WHEN MORE ALTERNATIVES LEAD TO LESS CHOICE*

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Abstract

One crucial decision firms or government agencies must make is the selection of the alternatives, e.g., the characteristics and/or the number of products, to offer to consumers or citizens. The selection of the set of alternatives to offer should not only take into account the potential preferences of the consumers and firms, but also the evaluation costs of the economic agents. This may lead to offering more alternatives not always being better for the firms than offering fewer alternatives. This paper shows that search/evaluation costs may lead consumers not to search and not to choose if too many or too few alternatives are offered. If too many alternatives are offered the consumer may have to engage in many searches/evaluations to find a satisfactory fit, which may be too costly and may dissuade the consumer from making a choice altogether. If too few alternatives are offered, a consumer may doubt that a satisfactory alternative is present, and therefore, also decide not to engage in search, and not to choose. These two forces may result in the existence of an interior optimal number of alternatives that should be offered in order to maximize the probability that a choice actually occurs. The number of alternatives offered may also signal the importance of fit in the market if there is asymmetric information about the importance of fit.
1. Introduction

A manager of an Internet Service Provider (ISP) was considering which DSL suppliers to provide as alternatives to its customers. When a customer was signing up to the service, the ISP offered a number of DSL suppliers as alternatives on a web page. The ISP had a list of up to fifteen DSL suppliers that it could present, and wondered what was the optimal number of DSL suppliers to actually present to their customers. The ISP experimented with lists of different sizes, from lists of the fifteen DSL suppliers down to lists of only one DSL supplier, and found that the number of DSL suppliers presented to the ISP customers that led to the most orders was four. The manager of the ISP conjectured that with too many alternatives customers were overwhelmed, while if only a few alternatives were provided customers may have felt that they did not have enough choice. Again, the answer was that four alternatives (not the minimum, not the maximum number of available alternatives) maximized the number of orders. The actual four alternatives that the ISP offered varied (randomly) across markets.\footnote{This example was described to the authors in an interview with the manager of the ISP.}

Examples where firms (or other economic agents) have to decide on how many offers to present to their customers are frequent. What is the optimal number of vacation options a travel agency should present in an email? How many brands/models should a retailer offer? How many options should a salesperson offer to a customer? How many health (or retirement) plans should a firm offer its employees? As the example above suggests, there seems to be an optimal number of alternatives to be offered, although the optimal number may vary from one situation to another.

This paper presents an explanation for this phenomenon based on search costs of evaluating each alternative. We will call the economic agent selecting an alternative the consumer of the alternative. If too many alternatives are offered, the consumer may have to engage in many searches to find a relatively good fit (Stigler, 1961, 1962). This may be too costly, and lead to no choice. If too few alternatives are offered, a consumer may believe that he will not find an alternative that is a relatively good fit, and therefore, decide also not to choose. These two forces may then result in the existence of an optimal and interior number of alternatives to be offered in order to maximize the probability that choice actually occurs. The paper presents this explanation in a model that endogeneizes both the search by the consumer, and the optimal number and design of the alternatives offered by the supplier. We also show that the number of alternatives offered may also signal the importance of fit in the market if there is asymmetric information, but still lead consumers with high search costs, or low general valuation for the product, not to search. We concentrate on the choice effects without including price-setting effects in order to simplify the
presentation. This case can be seen as relevant for some situations where prices are not present, when the supplier of alternatives does not have influence over prices, or when the price is observed prior to the consumers engaging in search.

Some experimental work (e.g., Iyengar and Lepper, 2000) has found that consumers are more likely to purchase and make a choice when confronted with a smaller choice set than when facing a larger choice set. This work has argued that consumers may feel overwhelmed or overloaded with too many alternatives or information. This idea goes back to the information overload literature (see, for example, Jacoby, 1977, and the references listed there), and the idea that decision-makers may only be able to process a limited amount of information, or have cognitive costs (e.g., Simon, 1955, Miller, 1956, Shugan, 1980, Gourville and Soman, 2005). In this regard, Hauser and Wernerfelt (1990) have argued that consumers may strategically limit their consideration sets (with search under fixed sampling) in order to limit evaluation costs at each consumption occasion. Another idea is that the product choices offered allow consumers to gain further information about their own preferences, and therefore affect preferences (Luce and Raiffa 1957, Sen 1993, Wernerfelt 1995, Kamenica 2005). In relation to those papers, this paper formalizes the costly product evaluation process. In this setting, consumers do not infer their preferences from the alternatives offered, but rather the number of alternatives offered affects the costs and benefits of that evaluation process.

The perspective presented in this paper is that the decision-maker is overwhelmed with alternatives because of search costs in evaluating the fit of each alternative. If many alternatives are offered the decision-maker may have to look through many alternatives in order to find one that provides a satisfactory fit, and therefore, the decision-maker may become discouraged of searching, and end up not searching, and not choosing. If fewer alternatives are provided (but not too few), the decision-maker may find that it is reasonable to search, and that within a “small” number of searches the decision-maker will find an alternative that provides a relatively good fit. As discussed in Section 3, it is crucial for this argument that the alternatives provided span the space of consumer preferences; that is, the locations of different alternatives in the space of consumer preferences are not independent, and an alternative does not have other alternatives located close by. This happens when the supplier of alternatives is strategic, which is a natural condition in choice situations faced by human beings. Note that even if this condition is not implemented in an experimental setting, the decision-makers may behave as if this condition is being implemented because they may per-

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2 Madrian and Shea (2001) show an example of when a firm automatically enrolls employees in the company 401(k) plan, there is an increase in retirement savings even though employees can easily opt out of the enrollment. See also the evidence in Benartzi and Thaler (2002), Choi et al. (2004), and Iyengar et al. (2004). This could potentially be seen as employees avoiding choice because of too many alternatives in the (401)k. See also Draganska and Jain (2005) for a discussion and estimation of the decreasing benefit of product line length.
ceive that the experimental setting is replicating the reality that they are familiar with. Another potential factor for consumers being overwhelmed with a greater number of alternatives may just be that a large number of alternatives creates overall confusion, independent of the search costs of evaluating each alternative. Note that if the locations of different alternatives were independent, this effect would still be present while the search costs effect would not have any impact.

Finally, it may also be that a greater number of alternatives may lead the decision-maker to delay choice (and not choose when the choice set is first presented) in order to gather more information on the choice problem (e.g., Dhar and Simonson, 2003). Possibly, this new information could come at a lower cost than the cost of evaluating different alternatives at the present. Note that this explanation could then be seen again as a search costs explanation for no choice when many alternatives are presented.³

The rest of the paper is organized as follows: The model is presented in the next section. Section 3 shows that the optimal number of alternatives is finite. Section 4 presents the search equilibrium for the case in which the firm offers up to three alternatives, and presents conditions under which offering two alternatives is better than offering either three or just one alternative. Section 5 considers the case where the supplier of alternatives has private information, Section 6 discusses several extensions, and Section 7 concludes.

2. Base Model

Consider a supplier of alternatives, which we call the firm, deciding on how many and which alternatives to offer to a set of decision-makers, who we will call the consumers. Before deciding whether to search, consumers observe the number \( n \) of alternatives being offered by the firm. The firm decides the number \( n \) of alternatives to offer, and the location of each of these alternatives on the segment \([0, 1]\). We denote the location of alternative \( i \) by \( z_i \), and without loss of generality assume that \( z_i \leq z_{i+1} \).⁴

After observing the number of alternatives offered (but not their locations), the consumer decides to evaluate an alternative if the expected utility of starting the search process is greater than both choosing an alternative at random (without evaluation) and not choosing any alternative. The utility of not choosing any alternative is normalized to zero. After evaluating one alternative,

³Note also that consumers could potentially prefer smaller choice sets because of self-control problems (see, for example, the discussion in Gul and Pesendorfer 2001, Bénaudeau and Tirole 2004, Fudenberg and Levine 2005).

⁴Several of the results presented here can be easily obtained for consumers distributed in a circle. That case would not allow us to consider what happens at the extreme points of the distribution of consumers (there are no extreme points), and the location of products would not be unique.
the consumer may decide to choose one of the alternatives already evaluated, choose one of the alternatives not yet evaluated, decide to evaluate an additional alternative, or decide not to choose.

Let the utility for a consumer of choosing an alternative \( i \) after evaluating \( m \) alternatives be

\[
U = v - t|x - z_i| - mc - K
\]  

(1)

where \( v \) is a general utility level of the ideal alternative, \( x \) is the location of the consumer ideal preferences, \( t \) is a dis-utility parameter of the ideal point of the consumer being different from alternative \( i \), \( c \) is the search cost incurred per alternative that is evaluated, and \( K \) is the cost incurred to enter the market. Once \( K \) is paid consumers learn \( x \). Consumers are heterogenous in \( v \) and \( x \) with \( v \) and \( x \) being independently distributed in the population, without mass points, and with supports \([0, \pi]\) and \([0, 1]\), respectively. Furthermore, we assume that \( x \) is distributed uniformly in order to simplify the analysis. All consumers have the same search cost \( c > 0 \) except for a mass \( \varepsilon \) of consumers, with \( \varepsilon \) close to zero, who have zero search costs (and have \( K = 0 \)). We focus on the case in which \( \varepsilon \to 0 \), and therefore, the expected profit for the firm, or expected payoff for the consumers, are always presented at the limit (when otherwise not noted). The role of this assumption is discussed in the next section. The consumers with zero search costs will always optimally choose the product that generates the best fit.

In the base model the consumer starts by knowing his own \( v \), but not his own \( x \). After observing \( v \) the consumer decides whether to incur the cost \( K \) and go ahead with the search process, choosing one alternative, or not to choose any alternative. When incurring the cost \( K \) the consumer learns his own \( x \). We assume \( K \) to be large enough, so that if in equilibrium a consumer incurs this cost, that consumer will end up choosing an alternative. From equation (1) the maximum net of \( K \) that a consumer can get is \( v \), so that for a consumer to buy in equilibrium it must be that \( v > K \). Then, since we allow consumers to choose an alternative at random without evaluation, if \( K > t \), after incurring the cost \( K \), a consumer always ends up choosing an alternative. The role of this assumption is to simplify the analysis so that whether a consumer decides not to choose depends only on \( v \) and not on both \( v \) and \( x \). In Section 6 we look at the case where \( K = 0 \), and consumers know both \( v \) and \( x \) before deciding whether to search. While consumers do not observe the locations of the products, they must form rational beliefs about their locations. This implies that consumers are able to infer the distribution of the product locations from the number of products offered. However, consumers remain uncertain about which product is which.

Suppose that a consumer observes the firm offering \( n \) alternatives and infers from this offer the location of the \( n \) alternatives being offered, \( \{z_i, i = 1, \ldots, n\} \). Consider the situation where the
consumer has evaluated $m$ alternatives, comprising a set which we denote by $I$. Then, the expected payoff of a consumer with preference characteristics $(v, x)$ after having evaluated the alternatives in this set $I$ and incurring the cost $K$ and search costs $mc$ is

$$V(v, x, I, n) = \max\{0, \max_{i \in I} v - t|x - z_i|, \sum_{i \in I} \frac{1}{n - m} (v - t|x - z_i|), -c + \sum_{i \in I} \frac{1}{n - m} V(v, x, I \cup \{i\}, n)\} \quad (2)$$

The right hand side of this equation represents the four possible options available to a customer who is searching. The first element in the max function represents the option of dropping out of the search process, and not choosing any alternative. The second element represents the option of stopping the search process and choosing the best alternative among the ones that have been evaluated. The third element represents the option of choosing an alternative at random among the alternatives that were not evaluated yet. Finally, the fourth element represents the option of evaluating one more alternative. All these options have the sunk cost of having evaluated $m$ alternatives, $mc + K$. As noted above, if $K > t$ the option of dropping out of the search process is never optimal for a consumer that incurs the cost $K$.

Note that the problem represented by (2) is a search problem with a finite and non-independent number of alternatives, with the possibility of recall, and with no replacement.

The expected utility net of $K$ for a consumer of preference characteristic $v$ of starting the search process can be written as

$$\int_0^1 V(v, x, \emptyset, n) \, dx = v - d(n), \quad (3)$$

where the function $d(n)$ represents the expected dis-utility of a consumer given that the firm offered $n$ alternatives. This expected dis-utility given $n$ offered alternatives is composed of the expected costs of searching plus the expected mis-fit of settling on an alternative that does not match the consumer preferences exactly. The marginal consumer $\hat{v}$ is then determined by $\hat{v} = K + d(n)$ (that is, $d(n) = \Delta$).

Suppose now that the payoff for the firm is equal to a fixed margin times the number of consumers that end up choosing one alternative. Then, the payoff is proportional to $\Pr[v \geq \hat{v}] = \Pr[v \geq K + d(n)]$, which is strictly decreasing in $d(n)$. Hence, the problem of the firm reduces to

$$\min_n d(n). \quad (4)$$

3. Finiteness of the Optimal Number of Alternatives

This section shows that the optimal number of alternatives offered is finite. That is, there is a
finite number of alternatives such that if the firm offers more, or less, alternatives the firm is worse off. To see this, we argue first that as the number of products \( n \) tends to infinity, the distribution of the products tends to the uniform distribution on \([0, 1]\). Then, we characterize the optimal search process of consumers under this continuous distribution of alternatives. Finally, we show that the firm can do better with a certain finite number of alternatives that we define.

3.1. Optimal Distribution of Alternatives

Suppose the firm offers \( n \) alternatives. Since \( K \) was assumed to be greater than \( t \), all consumers that incur the cost \( K \) choose an alternative. On the other hand, before incurring cost \( K \), uninformed consumers cannot find out the locations of any products. Therefore, the locations of the alternatives chosen by the firm do not determine whether the consumers with positive search costs decide to buy. That means that the locations chosen by the firm only affect the purchase decision whether to buy of the consumers with zero search costs. The number of zero search costs consumers that choose an alternative is maximized when the alternatives are equidistant on the segment \([0, 1]\), so that the subsegment of points that are closer to any given product than to any other product is of equal size across products. Therefore, the \( n \) products are located at \( \frac{2i-1}{2n} \) with \( i = 1, 2, ..., n \).\(^5\) When \( n \to \infty \) this distribution of locations of the alternatives converges to the continuous uniform distribution in the interval \([0, 1]\). That is, the limit when the number of products goes to infinity is a continuum of alternatives with a uniform distribution. This analysis shows that the uniform distribution is the only distribution for which the discrete approximation may consist of optimally-located products, given their number.

The next subsection considers the optimal search process when the firm offers a continuous and uniform distribution of alternatives.

3.2. Optimal Consumer Search Under a Continuous Uniform Distribution of Alternatives

Because there is an infinite number of products, the optimal search process is the same as a search process with replacement as in Diamond (1971).

The problem, generally defined, is the following: Let the alternatives be distributed with density \( f(x) \) (and cumulative distribution \( F(x) \)) on the line. It is well known that in such problems the optimal search process involves a stopping rule where the decision-maker keeps on searching until he

\(^5\)It can be shown that these locations also minimize \( d(n) \) for a given \( n \), if the products are such that for any given product there is a positive mass of uninformed consumers that continues to evaluate products until they find that particular product.
finds an alternative that provides a utility which is greater or equal to some reservation utility. In this particular set-up the problem is for the decision-maker to find a product that is sufficiently close to his ideal point. This means that a consumer located at \( x \) will have a reservation alternative located to his left, \( R_L(x) \leq x \), and a reservation alternative located to his right, \( R_R(x) \geq x \). If the product searched falls in \([R_L(x), R_R(x)]\), the consumer stops searching and buys that alternative; otherwise, the consumer keeps on searching. Note that under this search strategy the expected dis-utility of a consumer located at \( x \) is

\[
\int_{R_L(x)}^{R_R(x)} |y - x| \frac{dF(y)}{F(R_R(x)) - F(R_L(x))} + \frac{c}{F(R_R(x)) - F(R_L(x))}.
\]

Coming back to our problem of search with alternatives distributed uniformly on \([0, 1]\), one can see that for a consumer located close to the center of the segment \([0, 1]\) (where “close” is defined below) there are two reservation products located, one to the left and one to the right of the consumer’s location. The reservation product is defined by the condition that the marginal cost of searching an extra product, \( c \), is equal to the marginal expected benefit (in terms of better fit) of that search, given that a reservation product has just been found. This condition can be written for a consumer located at \( x \) as

\[
c = \int_{x-\delta}^{x+\delta} t \, |x - y| \, dy
\]

where \( \delta \) is the distance of the reservation product to the consumers location. This yields \( \delta = \sqrt{c/t} \); that is, if \( \sqrt{c/t} < x < 1 - \sqrt{c/t} \), the consumer can achieve the reservation utility with a product either on the right or the left of its location. This expected search costs plus dis-utility, for \( \sqrt{c/t} < x < 1 - \sqrt{c/t} \), is then \( t\delta = \sqrt{ct} \). We should compare this with the expected dis-utility if the consumer does not search and buys at random, which is \( \int_0^1 t \, |y - x| \, dy = t(x^2 - x + \frac{1}{2}) \). Note then that if \( \frac{c}{t} < \frac{1}{16} \), all consumers \( \sqrt{c/t} < x < 1 - \sqrt{c/t} \) engage in the search process. This means that if the search costs are low enough, or the importance of alternative fit is high enough, all consumers in the center of market that choose one alternative engage in the search process.

Consider now the case of \( x < \sqrt{c/t} \) or \( x > 1 - \sqrt{c/t} \). Then the reservation utility may only be obtained on one side of the consumers location. Consider the case of \( x < \sqrt{c/t} \). Then, the reservation product on the right of \( x \) is defined by

\[
c = \int_0^x t(x - y) \, dy + \int_x^{x+\tilde{\delta}(x)} t(y - x) \, dy
\]

where \( x + \tilde{\delta}(x) \) is the reservation product for the consumer located at \( x \). From this, one can obtain \( \tilde{\delta}(x) = \sqrt{2c/t - x^2} \), which is decreasing in \( x \), and such that \( \tilde{\delta}(\sqrt{c/t}) = \delta \).

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6The optimal stopping rule (with changing reservation utilities) is also optimal if the decision-maker does not know the distribution of the alternatives (Rothschild, 1974).
If the consumers engage in the search process, the expected search costs plus dis-utility of the product bought for a consumer located at \( x < \sqrt{c/t} \) is then \( t\delta(x) = t\sqrt{2c/t - x^2} \) (the case of \( x > 1 - \sqrt{c/t} \) is symmetric). Note that this expected search costs plus dis-utility of the product bought is concave in \( x \). That is, for \( x \) close to \( \sqrt{c/t} \), when the location of the consumer moves away from the center of the distribution of preferences, the expected search costs plus dis-utility of product bought increases “steeply” because the consumer is willing to accept a product that is relatively far away. However, when \( x \) is close to zero, when moving away from the center of the market, the expected dis-utility of searching and product mis-fit increases less steeply because the consumer is less and less willing to accept a product much further away.

In comparing with the expected dis-utility of choosing a product at random, \( t(x^2 - x + \frac{1}{2}) \), one can obtain that for \( \frac{c}{t} < \frac{1}{16} \), all consumers engage in the search process. Therefore, we have the following proposition:

**Proposition 1:** Suppose that the firm offers an infinite number of products and that \( c/t < 1/16 \). Then the consumer search strategy is the following: consumers with \( x \in (\sqrt{c/t}, 1 - \sqrt{c/t}) \) search until they find a product at most \( \sqrt{c/t} \) from them; consumers with \( x < \sqrt{c/t} \) search until they find a product at most \( \sqrt{2c/t - x^2} \) from them; finally, consumers with \( x > 1 - \sqrt{c/t} \) search until they find a product at most \( \sqrt{2c/t - (1 - x)^2} \) from them.

Consider now what happens with greater search costs (the complete analysis is presented in the Appendix). Comparing the expected dis-utility of buying a product at random with searching, we can obtain that if \( \frac{c}{t} > \frac{1}{8} \), then no consumer engages in the search process and all consumers buy at random. If \( \frac{c}{t} \in [\frac{1}{16}, \frac{1}{8}] \), and if search costs are not too high, we have a situation where consumers located in the center of the market buy at random, while the other consumers engage in the search process. This implies an interesting search strategy for the consumers depending on their preferences: Consumers that have relatively specific preferences (in the model, with a location close to zero or one) search, and consumers that have more generic preferences (around the center of the market) do not evaluate a product prior to purchase and buy at random.

In this case of greater search costs, one can also show that there are search costs such that the set of consumers that choose at random is not convex. That is, starting from the center of the market and going to either extreme, we have first consumers that choose at random, then consumers that engage in the search process, then consumers that choose at random, and, finally, again consumers that engage in the search process. In order to have some intuition for this possibility note that for \( x \) small the expected dis-utility is decreasing in \( x \) for both when a consumer engages in the search
process and when a consumer buys at random. As argued above, when a consumer engages in the search process, the expected search costs plus dis-utility of the product bought is concave in $x$. On the other hand, the expected dis-utility of the product bought when a consumer buys at random is convex in $x$; that is, the further away a consumer is from the center of the market the consumer is in an increasingly worse situation. This then allows for the possibility that for some search costs, there is an intermediate low region of $x$ where buying at random is better than engaging in the search process. Figure 1 illustrates this possibility with the comparison of the consumer payoffs under search and under choice at random.

Returning to the case $\xi t < \frac{1}{16}$, integrating over all $x$ we can get the expected dis-utility of the product bought as

$$d(\infty) = 2t\left[\left(\frac{1}{2} - \sqrt{c/t}\right)\sqrt{c/t} + \frac{c}{4t}(2 + \pi)\right] = \sqrt{ct} + \frac{c\pi - 2}{2}. \quad (7)$$

As noted previously, for $\xi t > \frac{1}{8}$ all consumers choose at random, and the expected utility across all consumers is $t \int_0^1 (x^2 - x + \frac{1}{2}) \, dx = \frac{7}{3}$. For $\xi t \in (\frac{1}{16}, \frac{1}{8})$ the expression for $d(\infty)$ is more complicated because some consumers engage in the search process while other consumers choose at random. However, by the principle of the optimum, it is easy to see that $d(\infty)$ is weakly increasing in $c$ and $t$ as $c$ or $t$ just represent a cost for the decision-maker. Figure 2 shows $d(\infty)$ as a function of $c$ for $t = 1$.

3.3. Optimality of a Finite Number of Alternatives

We now show that a firm offering a finite number of alternatives will do better than the firm offering an infinite number of alternatives. Consider $\xi t$ small ($\xi t < \frac{1}{16}$), and suppose that the firm offers $\hat{n}$ products located at $z_i = \frac{2(i-1)}{2\hat{n}}$, for $i = 1, 2, ..., \hat{n}$, where $\hat{n}$ is the smallest integer that is greater or equal to $\frac{1}{2\sqrt{c/t}}$, that is, $\hat{n} - 1 < \frac{1}{2\sqrt{c/t}} \leq \hat{n}$. We already know that these locations are uniquely optimal, and therefore will be expected by consumers who see that $\hat{n}$ products are offered. To show that offering these $\hat{n}$ products is better than offering an infinite number of products, it suffices to show that this set of products would generate lower consumer dis-utility as consumer search is restricted to some (not necessarily optimal) search rule. Specifically, consider the consumer’s search process which is to search until the consumer finds the alternative that is closest to him.

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7In order to get this expression, the Appendix shows that $\int_0^{\sqrt{\frac{c}{t}}} \sqrt{2\frac{c}{t} - x^2} \, dx = \frac{\pi}{4t}(2 + \pi)$.

8The case of $\xi t > \frac{1}{16}$ is dealt with in Section 4.
Let us denote the expected dis-utility under this search process by \( \hat{d}(\hat{n}) \geq d(\hat{n}) \), and compare it with the expected dis-utility \( d(\infty) \) given an infinite number of products.

With \( \hat{n} \) alternatives and with the proposed consumer search process the expected dis-utility of fit is \( \frac{1}{4n} \). The expected search costs for each consumer are

\[
c_1 \frac{1}{\hat{n}} + 2c \frac{\hat{n} - 1}{\hat{n}} - \frac{1}{\hat{n} - 1} + 3c \frac{\hat{n} - 1}{\hat{n}} - \frac{2}{\hat{n} - 2} + \cdots + (\hat{n} - 1)c_1 \frac{1}{\hat{n}} + (\hat{n} - 1)c_1 \frac{1}{\hat{n}} - \frac{1}{\hat{n} - 1} + \cdots + (\hat{n} - 1)c_1 \frac{1}{\hat{n}} = (\hat{n} - 1)(\frac{1}{2} + \frac{1}{\hat{n}})c. \tag{8}
\]

We then have \( \hat{d}(\hat{n}) = \frac{1}{4\hat{n}} + (\hat{n} - 1)(\frac{1}{2} + \frac{1}{\hat{n}})c \). By the definition of \( \hat{n} \) one can then obtain that \( d(\hat{n}) \leq \hat{d}(\hat{n}) < \frac{3}{4} \sqrt{ct} + c < \sqrt{ct} < d(\infty) \), because \( \frac{1}{4} < \frac{1}{16} \). That is, the expected dis-utility under \( \hat{n} \) alternatives is lower than in the expected dis-utility under an infinite number of alternatives. This means that the firm strictly prefers to offer a finite number of alternatives rather than infinite. Since as the number of alternatives increases to infinity the firm’s outcome tends to the inferior outcome of infinitely many alternatives, we obtain that there a \( N \) such that it is not optimal to have more than \( N \) alternatives. Therefore, the optimal number of products is finite, and we have the following proposition:

**Proposition 2:** When search costs are positive, the firm strictly prefers to offer a finite number of alternatives. That is, there is an optimal finite number of alternatives for a firm to offer.

The intuition is that by spreading out the location of the alternatives, and by offering a small number of alternatives, the firm allows the consumers to save on search costs. This is because, by having fewer alternatives to search through, a consumer can rule out areas of the product space that are less appealing for him. When \( c_t \) approaches zero, \( \hat{n} \) alternatives allows the consumers to save, in expected value, half of the search costs in the case of an infinite number of alternatives. This results also shows that if too many alternatives are offered the consumers realize that they will incur too many search costs, and will therefore prefer not to search – that is, not to choose.

When choosing the number of alternatives to offer, the firm faces the trade-off between potentially providing consumers with a better fit, and complicating their search process. The positive effect of better fit only holds when the number of products is not very large, while the search process is more and more costly as the number of alternatives increases to infinity. This is because when the number of alternatives is very large, consumers adopt a reservation rule strategy that never involves an exhaustive search for all alternatives, but rather search until the first product satisfying the reservation rule. When consumers adopt such a rule and the number of products is high enough so that consumers do not exhaustively search, increasing the number of the products
does not increase the expected fit between the first product found to satisfy the reservation rule and the consumer preferences. On the other hand, increasing the number of products, keeps increasing the expected search costs until the first product that satisfies the reservation rule. The intuition for that is that search with replacement is less efficient than search without replacement, and as the number of products tends to infinity, the search approaches search with replacement. Therefore, when the number of alternatives is large, there is only the negative effect and no positive effect of increasing the number of alternatives.

While we considered a particular distribution of consumers (uniform) and a particular functional form of the utility cost of misfit (linear travelling cost), the above intuition suggests that the result about the optimality of the finite number of alternatives would hold for a more general distribution of consumer preferences and when the utility cost of misfit is any decreasing function of the distance between the consumer’s ideal point $x$ and the product location.

4. Equilibrium Search and Optimal Behavior with Up to Three Alternatives

4.1. One Alternative

When the firm offers only one alternative, consumers do not need to search. The optimal product location for the firm is the center of the market, which also yields the lowest expected dis-utility. The expected dis-utility of the product bought across consumers is

$$d(1) = t \int_0^1 \frac{|x - \frac{1}{2}|}{4} \, dx = \frac{t}{4}. \quad (9)$$

Note that this is below the expected dis-utility in the case of random choice with an infinite number of products, $\frac{t}{3}$, because now consumers in the center of the market get dis-utility close to zero, and this was not the case with random choice with an infinite number of products.

4.2. Two Alternatives

In the case of only one product being offered no consumer incurs the search cost. Consider now the case of the firm offering two products, such that a positive mass of consumers will actually incur the search costs.

As discussed above, for $\varepsilon > 0$ (as assumed), the optimal locations for the two products are locations $\frac{1}{4}$ and $\frac{3}{4}$. This pair of locations is also the location of the products that minimizes the
expected dis-utility for consumers. To show this, consider general locations for the two alternatives. Denote by \( z_1 \) the location of the product that is located closer to zero, and \( z_2 \) as the location of the other product, with \( z_1 < z_2 \) (the case of \( z_1 = z_2 \) being the case of one product presented above). Suppose that \( z_2 - z_1 \geq 2c/t \). A consumer that searches one product gets his most preferred product (between the two products), because if the searched product is not the most preferred the consumer knows that the other product will be. Then, if a consumer located at \( x \) searches one product, he gets a total cost (search cost plus dis-utility of product bought) of \( c + t \min\{|x - z_1|, |z_2 - x|\} \). If a consumer chooses a product at random, he gets an expected dis-utility of \( t \left| x - z_1 \right| + \frac{t}{2} \left| z_2 - x \right| \).

From this one can see that if \( x \in \left( \frac{z_1 + z_2}{2} - \frac{c}{t}, \frac{z_1 + z_2}{2} + \frac{c}{t} \right) \), then a consumer prefers to choose an alternative at random, if \( x < \frac{z_1 + z_2}{2} - \frac{c}{t} \), then a consumer searches to find product \( z_1 \) and gets a search cost plus dis-utility of the product bought equal to \( c + t \left| x - z_1 \right| \), and finally, if \( x > \frac{z_1 + z_2}{2} + \frac{c}{t} \), then a consumer searches to find product \( z_2 \) and gets a search cost plus dis-utility of the product bought equal to \( c + t \left| x - z_2 \right| \).

The expected search costs plus dis-utility of the product bought across all consumers as a function of \( z_1 \) and \( z_2 \) is then equal to

\[
z_1(c + t\frac{z_1}{2}) + (1 - z_2)(c + t\frac{1 - z_2}{2}) + (\frac{z_1 + z_2}{2} - \frac{c}{t} - z_1)(c + t\frac{z_1 + z_2}{2} - \frac{c}{t} - z_1) + (z_2 - \frac{z_1 + z_2}{2} - c)[c + t\frac{2}{2}(z_2 - \frac{z_1 + z_2}{2} - \frac{c}{t})] + 2c\frac{z_2 - z_1}{t} \tag{10}
\]

which reduces to

\[
t\left\{\frac{c}{t} - \left(\frac{c}{t}\right)^2 + \frac{z_2^2}{2} + \frac{(1 - z_2)^2}{2} + \left(\frac{z_2 - z_1}{2}\right)^2\right\}. \tag{11}
\]

Minimizing with respect to the location of the products (for the firm to offer the best pair of alternatives) one gets \( z_1 = \frac{1}{4} \) and \( z_2 = \frac{3}{4} \), which are also the optimal locations with two products and zero search costs. As argued in Section 3 for the general case these are also the locations chosen by the firm given \( K \) large.

Substituting the equilibrium \((z_1, z_2) = (1/4, 3/4)\) into equation (11), one can obtain

\[
d(2) = \frac{t}{8} + c - \frac{c^2}{t} \tag{12}
\]

for \( c < \frac{t}{4} \) (for \( c > \frac{L}{4} \) consumers choose to buy at random, and then just having one product is optimal).

\[9\] Otherwise all consumers prefer to search at random (and we are back in the one alternative case).
The following proposition summarizes the above-derived consumer behavior when two products are offered.

PROPOSITION 3: Suppose that the firm offers two alternatives and that \( c/t < 1/4 \). Then consumers with \( x \in (1/2 - c/t, 1/2 + c/t) \) choose an alternative at random. The rest of consumers search once and choose the alternative that fits them better. Furthermore, the expected consumer dis-utility is defined by equation (12).

4.3. Three Alternatives

Consider now the case where the firm offers three alternatives. This case allows us to consider a situation where all consumers that buy a product engage in the search process (in contrast with the two alternative case above where some consumers that bought a product chose at random).

As argued above, the locations for the three products will be \( z_1 = \frac{1}{6}, z_2 = \frac{1}{2}, \) and \( z_3 = \frac{5}{6} \). As in the two-alternative case, it can be shown that these locations are also the ones that minimize the expected dis-utility of the consumers.

Consider the locations of consumers \( x \leq \frac{1}{2} \) (the case \( x > \frac{1}{2} \) is the symmetric case) and \( \xi < \frac{1}{6} \) (for \( \frac{\xi}{t} > \frac{1}{6} \), two alternatives is better than three alternatives as discussed below). Let us look first at the case where consumers buy at random. If \( x > \frac{1}{6} \) the expected dis-utility as a function of \( x \) is

\[
t[\frac{1}{3}(\frac{1}{6} - x) + \frac{1}{3}(\frac{1}{2} - x) + \frac{1}{3}(x - \frac{1}{6})] = t(\frac{7}{18} - \frac{2}{9}).
\]

If \( x < \frac{1}{6} \), and in the same way, we can find that the expected dis-utility as a function of \( x \) is \( t(\frac{1}{2} - x) \).

Consider now the case in which consumers search first one of the alternatives (restricting for now attention to \( x > \frac{1}{6} \)). Suppose that the consumer finds alternative \( z_1 = \frac{1}{6} \) first. Then, the consumer (i) can choose to buy this alternative, in which case the consumer gets a dis-utility of \( c + t(x - \frac{1}{6}) \), (ii) the consumer can choose to search once more, in which case the consumer can get his most other preferred alternative, \( z_2 = \frac{1}{2} \), for a dis-utility of \( 2c + t(\frac{1}{2} - x) \), or (iii) the consumer can choose to buy at random from among the other two alternatives, in which case the consumer gets an expected dis-utility of \( c + \frac{1}{2}(\frac{1}{2} - x) + \frac{1}{2}(\frac{5}{6} - x) = c + t(\frac{2}{3} - x) \). It can be easily seen that buying at random between the other two alternatives is worse that searching once more as long as \( \frac{\xi}{t} < \frac{1}{6} \), which was assumed. Finally, buying the alternative just searched, \( z_1 \), is better than searching once more if and only if \( x < \frac{1}{3} + \frac{c}{t} \). Note that this means that there are some consumers that even though they prefer \( z_2 \) to \( z_1 \) (if \( x > \frac{1}{3} \)) still keep alternative \( z_1 \) if they find it first, because of the additional search costs of trying to find \( z_2 \). Note that this also implies that if \( x < \frac{1}{6} \), if the consumer finds \( z_1 \) first, he will then naturally buy this alternative.
Suppose now that the consumer first finds alternative $z_2 = \frac{1}{2}$. If the consumer buys this alternative, he gets a dis-utility of $c + t\left(\frac{1}{2} - x\right)$. If the consumer searches once more, he finds his most preferred other alternative for a dis-utility of $2c + t(x - \frac{1}{6})$. It can be seen that the consumer buying at random between the other two remaining alternatives is dominated by either buying $z_2$, or searching once more. One can then see that buying the alternative just searched is better than searching once more if and only if $x > \frac{1}{3} - \frac{c}{27}$. Again, some consumers that prefer $z_1$ to $z_2$ will be happy to keep $z_2$ if they find it first.

Finally, suppose that the consumer first finds alternative $z_3 = \frac{5}{6}$. If the consumer buys this alternative, he gets a dis-utility of $c + t\left(\frac{5}{6} - x\right)$. If the consumer buys at random one of the other two alternatives, he gets an expected dis-utility of $c + \frac{1}{3}\left(\frac{1}{2} - x\right) + \frac{1}{2}(x - \frac{1}{6})$. If the consumer searches once more, he gets his most preferred alternative, with a dis-utility of $2c + t\left(\frac{1}{3} - x\right)$ if $x > \frac{1}{3}$ and a dis-utility of $2c + t(x - \frac{1}{6})$ if $x < \frac{1}{3}$. One can then obtain that buying alternative $z_3$ is always dominated by either buying at random or searching once more, and that buying at random of the two other remaining alternatives is better than searching once more if $x \in \left[\frac{1}{3} - \frac{5}{18}, \frac{1}{3} + \frac{5}{18}\right]$.

This discussion yields the optimal consumer search process in the case of three alternatives which is characterized in the Proposition 4.

**Proposition 4:** Suppose that the firm offers three alternatives, and that consumers search the first alternative. Then the consumers’ optimal search process is characterized by the following: If $x < \frac{1}{3} - \frac{5}{18}$ then the consumer keeps $z_1$ if he finds it first, and keeps on searching if he does not find $z_1$ first, for an expected dis-utility of $\frac{5}{3}c + t|x - \frac{1}{6}|$. If $x \in \left[\frac{1}{3} - \frac{5}{18}, \frac{1}{3} - \frac{5}{27}\right]$ the consumer buys $z_2$ if he finds it first, keeps on searching if he finds $z_2$ first, and buys at random if he finds $z_3$ first, for an expected dis-utility of $\frac{1}{3}c + t\left(\frac{7}{18} - \frac{1}{5}\right)$. If $x \in \left[\frac{1}{3} - \frac{5}{27}, \frac{1}{3} + \frac{5}{27}\right]$, the consumers buy $z_1$ or $z_2$ if he finds one of these alternatives in the first search, and, if he finds $z_3$ first, he buys at random one of the other two remaining alternatives, for an expected dis-utility of $c + \frac{1}{6}$. If $x \in \left[\frac{1}{3} + \frac{5}{27}, \frac{1}{3} + \frac{5}{18}\right]$, the consumer buys $z_2$ if he finds it first, keeps on searching (to find $z_2$) if he finds $z_2$ first, and buys at random one of the other two remaining alternatives if he finds $z_3$ first, for an expected dis-utility of $\frac{4}{3}c + t\left(\frac{7}{18} - \frac{2c}{3}\right)$. Finally, for $x \in \left[\frac{1}{3} + \frac{5}{18}, \frac{1}{2}\right]$, the consumer buys $z_2$ if he finds it first, and keeps on searching (to find $z_2$) if he finds $z_2$ or $z_3$ first, for an expected dis-utility of $\frac{5}{3}c + t\left(\frac{1}{2} - x\right)$.

This proposition completely characterizes the search process in a setting where all consumers buying a product engage in the evaluation of at least one product. The proposition illustrates again that consumers that are located in a certain range in between two alternatives are willing to take either alternative. The proposition also shows that this range is different whether the consumer has
searched one of these two alternatives, or if the consumer has searched all alternatives except these two alternatives. In the former, the consumer would benefit from searching by twice the distance to the mid-point. In the latter, the consumer buys at random between these two alternatives, and would benefit from searching by the distance to the expected product location, the mid-point between the two alternatives.

Figure 3 shows the expected dis-utility for each $x$ when the firm offers three alternatives and the consumers search the first alternative. As expected, the consumers located close to the alternatives’ locations do better.

However, consumers could also decide not to search the first alternative, and just to buy at random among the three alternatives. As stated in the following proposition (the proof is in the Appendix), if $\frac{c}{t}$ is sufficiently low, searching the first alternative is optimal for all consumers.

**Proposition 5:** Suppose that the firm offers three alternatives. If $\frac{c}{t} < \frac{2}{21}$, all consumers prefer to search a first alternative than to buy at random.

Note that, if $\frac{c}{t} \in \left(\frac{1}{6}, \frac{2}{21}\right)$, when faced with three alternatives, some consumers would buy at random while other consumers would engage in the evaluation of a first alternative. Figure 3 shows the expected dis-utility when consumers buy at random and when they engage in the evaluation of the first alternative for $c = \frac{1}{16}$ and $t = 1$, in which case all consumers engage in the evaluation of at least one alternative. Figure 4 shows the same curves when $c = \frac{1}{5}$ and $t = 1$, in which case some consumers engage in the evaluation of at least one alternative, while other consumers buy at random.

For the case in which all consumers that buy a product engage in the evaluation of the first alternative, $\frac{c}{t} < \frac{2}{21}$, one can compute (see Appendix) the expected dis-utility across all consumers as

$$d(3) = \frac{t}{12} + \frac{5}{3} c - \frac{c^2}{t}.$$  \hspace{1cm} (13)

### 4.4. Comparison Across Number of Alternatives

Comparing the expected dis-utility of one product with the case of an infinite number of products one can see that

$$d(1) < d(\infty) \text{ if and only if } \frac{c}{t} > \frac{1}{2(\sqrt{\pi} + \sqrt{2})^2}.$$  \hspace{1cm} (14)

where $\frac{1}{2(\sqrt{\pi} + \sqrt{2})^2}$ is close to .05 < $\frac{1}{16}$. If consumer search costs are sufficiently large, the firm is better off offering only one alternative and saving on the consumer search costs than offering an infinite
number of alternatives. This also illustrates the result in Proposition 2 that a finite number of alternatives (not necessarily just one alternative) is optimal with positive search costs.

Note again that for \( c_t > \frac{1}{8} \) consumers facing an infinite number of alternatives choose at random and get an expected dis-utility of the product bought, \( \frac{4}{3} \), that is greater than when the firm offers only one alternative. Because \( d(\infty) \) is weakly increasing in \( c \), it can also be immediately seen that, for \( \frac{1}{16} < \xi_t < \frac{1}{8} \), we have \( d(1) < d(\infty) \).

Comparing the case of two alternatives with the case of one alternative we have

\[
d(2) < d(1) \text{ if and only if } \frac{c_t}{t} < \frac{2 - \sqrt{2}}{4}, \tag{15}\]

that is, offering two products is better for the firm than offering one product if the search costs are low enough.

Comparing two products with the case of an infinite number of products we can see that

\[
d(2) < d(\infty) \text{ if and only if } \frac{c_t}{t} > \tilde{c}, \tag{16}\]

where \( \tilde{c} \) satisfies \( \tilde{c}^2 - (2 - \frac{\pi}{2})\tilde{c} + \sqrt{\tilde{c}} - \frac{1}{8} = 0 \). It can be easily seen that \( \tilde{c} \) is uniquely defined, and is close to \( .017 < \frac{1}{2(\sqrt{\pi}+\sqrt{2})^2} \).

Comparing three alternatives with an infinite number of alternatives, one can get that \( d(3) < d(\infty) \) if and only if \( \xi_t > c^{**} \) where \( c^{**} \) defined by \( c^{**2} - (\frac{8}{3} - \frac{\pi}{2})c^{**} + \sqrt{c^{**}} - \frac{1}{12} = 0 \) is below \( \frac{1}{16} \) and close to \( .009 \).

Comparing the optimal strategy for the firm among an infinite number of products, three products, two products, and just one product, we have then the following result (the proof is in the Appendix).

**Proposition 6:** Suppose that the options for the firm are to offer one, two, three, or an infinite number of products. Then, if \( \xi_t < c^{**} \), the firm prefers to offer an infinite number of products. If \( \xi_t \in [c^{**}, \frac{1}{16}] \), the firm prefers to offer three products. If \( \xi_t \in [\frac{1}{16}, 2-\sqrt{2}] \), the firm prefers to offer two products. Finally, if \( \xi_t > 2-\sqrt{2} \), the firm prefers to offer a single product.

This comparison illustrates that offering a finite number of products is optimal if the search costs are not too low, and suggests that lower search costs should lead the firm to offer more alternatives.

The above proposition also shows that if a firm offers too many alternatives some consumers may not make any choice and stay out of the market, because they understand that it will be too
costly (in terms of search costs) for them to find the alternative that best fits their preferences (that is, there would be a higher threshold \( \hat{v} \)). Similarly, if the firm offers too few alternatives, some consumers may also stay out of the market because they feel that the alternatives that are available may not fit well their preferences. In sum, given the existence of search costs there is an optimal (finite) number of alternatives to offer.

4.5. “Approximate” Optimal Number of Products

The analysis above suggest an approach to try to get at an approximate optimal number of products. Suppose that the firm chooses a number of products such that a set of consumers close to a product always keep on searching until they find the product closest to them. In addition, consider the approximation where all consumers are in this situation. That is, no consumer settles with the product that is not their most preferred product, or buys at random. As seen above, this may not be the optimal search process for some consumers.

Given this approximation, the expected dis-utility across all consumers of the firm offering \( n \) products is

\[
\tilde{d}(n) = \frac{t}{4n} + (n - 1)\left(\frac{1}{2} + \frac{1}{n}\right)c.
\]

The optimal number of products can then be obtained to be (without worrying about integer issues)

\[
n = \sqrt{\frac{t}{2c}} - 1.
\]

Figure 5 illustrates the optimal number of products as a function of \( \frac{c}{t} \) which illustrates that the optimal number of products reduces quickly when \( \frac{c}{t} \) increases. Figure 6 illustrates the expected dis-utility as a function of the number of products, and shows that the expected dis-utility increases beyond the optimal number of products, the information overload effect because of search costs, and that the expected dis-utility decreases for a lower number of products, because of less fit between the products offered and the consumer preferences.

This approximation does not account for the fact that consumers in a range in between two alternatives may be willing to accept either of the two alternatives. In the Appendix we derive an approximation for the optimal number of products (which is close to the one above) that accounts for this possibility.\(^10\)

\(^{10}\)In particular, the Appendix shows that when the search costs go to zero, the fraction of consumers that settle on the second-best alternatives is bounded away from zero, and derives an approximation of this fraction of consumers and the optimal number of the products under the condition that search costs are small.
5. Private Information of the Supplier of Alternatives

In some market situations, consumers learn something about the importance of alternative fit through the number of alternatives offered by the firm if they think that the firm might have more information about the importance of the fit. In the context of the model above, this would mean, for example, that the firm has private information on \( t \). Suppose that consumers have a prior on \( t \) that is distributed with cumulative distribution function \( G(t) \) with the support \([l, \bar{t}]\) and no mass points, and suppose that the firm is considering whether to offer one or two alternatives. In this section, to simplify the notation, we consider the case in which the consumer valuation \( v \) is uniformly distributed.

Consider first the case of no private information (complete information), explicitly considering that the set of consumers with zero search costs, \( \varepsilon \), is strictly greater than zero but less than \( 1/3 \). If only one alternative is offered, no consumer incurs search/evaluation costs and the expected dis-utility across all consumers is \( t \frac{4}{4} \). If two alternatives are offered, the expected dis-utility of the consumers with zero search costs is \( t \frac{8}{8} \), while the expected dis-utility for the consumers with positive search costs, for \( t > 4c \), is, as shown in the previous section, \( \frac{4}{8} + c - \frac{\varepsilon^2}{T} \), for an expected dis-utility across all consumers of \( \frac{4}{8} + (1 - \varepsilon)c - (1 - \varepsilon)^2 \). The firm is then indifferent between offering one and two alternatives if \( t = t^* \equiv 4(1 - \varepsilon)c + c\sqrt{8(1 - \varepsilon)(1 - 2\varepsilon)} \). If \( t < t^* \) the firm prefers only one alternative, as the cost of product mis-fit is not too large, and if \( t > t^* \), the firm prefers to offer two alternatives. As argued in the previous section, this is only valid as long as \( t^* > 4c \), which is true when \( \varepsilon < 1/3 \).

Consider now the case with private information. Suppose that there is a \( \hat{t} \) such that the firm offers one alternative if and only if \( t < \hat{t} \) (below, we confirm, that, in fact, the firm would follow such a rule). Then, if the firm offers one alternative, a consumer with positive search costs has the expected dis-utility \( \frac{E(t|t \leq \hat{t})}{4} \) prior to investing \( K \), where \( E(t|t \leq \hat{t}) \) is the expected value of \( t \) given the consumer prior and that \( t \leq \hat{t} \). If the firm offers two alternatives, the expected dis-utility of a consumer with positive search costs is \( \frac{E(t|t \geq \hat{t})}{8} + c - c^2 \frac{E(1|t \geq \hat{t})}{1} \). Given that the consumers with zero search costs can learn \( t \) prior to deciding whether to buy, the expected dis-utility (which determines the firm’s profits) across all consumers as a function of \( t \) is

\[
\varepsilon \frac{t}{4} + (1 - \varepsilon) \frac{E(t|t \leq \hat{t})}{4}
\]

\[ (17) \]

Similar results can be obtained for a general distribution.
when the firm offers only one alternative, and is
\[
\frac{\varepsilon t}{8} + \frac{1 - \varepsilon}{8} \left( \frac{E(t \mid t \geq \hat{t})}{8} \right) + (1 - \varepsilon)c - (1 - \varepsilon)c^2 E\left( \frac{1}{t} \mid t \geq \hat{t} \right)
\]  
when the firm offers two alternatives. Comparing (17) with (18) one can obtain that the firm prefers to offer two alternatives if and only if
\[
\frac{\varepsilon t}{8} + \frac{1 - \varepsilon}{8} \left[ 2E(t \mid t \leq \hat{t}) - E(t \mid t \geq \hat{t}) \right] - (1 - \varepsilon)c + (1 - \varepsilon)c^2 E\left( \frac{1}{t} \mid t \geq \hat{t} \right) \geq 0.
\]  
This condition is satisfied if and only if \( t \) is large enough, which justifies the assumption above that the firm only offers two alternatives if \( t \) is greater than some threshold \( \tilde{t} \). In fact, \( \tilde{t} \) is defined by \( t \) such that the expression in (19) is equal to zero. This characterizes the perfect Bayesian equilibrium, and we state this result in the following proposition.

**Proposition 7**: Suppose that the supplier of alternatives has private information on the preference fit parameter \( t \), and can offer one or two alternatives, suppose that \( t \) is low enough, and that \( \tilde{t} \) is high enough. Then, there is a partially-separating equilibrium where the firm offers two alternatives if and only if \( t > \tilde{t} \) and offers one alternative otherwise, where \( \tilde{t} \) is defined by making equation (19) equal to zero.

This result illustrates the idea that when a consumer sees more alternatives offered, he may infer that alternative-fit is more important. Notice that, even though the firm prefers the consumer to believe that alternative-fit is not too important, the firm may still end up offering more alternatives, and revealing the importance of alternative-fit, because of the consumers that can learn about the importance of alternative-fit in some other way.

This intuition raises the question of whether the firm may be tempted to offer fewer alternatives under private information than under the case in which there is no private information. Indeed, note that \( E(t \mid t \leq \tilde{t}) < \tilde{t}, E(t \mid t \geq \tilde{t}) > \tilde{t}, \) and \( E\left( \frac{1}{t} \mid t \geq \tilde{t} \right) < \frac{1}{\tilde{t}} \) for all \( \tilde{t} \in (\underline{t}, \overline{t}) \). Therefore, condition (19) with \( t = t^* \) is not satisfied. Thus, the expression in (19) evaluated with \( t = \tilde{t} \) is strictly below \( \frac{\varepsilon t}{8} - (1 - \varepsilon)c + (1 - \varepsilon)c^2 \frac{1}{\tilde{t}} \) for all \( t \). Then, because this latter expression is strictly increasing in \( t \) and is equal to zero when \( t = t^* \), we have that \( \tilde{t} > t^* \). That is, we have the following result.

**Corollary 1**: The supplier of alternatives offers a lower number of alternatives when she has private information than when she does not have private information.

This corollary states that private information is a force towards offering a lower number of alternatives. That is, when \( t \in (t^*, \tilde{t}) \) the firm offers two alternatives when the consumers know
as much as the firm about the preference-fit parameter \( t \), and only offers one alternative when the consumers know less than the firm about \( t \).

Note that if either \( \tilde{t} \) or \( \varepsilon \) is too small, condition (19) is never satisfied, and the firm offers only one alternative under private information, for all realizations of \( t \). Note also that if \( t \) is too high (above \( t^* \)), then the firm offers two alternatives for all the realizations of \( t \).

For the case when the distribution \( G(t) \) is uniform, one can obtain that \( \hat{t} \) is defined by

\[
\frac{16}{15}(1 + \varepsilon) - (1 - \varepsilon)c + \frac{16}{8}(\frac{\varepsilon}{t} - \frac{1}{2}) + \frac{4}{t^2} - \frac{16}{c} \log t = 0.
\]

Figures 7 and 8 plot \( \hat{t} \) for the uniform distribution case as a function of \( \varepsilon \) and \( \tilde{t} \), respectively.

6. Extensions

6.1. Knowledge of Preferences Prior to Deciding to Enter the Market

The analysis above is under the assumption that consumers need to invest a large amount \( K \) prior to searching, and only then they completely learn their preferences, learn \( x \) in addition to \( v \). This simplified the analysis because whether a consumer decided to buy a product only depended on \( v \), and not on both \( v \) and \( x \). Consider now the case in which \( K = 0 \), consumers know both \( v \) and \( x \) prior to deciding whether to search, or purchase a product at random. Denote also as \( H(v) \) the cumulative distribution function of \( v \), with density \( h(v) > 0, \forall v \in [0, \tilde{v}] \), and let us restrict attention to either one or two products.

Suppose first the case in which the firm offers only one product. Suppose also that the consumers infer the product location to be \( z_1 \). Without loss of generality consider \( z_1 \leq \frac{1}{2} \). No consumer will search because there is nothing to learn from incurring the search costs. Consumers just decide to buy or not the product depending on their preference parameters, \( v \) and \( x \). Then, consumers buy if \( v \geq t |z_1 - x| \). The total demand is then

\[
\int_0^{z_1} [1 - H(t(z_1 - x))] dx + \int_{z_1}^1 [1 - H(t(x - z_1))] dx
\]

which is maximized if \( z_1 = \frac{1}{2} \), which is also the equilibrium location if there is a mass \( \varepsilon > 0 \) of consumers with zero search costs.

Consider now the case in which the firm offers two products, located at \( z_1 \) and \( z_2 \), with \( z_1 < z_2 \). From the analysis in Section 4.2, we know that if the consumer decides to buy the product, buying at random is better if and only if \( x \in (\frac{z_1 + z_2}{2} - \frac{\varepsilon}{t}, \frac{z_1 + z_2}{2} + \frac{\varepsilon}{t}) \) (under the assumption that \( \frac{\varepsilon}{t} \leq \frac{z_2 - z_1}{2} \), otherwise all buying consumers buy at random). Then, the consumers buying the product are
determined by $v \geq c + t|z_1 - x|$ for $x \leq \frac{21+ta}{2} - \frac{c}{t}$, $v \geq t\frac{21+ta}{2}$ for $x \in (\frac{21+ta}{2} - \frac{c}{t}, \frac{21+ta}{2} + \frac{c}{t})$, and $v \geq c + t|z_2 - x|$ for $x \geq \frac{21+ta}{2} + \frac{c}{t}$. Then, the total demand is

$$\int_0^{z_1} [1 - H(c + t(z_1 - x))] \, dx + \int_{z_1}^{\frac{21+ta}{2} - \frac{c}{t}} [1 - H(c + t(x - z_1))] \, dx + \int_{\frac{21+ta}{2} - \frac{c}{t}}^{\frac{21+ta}{2} + \frac{c}{t}} [1 - H(t\frac{z_1 + z_2}{2})] \, dx + \int_{\frac{21+ta}{2} + \frac{c}{t}}^{z_2} [1 - H(c + t(z_2 - x))] \, dx + \int_{z_2}^{1} [1 - H(c + t(x - z_2))] \, dx. \quad (21)$$

This demand is maximized at $z_1 = 1 - z_2$ satisfying $H(t(\frac{1}{2} - z_1)) - H(c + tz_1) + ch(t(\frac{1}{2} - z_1)) = 0$. If the probability distribution on $v$ is uniform then this results in $z_1 = \frac{1}{4}$, as in Section 4.2, and which is also the equilibrium if there is a mass $\varepsilon > 0$ of consumers with zero search costs.

Note now that (21) is continuously decreasing in the search costs $c$, that at $\frac{c}{t} = \frac{1}{4}$ we have (21) smaller than (20), and that at $\frac{c}{t} = 0$ we have (21) greater than (20). Then, we have, as above, that if the search costs $c$ are low enough the firm chooses to offer two products, while if the search costs are high enough the firm prefers to offer only one product.

For the case in which $v$ is uniformly distributed, the comparison between offering one and two alternatives is relatively straightforward. The demand in the case of one alternative is $\frac{1}{3}(|c - 1\varepsilon|)$. The demand in the case of the firm offering two alternatives is $\frac{1}{3}(|c - \frac{1}{4}|) + \frac{2}{3}(\frac{1}{4} - \frac{c}{2})(|c - \varepsilon| - \frac{c}{2}) + \frac{2}{3}(|c - \frac{1}{4}|)$. Comparing the demand under two alternatives with the demand under one alternative one obtains that two alternatives is better than one alternative if and only if $\frac{c}{2} - c + \frac{c^2}{4} \geq 0$. Note that this is exactly the same condition as presented in Section 4.4 for $K$ large. This is because, with a uniform distribution for $v$, the density is constant for all $v$, and therefore demand is greater when the expected dis-utility, across all consumers, is smaller, which was exactly the decision criterion in the previous sections.

### 6.2. Consumer and Firm Uncertainty About the Market

In some markets, consumers, although knowing their own preferences, may not know where the preferences of the other consumers may be, while the firm may have information about the distribution of consumer preferences. In the context of the model, the firm could know where the unit segment of consumer ideal points is located, while each consumer knows only his own preference, the location of his own $x$. In such a case, a consumer does not know the location of the products with respect to his preferences. Consumers may then be willing to incur the search costs to evaluate a product in order to check if the product may be satisfactory. Some consumers could then engage in the search process, while ending up not purchasing any product.
Similarly, the firm may not know exactly where the consumers preferences lie, due to some non-exact market research. Then, consumers do not know where the products may be offered, which is a further incentive for consumers to incur search costs to evaluate products, and not be willing to purchase at random. Furthermore, some consumers may engage in the search process, only to find out that no product fits their preferences, and therefore, decide not to purchase any product.

7. Conclusion

This paper considers the decision of the number of alternatives to be offered by a firm or government agency. Offering more alternatives may potentially allow each consumer to get an alternative that best fits his preferences. However, if more alternatives are offered it is more costly for the consumers to find a relatively acceptable alternative matching their preferences. This results in there being an optimal number of alternatives to be offered. If the firm offers more alternatives, consumers incur too many search costs if they decide to enter the market. This leads some consumers to be “overwhelmed” with too many alternatives, and to decide not to choose any alternative. This can be seen as an explanation for the information overload (alternative overload, more specifically) effect that, with more information (more alternatives), consumers are less likely to make a choice.

Some of the results presented here may extend to a setting where more information about an alternative may lead a consumer to be less likely to choose that alternative. Another interesting issue that should be explored in future work is allowing for non-perfect evaluation of alternatives, and consumers being able to decide on the intensity of their evaluation of each alternative. It would also be interesting to investigate what happens when there is competition and price learning in a context where product attributes have to be evaluated for consumer fit.
APPENDIX

The case of an Infinite Number of Products, and \( \xi > \frac{1}{16} \): First, consider consumers with 
\[ x \in (\sqrt{c/t}, 1 - \sqrt{c/t}) \], i.e., those that are not too close to the ends of the segment. If \( \xi \in (\frac{1}{16}, \frac{3-2\sqrt{2}}{2}) \) consumers with 
\[ x \in \left( \frac{1 - \sqrt{4\sqrt{c/t} - 1}}{2}, \frac{1 + \sqrt{4\sqrt{c/t} - 1}}{2} \right) \]
buy at random, while the other consumers with 
\[ x \in (\sqrt{c/t}, 1 - \sqrt{c/t}) \] engage in the search process as described in the text. If \( \xi > \frac{3-2\sqrt{2}}{2} \), all consumers with 
\[ x \in (\sqrt{c/t}, 1 - \sqrt{c/t}) \] choose to buy at random. As noted in the text this implies an interesting search strategy for the consumers depending on their preferences: Consumers that have relatively specific preferences (in the model, with a location close to zero or one) search, and consumers that have more generic preferences (around the center of the market) do not evaluate a product prior to purchase and buy at random.

Now, consider consumers with \( x < \sqrt{c/t} \) (consumers with \( x > \sqrt{c/t} \) behave similarly to these). For \( \frac{1}{16} < \xi < \frac{1}{8} \), the expected dis-utility is decreasing in \( x \) for both when a consumer engages in the search process and when a consumer buys at random. Note first that, as argued above, when a consumer engages in the search process, the expected search costs plus dis-utility of the product bought is concave in \( x \).

On the other hand, the expected dis-utility of the product bought when a consumer buys at random is convex in \( x \), that is the further away a consumer is from the center of the market the consumer is in an increasingly worse situation.

Comparing the engaging in the search process strategy with buying at random, the condition on \( x \) that engaging in the search process is better than a random choice is 
\[ x^2 + \frac{1}{2} \geq \sqrt{2\frac{c}{t} - x^2} \]
which reduces to 
\[ f(x; \frac{c}{t}) = x^4 - 2x^3 + 3x^2 - x + \frac{1}{4} - 2\frac{c}{t} \geq 0. \]
This polynomial function is convex and decreasing in \( \xi \). Therefore, there is a \( c^* \) such that if \( \xi < c^* \) then \( f(x; \xi) = 0 \) has no solutions, if \( \xi > c^* \) then \( f(x; \xi) = 0 \) has two distinct solutions, and if \( \xi = c^* \) then \( f(x; \xi) = 0 \) has exactly one solution (\( c^* \) is close to 0.078). We also know that \( f(x; \frac{3-2\sqrt{2}}{2}) = 0 \) at \( x = \frac{2-\sqrt{2}}{2} \) and that \( \frac{3-2\sqrt{2}}{2} \) is the only \( \xi \) \( \in (\frac{1}{16}, \frac{1}{8}) \) in which \( x = \sqrt{\frac{c}{t}} \) satisfies \( f(x; \xi) = 0 \) and \( f(x; \xi) < 0 \) for \( \xi \in (\frac{3-2\sqrt{2}}{2}, \frac{1}{8}) \). Furthermore, \( f'(\frac{3-2\sqrt{2}}{2}, \frac{3-2\sqrt{2}}{2}) > 0 \) and \( f(0; \frac{3-2\sqrt{2}}{2}) > 0 \). Therefore, \( f(x; \frac{3-2\sqrt{2}}{2}) = 0 \) has another solution strictly greater than zero, and strictly smaller than \( \frac{2-\sqrt{2}}{2} \). This
implies that \( c^* < \frac{3 - 2\sqrt{2}}{2} \). Checking that \( f(x; \frac{1}{16}) - x^4 \) is always positive for \( x \in (0, \frac{1}{2}) \) we have that \( c^* > \frac{1}{16} \). We then can conclude the following. For \( \xi \in (\frac{1}{16}, c^*) \) all consumers with \( x \in (0, \sqrt{\xi}) \) engage in the search process. For \( \xi \in (c^*, \frac{3 - 2\sqrt{2}}{2}) \) we have that \( f(x; \frac{1}{16}) = 0 \) has two solutions, \( y_1(\xi) \) and \( y_2(\xi) \), with \( 0 < y_1(\xi) < y_2(\xi) < \sqrt{\xi} \), consumers with \( x \in [0, y_1(\xi)) \cup (y_2(\xi), \sqrt{\xi}) \) engage in the search process, and consumers with \( x \in (y_1(\xi), y_2(\xi)) \) choose at random. Finally, for \( \xi \in (\frac{3 - 2\sqrt{2}}{2}, \frac{1}{8}) \) there is only solution to \( f(x; \xi) = 0 \) that is below \( \sqrt{\xi} \), and consumers with \( x \in [0, y_1(\xi)) \) engage in the search process, and consumers with \( x \in (y_1(\xi), \sqrt{\xi}) \) choose at random.

Putting all these results together we have that there are \( \xi \in (c^*, \frac{3 - 2\sqrt{2}}{2}) \) such that the set of consumers that choose at random is not convex. That is, starting from the center of the market and going to either extreme, we have first consumers that choose at random, \( x \in (\frac{1}{4} - \sqrt{\xi} / (1 + \xi), \frac{1}{16} + \sqrt{\xi} / (1 + \xi)) \), then consumers that engage in the search process, \( x \in (\frac{1}{4} - \sqrt{\xi} / (1 + \xi), \frac{1}{16} + \sqrt{\xi} / (1 + \xi)) \), then consumers that choose at random,

\[
x \in (y_1(\xi), y_2(\xi)) \cup (1 - y_2(\xi), 1 - y_1(\xi)),
\]

and, finally, again consumers that engage in the search process, \( x \in [0, y_1(\xi)) \cup (1 - y_1(\xi), 1] \).

**Solution of the Integral \( \int_0^{\sqrt{\xi}} \sqrt{\frac{2c}{\xi} - x^2} \, dx \):**

Define the variable \( \tau \) as \( \sqrt{\frac{2c}{\xi} - x^2} = \tau + \sqrt{\frac{2c}{\xi}} \) which yields \( x = -\frac{2\sqrt{\xi}}{1 + \tau^2} \). Note also that

\[
\frac{dx}{d\tau} = -2\sqrt{\frac{2\xi}{1 + \tau^2}}.\]

Substituting variables one can then write

\[
\int_0^{\sqrt{\xi}} \sqrt{\frac{2c}{\xi} - x^2} \, dx = \int_{1 - \sqrt{\xi}}^{0} \frac{4c}{\xi} \frac{(1 - \tau^2)^2}{(1 + \tau^2)^3} \, d\tau. \quad (i)
\]

Noting now that \( \frac{(1 - \tau^2)^2}{(1 + \tau^2)^3} = \frac{4}{(1 + \tau)^2} - \frac{4}{(1 + \tau)^2} + \frac{1}{1 + \tau} \), and that \( \int \frac{1}{1 + \tau} \, d\tau = \arctan \tau \), \( \int \frac{1}{(1 + \tau)^2} \, d\tau = \frac{\tau + 3\tau^3}{8(1 + \tau)^2} + \frac{3}{8} \arctan \tau \), we have

\[
\int_0^{\sqrt{\xi}} \sqrt{\frac{2c}{\xi} - x^2} \, dx = 4\frac{c}{\xi} \frac{\tau(1 - \tau^2)}{2(1 + \tau^2)^2} + \frac{1}{2} \arctan \tau|_{1 - \sqrt{\xi}} = \frac{c}{4}\xi (2 + \pi). \quad (ii)
\]

**Proof of Proposition 5:** In order to prove the proposition we need to check the conditions under which the expected dis-utility of buying at random, \( t(1 - x) \) for \( x < \frac{1}{6} \), and \( t(\frac{7}{18} - \frac{3}{8} - x) \) for \( x > \frac{1}{6} \), is greater than the expected dis-utility of search a first alternative, as stated in Proposition 4. For \( x < \frac{1}{6} \) the condition is \( \xi > \frac{1}{6} \). For \( x \in [\frac{1}{6}, \frac{1}{3} - \frac{\xi}{2}] \) the condition is \( \xi < \frac{1}{3} \). For \( x \in [\frac{1}{3} - \frac{\xi}{2}, \frac{1}{3} - \frac{\xi}{2}] \)
and one obtains $\xi < \frac{2}{21}$. For $x \in [\frac{1}{3} - \frac{\xi}{21}, \frac{1}{3} + \frac{\xi}{21}]$ the condition results in $\xi < \frac{2}{21}$. Finally, for $x \in [\frac{1}{3} + \frac{\xi}{21}, \frac{1}{2}]$ the condition results in $\xi < \frac{3}{56}$. All these conditions are satisfied if $\xi < \frac{2}{21}$. QED

**Computation of Equation (13):** The expected dis-utility $d(3)$ across all consumers can be obtained to be $\frac{1}{5}(\frac{2c}{5} + t) + (\frac{1}{2} - 2\xi)(\frac{2c}{5} + t(\frac{1}{12} - \frac{\xi}{21})) + 2\xi(\frac{c}{9} + t(\frac{1}{6} - \frac{\xi}{21})) + (\frac{1}{3} - 2\xi)(\frac{2c}{5} + t(\frac{1}{12} - \frac{\xi}{21}))$ which reduces to equation (13).

**Proof of Proposition 6:** Comparing $d(3)$ with $d(2)$ for $\xi < \frac{2}{21}$ one obtains directly $d(3) < d(2)$ if and only if $\xi < \frac{1}{16}$. In order to complete the proof one has to consider what happens when $\xi \in [\frac{2}{21}, \frac{1}{6}]$. For $\xi \in [\frac{2}{21}, \frac{1}{6}]$ one obtains $d(3) = \frac{t}{12} + \frac{19}{9}c - \frac{14}{3}e^2$ and one obtains $d(3) < d(2)$ if and only if $\xi < \frac{20 - \sqrt{202}}{132} < \frac{1}{16}$, a contradiction. Finally, for $\xi \in [\frac{2}{21}, \frac{2}{15}]$ one obtains $d(3) = \frac{t}{12} + \frac{16}{9}c - \frac{49}{12}e^2$ and one obtains $d(3) < d(2)$ if and only if $\xi < \frac{20 - \sqrt{202}}{74} < \frac{1}{16}$, a contradiction. QED

"Approximate" Optimal Number of Products with the Possibility of Consumers Settling on the Second-Best Alternative: As we already know, the optimal distribution of $n$ products is such that the distance between neighboring products is $\frac{1}{3}$. Here, we estimate the approximate expected dis-utility of a consumer engaging in the optimal search, when some consumers always search until finding their first-best alternative, and consumers can settle in their second-best alternative. We then minimize this approximate expected dis-utility over $n$. This approximation is exact as $c$ goes to zero (and $n$ goes to infinity).\(^{12}\)

Since all but $\frac{1}{n}$ consumers are located between two products, we only consider the expected dis-utility of consumers conditional on them being located between some two products. We have assumed that $n$ is such that some consumers close to a product search until they find that particular product. However, consumers who are located almost at the mid-point between two adjacent products will search only until they find one of these two products. In other words, all consumers search until they find the first or second-best product. If a consumer finds the first-best before the second-best, he chooses the first-best. However, if he finds the second-best before finding the first-best, he chooses the second-best if he is close to the mid-point between the first-best and the second-best so that the expected cost of searching further until he finds the first-best is too high relative to the reduction in dis-utility.

The expected total cost of searching until the first-best is found is approximately $\frac{n}{2}c$ (the exact value is in Equation (8)). The expected cost of searching for the first-best after the second-best is

\(^{12}\)We can compute the exact expected dis-utility and exact optimal number of products under the assumption that some consumers close to one product always keep on searching until finding it. This computation is, however, more complicated than the approximation presented here for $n$ large, without yielding major new insights.
found depends on how many products have already been tried. Let \( k \) be the number of searches until the second-best is found and assume that the first-best was not yet found by the \( k \)th search, then the expected additional cost of searching until the first-best is found is \( \frac{n-k}{2}c \).

The optimal consumer strategy is the following. If the consumer is further than \( \frac{nc}{4t} \) from the mid-point between the first-best and second-best product, the consumer will search until the first-best is found, since even if he finds the second-best first, the cost of searching for the first-best (at most \( \frac{n}{2}c \)) is lower than the additional utility from the first-best, which is twice his distance from the mid-point times \( t \).

Consider now a consumer located within \( \frac{nc}{4t} \) of the mid-point between the first-best and second-best alternative. We have that this consumer searches at least until the first or second-best is found. The probability that he will find the first or second-best when at least \( q \) but less than \( q + dq \) fraction of the products (where \( q = \frac{k}{n} \) for some integer \( k \) and \( dq = \frac{1}{n} \)) are searched is asymptotically \( 2(1 - q)dq \). This is because the probability of finding one of the two best on the \( k \)th search is

\[
(1 - \frac{2}{n})(1 - \frac{2}{n-1}) \cdots (1 - \frac{2}{n-k+2}) \frac{2}{n-k+1} = \frac{2}{n-1}(1 - \frac{k}{n}) \approx 2(1 - q)dq.
\]

Once he found the first or second-best, he has spent \( qnc \) on search, and with probability \( 1/2 \) found the best product. The expected dis-utility of the best product across all consumers we are now considering is \( \frac{n^2c}{2t} \int_0^1 (ncq + \frac{1}{2}(\frac{t}{2n} - \frac{nc}{8})) \). Integrating the total expected dis-utilities of consumers located within \( \frac{nc}{4t} \) from the mid-point between their first-best and second-best choice over all values of \( q \), we obtain that the sum of their expected dis-utilities is

\[
\frac{n^2c}{2t} \int_0^1 (ncq + \frac{1}{2}(\frac{t}{2n} - \frac{nc}{8}))
\]

27
\[
+ \frac{1}{2} q \left( \frac{(1-q)nc}{2} + \frac{t}{2n} - (1-q)\frac{nc}{4} \right) + (1-q)\left( \frac{t}{2n} + \frac{(1-q)nc}{8} \right))2(1-q)\, dq,
\]

where \( \frac{n^2c}{2t} \) is the total number of such consumers. Let \( a \equiv \frac{n^2c}{2t} \). Then, the above expression reduces to \( at^5a + 4 \), and it is (asymptotically) the sum of dis-utilities from the fraction \( a \) of all consumers who can possibly stop the search process at the second-best. The remaining \( 1 - a \) fraction of the consumers always search until the best alternative is found. They incur each \( \frac{n^2c}{8t} \) expected search cost and product dis-utility, on average, of \( \frac{t}{4n} - \frac{n^2c}{8} = (1-a) \frac{t}{4n} \).

Hence, the total consumer dis-utility is \( t^2 + 8a - a^2 \). This expression is minimized at

\[
n = \frac{\sqrt{24 - 6\sqrt{10}}}{3} \sqrt{\frac{t}{c}} \approx 0.7473 \sqrt{\frac{t}{c}}.
\]

When \( n \) is chosen as above, asymptotically,

\[
\frac{n^2c}{2t} \int_0^1 \frac{1}{2} (1-q) 2(1-q)\, dq = \frac{n^2c}{6t} = 4 - \frac{\sqrt{10}}{9} \approx 0.093
\]

fraction of all consumers end up choosing the second-best product; the rest of the consumers end up choosing the first-best.
REFERENCES


Figure 1: Expected search costs plus dis-utility of product bought for each location $x$ for the case of an infinite number of products under purchase at random, and purchase with search for $c/t=.08$, $t=1$. $y_3=(1-(4c^{1/2}-1)^{1/2})/2$. 
Figure 2: Expected search costs plus dis-utility of the product bought as a function of the search costs when there is an infinite number of products. $c^*$ is defined in the text, and $c^{**}=(3-8^{1/2})/2$. $t=1$. 
Figure 3: Expected dis-utility of search first and buy at random for each $x$, for three alternatives, $t=1$, and $c=1/16$. Note $z_1=1/6$, $z_2=1/2$, $z_3=5/6$. If $z_3$ searched first then buy at random if $x$ in $[1/3-c/t,1/3+c/t]$. 
Figure 4: Expected dis-utility of search first and buy at random for each \( x \), for three alternatives, \( t=1 \), and \( c=1/9 \). Note \( z_1=1/6 \), \( z_2=1/2 \), \( z_3=5/6 \). Buy at random is better than search first for some \( x \).
Figure 5: "Approximate" optimal number of product as a function of c/t.
Figure 6: Expected dis-utility for each number of products under search until finding the closest product for $t=1$, $c=0.05$. 
Figure 7: Private information of the supplier of alternatives when $t$ is is distributed uniformly on $[0,10]$, and $c=0.2$. Figure shows $t^*$ and $t^\wedge$ as a function of $\varepsilon$. 
Figure 8: Private information of the supplier of alternatives when $t$ is uniformly distributed on $[0, tb]$, $c=1$, $\varepsilon=.2$. The figure shows $t^*$, $t^\wedge$, and $t^\wedge/tb$ as a function of $tb$. 