappear. This happens because the increase in the supply of capital will allow good firms to get new capital (the deep market effect). Anticipating reallocation profits to be high, an individual firm will then have the incentives to acquire information (the incentive effect). This moves the economy away from the zero information equilibrium, and towards a high information / high reallocation equilibrium. Thus, the effect of external finance on the allocation of capital can be even stronger than we suggested in section 2. By increasing incentives for information acquisition, external finance can expand the set of investment opportunities in the economy. As a result, we could have a substantial increase in the efficiency of capital allocation. While with internal finance all capital is allocated to projects of average profitability, with enough external finance capital will find its way to the most profitable users.

As in the previous model, the benefit of external finance relies on the externality it generates.[20] An entrepreneur who does not need external finance does not benefit from an external investor that forces liquidation. Entrepreneurs only seek external finance when they need it to set up their projects. Therefore, it is the *need* for external finance that is at the core of this positive externality.

Naturally, it is also possible that this increase in liquidation makes the social allocation of capital less efficient. This depends essentially on the degree of pledgeability. If pledgeability is low enough, good firms will not be able to attract capital irrespective of how deep the reallocation market is. In that case, excessive liquidation is just a private inefficiency but has no social benefit.

cash flows in the reallocation market for two reasons. First, good firms generate λY_2 for sure whereas uninformed firms generate λY_2 only with probability 1/2. And second, since firms that obtain information can pay investors in the bad state, their obligation in the good state is reduced.

[20] Our externality is a pecuniary externality (the effect of liquidation on the required rate of return). The reason why a pecuniary externality has real allocative effects here is because of the main inefficiency in the model (the capital market inefficiency). Because of limited pledgeability social values deviate from market prices, and thus there is room for price effects to have real effects on welfare.

In order to understand how these results come about, let us start from the an hypothetical equilibrium with zero information and no reallocation. As long as $\lambda Y_2 < x(K)$, this is an equilibrium when $Z = 0$ (internal finance case). Define $Z_{\min}$ as the value of Z satisfying:[21]

$$\lambda Y_2 = x(K + Z_{\min})$$

If the financing requirement of firms is above $Z_{\min}$, a fraction $Z \geq Z_{\min}$ of uninformed firms will be liquidated (since $l^*_{EF} = Z$). Now suppose $i = 0$. This means that the total supply of capital will be equal to $K + (1-i)Z = K + Z$. Thus, reallocation to a good firm is now possible. If an individual firm deviates and acquires information, it will be able to raise capital with probability 1. The payoff of acquiring information will then be equal to:

$$\begin{aligned} U_I &= \frac{1}{2}[Y_1 + Y_2 - x(K+Z)] + \frac{1}{2}x(K+Z) - c - Zx(K+Z) = \\ &= \frac{1}{2}[Y_1 + Y_2] - c - Zx(K+Z) \end{aligned}$$

while the payoff of continuing the project with no information acquisition is:

$$U_{NI} = (1-Z)\frac{Y_1}{2}$$

The difference in payoffs is:

$$U_I - U_{NI} = Z[\frac{Y_1}{2} - x(K+Z)] + \frac{1}{2}Y_2 - c > 0$$

Thus, an individual firm has incentives to deviate and acquire information. It is no longer an equilibrium to have zero information acquisition. Indeed, we can show the following result:

Proposition 9 *Suppose $\lambda Y_2 < x(K)$. If the external finance requirement is large enough ($Z \geq Z_{\min}$), there is a unique equilibrium given by the following:*

A) For $\lambda Y_2 \geq x(K + \frac{1}{2})$:

- *If $\omega^* \leq K + \frac{1}{2}\frac{Y_2 - 2c + Z(Y_1 - 2\lambda Y_2)}{Y_2(1-\lambda)}$, the equilibrium is $i^*_{EF} = 1$ and $r^*_{EF} = 1 - 2(\omega^* - K)$ with aggregate payoff of $\frac{1}{2}[Y_1 + r^*_{EF}Y_2] + \frac{1}{2}\lambda Y_2 - c$.*

[21] There always exists a value $Z_{\min} < \frac{1}{2}$, if $\lambda Y_2 > x(K + \frac{1}{2})$. The case when $\lambda Y_2 < x(K + \frac{1}{2})$ is essentially a case when pledgeability is too low for external finance to have social benefits, so we focus on the former case.

- *Otherwise* $0 < i^*_{EF} < 1$, $r^*_{EF} > 1-2(\omega^*-K)$ *and the aggregate payoff is* $(1-Z)\frac{Y_1}{2}+Z\lambda Y_2$.

B) For $\lambda Y_2 < x(K+\frac{1}{2})$

- *If* $Z[\frac{Y_1}{2} - x(K+\frac{1}{2})] + \frac{1}{2}x(K+\frac{1}{2}) - c \geq 0$, $i^*_{EF} = 1$, $r^*_{EF} = 0$ *and the aggregate payoff is* $\frac{1}{2}Y_1 + \frac{1}{2}x(K+\frac{1}{2}) - c$.

- *Otherwise* $0 < i^*_{EF} < 1$ *and the aggregate payoff is* $(1-Z)\frac{Y_1}{2} + Z\lambda Y_2$.

Thus, with external finance, there is always a unique equilibrium which involves some information acquisition. When is this equilibrium better than the one which obtains with internal finance? The next proposition establishes sufficient conditions.

Proposition 10 *If* $x(K+\frac{1}{2}) \leq \lambda Y_2 < x(K)$, *and* $\omega^* \leq K + \frac{1}{2}\frac{Y_2-2c+Z(Y_1-2\lambda Y_2)}{Y_2(1-\lambda)}$, *then the aggregate payoff when the financing requirement is above* $Z_{\min}$ *is higher than the payoff with internal finance* $(Z=0)$.

It can be shown that second inequality is satisfied for $\lambda > \overline{\lambda}$. So λ is in an intermediate range in the sense that the proposition applies to $\max\{\overline{\lambda}, x(1/2+K)/Y_2\} \leq \lambda < x(K)/Y_2$.

This proposition holds in the case A-first bullet of propositions 8 and 9. In this case, the allocation with external finance is better than that of the no information equilibrium, and is equivalent to the high information equilibrium of the internal financing case. One possible benefit of external finance is thus to eliminate the multiple equilibrium problem. In addition, there are parameter values for which the only equilibrium with internal finance is the no information one. In these cases, external finance is clearly superior.

Thus, keeping the other parameters fixed, the benefits of external finance come about when λ is in an intermediate range. If λ is very low, the welfare implication is reversed as the following proposition shows.

Proposition 11 *If* $\lambda Y_2 < x(K+\frac{1}{2})$, *and/or* $\omega^* > K + \frac{1}{2}\frac{Y_2-2c+Z(Y_1-2\lambda Y_2)}{Y_2(1-\lambda)}$. *In such cases, the aggregate payoff with internal finance* $(Z=0)$ *is higher than the payoff when the financing requirement is above* $Z_{\min}$

This proposition holds when $\lambda < \max\{\overline{\lambda}, x(1/2)/Y_2\}$ corresponding mainly with case B of Proposition 9. In this case, external finance causes an increase in liquidation. However, pledgeability is so low that even the increase in the supply of capital in the reallocation market is not sufficient for firms to be able to attract new capital. External finance is only costly in this case. The cost is that uninformed firms are not able to continue but have to liquidate. And the potential benefit of having a more active reallocation market does not materialize because pledgeability is so low.

5 Conclusions

We analyze the allocation of capital to investment projects in an equilibrium context. This allocation is affected by an imperfection in the market for capital: project-owners can only pledge a fraction of the returns generated by the additional investment. The required return on capital might be higher than the return that project-owners can pledge. In this case, an increase in the supply of capital to the market can improve the allocation if this additional supply drives down the required return to the point where project-owners can attract capital. In this paper, we study two scenarios: the need for external finance and deconglomeration. External finance increases the supply of capital in the reallocation market because external investors liquidate projects too often. Conglomeration affects the supply of capital because of the possibility of internal reallocation.

The benefits of external finance and deconglomeration come about because they increase activity in the external capital market, and improve its allocative role. Interestingly, these benefits occur even though external finance and deconglomeration are always privately inefficient. We show in the paper that whether these social benefits dominate private costs depends on the degree of pledgeability. When pledgeability is too low, the costs dominate the benefits. Pledgeability is one variable which will be related to the efficiency of reallocation, but in general any variables which affect reallocation will have a similar effect. This suggests the following conjectures.

One potential measure of the efficiency of reallocation is the flexibility of the market for corporate assets (mergers, divestitures, etc.). In countries where these markets are large, active and relatively flexible, it is more likely that external finance will have the kinds of benefits that we are describing here. Having more external finance will force firms to either acquire more assets or to divest, and this may lead to an improvement in the allocation of resources.

The increase in activity in external capital markets that generates our results is in a sense very similar to the notion that outside investors such as outside shareholders might be more 'short-termist' than inside shareholders, who are more willing to wait for a higher long term payoff. Short-termism might play the same role that external finance plays in the present model. Firms would never voluntarily choose to be short-termist, since this will bias decisions against long term projects. However, short-termism might be socially beneficial because it might force firms to liquidate investments instead of betting on a long term project. But this might improve the efficiency of the reallocation market because it increases the supply of funds to be reallocated, and may in fact lead to a higher social payoff.

We also show that an increase in the efficiency of reallocation can increase incentives for information acquisition. In fact, we can reinterpret our information acquisition technology as a very risky technology that can be very bad or very good (such as R&D investments), while no information can be reinterpreted as simply continuing with the 'old safe' technology (the no information case). The model would predict that external finance (and excessive liquidation) gives incentives for firms to undertake more risky investments such as R&D, in an equilibrium context. If pledgeability is high enough, this increase in risky investments will in turn improve the allocation of capital in the economy.

Our results also have implications for the literature on the boundaries of the firm. Bolton and Scharfstein (1998) and Stein (2001) have recently argued for a "capital-allocation-centric" view of the theory of the firm. The argument is essentially that a collection of assets (such as our projects) should reside under a single roof if internal capital markets do a better job of allocating capital to these projects than does the external capital market. Our model thus suggest that such privately optimal boundaries might not be socially efficient once we embed assets and firms in an equilibrium model for the allocation of capital.

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6 Appendix

Proof of Lemma 3

Step 0: In all the equilibria, $D_{1i} = 1$.

Suppose not, i.e., that in equilibrium a contract $\{q_H, D_{1H}, D_{2H}, q_M, D_{1M}, D_{2M}\}$ is offered with $D_{1i} < 1$ for $i = H$ or M. An entrepreneur can deviate an offer the same contract except for $D'_{1i} = 1$ and $q'_i = q_i D_{1i} \leq q_i$. Since $q'_i = q_i D_{1i}$ the first term inside the sum in equation 2 does not change and since q_i weakly decreases the second term weakly increases. Thus payoff to the investor weakly increases. The payoff of the entrepreneur conditional on productivity being i can be written as $-q_i D_{1i} R + q_i R + (1 - q_i)[Y_i + r_i(Y_i - D_{2i} - Q_i)]$. The first term does not change since $q'_i = q_i D_{1i}$. The last two terms are a weighted average of Y_i and $Y_i + r_i(Y_i - D_{2i} - R)$. Since $R < Y_i + r_i(Y_i - D_{2i} - Q_i)$, weakly increasing q_i,weakly increases the entrepreneur's payoff.

Step 1a: If $0 < r_i < 1$ then $Q_i + D_{2i} = \lambda Y_i$.

Suppose not, i.e., that $0 < r_i < 1$ and $Q_i + D_{2i} < \lambda Y_i$ (limited pledgeability and seniority of date t_0 claims rules out $Q_i > \lambda Y_i - D_{2i}$). Entrepreneur j's payoff conditional on state i is $q_i R + (1 - q_i)[Y_i + r_i(Y_i - D_{2i} - Q_i)]$. Consider the deviation by a entrepreneur j that offers $Q_i^j = Q_i + \varepsilon$. Since firms of productivity i get funds with positive probability by offering Q_i, firm j must receive one unit for sure, i.e., $r_i^j = 1$ (since there is a measure $p_i r_i$ being invested at Q_i, then surely there will be one infinitiesimal unit invested at $Q_i^j > Q_i$). The payoff of entrepreneur j conditional on state i is $q_i R + (1 - q_i)[Y_i + (Y_i - D_{2i} - Q_i - \varepsilon)]$. The change in payoff is $(1-q_i)[(1-r_i)(Y_i - D_{2i} - Q_i) - \varepsilon] > 0$ for small enough ε. In addition, the payoff to the date t_0 investors goes up since the r_i increases. This is a contradiction since the entrepreneur can deviate to the above contract and improve his payoff while still satisfying the date t_0 investor's participation constraint.

Step 1b: Suppose that $q_k > 0$ for $k = H$ or M. If $r_i = 1$, then $Q_i + D_{2i} = \lambda Y_i$.

Suppose $Q_i + D_{2i} < \lambda Y_i$, then the entrepreneur can increase D_{2i} and decrease q_k such that the date t_0 investor receives the same amount. This must benefit the entrepreneur since liquidation is inefficient.

Step 2: If $0 < r_i < 1$ then $D_{2i} = 0$

Suppose not, i.e., $0 < r_i < 1$ and $D_{2i} > 0$. Conditional on state i the payoff of entrepreneur j is $q_i R + (1 - q_i)[Y_i + r_i(Y_i - D_{2i} - Q_i)]$ and that of his date t_0 investor is $q_i R + (1 - q_i) r_i D_{2i}$. An entrepreneur j can deviate an offer $Q_i^j = Q_i + \varepsilon$ and $D_{2i}^j = D_{2i} - \varepsilon$. The constraint $Q_i \leq \lambda Y_i - D_{2i}$ still holds. Again, since there is a mass $p_i r_i$ invested at Q_i, then one (infinitesimal) unit must be invested at $Q_i^j > Q_i$ and so $r_i^j = 1$. The payoff of the entrepreneur conditional on state i increases and so does the payoff to the date t_0 investor. Contradiction.

Step 3: Suppose that $q_k > 0$ for $k = H$ or M. Then $r_H \geq r_M$ with equality only at $r_H = r_M = 0$ or $r_H = r_M = 1$

Suppose $0 < r_H < 1$. Since $r_H < 1$ then $D_{2H} = 0$ by step 2 and $Q_M = \lambda Y_M - D_{2M} < Q_H = \lambda Y_H$ by step 1. Therefore $r_M = 0$ since investors allocate capital first to the technology offering the highest return. If $r_H = 0$ then D_{2H} can be set to zero since the investor is not getting funds at date t_2 anyway. Thus, again $Q_M = \lambda Y_M - D_{2M} < Q_H = \lambda Y_H$ and $r_M = 0$.

Step 4: If $q_H > 0$ then $q_M = 1$

Suppose not, i.e., $q_H > 0$ and $q_M < 1$. The entrepreneur's payoff is $\sum_{i=H,M} p_i (1 - q_i^j)[Y_i + r_i^j (Y_i - D_{2i}^j - Q_i^j)]$ and the date t_0 investor's payoff is $\sum_{i=H,M} p_i \left\{ q_i^j R + (1 - q_i) r_i^j D_{2i}^j \right\}$. Can show that can reduce q_H and increase q_M to leave investor's payoff constant and that would increase intrepreneur's profits. This relies on $r_H > r_M$ and $Y_i - D_{2i}^j - Q_i^j = Y_i - \lambda Y_i$ which is higher in state H.

Step 5: Total liquidation at date t_0, $T = q_H p_H + q_M p_M$, satisfies $T \leq Z$.

Suppose not, i.e., $T > Z$. Then, there is i such that $q_i > 0$. Reducing q_i by ε, increases entrepreneur's profits since $R < Y_i + r_i(Y_i - D_{2i} - Q_i)$ and does not violate investors participation constraint.

Step 6: If $p_M \leq Z$ then $q_H = 0$

Suppose $q_H > 0$. Then by step 4, $q_M = 1$ and $T > Z$ which contradicts the result in step 5.

Step 7:If $D_{2H} > 0$ then $\lambda > \lambda_2^{EF}$

If $D_{2H} > 0$ then $r_H = 1$. Since $q_H = 0$, and amount p_H go to good firms and the rest $K + T - p_H$ (with $T < Z$ since at least something is paid out at date t_2) go to the general technology. Now $\lambda_2^{EF} Y_H = x(K + Z - p_H) < x(K + T - p_H) = Q_H = \lambda Y_H - D_{2H}$, and therefore $\lambda > \lambda_2^{EF}$. The second inequality follows from lemma 1.

Proof of Proposition 4

Since $Z \leq p_H$, then $K - p_H < K + Z - p_H \leq K < K + Z$ which implies that $\lambda_2^{IF} > \lambda_2^{EF} \geq \lambda_1^{IF} > \lambda_1^{EF}$. If $\lambda \leq \lambda_1^{EF}$, we are in case a) of lemmas 2 and 3 with $r_i = 0$ and all the capital allocated to the general technology. The payoffs with external and internal finance are:

$$\begin{aligned} U^{IF} &= p_H Y_H + p_M Y_M + K x(K) \\ U^{EF} &= p_H Y_H + (p_M - Z) Y_M + (K + Z) x(K + Z) \end{aligned}$$

and thus:

$$U^{IF} - U^{EF} = Z Y_M + K x(K) - (K + Z) x(K + Z) > 0 \tag{4}$$

The inequality follows from the fact that $Z > 0$ and that the general technology has decreasing returns to scale.

If $\lambda_1^{EF} < \lambda \leq \lambda_1^{IF}$, there will be reallocation with external finance, but not with internal finance (we are still in case a) of lemma 2, but in case b) of lemma 3). Reallocation with external finance is $r_H^{EF} = \frac{K+Z-\omega_{EF}^*}{p_H}$, where $\omega_{EF}^* \geq K$ satisfies $\lambda Y_H = x(\omega_{EF}^*)$. The payoff of internal finance is the same as above, but the payoff of external finance is now:

$$U^{EF} = p_H(Y_H + r_H^{EF} Y_H) + (p_M - Z) Y_M + \omega_{EF}^* x(\omega_{EF}^*)$$

so the difference $U^{IF} - U^{EF}$can be written as:

$$U^{IF} - U^{EF} = ZY_M - p_H r_H^{EF} Y_H + Kx(K) - \omega_{EF}^* x(\omega_{EF}^*) \tag{5}$$

As λ approaches λ_1^{EF}, r_H^{EF} converges to zero and ω_{EF}^* converges to $K + Z$ and we get 4. On the other hand, as λ approaches λ_1^{IF}, r_H^{EF} will converge to $\frac{Z}{p_H} \leq 1$ because $\lambda_1^{IF} Y_H = x(K)$, and thus ω_{EF}^* approaches K. Thus, the difference between payoffs converges to:

$$U^{IF} - U^{EF} = Z(Y_M - Y_H) < 0 \tag{6}$$

Now, notice that as λ increases r_H^{EF} increases monotonically and ω_{EF}^* decreases monotonically. Thus, the difference $U^{IF} - U^{EF}$ in equation 5 decreases monotonically with λ between λ_1^{EF} and λ_1^{IF}. It must then be the case that there exists a value of λ ($\lambda_{\min}$), such that for all $\lambda \leq \lambda_{\min}$ $U^{IF} - U^{EF} \geq 0$, and for all $\lambda_{\min} < \lambda \leq \lambda_1^{IF}$, we have $U^{IF} - U^{EF} < 0$.

Let us now show that for all $\lambda_1^{IF} < \lambda \leq \lambda_2^{EF}$ external finance dominates internal finance. This establishes the proposition. If this is the case, we are in case b) both in lemma 2 and in lemma 3. There will be reallocation to good firms in both cases. With internal finance, reallocation r_H^{IF} will be determined by:

$$\lambda Y_H = x(K - p_H r_H^{IF}) = x(\omega_{IF}^*)$$

while with external finance we have:

$$\lambda Y_H = x(K + Z - p_H r_H^{EF}) = x(\omega_{EF}^*)$$

We can thus write:

$$\begin{aligned} \omega_{EF}^* &= \omega_{IF}^* = \omega^* \\ r_H^{EF} &= r_H^{IF} + \frac{Z}{p_H} \leq 1 \end{aligned}$$

where the fact that $r_H^{EF} \leq 1$ comes from the fact that $\lambda \leq \lambda_2^{EF}$, and thus $\lambda Y_H \leq x(K+Z-p_H)$. The payoffs with internal and external finance will then be:

$$\begin{aligned} U^{IF} &= p_H(Y_H + r_H^{IF} Y_H) + p_M Y_M + \omega^* x(\omega^*) \\ U^{EF} &= p_H(Y_H + r_H^{EF} Y_H) + (p_M - Z) Y_M + \omega^* x(\omega^*) \end{aligned}$$

and the difference is given by:

$$U^{IF} - U^{EF} = -p_H(r_H^{EF} - r_H^{IF})Y_H + ZY_M = Z(Y_M - Y_H) < 0 \tag{7}$$

Proof of Proposition 5

First, we define the following thresholds for λ: $\lambda_1 Y_H = x(K + p_L)$, $\lambda_2 Y_H = x(K + p_L^2)$, $\lambda_3 Y_H = x(K + p_L - p_H)$, and $\lambda_4 Y_H = x(K + p_L - p_H - p_L p_M)$. Since $p_L > p_L^2 > p_L - p_H > p_L - p_H - p_L p_M$ (where the second inequality follows from assumption 4) and $x()$ is decreasing, it follows that $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$.

We write the aggregate payoff function for both the conglomerate equilibrium and the stand alone equilibrium in each of the regions defined by the tresholds of λ (case i: $\lambda < \lambda_1$, case ii: $\lambda_1 \leq \lambda < \lambda_2$, case iii: $\lambda_2 \leq \lambda \leq \lambda_3$ and case iv: $\lambda_3 < \lambda \leq \lambda_4$). As before the equilibrium in the reallocation market is denoted by R.

Stand alone equilibrium

There are p_L bad firms, p_M medium firms and p_H good firms. The bad firms supply their capital to market since $0 < R$. The medium firms supply their capital to the market whenever $R > Y_M$. However, because $x(0) < Y_M$ (assumption 2) and $x()$ is decreasing, as long as there is some capital going to the general technology we will have $R = x(\omega) < Y_M$. Therefore the total supply of capital in most of the cases we consider is $K + p_L$.

Case i: $\lambda < \lambda_1$

In this case all the capital is allocated to the general technology and the good firms cannot compete for it since $\lambda Y_H < \lambda_1 Y_H = x(K + p_L)$. Aggregate payoff is then: $U_{SA}(\lambda) = p_H Y_H + p_M Y_M + \omega x(\omega)$ where $\omega = K + p_L$.

Case ii and iii: $\lambda_1 \leq \lambda \leq \lambda_3$

In this case some capital is allocated to the good projects, every good project is allocated one unit with probability $r_{SA} = \frac{K+p_L-\omega^*(\lambda)}{p_H}$ where $\omega^*(\lambda)$ is defined as $\lambda Y_H = x(\omega^*)$. Since, in this region, $\omega^*(\lambda) \geq K + p_L^2 - p_H$, we have that $r_{SA} \leq 1$. The aggregate payoff is then $U_{SA}(\lambda) = p_H[Y_H + r_{SA} Y_H] + p_M Y_M + \omega^* x(\omega^*)$.

Case iv: $\lambda_3 < \lambda \leq \lambda_4$

In this region, all good projects receive one unit ($r_{SA} = 1$). The medium projects might receive one unit depending on the parameters. Letting $\omega^{**}(\lambda)$ be such that $\lambda Y_M = x(\omega^{**})$, $s = \max\{\frac{K+p_L-p_H-\omega^{**}(\lambda)}{p_M}, 0\}$ units. The aggregate payoff is then $U_{SA}(\lambda) = p_H[2Y_H] + p_M[Y_M + sY_M] + \omega x(\omega)$ where $\omega = K + p_L - p_H - sp_M$. Since $x()$ is decreasing, we have $\omega^*(\lambda) < \omega^{**}(\lambda)$.

Conglomerate equilibrium

There are six types of conglomerates at date t_1 which we refer as (i, j) where $i, j = H, M$,or L is the type of project. There are a number $\frac{1}{2}p_H^2$ of type (H, H), $\frac{1}{2}p_M^2$ of type (M, M), $\frac{1}{2}p_L^2$ of type (L, L), $p_H p_M$ of type (H, M), $p_H p_L$ of type (H, L) and a number $p_M p_L$ of type (M, L).We need to specify what each type of conglomerate does as a function of the interest rate R. As in the stand alone case, it is always the case that $R = x(\omega) < Y_M$ and therefore conglomerates with an M project will not supply this unit of capital to the market. First, the (H, H) and the (M, M) conglomerates will always try to raise funds and will never supply funds to the market. Second, the (L, L) conglomerate will always supply funds to the market. Third, the (H, M) conglomerate will try to raise funds from the market. If it raises one unit it will invest it in the H project. If it raises two units, it will invest one in each project. However, if it raises no units, it will internally reallocate the unit in the M project to the H project. Fourth, the (H, L) conglomerate will always allocate the unit in the L project to the H project. This conglomerate will not try to raise funds as it will have no use for them. And finally, the (M, L) conglomerate will allocate one unit from the L project to the M project as this has a higher payoff than supplying the unit to the market ($R = x(\omega) < Y_M$).

From the above discussion, the capital available in the external reallocation market is $K+p_L^2$. Note we refer to the external reallocation market as the reallocation market. Of course, there is capital being reallocated internally in the conglomerates in addition to the $K + p_L^2$.

<u>Case i and ii: $\lambda < \lambda_2$</u>

In this case all the capital supplied to the market is allocated to the general technology. Conglomerates with good projects cannot compete for funds since $\lambda Y_H < \lambda_2 Y_H = x(K + p_L^2)$. Aggregate payoff is then:

$$U_C(\lambda) = p_H^2 Y_H + p_M^2 Y_M + p_H p_M[2Y_H] + p_H p_L[2Y_H] + p_M p_L[2Y_M] + \omega x(\omega)$$

where $\omega = K + p_L^2$.

Case iii and iv: $\lambda_2 \leq \lambda \leq \lambda_4$

In this region, some capital is allocated (externally) to the good projects (if all the capital supplied to the market went to the general technology then $\lambda Y_H \geq \lambda_2 Y_H = x(K + p_L^2)$ which is inconsistent since good firms would be able to attract some capital). The probability r_C that a good project receives one unit of capital is $r_C = \frac{K+p_L^2-\omega^*(\lambda)}{p_H^2+p_H p_M}$ since $K + p_L^2 - \omega^*(\lambda)$ is the amount available after the necessary amount has gone to the general technology and $p_H^2 + p_H p_M$ is the number of projects requiring finance (the (H, H) conglomerate demands 2 units and the (H, M) conglomerate demands one unit). Given the range of λ, it follows that $\omega^*(\lambda) \leq K + p_L - p_H - p_L p_M$ with equality at $\lambda = \lambda_4$ and therefore $r_C \leq \frac{K+p_L^2-(K+p_L-p_H-p_L p_M)}{p_H^2+p_H p_M} = 1$ with equality at $\lambda = \lambda_4$. The aggregate payoff is

$$
\begin{aligned}
U_C(\lambda) &= p_H^2[Y_H + r_C Y_H] + p_M^2 Y_M + p_H p_M[r_C(2Y_H + Y_M) + (1 - r_C)2Y_H] + \\
&\quad + p_H p_L[2Y_H] + p_M p_L[2Y_M] + \omega^* x(\omega^*)
\end{aligned}
$$

Comparison of payoffs

Case i

$$
\begin{aligned}
U_{SA} - U_C &= p_H Y_H + p_M Y_M + (K + p_L)x(K + p_L) - p_H^2 Y_H - p_M^2 Y_M - \\
&\quad - p_H p_M[2Y_H] - p_H p_L[2Y_H] - p_M p_L[2Y_M] - (K + p_L^2)x(K + p_L^2) \\
&= Y_H[p_H - p_H^2 - 2p_H p_M - 2p_H p_L] + Y_M[p_M - p_M^2 - 2p_M p_L] + \\
&\quad + (K + p_L)x(K + p_L) - (K + p_L^2)x(K + p_L^2) \\
&= -p_H p_M[Y_H - Y_M] - p_H p_L Y_H - p_M p_L Y_M + (K + p_L)x(K + p_L) - \\
&\quad - (K + p_L^2)x(K + p_L^2) \\
&< -p_H p_M[Y_H - Y_M] - p_H p_L Y_M - p_M p_L Y_M + (K + p_L)x(K + p_L^2) - \\
&\quad - (K + p_L^2)x(K + p_L^2) \\
&= -p_H p_M[Y_H - Y_M] - Y_M p_L(p_H + p_M) + (p_L - p_L^2)x(K + p_L^2) \\
&= -p_H p_M[Y_H - Y_M] - (p_L - p_L^2)[x(K + p_L^2) - Y_M] < 0
\end{aligned}
$$

Case ii

$$
\begin{aligned}
U_{SA} - U_C &= p_H[Y_H + r_{SA}Y_H] + p_M Y_M + \omega^* x(\omega^*) - p_H^2 Y_H - p_M^2 Y_M - \\
&\quad - p_H p_M[2Y_H] - p_H p_L[2Y_H] - p_M p_L[2Y_M] - \\
&\quad -(K + p_L^2)x(K + p_L^2) \\
&= -p_H p_M[Y_H - Y_M] - p_H p_L Y_H - p_M p_L Y_M + \omega^* x(\omega^*) - \\
&\quad (K + p_L^2)x(K + p_L^2) + p_H r_{SA} Y_H
\end{aligned}
$$

Now at $\lambda = \lambda_1$, $\omega^*(\lambda) = K + p_L$ and therefore $r_{SA} = 0$. In this case $U_{SA} - U_C$ reduces to the expression in case i and is negative. As λ goes to λ_2, $\omega^*(\lambda) \to K + p_L^2$ and $r_{SA} \to \frac{p_L - p_L^2}{p_H} < 1$. Therefore:

$$
\begin{aligned}
U_{SA} - U_C &\to -p_H p_M[Y_H - Y_M] - p_H p_L Y_H - p_M p_L Y_M + (p_L - p_L^2) Y_H \\
&= Y_H[p_L - p_L^2 - p_H p_M - p_H p_L] - Y_M[p_M p_L - p_H p_M] \\
&= p_M(p_L - p_H)(Y_H - Y_M) > 0
\end{aligned}
$$

Since over this range $\frac{\partial}{\partial\lambda}[U_{SA} - U_C] = [\omega^* x'(\omega^*) + x(\omega^*) - Y_H]\frac{\partial\omega^*}{\partial\lambda} > 0$ (because $x' < 0$, $x(\omega^*) - Y_H < 0$ and $\frac{\partial\omega^*}{\partial\lambda} < 0$) there is a unique $\overline{\lambda}_1 \in (\lambda_1, \lambda_2)$ such that $U_{SA} - U_C < 0$ for $\lambda < \overline{\lambda}_1$ and $U_{SA} - U_C > 0$ for $\overline{\lambda}_1 < \lambda \leq \lambda_2$.

Case iii

$$
\begin{aligned}
U_{SA} - U_C &= p_H[Y_H + r_{SA}Y_H] + p_M Y_M - p_H^2[Y_H + r_C Y_H] - p_M^2 Y_M - \\
&\quad - p_H p_M[r_C(2Y_H + Y_M) + (1 - r_C)2Y_H] - p_H p_L[2Y_H] - \\
&\quad - p_M p_L[2Y_M] \\
&= -p_H p_M[Y_H - Y_M] - p_H p_L Y_H - p_M p_L Y_M + p_H r_{SA} Y_H - \\
&\quad - r_C[p_H^2 Y_H + p_H p_M Y_M]
\end{aligned}
$$

Over all this range $U_{SA} - U_C > 0$. First, at $\lambda = \lambda_2$, $r_{SA} = \frac{p_L - p_L^2}{p_H}$, and $r_C = 0$ and thus $U_{SA} - U_C = p_M(p_L - p_H)(Y_H - Y_M) > 0$.And second, $\frac{\partial}{\partial\lambda}[U_{SA} - U_C] = [-Y_H + \frac{p_H^2 Y_H + p_H p_M Y_M}{p_H^2 + p_H p_M}]\frac{\partial\omega^*}{\partial\lambda} > 0$ since $-Y_H + \frac{p_H^2 Y_H + p_H p_M Y_M}{p_H^2 + p_H p_M} < -Y_H + \frac{p_H^2 Y_H + p_H p_M Y_H}{p_H^2 + p_H p_M} = 0$ and $\frac{\partial\omega^*}{\partial\lambda} < 0$.

Case iv

$$
\begin{aligned}
U_{SA} - U_C &= -p_H p_M[Y_H - Y_M] - p_H p_L Y_H - p_M p_L Y_M + p_H Y_H + s p_M Y_M - \\
&\quad - r_C[p_H^2 Y_H + p_H p_M Y_M] + \\
&\quad + (K + p_L - p_H - s p_M) x(K + p_L - p_H - s p_M) - \omega^* x(\omega^*)
\end{aligned}
$$

First as λ goes to λ_3, $s = 0$ (since $K + p_L - p_H - \omega^{**}(\lambda_3) < K + p_L - p_H - \omega^*(\lambda_3) = 0$) and the expression for $U_{SA} - U_C$ is the same as the one for case iii evaluated at $\lambda = \lambda_3$ which is positive. Second at $\lambda = \lambda_4$, $r_C = 1$ and

$$
\begin{aligned}
U_{SA} - U_C &= [s p_M Y_M + \\
&\quad + (K + p_L - p_H - s p_M) x(K + p_L - p_H - s p_M)] - \\
&\quad - [(K + p_L - p_H - p_L p_M) x(K + p_L - p_H - p_L p_M) + \\
&\quad + p_M p_L Y_M].
\end{aligned}
$$

Now we show that this expression is negative, i.e.,$U_{SA}-U_C < 0$. First, $s = \max\{\frac{K+p_L-p_H-\omega^{**}(\lambda)}{p_M}, 0\} \leq \max\{\frac{K+p_L-p_H-\omega^*(\lambda)}{p_M}, 0\} \leq$

$\leq \max\{\frac{K+p_L-p_H-(K+p_L-p_H-p_L p_M)}{p_M}, 0\} \leq p_L$ where the second inequality follows because $K + p_L - p_H - p_L p_M$ is the lowest value that ω^* can take in this case. Second, each of the two terms in the expression for $U_{SA} - U_C$ can be interpret as the productive of an amount $K+p_L-p_H$ of funds to be distributed between the general technology and the M project. Since the M project is more productive the first term is lower as less funds are allocated to the M project ($s p_M < p_L p_M$).

[need to show that it crosses the axis only once, i.e., that it is monotonic] Therefore, there is a $\overline{\lambda}_2$ such that for $\lambda_3 < \lambda < \overline{\lambda}_2$, $U_{SA} - U_C < 0$ and for $\overline{\lambda}_2 < \lambda < \overline{\lambda}_3 \equiv \lambda_4$.

Proof of Proposition 6

If $\lambda Y_2 > x(K)$,then the only possible case of the financial market equilibrium is case III. In order to get $\omega < K$ we would need to have $s > 0$, but this can only occur when $r = 1$. Thus, $r = 1$.

Next we show that $l = 0$. Consider the case in which $\frac{\lambda Y_2}{2} < x(K+(1-i)l)$. As discussed above, this implies that $s = 0$, $\omega = K + (1-i)l$, and $R = x(\omega)$. Since $U_{NI}^l = x(\omega) < \frac{Y_1}{2} = U_{NI}^c$, $l = 0$ in this case. If $\frac{\lambda Y_2}{2} \geq x(K + (1-i)l)$,then $s > 0$ and $R \leq \frac{\lambda Y_2}{2}$.In any case, $U_{NI}^l \leq \frac{\lambda Y_2}{2} < \frac{Y_1}{2} = U_{NI}^c$, since $Y_1 > Y_2$. Thus, $l = 0$.

Since $r = 1$ and $l = 0$, we have $\omega = K - (1-i)s$, and:

$$
\begin{aligned}
U_I &= \frac{1}{2}[Y_1 + Y_2] - c \\
U_{NI}^c &= \frac{1}{2}Y_1 + \frac{s}{2}(Y_2 - 2R)
\end{aligned}
$$

we do not have to consider the liquidation payoff because we have already shown that $l = 0$. We now show that $U_I > U_{NI}^c$ such that $i = 1$. There are a few cases to consider. When $\frac{\lambda Y_2}{2} < x(K)$, then $s = 0$ and $U_I > U_{NI}^c$ because $\frac{1}{2}Y_2 - c > 0$. On the other hand, if K is high enough we could get $s = 1$. In such a case we have $R \geq x[K - (1-i)]$, and thus $U_I - U_{NI}^c = R - c$ $\geq x[K - (1-i)] - c \geq x(K) - c \geq \frac{\partial[\omega x(\omega)]}{\partial \omega}|_{\omega=K} - c > 0$ by assumption 8. Finally, if $s \in (0,1)$, then we have $R = x[K - (1-i)s] = \frac{\lambda Y_2}{2}$ and $U_I - U_{NI}^c = (1-s)\frac{Y_2}{2} + sx[K - (1-i)s] - c =$ $(1-s)\left(\frac{Y_2}{2} - \frac{\lambda Y_2}{2}\right) + x[K - (1-i)s] - c > 0$. Thus, we have $i = 1$.■

<u>Proof of Proposition 7</u>

First, notice that if $\lambda Y_2 < x(K)$ the equilibrium cannot be in case III of the financial market equilibrium. Suppose the equilibrium falls in case III of the financial market equilibrium. Then $r = 1$. Following the same steps as the previous proof, we can show that $l = 0$. But if we are in case III of the financial market equilibrium and $l = 0$, it must be that $\lambda Y_2 > x(K)$ which is a contradiction.

We look for equilibria in case I and II of the financial market equilibrium. The equilibrium of proposition 7 will be in case I, where $r = s = 0$. Therefore, the payoff of getting information, of not getting information and liquidating and of not getting information and continuing are respectively $U_I = \frac{Y_1}{2} + \frac{x(\omega)}{2} - c$, $U_{NI}^l = x(\omega)$, and $U_{NI}^c = \frac{1}{2}Y_1$. But $\frac{1}{2}Y_1 > x(0) \geq x(\omega)$ so no uninformed firm liquidates, i.e., $l = 0$. Finally, $\frac{1}{2}Y_1 + \frac{1}{2}x(\omega) - c \leq \frac{1}{2}Y_1 + \frac{1}{2}x(0) - c < \frac{1}{2}Y_1$. Therefore, no firm gets information. This equilibrium always exists when $\lambda Y_2 \leq x(K)$.■

<u>Proof of Proposition 8</u>

Consider the first part of the proposition ($x(K+\frac{1}{2}) \leq \lambda Y_2$). The equilibrium is in case II, at which informed firms get some funds ($r > 0$), and uninformed firms that continue get no extra capital ($s = 0$) because not all informed firms get new capital ($r < 1$). Thus, if an uninformed firm liquidates, it gets $x(\omega)$ and if it continues, it gets$\frac{1}{2}Y_1$. Therefore, all uninformed firms continue, i.e., $l = 0$. Note that in case II, the amount of capital going to the general technology is ω^*, where ω^* satisfies: $x(\omega^*) = \lambda Y_2$. The rest of the capital supplied, i.e., $K + 0.5i - \omega^*$ go to the good firms. Thus, the probability that a good firm receives one unit of capital is $r(i) = 1 - \frac{2(w^*-K)}{i}$.

An equilibrium with $i = 1$ requires that $U_I > U_{NI}$ that is

$$(1/2)[Y_1 + r(1)(Y_2 - \lambda Y_2)] + (1/2)\lambda Y_2 - c > Y_1/2 \tag{8}$$

That is, $r(1) > [2c - \lambda Y_2]/[Y_2 - \lambda Y_2]$. But we also know that $r(1) = 1 - 2(w^* - K)$. Therefore, for this equilibrium to exist, it must be that

$$1 - 2(w^* - K) > [2c - \lambda Y_2]/[Y_2 - \lambda Y_2]$$

what can also be written as:

$$2(\omega^* - K)/[1 - (2c - \lambda Y_2)/(Y_2 - \lambda Y_2)] \leq 1$$

or:

$$\omega^* \leq K + \frac{1}{2}\frac{Y_2 - 2c}{Y_2(1-\lambda)}$$

The aggregate payoff in the equilibrium is equal to U_I, which is bigger than $\frac{Y_1}{2}$ by construction.

An equilibrium with $i < 1$ requires that $U_{NI} = U_I$ that is

$$(1/2)[Y_1 + r(i)(Y_2 - \lambda Y_2)] + (1/2)\lambda Y_2 - c = Y_1/2$$

That is, $r(i) = [2c - \lambda Y_2]/[Y_2 - \lambda Y_2]$, which since $Y_2 > 2c$, is less than 1. But we also know that $r(i) = 1 - \frac{2(w^* - K)}{i}$, so i must be:

$$i^{INT} = 2(\omega^* - K)/[1 - (2c - \lambda Y_2)/(Y_2 - \lambda Y_2)]$$

This equilibrium exists when $i^{INT} \leq 1$, which is the same condition as for the equilibrium with $i = 1$ to exist.

Finally, if i is kicked just above i^{INT}, then the payoff to acquiring information is higher than not and so the i goes to $i = 1$. The opposite happens when i is kicked so that is below i^{INT}, it goes to 0 (the other equilibrium).

Consider now case ii). In such a situation, reallocation to good firms is impossible even when $r = 0$ and $i = 1$, since $x(K + \frac{1}{2}) > \lambda Y_2$ Since there is no reallocation, no firm acquires information and thus we are back to the equilibrium described in proposition 7.

■

Proof of Proposition 9

First, we discuss a few preliminary results which we use in the proof. Let ω^* be such that $x(\omega^*) = \lambda Y_2$. Let $i_l = K + Z - \omega^*$ and $i_h = 2i_l$. We have $l = Z$. The total amount of capital available for reallocation is $K + \frac{1}{2}i + Z(1 - i)$. There are three cases in the financial market equilibrium. They are comparable to those in the case of internal finance of the initial investment, so we just state the results.

Case I: if $i \geq i_h$ then $\lambda Y_2 \leq x(K + \frac{1}{2}i + Z(1 - i))$. In this case $r = 0$, $\omega = K + \frac{1}{2}i + Z(1 - i)$, and $R = x(\omega)$.

Case II: if $i_l \leq i < i_h$ then $x(K + \frac{1}{2}i + Z(1 - i)) < \lambda Y_2 \leq x[(K + Z(1 - i)]$.In this case $r(i) = 1 - \frac{\omega^* - K - Z(1-i)}{\frac{1}{2}i}$ and $R = \lambda Y_2$

Case III: if $i < i_l$ then $x[K + Z(1 - i)] < \lambda Y_2$. In this case $r = 1$, $\omega = K + Z(1 - i)$ and $R = x(\omega)$

The way to solve for the equilibrium will be to assume a level of i and then check the difference between U_I and U_{NI}. First of all, notice that if $i = 0$, $U_I - U_{NI} > 0$. This is because, for small values of i $(i < i_l)$, $U_I - U_{NI} = Z[\frac{Y_1}{2} - x(\omega)] + \frac{1}{2}Y_2 - c$.Thus, we cannot have an equilibrium with $i = 0$.

Next, we show that the difference $U_I - U_{NI}$ is decreasing in i. Therefore, if for some i, $U_I - U_{NI} = 0$ then that particular i is the equilibrium i. Finally if, for all i, $U_I - U_{NI} > 0$, then the equilibrium is one with $i = 1$.

For small values of i $(i < i_l)$, $U_I - U_{NI} = Z[\frac{Y_1}{2} - x(\omega)] + \frac{1}{2}Y_2 - c$. Also, $\frac{\partial(U_I - U_{NI})}{\partial i} = -\frac{1}{2}Zx'(\omega)(-1) < 0$. Finally as $i \to i_l$, $x(\omega) \rightharpoondown \lambda Y_2$ and $U_I - U_{NI} \rightharpoondown Z[\frac{Y_1}{2} - \lambda Y_2] + \frac{1}{2}Y_2 - c$. For intermediate values of i, $(i_l \leq i < i_h)$, $U_I - U_{NI} = \frac{1}{2}[Y_1 + r(i)(Y_2 - \lambda Y_2)] + \frac{1}{2}(1 - 2Z)\lambda Y_2 - c - (1 - Z)\frac{Y_1}{2}$ and $\frac{\partial(U_I - U_{NI})}{\partial i} = \frac{1}{2}r'(i)(Y_2 - \lambda Y_2) < 0$. Also, as $i \to i_l$, $r(i) \rightharpoondown 1$ and $U_I - U_{NI} \rightharpoondown Z[\frac{Y_1}{2} - \lambda Y_2] + \frac{1}{2}Y_2 - c$ so $U_I - U_{NI}$ is continuous at i_l. Finally, as $i \to i_h$, $r(i) \rightharpoondown 0$, and $U_I - U_{NI} \rightharpoondown Z[\frac{Y_1}{2} - \lambda Y_2] + \frac{1}{2}\lambda Y_2 - c$. For high values of i $(i \geq i_h)$, $U_I - U_{NI} = Z[\frac{Y_1}{2} - x(\omega)] + \frac{1}{2}x(\omega) - c$ and $\frac{\partial(U_I - U_{NI})}{\partial i} = -\frac{1}{2}x'(\omega)(-1) < 0$. Finally as $i \to i_h$, $x(\omega) \rightharpoondown \lambda Y_2$, and $U_I - U_{NI} \rightharpoondown Z[\frac{Y_1}{2} - \lambda Y_2] + \frac{1}{2}\lambda Y_2 - c$ so $U_I - U_{NI}$ is continuous at $i = i_h$.

This proves that $U_I - U_{NI}$ is decreasing in i.

The region where $i \geq i_h$ does not exist if $x(K + \frac{1}{2}) < \lambda Y_2$. In this case, when $i = 1$ we will be in case II above. The condition to check whether the equilibrium is at $i = 1$ is then $\frac{1}{2}[Y_1 + r(1)(Y_2 - \lambda Y_2)] + \frac{1}{2}(1 - 2Z)\lambda Y_2 - c - (1 - Z)\frac{Y_1}{2} > 0$. This condition is equivalent to:

$$\omega^* \leq K + \frac{1}{2}\frac{Y_2 - 2c + Z(Y_1 - 2\lambda Y_2)}{Y_2(1 - \lambda)} \tag{9}$$

so if 9 holds the unique equilibrium has $i^* = 1$, ω^* and $r^* = 1 - 2(\omega^* - K)$. The aggregate payoff is $U_I = \frac{1}{2}[Y_1 + r(1)(Y_2 - \lambda Y_2)] + \frac{1}{2}\lambda Y_2 - c$.

If 9 does not hold, then i^* decreases (since $U_I - U_{NI}$ is decreasing in i). In the equilibrium we must have $U_I - U_{NI} = 0$, so the aggregate payoff is $U_I = (1 - Z)\frac{Y_1}{2}$.

If conversely $x(K + \frac{1}{2}) > \lambda Y_2$, then the region where $i \geq i_h$ exists and the condition for $i = 1$ to be an equilibrium is $Z[\frac{Y_1}{2} - x(\omega)] + \frac{1}{2}x(\omega) - c \geq 0$.In this equilibrium there is no reallocation and $\omega = K + \frac{1}{2}$. If $Z[\frac{Y_1}{2} - x(\omega)] + \frac{1}{2}x(\omega) - c < 0$, we will get an interior equilibrium $i^* < \frac{1}{2}$ where $U_I = U_{NI} = (1 - Z)\frac{Y_1}{2}$.■

Proof of Proposition 10

If the conditions on the proposition hold, by proposition 9 the unique equilibrium with external finance is ($i = 1, \omega^*$) where ω^* is defined by $\lambda Y_2 = x(\omega^*)$. This is always the best possible equilibrium with internal finance if $\lambda Y_2 < x(K)$, so this shows that external finance achieves at least the same payoff. Furthermore, by proposition 8 the condition for the high information equilibrium ($i = 1, \omega^*$) to exist with internal finance is:

$$\omega^* \leq K + \frac{1}{2}\frac{Y_2 - 2c}{Y_2(1-\lambda)}$$

This condition is stronger than the condition on the statement of the proposition:

$$\omega^* \leq K + \frac{1}{2}\frac{Y_2 - 2c + Z(Y_1 - 2\lambda Y_2)}{Y_2(1-\lambda)}$$

This is because $Y_1 > 2x(K) > 2\lambda Y_2$. Therefore, we could have a case where the high information equilibrium only exists with external finance. If this is the case, the payoff with external finance is strictly higher:

$$U_I = U_I = \frac{1}{2}[Y_1 + r(1)Y_2] + \frac{1}{2}\lambda Y_2 - c > \frac{Y_1}{2}$$

In the other cases, the high information equilibrium is unique with external finance, while with internal finance we cannot predict a high information equilibrium since the no information equilibrium always exists.■

Proof of Proposition 11

For these parameters, there is a unique equilibrium with internal finance at which $i = 0$ and $\omega = 0$. The payoff is $U_{NI} = \frac{Y_1}{2}$. These conditions essentially mean that we can no longer have reallocation when all firms acquire information. Thus, with external finance we either have an equilibrium with full information and no reallocation, or an equilibrium with partial information (see proposition 9). In the former case, the payoff is:

$$U_I = \frac{1}{2}Y_1 + \frac{1}{2}x(\frac{1}{2} + K) - c < \frac{1}{2}Y_1$$

In the latter case the payoff of information acquisition must be the equal to the liquidation payoff and thus we have:

$$U_I = (1-Z)\frac{Y_1}{2} + Zx(\frac{1}{2} + K) < \frac{1}{2}Y_1$$

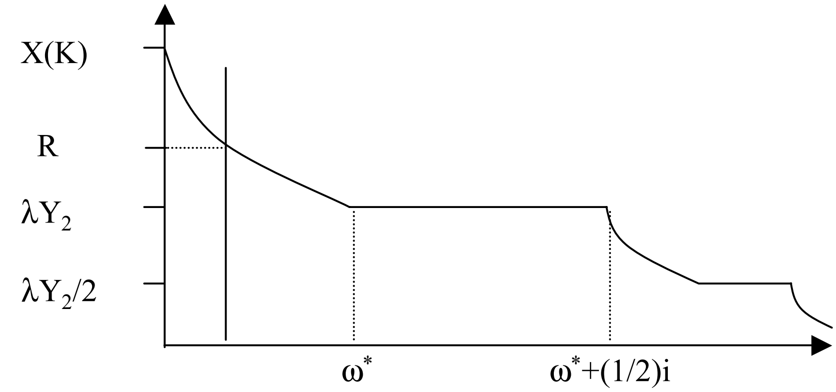

Figure 1 – Panel A

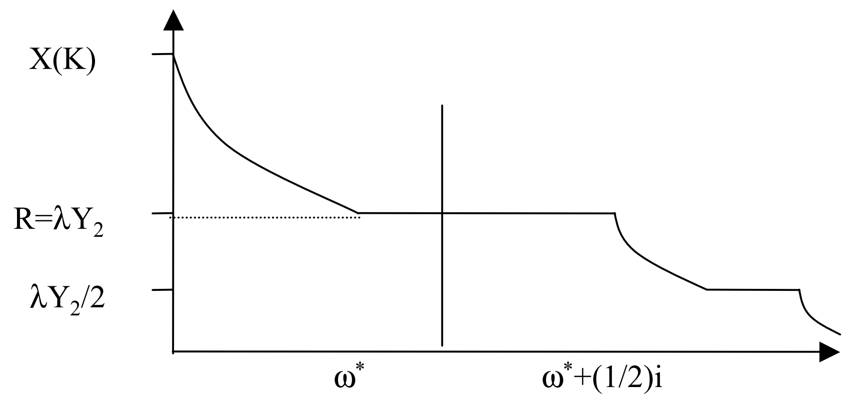

Figure 1 – Panel B

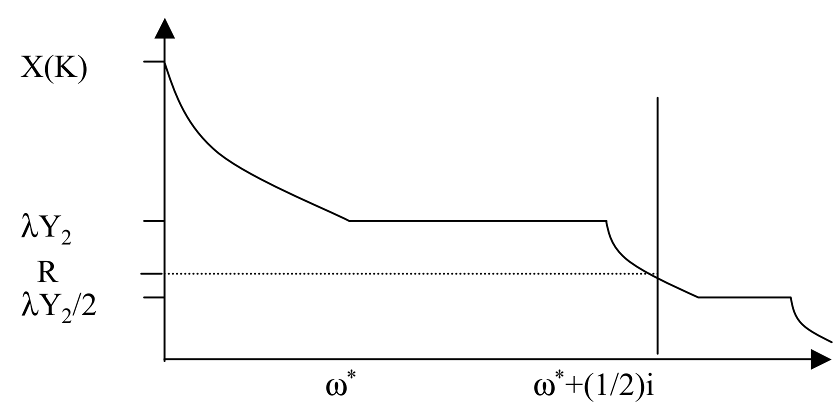

Figure 1 – Panel C

Figure 2

Technology - information acquired

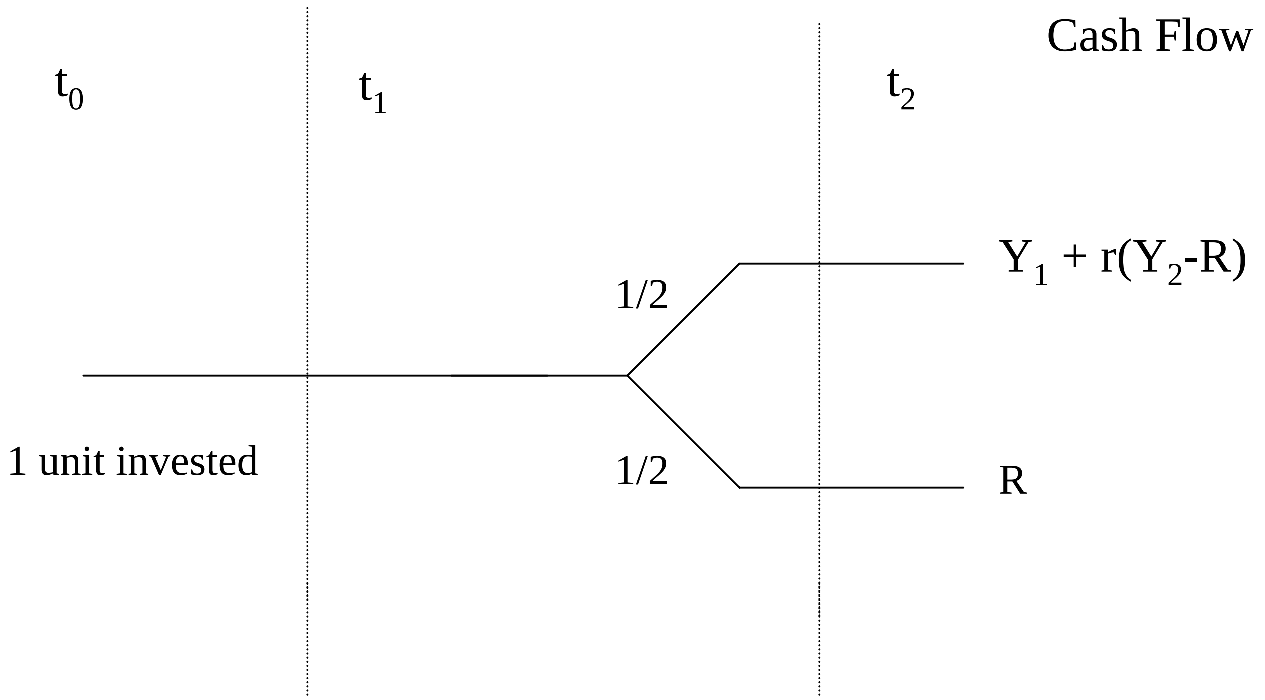

Figure 3

Technology - no information