Segmentation and Competition: An Application to Rebates

Kutsal Dogan      Ernan Haruvy      Ram C. Rao

University of Texas at Dallas

September 2005
Abstract

We investigate segmentation in a market with two asymmetric firms and two heterogeneous consumer segments that differ in the importance of price and product attributes. We explore whether it would be profitable for either or both firms to pursue a rebate-based segmentation strategy. We find that segmentation lessens competition for the less price-sensitive consumer segment and leads to higher profits. We identify a role for segmentation that is distinct from pure price discrimination by showing that in our model a monopolist would not gain by rebating, yet under competition rebating can be profitable. We also show that if firms are asymmetric in their attractiveness to consumers, the disadvantaged firm is more likely to pursue a segmentation strategy than its rival. We find that the model’s predictions are supported by price and rebate data for printers sold by a leading retailer.

Key Words: Segmentation; Competition; Game Theory; Pricing, Rebates; Printers
1. Introduction

Market segmentation is an important part of developing a marketing strategy. Managers recognize that by correctly segmenting a market, they can capture more consumer surplus when faced with consumer heterogeneity. For example, if consumers value quality differentially, a firm would want to offer a product line with vertically differentiated products rather than a single product. This would allow the firm to charge differential prices and earn higher profits (e.g., Moorthy, 1984). In general, price discrimination involves segmentation entailing different offers to different groups of consumers. For example, segmentation based on price sensitivity can be accomplished often by the use of coupons if price sensitive consumers also happen to have a low cost of time (Narasimhan, 1984). Thus, coupons serve as a mechanism to engage in price discrimination. The usefulness of coupons in this context has been explored by numerous researchers: Gerstner and Hess (1991), Shaffer and Zhang (1995), Moraga-Gonzalez and Petrakis (1999), and Krishna and Zhang (1999). The key driver of profitable segmentation for price discrimination is consumer heterogeneity.

In this paper we ask if segmentation can be used as a strategy to lessen competition. To this end, we explore a market with two firms that compete for two consumer segments of consumers. The two segments are heterogeneous in how they make a trade-off between their desired combination of attributes and price. We capture the competition between the firms in a Hotelling framework. We then ask if it would be profitable for the firms to segment the market. Specifically, we assume that segmentation can be implemented by the mechanism of rebates. Our choice of rebates is motivated by the fact that some consumers place a higher value on delayed rewards than others. We solve for equilibrium pricing and rebate strategies and then derive the profits with segmentation by one firm, both firms, or neither firm. In other words, either, both or neither firm can implement a segmentation mechanism based on rebates. We obtain an interesting insight into how segmentation affects competition. Segmentation softens competition for one segment of consumers that value attributes more than price, while simultaneously increasing the competition for the other segment. What is more interesting is that the net effect of

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1 Segmentation can be thought of as a problem in mechanism design.
segmentation is higher profits for both firms. In this way we provide new insights into the strategic consequences of segmentation. We use our model to analyze asymmetries among firms. Specifically, if firms are asymmetric in their attractiveness, the firm whose attractiveness is lower is more likely to pursue a segmentation strategy, through the use of rebates, than its rival. We then test the proposition that the firm with less attractiveness is more likely to offer a rebate. We do this using data on rebates in the printer market that we were able to obtain. Our analysis shows support for our proposition.

The challenge of lessening competition has been approached in other ways. For example, Kim, Shi and Srinivasan (2001), following the arguments of Klemperer (1987), show that frequency based loyalty programs, effectively segmenting the market into heavy and light users, can soften price competition by introducing endogenous switching costs. Turning to the case in which some consumers are brand loyal and others willing to switch, competitive pricing often leads to price promotions (Narasimhan, 1988; Raju, Srinivasan and Lal 1990; Rao, 1991). These temporary promotions, arising from mixed strategies, do not involve a constant low price targeted at switchers because that would leave money on the table for loyal consumers whose brand choice is unaffected by price promotions. In this way price promotions can be seen as a way to lessen price competition.

The rest of the paper is organized as follows. In section 2 we describe our consumer model and how firms compete both on segmentation strategy and pricing strategy. In section 3 we analyze our model and derive the equilibrium segmentation and pricing strategies. We present our central result contained in propositions 2 and 4. In section 4 we analyze the printer market and present our empirical results. We also discuss alternative explanations for our empirical findings in this section. We end with concluding comments in section 5.

2. The Model

**Firms:** In our model there are two firms, A and D. The two firms offer products that are differentiated. Differentiation is based on two factors. The first is a combination of attributes, represented by a one-dimensional space. We posit a spatial model of differentiation in a one dimensional space a la Hotelling, following Shaffer and Zhang (1995) among others. The two firms are assumed to be located at the ends of
a line segment $DA$ of unit length, as shown in Figure 1. The second basis of differentiation is the brand itself. This might capture “inimitable” characteristics that consumers might value. In particular, we assume that all consumers agree that one of the brands is superior on this second factor. The firm that is superior in this sense is called the advantaged firm ($A$) and the other firm is called the disadvantaged firm ($D$). Finally, firm $i$, $i = A, D$, is assumed to incur a constant marginal cost $C_i$ on its product.

$$D \quad r \quad A$$

**Figure 1:** A spatial depiction of the competitive environment

**CONSUMERS:** In the spirit of the Hotelling model, we assume that consumers are located on the line segment $DA$. The location of a consumer defines the ideal combination of attributes desired by her. The utility of a product to a consumer $t$ then consists of three components: the utility for his or her ideal product, $v$; the disutility of a non-ideal product by virtue of its location far away from the consumer’s ideal point at a distance $d_{ti}$, $i = A, D$; and a brand value $b_i$ for firm $i$’s product arising from the inimitable characteristics of firm $i$. Note that $v$ and $b_i$ are assumed to be the same for all consumers, and as a result consumer heterogeneity arises purely from considerations of location in the product space. We assume that the disutility due to a product’s non-ideal location is linear in the distance of the product from the consumer’s ideal point. Specifically, we model the disutility of a consumer $t$, located at a distance $t$ from $D$ and $(1-t)$ from $A$, as

$$d_{id} = a \cdot t$$ and $$d_{ia} = a \cdot (1-t).$$

If $a$ is high, consumers are less willing to sacrifice their desired attribute combination and we say that the product is more salient to consumers. On the other hand if $a$ is low, price is more salient to consumers. Define the valuation of each brand as $V_i = v + b_i$. Then the utility of brand $i$ to consumer $t$ is $(V_i - d_{ti})$.

Finally, we assume that consumers are distributed uniformly over $DA$.

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2 As an example, vitamin tablets shaped like the *Flintstones* would possess an inimitable characteristic. Reputation, in the short run, would also be inimitable.
**CONSUMER SEGMENTS:** In our model, we assume that there are two segments of consumers, each distributed uniformly over DA. We denote the location at which consumers of type $k$ are indifferent over the two firms by $\tau_k$, $k = 1, 2$. The two segments differ in whether the product is more salient to them or price is more salient. Specifically, we assume that $a_1 > a_2$. This implies that for consumers in segment 1 the product is more salient while for those in segment 2, price is more salient. One way to interpret this is to say that consumers in segment 2 are more price oriented than consumers in segment 1, as in Shaffer and Zhang (1995) and Lal and Rao (1997). This, in turn, would imply that consumers in segment 2 would also value rebates more than consumers in segment 1. To capture this, we assume that consumers in segment 1 discount future cash flows more while those in segment 2 discount them less. Thus, denoting $d_k$ to be the discount factor used by consumers in segment $k$, we assume $d_1 < d_2$. Given this inequality of discount factors, a rebate would represent a higher value to consumers in segment 2 who are more “patient” than to those in segment 1 who are less patient. We refer to consumers in segment 1 as high type consumers and to consumers in segment 2 as low type consumers. Foreseeing the possibility of firms offering rebates, we assume that there is a cost to redeem a rebate, denoted by $e$ that consumers incur. We assume that the high type segment (segment 1) is of size $\gamma$ and the low type segment (segment 2) is of size $(1-\gamma)$, where $0 < \gamma < 1$.

**DYNAMICS:** We assume that there is a time lag between purchase and rebate payment. This time lag is assumed to be specified exogenously, reflecting the technology of rebate processing. Thus, the time to redemption is not a strategic variable in our model. We denote time by $t$. Finally, we assume that both firms discount future cash flows by a discount factor $d_F$. To keep the exposition simple, we assume that $d_F = d_2$. This is not a crucial assumption and has no effect on the generality of our results.

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3 A higher discount factor is associated with a lower discount rate.

4 We assume that the cost of redemption is constant across consumers. One could alternatively let this cost vary in the consumer population. We chose to let the consumers’ patience vary and keep this cost constant to be able to isolate the effect of time on the rebate use decision.

5 One could allow all three discount rates to differ. This would make the problem less tractable with little or no additional insight. Alternatively, one could have $d_F = d_1$ and leave type 2 as the “unusually patient” type. This would result in one or both firms targeting that type with rebates that mature at $t \rightarrow \infty$. 
GAME: The game progresses in two stages. In stage 1, each firm decides whether to offer a rebate or not and announces that decision. In stage 2, firms decide on their prices and rebate amounts that maximize their respective profits. The profit function then is:

\[ \Pi_D = \gamma P_D \tau_1 + (1 - \gamma)(P_D - \delta_D R_D)\tau_2 \]  

and

\[ \Pi_A = \gamma P_A (1 - \tau_1) + (1 - \gamma)(P_A - \delta_A R_A)(1 - \tau_2) \]  

The first term in (1) and (2) is the profit from high types and the second term is the profit from the low types. The two stage structure is standard in decisions that involve a message by the firm to be communicated prior to setting other variables (see, for example, Lal and Rao, 1997). The message in this case is the rebate offer. The structure of the decision process is shown in Figure 2.

**Figure 2:** Structure of Two-Stage Game

If a firm decides to offer a rebate, it must satisfy incentive compatibility (IC) and participation (PC) constraints such that consumers of each type pick the offering targeted at them. Specifically, rebates are not targeted at high type consumers in segment 1. So we require that the value of the rebate be less than the cost of obtaining it for these consumers, which implies the following inequality:

\[ \delta_1^i R_i \leq e \]  

IC for high type (3)

And the rebates must yield positive surplus to low type consumers in segment 2. Mathematically,

\[ \delta_2^i R_i \geq e \]  

for all \( R_i > 0 \)  

IC for low type (4)
Consumers of each type must also obtain non-negative surplus if they decide to purchase the product. For the marginal consumer the two products must yield equal surplus. Therefore, for the marginal consumer who is indifferent to the two products, we have, for consumers in each segment the following conditions:

\[
V_D - \alpha_1 \tau_1 - P_D = V_A - \alpha_1 (1 - \tau_1) - P_A \geq 0 \quad \text{PC for high type (5)}
\]

\[
V_D - \alpha_2 \tau_2 - P_D + \delta_2 R_D - e = V_A - \alpha_2 (1 - \tau_2) - P_A + \delta_2 R_A - e \geq 0 \quad \text{PC for low type (6)}
\]

In equation (6) we assume that both firms offer rebates. If one firm, say firm i, does not offer a rebate, the corresponding value of \(\delta_2 R_i - e\) drops out of equation (6). Thus, the constraints change depending on whether one firm, both or neither firm offers rebates. Depending on the decisions in stage 1, firms in stage 2 find themselves in one of four scenarios: (1) neither firm provides a rebate, (2) only firm \(D\) provides a rebate, (3) only firm \(A\) provides a rebate, and (4) both firms provide a rebate. To solve the problem, we find the equilibrium in stage 2 for each scenario and then work backwards to find the profit maximizing decisions in stage 1. We characterize the equilibrium in each scenario (sub-game) and then check whether that outcome is an equilibrium for the entire game. Thus, our equilibrium is sub-game perfect. To keep the analysis short, we set \(C_D = C_A = 0\) and limit our attention to the more general case of asymmetry in the consumer valuation through asymmetry in \(b\).

3. Analysis

Before we proceed with the analysis it is useful to consider what would happen if there were only one firm in our model. In this case of monopoly, we can readily see that since the low type consumers in segment 2 are willing to pay more for any combination of product attributes than high type consumers, offering a rebate to the low types is not optimal. In other words, in our model, a price discriminating monopolist would not offer rebates. The question we answer is: if rebates do not help to capture additional consumer surplus, what role might they play in a segmented market under competition?

Our analysis proceeds in the usual way of solving the game by backward induction. We begin with stage 2 of the game, and in section 3.1 we characterize the price and rebate amount in each of the four scenarios. In particular, we show that the disadvantaged firm \(D\) charges a lower price. This is
Proposition 1. We then proceed to stage 1, where we solve a 2x2 normal form game. In section 3.2.1 we first show that the greater the valuation difference, $V_A - V_D$, the greater the attractiveness of rebates to firm $D$. This is Proposition 2, and we use this result to develop the main intuition for the role that rebates play in our model. We then show, in Proposition 3, that there exists no pure strategy equilibrium in which $A$ offers a rebate but $D$ does not when the required consumer effort is sufficiently small. Following that, in section 3.2.2 we show that in any mixed strategy equilibrium, firm $D$ has a higher probability of offering a rebate. This is Proposition 4. Taken together, propositions 3 and 4 suggest that in practice we are likely to see rebates offered by disadvantaged firms rather than advantaged ones. This will be the basis for our examination of data in section 4.

3.1. Stage 2 analysis

There are four possible scenarios in stage 2: (1) Neither firm rebates, (2) only firm $D$ rebates, (3) only firm $A$ rebates and (4) both firms rebate. In each scenario, the two firms simultaneously maximize profits subject to incentive compatibility and participation constraints. The derivation for the four scenarios is provided in the appendix and the firm profits are given in Table 1.

Having solved for the equilibrium prices and rebates in stage 2, we can now represent stage 1 in normal-form as in Table 1. Note that in all scenarios, the advantaged firm’s price is higher than its rival. The following proposition states this finding formally.

Proposition 1: When competing firms are asymmetric in consumer valuations, the disadvantaged firm’s price is always lower than its rival’s unless only the disadvantaged firm offers a rebate. When only the disadvantaged firm offers a rebate, its price is lower only if the difference between the firms’ consumer valuations is higher than a threshold value ($u^*$), given by

$$u^* = \frac{\alpha_1(1-\gamma)[3(\alpha_1-\alpha_2)-e]}{4[(1-\gamma)\alpha_1 + \gamma\alpha_2]}.$$

Proof: All the proofs are relegated to the appendix.
3.2. Stage 1 analysis

Examining the profit solutions given in Table 1 reveals that the equilibrium clearly depends on the parameters of the model. To compare all possible equilibria in this setting we first characterize the attractiveness of some equilibria and rule out others.

3.2.1 Rebating and Segmentation in Asymmetric Duopoly

We investigate how asymmetry in firm valuations by consumers affects firms’ decision on rebates. First we note that when the asymmetry is sufficiently small, there will be no asymmetric equilibrium wherein one firm rebates and the other does not. A closer examination of the values of incremental profits indicates that the attractiveness of offering a rebate increases in the difference of firm valuations. Suppose firm $D$ is weakly disadvantaged against its rival ($A$), i.e. $V_A - V_D = 0$. We find that the disadvantaged firm’s “incremental” profits from offering a rebate increases as it becomes more disadvantaged vis-à-vis its rival. Therefore, offering a rebate becomes more attractive for the disadvantaged firm. At the same time, the exact opposite result is observed for the advantaged firm. The advantaged firm benefits less from offering rebates if its advantage is higher. As the difference between consumer valuations for the two firms widens we can expect rebates to be provided by the disadvantaged firm rather than the advantaged one. We formalize this result in Proposition 2 as follows:

Proposition 2: The incremental profits the disadvantaged (advantaged) firm can generate by offering rebates increase (decrease) the more disadvantaged (advantaged) the firm becomes relative to its rival.

It is useful to interpret proposition 2. When only the disadvantaged firm $D$ offers a rebate to the low type consumers, in effect its pricing is determined by its desire to compete for the high type consumers. Since the high types are not as price oriented as the low types, moving competition
Table 1: Normal-form rebating game

<table>
<thead>
<tr>
<th>Firm D – No Rebate</th>
<th>Firm A -- No Rebate</th>
<th>Firm A -- Rebate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi_{no}$ = $\frac{(V_n - V_d) - 3(\gamma(\alpha_d + (1 - \gamma)\alpha_A))}{18(\gamma\alpha_A + (1 - \gamma)\alpha_d)}$</td>
<td>$\Pi_{no} = \frac{(V_n - V_d) - 3(\gamma(\alpha_d + (1 - \gamma)\alpha_A))}{18(\gamma\alpha_A + (1 - \gamma)\alpha_d)}$</td>
<td>$\Pi_{no} = \frac{V_n - V_d - 3(\gamma(\alpha_d + (1 - \gamma)\alpha_A))}{18(\gamma\alpha_A + (1 - \gamma)\alpha_d)}$</td>
</tr>
<tr>
<td>$\Pi_{so}$ = $\frac{(V_n - V_d) - 3(\gamma(\alpha_d + (1 - \gamma)\alpha_A))}{18(\gamma\alpha_A + (1 - \gamma)\alpha_d)}$</td>
<td>$\Pi_{so} = \frac{(V_n - V_d) - 3(\gamma(\alpha_d + (1 - \gamma)\alpha_A))}{18(\gamma\alpha_A + (1 - \gamma)\alpha_d)}$</td>
<td>$\Pi_{so} = \frac{V_n - V_d - 3(\gamma(\alpha_d + (1 - \gamma)\alpha_A))}{18(\gamma\alpha_A + (1 - \gamma)\alpha_d)}$</td>
</tr>
</tbody>
</table>

For Firm D – Rebate:

$\Pi_{so} = \frac{V_n - V_d - 3(\gamma(\alpha_d + (1 - \gamma)\alpha_A))}{18(\gamma\alpha_A + (1 - \gamma)\alpha_d)}$

For Firm A – Rebate:

$\Pi_{so} = \frac{V_n - V_d - 3(\gamma(\alpha_d + (1 - \gamma)\alpha_A))}{18(\gamma\alpha_A + (1 - \gamma)\alpha_d)}$
away from the low types leads to higher prices, benefiting both firms even if only one firm offers a rebate. Of-course, when only $D$ offers a rebate, some low type consumers would still be attracted to $A$, depending on the location of their ideal product. In this case, the non-zero rebate by $D$ does not completely separate the two types of consumers, but changes the customer mix more towards high types so that “average” price salience is lower. This separation of the two types of consumers is a segmentation strategy, but with one crucial difference. As we already noted, a monopolist would gain nothing from a segmentation strategy in our model. But under competition, firms would like to compete separately for the two segments. In particular, for the low types price is more salient, and so competing for them lowers prices to high types. This can be avoided if the low types could be addressed using a tool other than price – rebates in this case.

When only one firm offers a rebate, segmentation is partial and prices are higher relative to the no rebate case. When both firms offer rebates, segmentation is complete, and prices are highest. Thus we can see that segmentation, whether partial or full, benefits both firms by enabling them to extract higher surplus from the high type consumers. The idea that segmentation enables firms to compete for high type consumers without having to provide them a “subsidy” resulting from the presence of low types is an important one. Employing a blunt pricing strategy in the presence of consumer heterogeneity has the effect of providing such a subsidy. Segmentation can have the beneficial consequence of lessening or eliminating this subsidy. This has been shown to be the case by Kumar and Rao (2005) who find that complete retail segmentation by offering rewards eliminates subsidy to one segment of consumers when segments are heterogeneous in their shopping baskets. Thus, we see that proper targeting of rebates and other promotions can indeed increase profits under competition if consumers are heterogeneous. There are situations, as noted by Shaffer and Zhang (1995), where targeting of coupons is not profitable under competition while it would be profitable for a monopolist. In contrast, we have identified conditions under which segmentation is profitable under competition but not under monopoly.
We can get a clear picture of how the equilibria change with the extent of asymmetry between the firms by investigating the equilibria numerically. With this in mind, we computed the equilibria by varying \( u = V_A - V_D \) over the interval \([0, 5]\); holding all other parameters constant\(^6\). Figure 3 depicts how the equilibria change. For low values of asymmetry (region I) we observe multiple equilibria. In region I, both firms offering rebates is an equilibrium as also neither firm offering rebates. There is also a mixed strategy equilibrium in this range and we derive it in the next section. The equilibrium in which both firms offer rebates results in higher profits than the other two equilibria for low values of asymmetry. So, it is preferable for both firms, and so firms face a coordination problem. For high values of asymmetry (region III), on the other hand, both firms improve their profits if only firm \( D \) offers a rebate as Proposition 2 suggests. Thus, in this example, the best equilibrium is for both firms to offer rebates for small asymmetries, and only firm \( D \) offering a rebate for larger asymmetries. In the intermediate range of asymmetry (region II) we observe a Prisoners’ Dilemma and neither firm offering a rebate emerges as the unique equilibrium. This is because firm \( A \) prefers only firm \( D \) to offer a rebate in this intermediate range; however, firm \( D \) prefers both firms to offer rebates. Even though both firms offering rebates is a better outcome for both than neither firm offering rebates, it is not equilibrium.

![Figure 3](image-url)

**Figure 3:** Equilibria with changing asymmetry in consumer valuations \((u)\)

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\(^6\) We set \( \alpha_1 = 5, \alpha_2 = 2.5, d_2 = 0.9, d_1 = 0.7, \gamma = 0.4, e = 0.8, t = 8. \)
In the above numerical example, we found no equilibrium in which only firm A offers a rebate. Is this result generalizable? We find that for sufficiently small values of consumer effort \( e \), there is no pure strategy equilibrium in which only firm A offers a rebate. This finding is stated in Proposition 3. The result makes intuitive sense, with its higher valuation the advantaged firm (A) prefers to focus on the high types.

**Proposition 3:** When \( e < e^* \), there is no pure NE where the advantaged firm rebates and the disadvantaged firm does not, where

\[
e^* = \sqrt{\frac{16(\alpha_1 (1 - \gamma) + \gamma \alpha_2)^2 u^2 + 36 \gamma \alpha_2 (\alpha_1 - \alpha_2) (\alpha_1 (1 - \gamma) + \gamma \alpha_2) u - 8(\alpha_1 (1 - \gamma) + \gamma \alpha_2) u - 9 \gamma \alpha_2 (\alpha_1 - \alpha_2)}{27 (\alpha_1 - \alpha_2)^2 (2 \alpha_1 (1 - \gamma) + 3 \gamma \alpha_2)}}.
\]

We saw earlier that for high values of asymmetry the unique equilibrium is only firm D offering a rebate. We next examine how the sizes of the consumer segments affect the uniqueness of equilibrium in region III. We set the value of asymmetry equal to 4.5 and vary the size of the high type segment over the interval \([0, 1]\). Figure 4 shows the equilibria. We see that only one firm offering a rebate emerges as the unique equilibrium when the size of the high type segment is intermediate (region II). When the proportion of high type consumers is moderate, both firms will have much to gain by separating the segments. However, partial segmentation entails one firm taking the less lucrative low type segment. Though both firms benefit from segmentation relative to no segmentation, each firm would like to be the one with the high type segment. That is, each firm would like the other firm to offer the rebate. Even when it is profitable for a firm to offer a rebate, it is more attractive if the other firm offers a rebate as well.\(^7\)

In equilibrium, the advantaged firm, who would win out in the competition for the high type segment, has less to gain from offering a rebate. Hence, whenever only one firm offers a rebate in

\(^7\)A similar strategic complementarity is shown in Kumar and Rao (2005), where a firm benefits more from a data-analytics program that allows it to segment the market if the competing firm also obtains the data-analytics program.
equilibrium (only one firm finds the benefit from segmentation to exceed the cost of foregoing the lucrative high type segment), it will be the disadvantaged firm.

When the size of the high type segment is small (region I), i.e. the size of the low type segment is large, the firms once again face a Prisoners’ Dilemma. Each firm prefers the other to offer rebates. Even though both firms would profit if they both offered rebates, it is not equilibrium. Finally, when the size of the high type segment is large (region III), both firms offering rebates is the unique equilibrium. We find that for a given value of asymmetry in the interval \([\mu_2^*, 5]\), the sizes of segments impact the nature of the equilibrium. However, the uniqueness of the equilibrium is maintained.

![Figure 4: Equilibria with changing proportion of high type consumers (γ).](image)

**3.2.2 The Mixed Strategy Equilibrium**

Under certain conditions, stage 1 equilibrium is in mixed strategies, in addition to equilibria in pure strategies. Given our interest in examining data on rebates, it is useful to characterize the mixed strategy equilibrium also, since in practice many firms in the same industry offer rebates.\(^8\) Since rebates are temporary price reductions, they are properly viewed as promotions. And many kinds of promotions have been identified by mixed strategies in prior

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\(^8\) In fact we will see in the next section that in the printer market we observe rebate promotions by virtually all manufacturers and these promotions appear to be uncorrelated, with the exception of seasonal promotion such as end of year sales. This seems to indicate that the mixed strategy equilibrium would be a good candidate to describe empirical observation.
analytical work by Varian (1980) and Narasimhan (1984), and in empirical work by Rao, Arjunji and Murthi (1995) and Villas-Boas (1995). Indeed, Rao, Arjunji and Murthi offered the generalization that competitive promotions are mixed strategies based on empirical findings in many product categories. In our model, we find that that the disadvantaged firm would rebate more frequently than its advantaged rival in any mixed strategy Nash equilibrium. This is consistent with propositions 2 and 3, which show that the disadvantaged firm has more to gain from offering rebates and will never find itself not offering a rebate in response to a competitor’s rebate. This finding is formalized in the following proposition.

**Proposition 4:** In any mixed strategy Nash equilibrium, the disadvantaged firm offers a rebate with higher probability than the advantaged firm.

### 4. Analysis of Rebates in the Printer Category

In this paper we examine the printer category. The printer category is a dynamic category, with firms continually devising pricing, promotion and new product initiatives as part of their competitive strategy (Shankar, 2005). There are three major subcategories in the printer category: Inkjet, Laser, and Multi-function (printer with other functions such as photo printing, copying, and fax functions) with a handful of firms dominating each sub-category. While *Hewlett-Packard* has long established itself as the leader in all printer categories, there is fierce competition for second place. Given our interest in rebates, we want to ensure that rebates occur with sufficient frequency in the product category we choose to examine. In the printer category, looking at all brands, in any given week 30% of SKU’s carry a rebate on average. For all these reasons, we chose to study the printer category.

#### 4.1 Description of Data

We collected data on printers from the website of a major retailer specializing in office equipment. The time period for the data is November 2004 to August 2005. We gathered daily price and rebate information for each SKU that the store carries. We can identify the brand name for each SKU. For purposes of analysis we decided to aggregate daily data over a week.
reason for this is that most rebates run for a week. Table 2 gives a frequency distribution of the duration of rebates. We can see that over 75% of the rebates are offered for one week. The price variable was the average price in the week. In addition we gathered data on 2004 market share for each brand in the laser sub-category from International Data Corporation (IDC).

<table>
<thead>
<tr>
<th>Rebate length (weeks)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>199</td>
<td>28</td>
<td>8</td>
<td>5</td>
<td>14</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>%</td>
<td>75.67</td>
<td>10.65</td>
<td>3.04</td>
<td>1.90</td>
<td>5.32</td>
<td>2.28</td>
<td>0.38</td>
<td>0.38</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Table 2: Rebate length distribution

4.2 Hypothesis

We want to test our main result that the disadvantaged firm is more likely to offer rebates. One thing to note in our data is that all brands and most SKU’s offer a rebate. This might suggest a mixed Nash strategy equilibrium as in our model. As a first step, we wanted to see if the retailer might act in such a way as to coordinate the rebates across brands, Lal (1990). So we tested for independence of rebate offers across pairs of brands using a Chi-squared test. This test is conservative for our purpose. Table 3 contains the test statistics for pair-wise independence of rebates. We find that in all but four cases, the hypothesis of independence cannot be rejected at the 5% level. Given the conservative nature of the test, we would be justified in rejecting any coordination of rebate offers by competitors. We will therefore assume that rebates result from mixed strategies. Now, invoking proposition 4 of our analysis, we should expect that the disadvantaged firm would offer rebates with greater probability than the advantaged firm. This is what we wish to test next. To render this into a specific hypothesis it is necessary to map a firm’s

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9 See Rao (1996), for example.
advantage to observable variables. This is challenging because it is not easy to determine who exactly is an advantaged (or disadvantaged) firm. We met this challenge by conducting our analysis by using several proxies.

<table>
<thead>
<tr>
<th>Firm</th>
<th>Brother</th>
<th>HP</th>
<th>Konica</th>
<th>Lexmark</th>
<th>Okidata</th>
<th>Xerox</th>
<th># of Rebates$^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brother</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>HP</td>
<td>0.01</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22</td>
</tr>
<tr>
<td>Konica</td>
<td>0.68</td>
<td>0.20</td>
<td>0</td>
<td></td>
<td>0.01</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Lexmark</td>
<td>0.003</td>
<td>0.02</td>
<td>0.88</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>Okidata</td>
<td>0.12</td>
<td>0.09</td>
<td>0.39</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>Xerox</td>
<td>0.49</td>
<td>0.39</td>
<td>0.18</td>
<td>0.46</td>
<td>0.46</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

*Table 3:* Test statistics (p-values) from tests for independence

We analyzed the laser printer market at the brand level using two proxies: market share and price. A firm with a higher market share in equilibrium can reasonably be argued to have greater brand equity, and so would be the advantaged firm. Turning to price, a brand with a higher consumer valuation would charge a higher price (by Proposition 1). As such, a brand that is priced higher in equilibrium would be the advantaged firm. In addition to the brand level analysis, we examined the relationship between price and the likelihood of a rebate at the SKU level in each of the three sub-categories. Finally, we also conducted an analysis across sub-categories relating price to rebate frequency. The inkjet category is the most disadvantaged and the laser category the most advantaged. As such the laser category should carry rebates least often and inkjet the most$^{11}$. In this way, we want to see how robust our findings are at the brand level, SKU level and sub-category level. We now formally state our hypothesis, corresponding to our proxy for the advantaged firm:

**Hypothesis 1a:** Brands with higher market share will offer rebate less frequently.

$^{10}$To be precise, this is how often *at least* one SKU in the brand carries a rebate in a given week.

$^{11}$In theory, we could use features to assess quality. The problem there is that the number of features is large relative to the number of SKU’s and these features are highly correlated with each other as well as with brand names.
**Hypothesis 1b:** Brands and/or SKUs with higher average price will offer rebate less frequently.

**Hypothesis 1c:** Subcategories with higher price will rebate less frequently.

### 4.3 Results and Discussion

Our first analysis is at the brand level in the laser sub-category. The brands are ordered, from highest to lowest, by market share as follows: HP, Xerox, Konica, Brother, Lexmark. The ordering, by price, is: Xerox, HP, Konica, Lexmark, and Brother. We can see in the histogram in Figure 5 that displays the frequency of rebates along with market share and price. Rebate frequency is operationalized as the proportion of SKU’s promoted each period by the brand given that our analysis is at the brand level.

![Figure 5: Price, Rebate Frequency and Market Share Data for Laser Printer Category](image)

As we can see there is a negative relationship between price and rebate frequency. There is also a negative relationship between market share and rebate frequency, with the possible exception of Xerox. The brand with the highest market share and the second highest price, HP, has the lowest rebate frequency, while the brand with the lowest price and the second lowest market share, Brother, has the highest rebate frequency.

To get a better feel for this data we conducted paired comparisons for these brands. The *t*-test results for these comparisons are shown in Table 4. In the first column, each pair is ordered
with the first brand being the advantaged brand relative to the second with two exceptions: HP-Xerox and Brother-Lexmark. Our proxies have different predictions for these two pairs. The dependent variable is the difference in the rebate frequencies of the first and the second brands in any given week. Therefore, according to our hypothesis this difference should be negative for all but two aforementioned cases. In those two cases, our prediction is positive for one of the proxies whereas it is negative for the other. The number of observations for each hypothesis is the number of weeks in our sample. In the two right-most columns, we provide the predictions of hypothesis 1a and 1b. In the last row, we display a count of the number of predictions which are inconsistent with our hypothesis.

| Comparison     | DF | t Value | Pr > |t| | 1a Prediction | 1b Prediction |
|----------------|----|---------|------|---|---------------|---------------|
| HP-Brother     | 37 | -6.93   | < 0001|    | Negative      | Negative      |
| HP-Lexmark     | 37 | -3.02   | 0.0045|    | Negative      | Negative      |
| HP-Konica      | 37 | -3.25   | 0.0025|    | Negative      | Negative      |
| HP-Xerox       | 37 | 7.01    | < 0001|    | Negative      | Positive      |
| Konica-Brother | 37 | 0.65    | 0.5175|    | Negative      | Negative      |
| Konica-Lexmark | 37 | 0.77    | 0.4481|    | Negative      | Negative      |
| Xerox-Konica   | 37 | -4.68   | < 0001|    | Negative      | Negative      |
| Xerox-Brother  | 37 | -9.41   | < 0001|    | Negative      | Negative      |
| Xerox-Lexmark  | 37 | -4.50   | < 0001|    | Negative      | Negative      |
| Brother-Lexmark| 37 | 0.41    | 0.6820|    | Negative      | Positive      |

Table 4: Paired comparisons of the proportion of SKU’s promoted in the laser printer category.

From Table 4 we see that 13/14 significant cases support our hypothesis 1a and 1b that the advantaged brand offers rebates less frequently. The exceptions involve Xerox which has low market share and a high price and offers few SKUs that are high volume printers. As such, the comparisons in that case may be less valid. The results in Table 4 confirm the general picture that Figure 5 depicts.

We think it would be worthwhile to examine the data at the SKU level since brands vary in the number of SKUs they offer. Therefore, we next look at SKUs within each of the three sub-
categories, separately. To this end, we formulated a simple model for the probability of an SKU carrying a rebate. Define $W_{jt}$ to be value of a rebate to an SKU $j$ in week $t$. We model $W_{jt}$ as

$$W_{jt} = \alpha_{bj} + \beta P_{jt} + \mu_t + \epsilon_{jt},$$

where $B_j$ is the brand of SKU $j$, $P_{jt}$ is the price of SKU $j$ at time $t$, $\mu_t$ is a common shock in week $t$ and $\epsilon_{jt}$ is the idiosyncratic shock for SKU $j$ in week $t$. The common shock should take care of any effects in a given week that affects all printers, such as the introduction of new products, the state of the economy and possible seasonality. We model it as normally distributed, with mean 0 and standard deviation $\sigma$. The idiosyncratic shock is assumed to be i.i.d, following an extreme value distribution, yielding a binary logit model for estimation. Our interest is in the sign of the price parameter. The estimation results are shown in Table 5.

<table>
<thead>
<tr>
<th>Subcategory</th>
<th>Inkjet</th>
<th>Multifunction</th>
<th>Laser</th>
</tr>
</thead>
<tbody>
<tr>
<td># of obs.</td>
<td>724</td>
<td>1708</td>
<td>1947</td>
</tr>
<tr>
<td># of weeks</td>
<td>38</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td># of sku’s</td>
<td>25</td>
<td>60</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>Estimate</td>
<td>t-stat</td>
<td>Estimate</td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.35</td>
<td>-5.48</td>
<td>-1.96</td>
</tr>
<tr>
<td>Brother</td>
<td>NA</td>
<td>NA</td>
<td>-0.80</td>
</tr>
<tr>
<td>HP</td>
<td>1.41</td>
<td>3.34</td>
<td>-0.13</td>
</tr>
<tr>
<td>Canon</td>
<td>-13.50</td>
<td>-0.03</td>
<td>0.35</td>
</tr>
<tr>
<td>Konica</td>
<td></td>
<td></td>
<td>0.71</td>
</tr>
<tr>
<td>Lexmark</td>
<td>1.03</td>
<td>2.22</td>
<td>-0.21</td>
</tr>
<tr>
<td>Sharp</td>
<td>NA</td>
<td>NA</td>
<td>-1.26</td>
</tr>
<tr>
<td>Okidata</td>
<td>NA</td>
<td>NA</td>
<td></td>
</tr>
<tr>
<td>Epson</td>
<td>Benchmark</td>
<td>Benchmark</td>
<td>NA</td>
</tr>
<tr>
<td>Xerox</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Price</td>
<td>-0.0035</td>
<td>-6.15</td>
<td>-0.0009</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.28</td>
<td>1.17</td>
<td>2.25</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>230.89</td>
<td>535.17</td>
<td>496.61</td>
</tr>
</tbody>
</table>

Table 5: Rebate Frequency at SKU level

From Table 5 we see that the price coefficient is significant for all three sub-categories, and has the right, negative, sign. The negative sign means that higher priced SKUs have a lower probability of carrying a rebate. This supports our hypothesis 1b. We also see that the brand effects vary by sub-category, because some brands are strong in one sub-category but not
necessarily in another. For the category level data we chose not to perform any statistical analysis. Instead we present a visual depiction of the data in Figure 6.

![Figure 6: Price vs. Rebate Frequency for Printer Categories](image)

We can see from Figure 6 that laser printer sub-category is the most expensive and yet has the lowest rebate frequency. This could be interpreted as supporting our hypothesis 1c.

Summarizing our results, we see that the advantaged competitor in our data offers rebates less often than the disadvantaged one. In this way we find support for our proposition 4 which says that in any mixed strategy equilibrium the advantaged firm has a lower probability of offering a rebate.

4.4 Robustness of our Findings

Recent work (Chen, Moorthy and Zhang, 2001; Moorthy and Lu, 2004; Soman, 1998) proposes that firms could benefit from using rebates because consumers often make purchases intending to redeem rebates but end up not redeeming them. This is known as “slippage”. Moorthy and Lu (2004) point to the growing popularity of free-after-rebate promotions as one type of evidence. They report that consumers fail to redeem 50% of the rebates even for free-after-rebate offers\textsuperscript{12}. The slippage is higher for smaller rebates. Empirically, it would appear that slippage is an established phenomenon. Thus, even if firms use rebates as segmentation devices, they could potentially benefit from the use of rebates because of slippage. However, from our

\textsuperscript{12} While it is possible that failure to redeem “free-after-rebate” offers is due to slippage, one cannot rule out that some people may have bought the items without the intention of ever redeeming the rebate.
perspective, the more relevant question is: can the observed rebate data be explained by a
different motivation for rebates? So, we would like to investigate whether in the presence of
slippage, a disadvantaged firm is more likely to use rebates than an advantaged firm as the printer
market data suggests.

We incorporate slippage in our model by assuming that consumers for whom price is
salient purchase the product with the intention of redeeming the rebate but a fraction, $s$, of these
fails to redeem the rebate after purchase. Consumers for whom product attributes are more salient
are assumed to be unaffected by rebates. To isolate the effects of slippage, we assume no
differences among consumers in terms of their discount factors. We simplify the exposition by
assuming no discounting and set marginal costs equal to zero. We retain the rest of the framework
and analyze the four scenarios based on whether one firm, both firms, or neither firm offers
rebates. We illustrate how we incorporate slippage into our model by considering, for example,
scenario 4 in which both firms offer a rebate. The profit of a firm in this case is calculated as the
sum of the profit from the price-sensitive segment $(1-\gamma)$ where only a fraction $(1-s)$ of consumers
redeem the rebate and the profit from the price insensitive segment $(\gamma)$. The market share of firm
$D$ in the price sensitive (price insensitive) segment is given by $\tau_D (\tau_A)$.

$$\Pi_D = (1-\gamma)\tau_D (P_D - (1-s)R_D) + \gamma\tau_D P_D$$

$$\Pi_A = (1-\gamma)(1-\tau_A)(P_A - (1-s)R_A) + \gamma (1-\tau_A)P_A$$

The solutions of the four scenarios for our slippage model are presented in the appendix.

Comparing these results to our propositions 2, 3 and 4 we see that qualitatively they make
different predictions (Proposition 5 and Corollary 1).

**Proposition 5:** In equilibrium both firms offer a rebate and the rebate amount for the advantaged
firm is higher than the rebate amount for the disadvantaged firm.

---

13 It is easy to endogenize consumers’ use of rebates and make that equivalent to our model by assuming a
sufficiently high cost of rebate redemption for consumers for whom attributes are more salient.
Proposition 5 shows that in equilibrium both firms offer rebates, and the advantaged firm offers a higher rebate amount than the disadvantaged firm. We note that in reality there is a non-negligible cost of a rebate program and when that cost exceeds the marginal benefit of rebating, no rebate will be offered. Given that the marginal revenue from rebating is higher for the advantaged firm when neither firm rebates (see proof to proposition 5), we can argue that there will be instances where the disadvantaged firm will not offer a rebate but the advantaged firm will. Another way to argue this point is to show that the rebate amount offered by the disadvantaged firm approaches zero as the asymmetry grows, whereas it does not decline for the advantaged firm. The corollary to proposition 5 proves this claim.

*Corollary 1: As the asymmetry between firms increases, the disadvantaged firm’s rebate decreases and eventually approaches zero and the advantaged firm’s rebate increases.*

We can contrast this result with the corresponding result in our model of segmentation. In the segmentation setting, the disadvantaged firm is more likely to rebate in equilibrium and the likelihood of rebate increases with the asymmetry. In the present setting, the opposite holds. The main point to note is that slippage makes rebates attractive to both advantaged and disadvantaged firms in competing for the low types. The price can then be raised reducing the subsidy for high types. Indeed, the higher the desired price for the high types, the greater the rebate that would be offered. Since the advantaged firm targets high types more, under slippage the effect of rebates on the advantaged firm’s profits is greater. As such, if slippage were the only effect of rebates we should observe more frequent rebate offers by the advantaged firm and not by the disadvantaged firm as we see in the printer market. We therefore conclude that in addition to providing gains
from slippage, rebates also have a segmentation role. In particular, the empirical finding that the disadvantaged firm offering rebates more frequently in the printer market lends support to our result that rebates, in their role as segmentation devices, help to lessen the erosion of profit margins under competition.

5. Conclusions

Our main result is that segmentation is a way to soften the competition for the more profitable consumers and thus increase profits. Without segmentation the equilibrium price tends to be too low for the more profitable consumers and too high for the less profitable consumers. This in turn means that without segmentation some money is left on the table for the more profitable consumers, who in effect receive a “subsidy”. In general, this is not offset by the possibility of higher prices for the less profitable segment that results in the absence of segmentation. What segmentation does is reduce or eliminate the subsidy, simultaneously lowering prices for the less profitable segments. This is more than compensated for by the reduced subsidy. We have also shown that in our model a monopolist would not gain by segmenting the market, but under competition, segmentation increases firms’ profits. Thus, we make an important contribution to the understanding of marketing strategy through segmentation by identifying a role for segmentation in addition to its being a price discrimination device. In both cases, segmentation requires a mechanism.

One mechanism for segmentation is a rebate offer that is a common promotion tool in the marketing of big ticket items. We model the rebate decision in a competitive duopoly setting, with price and rebate as decision variables and heterogeneous consumers of different price orientations. Our analysis provides important insights in formulating a firm’s rebate strategy. The main insight is that rebates when used to segment a market lessen competition that would result if firms were competing on prices separately over each segment. A second important insight is that rebates would typically be offered by the disadvantaged firm. In other words, the availability of
rebates does something to level the playing field for disadvantaged competitors. Indeed, the greater the disadvantage of the firm relative to its rival, the more beneficial rebates become to the firm’s profits. Third, if consumer effort involved in obtaining rebates is not too high, the disadvantaged firm will typically offer a rebate, but the advantaged competitor would not. Finally, we also identify a mixed strategy Nash equilibrium of rebate offers. In such an equilibrium the disadvantaged firm always offers rebates more frequently than the advantaged firm.

The current analysis does not capture all aspects of a rebating strategy. We believe that distribution costs are a major determinant in the rebate decision. Specifically, if coupons are also available as a segmentation device, the decision between rebates and coupons would depend on costs of administering coupon or rebate programs. Future research should also examine the role of the channel, specifically, who in the channel is most likely to offer a rebate and who stands to benefit the most. Finally, the framework used here could be extended to coupons with some changes in the dimensions of focus. Examining coupon strategies from segmentation under competition as opposed to a pure price discrimination perspective could offer new insights.
References

Appendix

Stage Two Solutions

There are four possible scenarios in stage 2.

Scenario 1: Neither firm provides a rebate

This case is the standard differentiated duopoly model and the equilibrium prices can be shown to be:

\[
P_D = \frac{1}{3} (V_D - V_A) + \frac{\alpha_1 \alpha_2}{(1-\gamma)\alpha_1 + \gamma \alpha_2} \tag{A.1}
\]

\[
P_A = \frac{1}{3} (V_A - V_D) + \frac{\alpha_1 \alpha_2}{(1-\gamma)\alpha_1 + \gamma \alpha_2} \tag{A.2}
\]

Scenario 2: Only firm D provides a rebate

This calls for maximization subject to constraints by firm D. In this case firm D’s profit is:

\[
\Pi_D = \gamma P_D \tau_D + (1-\gamma)(P_D - \delta_D R_D) \tau_D
\]

Firm D chooses \( P_D \) and \( R_D \) to maximize the profit subject to the following incentive compatibility and participation constraints:

\[
\delta_D R_D \leq e \\
\delta_D R_D \geq e \\
V_D - \alpha_1 \tau_1 - P_D = V_A - \alpha_1 (1-\tau_1) - P_A \\
V_D - \alpha_2 \tau_2 - P_D + \delta_D R_D - e = V_A - \alpha_2 (1-\tau_2) - P_A
\]

Solving for the participation constraints we find the market shares of firm D in two consumer segments as:

\[
\tau_1 = \frac{V_D - V_A + P_A - P_D + \alpha_1}{2\alpha_1} \text{ and } \tau_2 = \frac{V_D - V_A + P_A - P_D + \delta_D R_D - e + \alpha_2}{2\alpha_2}
\]

Substituting these into the profit function verifies that participation constraints are indeed satisfied. We then form the Lagrangian to solve the firm’s constrained optimization problem, which is now subject to incentive compatibility constraints.

\[
L_D = \Pi_D - \lambda_1 (\delta_D R_D - e) - \lambda_2 (-\delta_D R_D + e) \tag{A.3}
\]

and the Kuhn-Tucker conditions are:

\[
\{ \lambda_1 = 0 \text{ and } (\delta_D R_D < e) \} \text{ or } \{ \lambda_1 > 0 \text{ and } (\delta_D R_D = e) \} \tag{A.4}
\]

\[
\{ \lambda_2 = 0 \text{ and } (\delta_D R_D > e) \} \text{ or } \{ \lambda_2 > 0 \text{ and } (\delta_D R_D = e) \} \tag{A.5}
\]

Firm A’s problem in this case is to maximize its profit given by

\[
\Pi_A = P_A [\gamma (1-\tau_1) + (1-\gamma)(1-\tau_2)]
\]

Solving the two maximization problems simultaneously, we find that the solution depends on parameter values that could be in one of three distinct ranges. We find that only one of these ranges corresponds to the case in which firm D offers a rebate, and that solution is an interior one. For the incentive compatibility conditions to hold, it is necessary that the exogenous time to fulfill a rebate claim, \( t \), must satisfy:
\[
\ln\left(\frac{e}{R_d}\right) + \ln\left(\frac{e}{R_d}\right) < t < \ln\left(\frac{e}{R_d}\right) \quad (A.6)
\]

Then, the optimal rebate in the interior solution can be shown to be
\[
R_d = \frac{\alpha_i - \alpha_2 + \epsilon}{2\delta_2^2} \quad (A.7)
\]

The corresponding equilibrium prices are:
\[
P_d = \frac{1}{3}(V_d - V_A) + \frac{\alpha_i(1-\gamma)e}{6(1-\gamma)\alpha_i + \gamma\alpha_2} + \frac{\alpha_i(1-\gamma)e}{2(1-\gamma)\alpha_i + \gamma\alpha_2} \quad (A.8)
\]

and
\[
P_A = \frac{1}{3}(V_A - V_d) + \frac{\alpha_i(1-\gamma)e}{3(1-\gamma)\alpha_i + \gamma\alpha_2} + \frac{\alpha_i(1-\gamma)e}{(1-\gamma)\alpha_i + \gamma\alpha_2} \quad (A.9)
\]

Scenario 3, where only firm A offers a rebate, has a similar setup and first order conditions to scenario 2, and so we do not develop it in more detail.

**Scenario 4: Both firms offer rebates**

In this case, both firms solve constrained maximization problems. Moreover, one of the PC conditions is now different. The constraints become:
\[
\delta_1^{(i)}R_i \leq e \quad (A.10)
\]
\[
\delta_2^{(i)}R_i \geq e \quad (A.11)
\]
\[
V_i - \alpha_i \tau_i - P_i = V_i - \alpha_i (1-\tau_i) - P_i \quad (A.12)
\]
\[
V_i - \alpha_i \tau_i - P + \delta_2^{(i)}R_i - e = V_i - \alpha_i \tau_i - P_i + \delta_2^{(i)}R_i - e \quad (A.13)
\]

Solving for market share figures for firm D in each segment gives
\[
\tau_1 = \frac{V_d - V_A + P_A - P_D + \alpha_i}{2\alpha_i} \quad \text{and} \quad \tau_2 = \frac{V_d - V_A + P_A - P_D + \delta_1^{(D)}R_D - \delta_2^{(D)}R_D + \alpha_2}{2\alpha_2}.
\]

The profit for firm D is
\[
\Pi_d = \gamma P_d \tau_1 + (1-\gamma)(P_d - \delta_1^{(D)}R_D)\tau_2.
\]

And it is subject to
\[
\delta_1^{(D)}R_D \leq e, \quad \delta_2^{(D)}R_D \geq e.
\]

The profit for firm A is
\[
\Pi_A = \gamma P_A (1-\tau_i) + (1-\gamma)(P_A - \delta_1^{(A)}R_A)(1-\tau_2)
\]

subject to
\[
\delta_1^{(A)}R_A \leq e, \quad \delta_2^{(A)}R_A \geq e.
\]

We now have two Lagrangians:
\[
L_d = \Pi_d - \lambda_1 (\delta_1^{(D)}R_D - e) - \lambda_2 (\delta_2^{(D)}R_D + e) \quad (A.14)
\]
\[
L_A = \Pi_A - \lambda_1 (\delta_1^{(A)}R_A - e) - \lambda_2 (\delta_2^{(A)}R_A + e) \quad (A.15)
\]

Solving the Kuhn-Tucker conditions, we now get 9 cases. Only one of these, however, corresponds to scenario 4. That is, the effort of obtaining a rebate, \(\epsilon\), must lie in the interval \([\frac{\delta_1^d(\alpha_i - \alpha_2)}{2\delta_1^d - \delta_2^d}, (\alpha_i - \alpha_2)]\) for both firms. We then get equilibrium prices and rebates as:
Proof of Proposition 1

Let \( D_1 \) (\( A_1 \)) denote the improvement in firm \( D \)'s \( A \)'s profits if firm \( D \) \( A \) chooses to rebate given firm \( A \) \( D \) does not rebate. Also let \( D_2 \) (\( A_2 \)) denote the improvement in firm \( D \)'s \( A \)'s profits if firm \( D \) \( A \) chooses to rebate given firm \( A \) \( D \) rebates as well. Using the definitions of the profits given in Table 1 we formulate these four values as follows:\(^{14}\):

\[
P_D = \frac{1}{3} (V_D - V_A) + \alpha_1
\]

\[
P_A = \frac{1}{3} (V_A - V_D) + \alpha_1
\]

\[
R_D = \frac{\alpha_1 - \alpha_2}{\delta^D_2}
\]

\[
R_A = \frac{\alpha_1 - \alpha_2}{\delta^A_2}
\]

Assume firm \( A \) is advantaged and firm \( D \) is disadvantaged, i.e. \( V_A - V_D > 0 \). We start with the second part of the proposition. Only in scenario 2 does firm \( D \) rebate alone. In this scenario combining equations A.8 and A.9 gives

\[
P_A - P_D = \frac{2}{3} (V_A - V_D) - \frac{\alpha_1 (1-\gamma) [3(\alpha_1 - \alpha_2) - e]}{6(1-\gamma) \alpha_1 + \gamma \alpha_2}
\]

Hence, \( P_A > P_D \) if

\[
V_A - V_D > \frac{\alpha_1 (1-\gamma) [3(\alpha_1 - \alpha_2) - e]}{4(1-\gamma) \alpha_1 + \gamma \alpha_2}
\]

The right hand side of this inequality is \( u^* \).

To prove the first statement we look at each of the other three scenarios. In scenario 1, using equations A.1 and A.2, we find

\[
P_A - P_D = \frac{2}{3} (V_A - V_D) > 0
\]

In scenario 3, which is symmetric reflection of scenario 2, we have

\[
P_A - P_D = \frac{2}{3} (V_A - V_D) + \frac{\alpha_1 (1-\gamma) [3(\alpha_1 - \alpha_2) - e]}{6(1-\gamma) \alpha_1 + \gamma \alpha_2}
\]

In this case \( P_A - P_D > 0 \) since \( \gamma < 1 \) and \( \alpha_1 - \alpha_2 > e \). Finally in scenario 4, \( P_A - P_D = \frac{2}{3} (V_A - V_D) > 0 \). Q.E.D.

Proof of Proposition 2

Suppose firm \( A \) is weakly advantaged: \( V_A - V_D = 0 \). Let \( u = V_A - V_D \). Thus, firm \( D \) \( A \) becomes increasingly disadvantaged (advantaged) as \( u \) increases (decreases). Then it remains to show that \( D_1 \) and \( D_2 \) increase in \( u \) whereas \( A_1 \) and \( A_2 \) decrease. First we find the values of \( D_1, D_2, A_1 \) and \( A_2 \) from the definitions given in the text and get the following values:

\[
D_1 = \frac{(1-\gamma) \left[ 4(1-\gamma) \alpha_1 + 9\gamma \alpha_2 \right] e^2 + [8((1-\gamma) \alpha_1 + \gamma \alpha_2) u - 6 \alpha_1 ((4-3\gamma) \alpha_1 + 3\gamma \alpha_2)] e + 9 \alpha_2 \gamma (\alpha_1 - \alpha_2)^2}{72 \alpha_2 ((1-\gamma) \alpha_1 + \gamma \alpha_2)}
\]

\[
D_2 = \frac{(1-\gamma) \left[ -\alpha_1 (1-\gamma) e^2 + [2((1-\gamma) \alpha_1 + \gamma \alpha_2) u + 6 \alpha_1 \alpha_2] e + 9 \alpha_2 \gamma (\alpha_1 - \alpha_2)^2 \right]}{18 \alpha_2 ((1-\gamma) \alpha_1 + \gamma \alpha_2)}
\]

\(^{14}\)The values of these four quantities are given in the appendix.
\[ A_1 = \frac{(1-\gamma)\left[ (4(1-\gamma)\alpha_1 + 9\gamma\alpha_2)e^2 + [-8((1-\gamma)\alpha_1 + \gamma\alpha_2)u - 6\alpha_2 ((4 - 3\gamma)\alpha_1 + 3\gamma\alpha_2) e + 9\alpha_2 \gamma(\alpha_1 - \alpha_2)^2 \right]}{72\alpha_2 ((1-\gamma)\alpha_1 + \gamma\alpha_2)} \]

\[ A_2 = \frac{(1-\gamma)\left[ -\alpha_1 (1-\gamma)e^2 + [-2((1-\gamma)\alpha_1 + \gamma\alpha_2)u + 6\alpha_1 \alpha_2 e \gamma + 9\alpha_2 \gamma(\alpha_1 - \alpha_2)^2 \right]}{18\alpha_2 ((1-\gamma)\alpha_1 + \gamma\alpha_2)} \]

The derivatives of these values with respect to \( u \) are:

\[ \frac{\partial D_1}{\partial u} = \frac{(1-\gamma)e}{9} \quad \text{and} \quad \frac{\partial A_1}{\partial u} = \frac{(1-\gamma)e}{9} \]

Clearly the first term is strictly positive and the second term is strictly negative for all strictly positives values of \( e \) and \( \gamma \). Q.E.D.

**Proof of Proposition 3**

Suppose firm \( A \) is strictly advantaged: \( V_{A} < V_{D} \). It is sufficient to show that if firm \( A \) is rebating, firm \( D \) will choose to rebate as well. That is, we aim to show that \( D_2 \) will be positive if \( A_1 > 0 \) when \( e < e^* \). Solving \( D_2 - A_1 > 0 \) we find the unique \( e^* \) that is reported in the paper. Q.E.D.

**The derivation of the Mixed Strategy Equilibrium**

Let \((P_{RD}, P_{RA})\) be a mixed strategy equilibrium profile of the normal form game given in Table 1, where \( P_{ri} \) is the probability that firm \( i \) will offer a rebate. Since in any mixed strategy equilibrium each firm will rebate with probability so that its rival is indifferent between its two pure strategies, the following two equations must be satisfied:

\[ (1)(1) \quad r_{ADR} r_{AD} r_{AD} + r_{AD} r_{AR} P_{PP} - \Pi + \Pi = -\Pi + \Pi \]

\[ (1)(1) \quad r_{DAR} r_{DA} r_{DA} + r_{DA} r_{DR} P_{PP} - \Pi + \Pi = -\Pi + \Pi \]

Solving these two equations simultaneously we get the following unique solution:

\[ P_{RD} = \frac{8e((1-\gamma)\alpha_1 + \gamma\alpha_2)(V_{A} - V_{D}) - 9\gamma\alpha_2 (e + \alpha_1 - \alpha_2)^2 - 4\alpha_2 e((1-\gamma)e - 6\alpha_2)}{9\gamma\alpha_2 (e + 3(\alpha_1 - \alpha_2)) - 8\alpha_1 (1-\gamma)e^2} \]

\[ P_{RA} = \frac{8e((1-\gamma)\alpha_1 + \gamma\alpha_2)(V_{D} - V_{A}) - 9\gamma\alpha_2 (e + \alpha_1 - \alpha_2)^2 - 4\alpha_2 e((1-\gamma)e - 6\alpha_2)}{9\gamma\alpha_2 (e + 3(\alpha_1 - \alpha_2)) - 8\alpha_1 (1-\gamma)e^2} \]

**Proof of Proposition 4**

Suppose firm \( A \) is advantaged and firm \( D \) is disadvantaged, i.e. \( V_{A} - V_{D} > 0 \). Using the probabilities given for the mixed Nash equilibrium case, we find

\[ P_{RD} - P_{RA} = \frac{16e((1-\gamma)\alpha_1 + \gamma\alpha_2)(V_{A} - V_{D})}{9\gamma\alpha_2 (e + 3(\alpha_1 - \alpha_2)) - 8\alpha_1 (1-\gamma)e^2}. \]

The denominator has to be positive to have positive rebating probabilities. Therefore \( P_{RD} - P_{RA} > 0 \) if \( 16e((1-\gamma)\alpha_1 + \gamma\alpha_2)(V_{A} - V_{D}) > 0 \). This inequality clearly holds for all \( \gamma < 1 \). Q.E.D.

**Solutions for the slippage scenarios**

The four slippage scenarios lead to maximization of the profit functions defined in equations 7 and 8. We follow a two-stage solution procedure using backward induction mechanisms earlier. First, we derive the market share figures for firm \( D \) in segments 1 and 2 for each scenario (scenario 4 is shown):

\[ \tau_1 = \frac{1 - u + \alpha - P_{D} + R_{D} + P_{A} - R_{A}}{2} \quad \text{and} \quad \tau_2 = \frac{1 - u + \alpha - P_{D} + P_{A}}{2} \]

where \( u \) is defined as \( V_{A} - V_{D} \). The solutions for the four scenarios are given below. We use the superscript \( s \) to denote the solutions for the slippage cases.

**Scenario 1: Neither firm rebates**

\[ \Pi_{DO}^s = \frac{u^2 - \frac{u}{2} + \alpha}{18} \quad \Pi_{AO}^s = \frac{u^2}{18} + \frac{u}{3} + \frac{\alpha}{2} \]
\[ P_{\text{DD}}' = \alpha - \frac{1}{3}u \]
\[ P_{\text{AO}}' = \alpha + \frac{1}{3}u \]

Scenario 2: Only Firm D rebates
\[ \Pi_{\text{D}} = -\frac{1}{2} \gamma ((s^3 - 5s^2 + 8s - 4)\gamma + s^2 - s^3)u^2 - \frac{1}{2} \gamma ((3s^3\alpha - 48\alpha\alpha - 6s^3\alpha + 24\alpha)\gamma - 6s^2\alpha + 6s^3\alpha)u \]
\[ \Pi_{\text{A}} = \frac{1}{2} \gamma ((-36\alpha^2 + 72s\alpha^2 - 45s^2\alpha^2 + 9s^3\alpha^2)\gamma + 9s^2\alpha^2 - 9s^3\alpha^2) \]
\[ \Pi_{\text{DR}} = \frac{2}{2} ((6 - 6s + s^2)\gamma - s^3)^2 \alpha \]
\[ \Pi_{\text{ADR}} = \frac{2}{2} ((-s^2\gamma + s^2\gamma \alpha + \gamma u - \gamma s + 3\alpha\gamma s^2)^2 \]
\[ \Pi_{\text{PR}} = \frac{(3\alpha - u)\gamma - 3\alpha + u) s^2 + ((-9\alpha + 3u)\gamma + 3\alpha - u) s + (6\alpha - 2u)\gamma}{(\gamma - 1)s^2 - 6\gamma s + 6\gamma} \]
\[ R_{\text{IR}} = \frac{(3\alpha - u)s}{(\gamma - 1)s^2 - 6\gamma s + 6\gamma} \]
\[ P_{\text{ADR}} = \frac{(-\alpha + \gamma s)\gamma s + (-3\alpha - u)\gamma s + (u + 3\alpha)\gamma}{(\gamma - 1)s^2 - 6\gamma s + 6\gamma} \]

Scenario 3: Firm A rebates
\[ \Pi_{\text{AR}} = \frac{1}{2} \gamma ((s^3 - 5s^2 + 8s - 4)\gamma + s^2 - s^3)u^2 + \frac{1}{2} \gamma ((3s^3\alpha - 48\alpha\alpha - 6s^3\alpha + 24\alpha)\gamma - 6s^2\alpha + 6s^3\alpha)u \]
\[ \Pi_{\text{RAR}} = \frac{1}{2} \gamma ((-36\alpha^2 + 72s\alpha^2 - 45s^2\alpha^2 + 9s^3\alpha^2)\gamma + 9s^2\alpha^2 - 9s^3\alpha^2) \]
\[ \Pi_{\text{DR}} = \frac{2}{2} ((6 - 6s + s^2)\gamma - s^3)^2 \alpha \]
\[ \Pi_{\text{ADR}} = \frac{2}{2} ((-s^2\gamma + s^2\gamma \alpha + \gamma u - \gamma s + 3\alpha\gamma s^2)^2 \]
\[ \Pi_{\text{AR}} = \frac{(3\alpha + u)\gamma - 3\alpha - u) s^2 + ((-9\alpha + 3u)\gamma + 3\alpha + u) s + (6\alpha + 2u)\gamma}{(\gamma - 1)s^2 - 6\gamma s + 6\gamma} \]
\[ R_{\text{AR}} = \frac{s(3\alpha + u)}{(1 + \gamma)s^2 - 6\gamma s + 6\gamma} \]

Scenario 4: Both firms rebate
\[ \Pi_{\text{DD}} = \frac{(1 - s)\gamma u^2 - \alpha u((6 - 6s + 2s^2)\gamma - 2s^3) + \alpha^2(9\gamma(1 - s) - 2s^2(1 - s))}{2\alpha(9\gamma(1 - s) - 2s^2(1 - s))} \]
\[ \Pi_{\text{AD}} = \frac{(1 - s)\gamma u^2 + \alpha u((6 - 6s + 2s^2)\gamma - 2s^3) + \alpha^2(9\gamma(1 - s) - 2s^2(1 - s))}{2\alpha(9\gamma(1 - s) - 2s^2(1 - s))} \]
\[ R_{\text{DD}} = \frac{(1 - s)s\gamma u + \alpha s(9\gamma(1 - s) - 2s^2(1 - s))}{\gamma(1 - s)(9\gamma(1 - s) - 2s^2(1 - s))} \]
Proof of proposition 5

Using equations 7 and 8 and substituting the values of market shares (τ₁ and τ₂), we find the following two partial derivatives of the profit functions with respect to rebate amounts:

\[
\frac{\partial \Pi}{\partial R_A} = (1 - \gamma)(1 - s)(u - \alpha) - (1 - s)P_A + (1 - s)R_A + (2 - s)P_D - 2(1 - s)R_D, \quad \text{and}
\]

\[
\frac{\partial \Pi}{\partial R_D} = (1 - \gamma)(1 - s)(u + \alpha) - (1 - s)P_D + (1 - s)R_D + (2 - s)P_A - 2(1 - s)R_A.
\]

Setting the rebate amounts equal to zero and prices to their no rebate scenario (1) gives

\[
\left. \frac{\partial \Pi}{\partial R_A} \right|_{R_A = 0, R_D = 0} = \frac{s(1 - \gamma)(3\alpha - u)}{6\alpha} \quad \text{and} \quad \left. \frac{\partial \Pi}{\partial R_D} \right|_{R_A = 0, R_D = 0} = \frac{s(1 - \gamma)(3\alpha + u)}{6\alpha}.
\]

It is readily seen that the second derivative is non negative for all positive values of the model parameters while the first term is non negative if the price of the disadvantaged firm is positive when neither firm offers rebates (P_D ≥ 0).

Setting the rebate values equal a common (R) value

\[
\frac{\partial \Pi}{\partial R_A} = (1 - \gamma)(1 - s)(u - \alpha) - (1 - s)P_A - (1 - s)R + (2 - s)P_D, \quad \text{and}
\]

\[
\frac{\partial \Pi}{\partial R_D} = (1 - \gamma)(1 - s)(u + \alpha) - (1 - s)P_D - (1 - s)R + (2 - s)P_A.
\]

Simultaneously maximizing firm profits leads to the following prices:

\[
P_D^* = \operatorname{argmax}(\Pi_D)_{R_A = 0, R_D = P_D} = R(1 - s)(1 - \gamma) + \alpha - \frac{u}{3} \quad \text{and}
\]

\[
P_A^* = \operatorname{argmax}(\Pi_A)_{R_A = 0, R_D = P_D} = R(1 - s)(1 - \gamma) + \alpha + \frac{u}{3}.
\]

Substituting these optimal prices into the functions of the partial derivatives with respect to rebate amounts and calculating the difference results in:

\[
\left[ \frac{\partial \Pi_A}{\partial R_A} - \frac{\partial \Pi_D}{\partial R_D} \right]_{R_A = 0, R_D = P_D} = \frac{(1 - \gamma)su}{3\alpha}.
\]

Note that the difference between the two derivatives is strictly positive for all positive values of model parameters. Therefore, the advantaged firm stands to gain more from offering a rebate. In equilibrium both firms rebate and the advantaged firm’s rebate is larger. Q.E.D.

Proof of corollary 1

The proof follows directly from the solutions of scenarios 2,3 and 6. The derivatives of the rebate amounts with respect to asymmetry (u) are:
\[
\frac{\partial R^*_{BR}}{\partial u} = \frac{-s}{(\gamma - 1)s^2 - 6\gamma s + 6\gamma} < 0
\]
\[
\frac{\partial R^*_{IE}}{\partial u} = \frac{s}{(-1 + \gamma)s^2 - 6\gamma s + 6\gamma} > 0
\]
\[
\frac{\partial R^*_{DD}}{\partial u} = \frac{-(1 - s)s\gamma}{\gamma(1 - s)(9\gamma(1 - s) - 2\gamma^2(1 - \gamma))} < 0
\]
\[
\frac{\partial R^*_D}{\partial u} = \frac{(1 - s)s\gamma u + \alpha s(9\gamma(1 - s) - 2\gamma^2(1 - \gamma))}{\gamma(1 - s)(9\gamma(1 - s) - 2\gamma^2(1 - \gamma))} > 0
\]

The denominator values for all the derivatives above are strictly positive because their second order conditions for maximization require the determinant of the Hessian matrices to be non-negative. We omit the derivation of the Hessian matrices. Given the conditions, the inequalities above are easily verified.

Q.E.D.