Vertical Information Sharing in a Volatile Market

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Abstract

When demand is uncertain, manufacturers and retailers often have private information on future demand, and such information asymmetry impacts strategic interaction in distribution channels. In this paper, we investigate a channel consisting of a manufacturer and a downstream retailer facing a product market characterized by short product life, uncertain demand, and price rigidity. Assuming the firms have asymmetric information about the demand volatility, we examine the potential benefits of sharing information and contracts that facilitate such cooperation. We conclude that under a wholesale price regime, information sharing may not improve channel profits when the retailer underestimates the demand volatility but the manufacturer does not. Although information sharing is always beneficial under a two-part tariff regime, it is in general not sufficient to achieve information sharing and additional contractual arrangements are necessary. The contract types we consider to facilitate sharing are profit sharing and buy back contracts.

Keywords: Channels of Distribution, Decisions under Uncertainty, Game Theory, Supply Chains
If he had 2003 to do over again, Mr. Marineau (CEO of Levi Strauss) said, he wouldn’t have launched the company’s hyped “Type 1” jeans, which adorned the cover of the company’s 2002 annual report. ... Levi marketing materials promised “the boldest, most provocative Levi’s jeans in decades.”... But the line was too cutting-edge for mainstream consumers and the company was slow to correct fashion mistakes when demand failed to materialize, Levi executives said, after posting its largest annual loss.


1. Introduction

Demand uncertainty is ubiquitous in essentially any marketplace. Understanding it is one of the constant challenges firms face on the road towards success. As the example illustrates, failure to do so often has devastating effects on the bottom line. Levi Strauss, like many other manufacturers of apparel, introduces new fashions every year. Given its years of experience, Levi Strauss has a relatively clear picture on what the demand will be on average. The actual demand realization, however, is uncertain. Some of the numerous factors that influence the demand realization are weather, opinion leaders’ attitudes, popularity of competing products, etc. Put differently, there is a range of possibilities surrounding the average demand so that the actual demand realization can be either higher or lower than the average demand. Although the manufacturer has a good sense of the average demand, the range of the actual demand realization is impacted by so many different factors beyond the manufacturer’s control that the only appropriate notion is that it is determined by nature. We can repeat the above scenario by replacing the term manufacturer with retailer; we may also replace fashion apparel with many other highly perishable goods such as fresh produce, movies.

In distribution channels, information asymmetry across channel members with respect to demand uncertainty further complicates matters. In industries ranging from grocery to aerospace,
manufacturers and retailers often make independent demand estimates with different degrees of accuracy. Many factors, such as the proximity to consumers, the availability and quality of customer databases, sophistication of decision support tools, and experience, contribute to the accuracy of the demand estimate. Consistent with the opening example, we motivate the core issue of this paper in the context of a distribution channel of fashion apparel. Before the season begins, the retailer needs to place an order from the manufacturer. The retailer knows that if she orders too late (say, after the season begins), she will forgo sales. At this point, the only information available is the demand estimates (private information) of the manufacturer and the retailer and each party’s profile (public information). However, neither the manufacturer nor the retailer knows the other party’s estimate unless it is purposely revealed. Is there any value in sharing the two parties’ demand estimates? If the answer is positive, what mechanism(s) can help accomplish information sharing?

There is ample evidence that members of a distribution channel can benefit greatly by sharing demand information. According to an A.T. Kearney report (Field, 2005), the average manufacturer has enjoyed benefits equivalent to $1 million in savings for every $1 billion of sales by synchronizing their demand databases with their retailers. However, it is not clear that information sharing is always feasible in decentralized channels as it is marred by incentive and credibility problems. In the PC industry, manufacturers often suspect their distributors of submitting “phantom orders”, i.e., forecasts of high future demand that do not materialize (Zarley and Damore, 1996). Therefore, the value of information sharing and the mechanisms for facilitating information sharing are topics of great managerial importance and have received considerable attention in the literature (e.g., Cachon and Fisher, 2000; Cachon and Lariviere, 2001; Chen, 2003; Gal-Or, 1986; Gu and Chen, 2005; Kulp, Lee and Ofek, 2004; Lee, Padmanabhan and Whang, 1997; Lee, So and Tang, 2000; Li, 2002; Niraj and Narasimhan, 2002; Villas-Boas, 1994; Vives, 1984).

In this paper, we focus on a product market characterized by short product life, uncertain demand, and price rigidity (lack of price promotion). A catalogue marketer of fashion goods is a prototypical
example of such a market. Due to the inherent uncertainty about consumer tastes, fashion goods are synonymous with rapid change and subject to significant demand variability. We investigate a distribution channel with one manufacturer and one retailer, each with private information about the distribution of future demand. Our objective is to examine the benefits of sharing information about future demand volatility and contracts that facilitate such cooperation. More precisely, we consider multiplicative demand uncertainty, uniformly distributed between an upper and a lower bound about which the members of the channel have asymmetric information. To capture the impact of uncertainty, we model the retailer as a newsvendor problem with endogenous pricing. This allows us to balance the expected cost of ordering too much with the expected opportunity cost of ordering too little, when maximizing the retailer profits. The manufacturer is assumed to operate under a constant marginal cost, without any capacity constraints.

Several notable features emerge from our model setup. First, the environment under consideration is a one-shot, highly volatile setting; firms engage in short-term relationships and cannot refine their demand estimates by using historical data. Second, firms’ demand estimates may either overshoot or undershoot the true demand volatility because the demand is intrinsically stochastic, and the correlation between the firms’ demand estimates is unknown. Thus, there is no obvious way to combine or pool them into a more accurate estimate.

We find that sharing information about market demand volatility does not always improve channel profits. In particular, under a wholesale price only regime, when the retailer underestimates the demand variability but the manufacturer does not, sharing can lead to lower channel profits. In addition, we show that the feasibility of information sharing not only depends on the asymmetric estimates of the demand volatility but also on the channel members’ knowledge of the quality of each member’s forecast. Specifically, we demonstrate that a profit sharing contract is a viable mechanism when each channel member is aware of the quality of the other member’s demand forecast relative to its own,
whereas buy back contracts can improve channel coordination when only one member knows whose
demand forecast is more accurate.

We also find that under a two-part tariff regime, sharing information about market demand
volatility always improves channel profits, but additional contractual arrangements are needed for
sharing to occur. In contrast to the symmetric case, under asymmetric information about demand
volatility, a two-part tariff does not always coordinate the channel.

Three streams of literature are relevant to our study: the literature on channel coordination, the
literature on newsvendor models with endogenous pricing decisions, and the emerging literature on
information sharing. The extant channel literature generally assumes deterministic demand (e.g.,
Jeuland and Shugan, 1983; McGuire and Staelin, 1983). The limited channel literature that
incorporates demand uncertainty usually assumes that uncertainty is a white noise and has no
consequence as firms maximize expected profits (e.g., Lal, 1990), or models uncertainty as consisting
of a high state and a low state (e.g., Biyalogorsky and Koenigsberg, 2004; Desai, 2000; Padmanabhan
and Png, 1997; Gu and Chen, 2005). These papers primarily consider the impact of uncertainty in the
demand trend (i.e., the mean), but do not focus on the impact of asymmetric information about demand
volatility. Such treatment allows researchers to obtain a high level of parsimony, which facilitates the
analysis of complex issues such as channel structure, upstream/downstream competition, multi-period
interaction, etc. However, this parsimony is obtained at the cost of generality. In particular, the actual
demand realization may be either above or below the demand trend and firms incur costs in either case.
By focusing on demand trend but ignoring volatility, one assumes that the costs associated with
undershooting and overshooting are identical (we shall revisit this point in details in Section 3.3). This
assumption becomes very troublesome when demand volatility is large, as in the case of highly
perishable goods. We complement this literature by looking at demand uncertainty from a different
perspective, focusing on the impact of asymmetric estimates of the demand volatility in a newsvendor
setting with uniformly distributed demand uncertainty.
The classical newsvendor approach offers an intuitive way to incorporate the effect of demand volatility into the analysis of channel behavior. In a one-period setting, the retailer (newsvendor) trades off the marginal cost of understocking with that of overstocking when determining how much to order. In its traditional formulation, the newsvendor model determines the quantity to order under the assumption that price is an exogenously set parameter which does not affect demand (See Porteus (1990) for a comprehensive review of general newsvendor problems). The impact of adding pricing decisions to the newsvendor problem was first considered by Whitin (1955). Mills (1959) considers this problem under the assumption of additive demand uncertainty, while Karlin and Carr (1962) assume multiplicative demand uncertainty. For a comprehensive review of the newsvendor problem with endogenous pricing we refer to Petruzzi and Dada (1999), who propose a unified view of the above mentioned approaches.

Demand uncertainty and the use of local demand information often lead to severe distortion and loss of channel profits. A related issue is the inherent amplification of demand variability as local demand information is transmitted along a distribution channel. The phenomenon, which is due to successive distortion of the demand information, is often referred to as the “bullwhip effect.” It was first illustrated in Forrester (1958) and more recently analyzed by Lee, Padmanabhan and Whang (1997), the latter spawning a large number of publications on this issue, for which one remedy is information sharing.

Lee, So and Tang (2000) study information sharing in supply chains with long-term relationships and find that the value of information sharing is high when the demand correlation over time is high and when the demand variance within each time period is high. However, information sharing is complicated by incentive and credibility problems that arise from information asymmetry. Cachon and Lariviere (2001) explore capacity decisions in a supply chain with one manufacturer and one supplier (retailer). They show that the use of forcing contracts (involuntary compliance) vs. self-enforcing contracts (voluntary compliance) significantly impacts the outcome of demand forecast sharing. Li
(2002) examines the incentives for firms to share information vertically in a supply chain with one manufacturer and many retailers. He shows that voluntary information sharing is often infeasible when the retailers are engaged in a Cournot competition and are endowed with some private information. For excellent reviews of information sharing and contracts in supply chain coordination, readers are referred to Cachon (2003) and Chen (2003).

Our paper differs from the above mentioned literature in two aspects. First, we study the value of information sharing in a distribution channel in which the members have asymmetric information about demand volatility. Second, we examine the mechanisms for facilitating information sharing in the context of short-term relationships, where cooperative behavior is rare and forcing contracts of any kind are difficult to implement.

The remainder of the paper is organized as follows. Section 2 lays out the key elements of the model. Section 3 analyzes the basic model, in which a manufacturer and a retailer with asymmetric information about demand volatility operate under a wholesale price only regime without information sharing. Section 4 explores the value of sharing this type of information, and how to facilitate sharing using different contractual agreements. Section 5 considers the impact of sharing information under a two-part tariff regime instead of a wholesale price mechanism. Section 6 concludes.

2. Assumptions and model set-up

The distribution channel we consider consists of a manufacturer and a downstream retailer; both firms are risk neutral and maximize their expected profits. Given that \( p \) represents the retail price, end-customer demand is given by \( D(p) = y(p)\varepsilon \), where \( y(p) = ap^{-b} \) (defined for \( a > 0, b > 1 \)) is a price dependent deterministic function, and \( \varepsilon \) is a uniformly distributed random variable with mean of 1. More precisely, \( \varepsilon \in U[1-L,1+L] \) for \( 0 \leq L \leq 1 \), with \( f(.) \) and \( F(.) \) denoting its pdf and cdf. Hence, end-customer demand is determined by a random draw from the distribution \( U[1-L,1+L] \), where nature
determines the extent of uncertainty $L$. The demand is intrinsically stochastic with an average of $y(p)$, a standard deviation of $y(p)L/\sqrt{3}$, and a volatility which decreases (increases) as $L$ tends to 0 (1). It follows that the average demand $y(p)$ tends to 0 as $p$ grows very large, and goes to infinity as $p$ approaches 0. This property is innocuous and is equivalent to the usual assumption that the demand reaches the market potential (usually a constant) when the product is free. The parameter $b$ measures price sensitivity. We assume that the price elasticity of the average demand $b$ is common knowledge and that $b > 1$ so that the demand is sufficiently elastic.

In our model, the random variable $\varepsilon$ captures demand uncertainty, the functional form of its distribution (uniform) and its mean are common knowledge. The only unknown is the variance of $\varepsilon$, which is characterized by the lower and upper bounds of the distribution $1-L$ and $1+L$. As such, the manufacturer and the retailer need to form estimates of $L$ independently.

The steps in our model are as follows. First, the manufacturer determines the wholesale price $w$, using its private demand estimate, $\varepsilon_m$, to predict the retailer’s reaction. We assume that the manufacturer has a constant marginal cost, $c$, unknown to the retailer, and does not have any capacity constraints. (The assumption that $c$ is private information is motivated by the fact that the marginal costs are highly dependent upon the manufacturer’s organizational capabilities, which in general are unknown to the retailer.) Subsequently, the retailer, with the wholesale price $w$ at hand, sets the retail price, $p$, and places an order, $q$, using its own demand estimate $\varepsilon_r$. Finally, the demand $D(p)$ is realized. If $D(p) > q$, the retailer experiences a shortage and incurs an opportunity cost in terms of profits lost, $(p-w)[D(p)-q]$; if $D(p) < q$, the retailer has leftover products over which it incurs a loss, $w[q-D(p)]$. We assume that neither the retailer nor the manufacturer incurs any goodwill cost in the event of shortage, and that any leftover products have zero salvage value. However, it would be possible to extend our model to incorporate these.
The resulting framework can be viewed as a one-period game of incomplete information, and the appropriate solution concept in this setting would be the Bayesian-Nash equilibrium. Specifically, the firms’ optimal strategies need to be consistent with their beliefs derived from Bayes rule. In our framework, however, the beliefs do not impact our results. This is because (i) as we will show in Section 3.2, the optimal wholesale price \( w \) is independent of \( L_m \) and does not convey any useful information about the manufacturer’s estimate to the retailer, and (ii) even though the manufacturer may deduce the retailer estimate \( L_r \), this has no consequence on the optimal wholesale price, and the equilibrium solution remains unchanged. Therefore, it is rational for a firm to use its own estimate. Formally, the non-Bayesian nature of the game is illustrated by the following claims:

**Claim 1:** The manufacturer and the retailer form point estimates of \( L \).

Proof: All proofs are in the Appendix.

**Claim 2:** The manufacturer’s wholesale price \( w \) is uninformative.

**Claim 3:** Without information sharing (\( L_m \) is unknown to the retailer), it is rational for the retailer to use her own estimate \( L_r \); with information sharing, a known \( L_m \) triggers an all-or-nothing update.

Now suppose that the manufacturer and the retailer each form a point estimate of \( L \), their beliefs on \( \varepsilon \) are then given by \( \varepsilon_m \in U[1-L_m, 1+L_m] \) and \( \varepsilon_r \in U[1-L_r, 1+L_r] \), respectively. Note that it is possible that \( L_i \leq L \) or \( L_i \geq L \), where \( i = r,m \). In words, the manufacturer and the retailer can either overestimate or underestimate \( L \). Since the mean of the demand distribution is known, a firm’s demand estimate is superior if its estimated variance is closer to the true state. According to our definition, \( L_i \) is superior to \( L_j \) if \( |L - L_i| < |L - L_j| \). At the same time, the manufacturer and the retailer each receive a signal \( \sigma_i \) that reveals the type of the other player. For simplicity, we assume that each player can be of three types \( \{h, \emptyset, l\} \), where \( h \) type is superior, \( \emptyset \) type is the same as the other player or unknown, \( l \)
type is inferior. In sum, the information set of each player is given by \( \{L_i, \sigma_i\} \) (see Figure 1). While information about the firm’s own demand volatility estimate \((L_m \text{ or } L_r)\) is always private, information about which estimate is superior (if it exists) can be either private or common knowledge. Put differently, a firm’s demand estimate is only known to itself, but the quality of this estimate may be known to the other firm. For example, when forecasting the sales of a new novel, a national book store such as Amazon may have a demand estimate of \( n \pm x \) units, while a regional publisher’s demand estimate is \( n \pm y \) units. This information is strictly private unless purposely revealed. On average, one would expect Amazon’s estimate to be superior because of its sophisticated CRM (customer relationship management) solutions and national presence (thus \( \sigma_r = h, \sigma_m = l \)). However, the regional publisher can generate a superior estimate through proprietary marketing research (this may or may not be known to Amazon, thus \( \sigma_r = l, \sigma_m = h \) or \( \emptyset \)). If the regional publisher deals with a local bookstore that has similar marketing capability as itself, \( \sigma_r \) dictates that the manufacturer is of type \( \emptyset \) and vice versa, etc. Note that the cases where \( \sigma_r = \sigma_m = l \), \( \sigma_r = \sigma_m = h \) are equivalent to \( \sigma_r = \sigma_m = \emptyset \). We summarize the alternative information structure implied by the signal generating process of \( \sigma_i \) in Table 1.
Table 1: Alternative information structures

<table>
<thead>
<tr>
<th>Whose estimate of demand volatility is superior</th>
<th>Only M knows</th>
<th>Only R knows</th>
<th>Neither firm knows</th>
<th>Both firms know</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Scenario 1a</td>
<td>Scenario 2a</td>
<td>Scenario 3</td>
<td>Quality is common knowledge</td>
</tr>
<tr>
<td>$\sigma_m = l, \sigma_r = \emptyset$</td>
<td>$\sigma_m = \emptyset, \sigma_r = h$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>Scenario 1b</td>
<td>Scenario 2b</td>
<td>$\sigma_i = \sigma_j = \emptyset$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_m = h, \sigma_r = \emptyset$</td>
<td>$\sigma_m = \emptyset, \sigma_r = l$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When the quality of the demand estimates is private information, there are three possible scenarios (see Table 1): (1) the manufacturer knows whose estimate is superior but the retailer does not, (2) the retailer knows whose information is superior but the manufacturer does not, and (3) neither firm knows who has better demand estimate. As the model unfolds, we shall see that these alternative information structures generate various incentive and credibility issues and hence impact the feasibility of information sharing. We will explore these implications in details in Section 4.

3. The Basic Model

The basic model examined in this section focuses on situations where the manufacturer and the retailer interact using a wholesale price only regime. Situations in which the manufacturer and the retailer interact using a two-part tariff mechanism are analyzed in Section 5. The wholesale price only contract implies that the only interaction between the retailer and the manufacturer occurs when the manufacturer posts its wholesale price and when the retailer places its order. As such, our basic model considers the impact of demand volatility and asymmetric information when information sharing does not occur. The results obtained here serve as the basis for analyzing the value and feasibility of information sharing in Section 4.

To analyze the framework and determine the equilibrium decisions we use backward induction. To illustrate the impact of demand volatility, we first consider the case in which both the retailer and the
Section 3.1 derives the retailer’s profit maximizing strategy for this case, given the wholesale price determined by the manufacturer. Based on these results, the manufacturer’s decision problem is analyzed in Section 3.2. Subsequently, section 3.3 provides insights as to the impact of demand volatility on the equilibrium strategies. Finally, section 3.4 discusses the impact of information asymmetry on the equilibrium strategies. Although Sections 3.1 and 3.2 are integral components of our analysis, they focus on the technical aspects of the model and are relatively self-contained. Readers who are not interested in the underlying mathematics of the model development can go directly to Sections 3.3 and 3.4 to obtain the key insights from the model.

3.1 Symmetric Information: Retailer Profit Maximizing Strategy

The retailer profit for the period, \( \pi_r \), is the difference between its sales revenues and purchase costs. Assuming a wholesale price \( w \), a retail price, \( p \), an order quantity, \( q \), and the demand \( D(p) \), \( \pi_r \) can be expressed as follows:

\[
\pi_r(q, p, w) = \begin{cases} 
   pD(p) - wq & \text{for } D(p) \leq q \\
   (p-w)q & \text{for } D(p) > q 
\end{cases}
\]  

(1)

Following the approach outlined, for example, in Petruzzi and Dada (1999), we express all quantities relative to the deterministic price-dependent component \( y(p) \) by defining what we refer to as the mean-adjusted order quantity \( z = q/y(p) \). This results in an alternative mean-adjusted representation, which clarifies the impact of demand uncertainty and simplifies the analysis to come.

\[
\pi_r(z, p, w) = y(p) \cdot \begin{cases} 
   (p-w)e - w(z-e) & \text{for } e \leq z \\
   (p-w)e - (p-w)(e-z) & \text{for } e > z 
\end{cases}
\]  

(2)

If the retailer has accurate information ( \( L_r = L \) ), the expected profit that the retailer attempts to maximize when deciding \( p \) and \( z \) (or equivalently \( q \)) can be expressed as


\[
E_\varepsilon \{\pi_r(z, p, w)\} = y(p) \left[ \int_{1-L}^{1+L} (p-w)xf(x)dx \right.
- w \int_{1-L}^{1+L} (z-x)f(x)dx - (p-w) \int_{z}^{1+L} (x-z)f(x)dx \bigg] \tag{3}
\]

Defining

\[
\Psi(p,w) = \Psi(p)(p-w) \quad \text{and} \\
\Lambda_\varepsilon(z, p, w) = y(p) \left[ w \int_{1-L}^{1+L} (z-x)f(x)dx + (p-w) \int_{z}^{1+L} (x-z)f(x)dx \right]
= y(p) \left[ LpF^2(z) + (p-w)(1-z) \right]
\]

we can also write

\[
E_\varepsilon \{\pi_r(z, p, w)\} = \Psi(p, w) - \Lambda_\varepsilon(z, p, w) \tag{4}
\]

Hence, the expected profit can be expressed as the riskless profit, \(\Psi(p,w)\), which represents the profit obtained by the retailer in the deterministic problem (e.g. \(L=0\)), less the expected loss that results from the presence of uncertainty, expressed by the loss function \(\Lambda_\varepsilon(z, p, w)\). Note that the expected loss equals the sum of the expected cost of ordering too much and the expected opportunity cost of ordering too little.

To determine the optimal price \(p_r^*(w)\) and mean-adjusted order quantity \(z_r^*(w)\) that maximize (3) and (4) for a given \(w\), we consider the first-order optimality conditions and proceed by taking the first partial derivatives of \(E_\varepsilon \{\pi_r(z, p, w)\}\) with respect to \(z\) and \(p\):

\[
\frac{\partial E_\varepsilon \{\pi_r(z, p, w)\}}{\partial z} = - \frac{\partial \Lambda_\varepsilon(z, p, w)}{\partial z} = y(p) \left[ (p-w) - pF(z) \right]
\]

\[
\frac{\partial E_\varepsilon \{\pi_r(z, p, w)\}}{\partial p} = y(p) \left[ z(1 - b \frac{p-w}{p}) + (b-1)LF^2(z) \right]
\]

Observe that for any given \(p\) the optimal mean-adjusted order quantity \(z_r^*(p,w)\) corresponds to the standard newsvendor result, that is,
\[ z^*_r(p, w) = F^{-1}(\frac{p - w}{p}) = 1 - L + 2L \frac{p - w}{p}. \]  

(5)

It is easy to see from (5) that \( z^*_r(p, w) \) is increasing in \( p \). Intuitively, the explanation for this is that the opportunity cost for every unsatisfied demand \( (p - w) \) increases with \( p \), while the cost of every surplus item remains constant at \( w \).

Similarly, for any given \( z \) we can determine the optimal price \( p^*_r(z, w) \):

\[ p^*(z, w) = w \left( \frac{b}{b-1} \right) \left( \frac{z}{z - LF^2(z)} \right) \]  

(6)

Observe that whenever demand is deterministic (e.g. \( L = 0 \)), the optimal riskless price \( p^*_r(w) = \frac{w}{b-1} \).

Moreover, we note that \( z - LF^2(z) < z \) since both \( L \) and \( F(z) \) are non-negative, and that \( z - LF^2(z) > 0 \) since \( L \leq 1 \) and \( F^2(z) \leq F(z) \leq z \). Thus, equation (6) illustrates the well-known result (Karlin and Carr, 1962) that the introduction of uncertainty will yield a price that is larger than the optimal riskless price, i.e. \( p^*_r(w) \geq \frac{w}{b-1} \).

Using (5) (or (6)), the retailer’s profit maximization problem can be reduced to an optimization problem over a single variable. Substituting the expression for \( z^*_r(p, w) \) into the expected profit function (4) renders the expression

\[ E_e \{ \pi_r(z^*_r(p, w), p, w) \} = y(p) \left( (p - w) \left( 1 - L \frac{w}{p} \right) \right), \]

which is decreasing in the demand volatility \( L \). Using the first order optimality condition with respect to \( p \), renders \( p^*_r(w) \) as specified in Lemma 1.

**Lemma 1:** For any given \( w \), the optimal retail price

\[ p^*_r(w) = \frac{w}{b-1} H(b, L), \]  

(7)
where

\[ H(b, L) = \frac{1 + L}{2} + \sqrt{\left(\frac{1 - L}{2}\right)^2 + \frac{L}{b^2}} > 1. \] (8)

Note that \([b/(b - 1)]H(b, L)\) can be viewed as a generalized double marginalization factor, which is minimized for the special case of deterministic demand (i.e., \(L = 0\)) where it degenerates to \(b/(b-1)\).

From Lemma 1 it is easy to determine the optimal mean adjusted order quantity \(z_r^*(w)\) by substituting \(p_r^*(w)\) for \(p\) in (5).

\[ z_r^*(w) = F^{-1}\left(1 - \frac{b - 1}{bH(b, L)}\right) = 1 - L + 2L \left(1 - \frac{b - 1}{bH(b, L)}\right) \] (9)

The corresponding actual order quantity, \(q_r^*(w)\), follows directly as \(q_r^*(w) = z_r^*(w)y(p_r^*(w))\).

3.2 Symmetric Information: Manufacturer Profit Maximizing Strategy

The manufacturer has to determine an optimal wholesale price \(w_m^*\). Given that, in the current case, the retailer and manufacturer have symmetric and accurate information, the manufacturer can predict the retailer’s response to any given order quantity he chooses. Consequently, the retailer’s profit function, as a function of the wholesale price, can be expressed as

\[ \pi_m(w) = (w - c)q_r^*(p_r^*(w), w) \]
\[ = (w - c) \left(1 + L - 2L \frac{b - 1}{H(b, L)} \right) \left(\frac{b}{b - 1} H(b, L)\right)^{-b} y(w) \] (10)

Note that \(\pi_m(w)\) is a deterministic function, that is, the manufacturer’s profit is determined by how much the retailer orders instead of actual sales during the period. This implies that the manufacturer does not expose itself to any risk associated with demand uncertainty. To determine the manufacturer’s optimal wholesale price \(w_m^*\), we use the first order optimality condition for \(\pi_m(w)\) with respect to \(w\),

\[ \frac{\partial \pi_m(w)}{\partial w} = \left(1 - b \frac{w - c}{w}\right) \left(1 + L - 2L \frac{b - 1}{bH(b, L)} \right) \left(\frac{b}{b - 1} H(b, L)\right)^{-b} y(w) = 0. \]
Solving for \( w \) yields

\[
    w_m^* = \frac{b}{b-1} c, \quad (11)
\]

It is interesting to note that \( w_m^* \) is in fact independent of the demand volatility \( L \), and worth pointing out that this is unlikely to hold if the manufacturer faces constrained capacity, which typically creates a complex relationship between \( c \) and the quantity ordered. Finally, we also note that in the case of deterministic demand \( (L = 0) \), (11) together with (6) illustrates the well-known double-marginalization principle.

### 3.3 Impact of Demand Volatility

From the above, we conclude that for given estimates of the demand volatility \( L \), the profit maximizing strategy for the manufacturer is to set a wholesale price \( w_m^* \) and the optimal response for the retailer is to order \( q_r^*(w_m^*) \) units which are sold at a retail price of \( p_r^*(w_m^*) \). To analyze the value of information sharing, an important question is how changes in the demand volatility estimates impact these decisions and the associated profits. In the remainder of this section we provide some structural properties that help answer these questions. First, however, we observe that since the manufacturer’s optimal wholesale price \( w_m^* \) is independent of \( L \) (see (11)), the profit maximizing strategies are unaffected by changes in the manufacturer’s estimate of the demand volatility. We therefore concentrate on the impact of demand volatility changes on the retailer’s strategy.

In the presence of demand volatility, the order quantity \( q_r^*(w) \) generally differs from the average demand \( y(p_r^*(w)) \) due to the asymmetry between the costs associated with understocking and overstocking.

**Lemma 2:** For any given \( L \), \( q_r^*(w) > y(p_r^*(w)) \) if and only if \( H(b, L) > 2 \left( \frac{b-1}{b} \right) \).
This is illustrated in Figure 2, which shows both the mean-adjusted and the absolute order quantity as a function of $L$. Clearly, $q^*_r(w) = y(p^*_r(w))$ when $L = 0$. However, as $L$ increases $q^*_r(w)$ will initially decrease below 1 (and therefore $q^*_r(w) < y(p^*_r(w))$), until we reach the value of $L$ for which $H(b, L) = 2\left(\frac{b-1}{b}\right)$. For larger values of $L$, $q^*_r(w)$ will be larger than 1.

In addition, the retailer can fine-tune the costs of overstocking and understocking since it has the ability to set prices, which will cause the retail price $p^*_r(w)$ to differ from the optimal riskless price $p^*_r(w) = w\frac{b}{b-1}$. As discussed in the introduction of this paper, the conventional approach to demand uncertainty focuses on the mean, but fails to account for these additional trade-offs that are caused by the introduction of demand volatility. This may become problematic when demand volatility is an important factor. We note that while a two-point distribution can also account for the effect of demand volatility, it is impossible to disentangle the effect of the mean from that of variance in such distributions, since any change in the variance will also imply a change in the mean.

To illustrate the impact of demand volatility, we first consider its impact on the optimal retail price.

**Proposition 1**: The profit maximizing retail price $p^*_r(w)$ is strictly increasing in $L$. 

Figure 2: Impact of demand volatility on order quantity.
Intuitively, by increasing the retail price, the retailer can reduce the demand variability (recall that its standard deviation is \( y(p)L/\sqrt{3} \) and that \( y(p) \) is decreasing in \( p \)) and counter the effect of an increase in \( L \) on the loss term \( \Lambda_r(z, p, w) \). Of course, an increase in the retail price also reduces the mean demand, \( y(p) \), which implies a reduction in the riskless profit \( \Psi(p, w) \).

To illustrate how demand volatility impacts the optimal order quantity \( q_r^*(w) \) we first observe that, while the mean-adjusted order quantity \( z_r^*(p, w) \) is increasing in \( p \), the actual order quantity \( q_r^*(p, w) = y(p)z_r^*(p, w) \) will have a maximum. Intuitively, this follows since any increase in \( z_r^*(p, w) \) are countered by a decrease in the mean demand \( y(p) = ap^{-b} \) as \( p \) increases.

**Lemma 3:** For any given \( w \), \( q_r^*(p, w) \) strictly increases with respect to \( p \) when
\[
p \leq w \left( \frac{b}{b+1} \right) \left( \frac{2L}{1+L} \right)
\]
and strictly decreases with respect to \( p \) when \( p \geq w \left( \frac{b}{b+1} \right) \left( \frac{2L}{1+L} \right) \).

**Corollary 1:** The profit maximizing order quantity \( q_r^*(w) \) is strictly decreasing in \( L \).

Observe that corollary 1 immediately follows from Proposition 1 and Lemma 3, since
\[
p^*(w) \geq w \frac{b}{b-1} > w \left( \frac{b}{b+1} \right) \left( \frac{2L}{1+L} \right).
\]

### 3.4 Impact of Information Asymmetry

Let us now consider the situation where the retailer and the manufacturer have asymmetric demand estimates, which may both be different from the true demand volatility \( L \). Thus, the retailer’s estimate \( L_r \) may be different from the manufacturer’s estimate \( L_m \).

In this case, the basic model is again derived from backward induction. However, when the retailer makes its decisions regarding retail price and order quantity to maximize its expected profits (given \( w \)), it does so *ex ante* without knowing \( D(p) \). Instead, it has to rely on its best demand estimate \( D_r(p) \). The associated profit function, and consequently the expected profit, are equivalent to (2) and (3) after
substituting $D_r(p)$ for $D(p)$ and $\varepsilon_r$ for $\varepsilon$, respectively. In the asymmetric case, the optimal retail price is given by $p^*_r(w) = w \frac{b}{b-1} H(b, L_r)$ whereas the optimal mean adjusted order quantity $z^*_r(w) = 1 - L_r + 2L_r \left(1 - \frac{b-1}{bH(b, L_r)}\right)$. In Section 3.2, we showed that the manufacturer’s optimal strategy is in fact independent of its demand estimate. Thus, even with asymmetric information the optimal wholesale price $w_m^* = \frac{b}{b-1} c$.

To analyze the impact of information asymmetry, we first observe that Proposition 1 and Corollary 1 imply that whenever the retailer underestimates demand volatility, the result will be a retail price that is too low and an order quantity that is too high. Conversely, if the retailer overestimates the demand volatility, the retail price will be too high and the order quantity too low. The impact on channel profits, however, is somewhat more complicated. After the profit maximizing strategies $(w_m^*, q_r^*(w_m^*))$ and $p_r^*(w_m^*)$ are revealed, the manufacturer’s profit, $\pi_m(w_m^*) = (w_m^* - c)q_r^*(w_m^*)$, is unambiguously defined. Note that $\pi_m(w_m^*)$ depends on $L_r$ via $q_r^*(w_m^*)$, but that it is independent of $L$ and $L_m$. The retailer’s expected profit, on the other hand, is a bit more ambiguous. From the retailer’s perspective, the demand estimate, $\varepsilon_r$, is correct. Under this demand estimate, its perceived expected profit is $E_{\varepsilon_r}\{\pi_r(z_r^*(w_m^*), p_r^*(w_m^*), w_m^*)\}$ as defined in (3) and (4). However, objectively, the true demand uncertainty is $\varepsilon$ (unknown to the retailer). Under the true demand distribution the retailer’s actual expected profit is $E_\varepsilon\{\pi_r(z_r^*(w_m^*), p_r^*(w_m^*), w_m^*)\}$. In analogy with (4) we have

$$E_\varepsilon\{\pi_r(z_r^*(w_m^*), p_r^*(w_m^*), w_m^*)\} = \Psi\left(p_r^*(w_m^*), w_m^*\right) - A_\varepsilon\left(z_r^*(w_m^*), p_r^*(w_m^*), w_m^*\right)$$ (12)

where

$$A_\varepsilon(z, p, w) = y(p) \left[ G(L, z) \int_{L-L}^{L+L} (z-x) f(x) dx + (p-w) \int_{G(L, z)} f(x) dx \right],$$ (13)
and

\[
G(L, z) = \begin{cases} 
1 - L & \text{if } z < 1 - L \\
z & \text{if } 1 - L \leq z \leq 1 + L \\
1 + L & \text{if } z > 1 + L 
\end{cases}
\]  

Clearly, if the retailer’s estimate is correct, i.e., \( \varepsilon_r = \varepsilon \) and \( L_r = L \), the retailer’s perceived expected profit coincides with its actual expected profit.

**Proposition 2**: Under the equilibrium strategies \( w_m^*, q_r^*(w_m^*) \) and \( p_r^*(w_m^*) \):

(i) The manufacturer’s profit, \( \pi_m(w_m^*) \), and the retailer’s perceived expected profit, 

\[
E_{\varepsilon_r}\{\pi_r(z_r^*(w_m^*), p_r^*(w_m^*), w_m^*)\},
\]

are decreasing in \( L_r \).

(ii) The retailer’s actual expected profit, 

\[
E_{\varepsilon r}\{\pi_r(z_r^*(w_m^*), p_r^*(w_m^*), w_m^*)\},
\]

is maximized when the retailer’s estimate of the demand distribution is correct, i.e., when \( L_r = L \).

(iii) When \( L_r = L \) and the retailer’s demand estimate is correct, the retailer’s actual expected profit is decreasing in the demand volatility \( L \).

From Proposition 2 we conclude that if the retailer overestimates the demand volatility (\( L_r > L \)), it has a negative impact on the profits of both the retailer and the manufacturer. On the other hand, an underestimation (\( L_r < L \)), will benefit the manufacturer. Proposition 2 also asserts that increased volatility, even when correctly estimated, reduces profits of both the retailer and manufacturer. The implications of this proposition on the potential benefits for the channel to share information about demand volatility are further discussed in Section 4.

4. Information Sharing in a Wholesale Price Only Regime

In the information sharing scenarios we consider, each firm knows its own estimate of demand volatility (\( L_m \) or \( L_r \)). In addition it may also have information about the quality of the other firm’s estimate relative to its own. More precisely, a firm might know which estimate is superior, that is,
whether \( L_m \) or \( L_r \) is closer to the true state \( L \). However, as discussed in Section 2, \( L \) as well as the correlation between the estimates are unknown. Under this information asymmetry, the firm with a superior estimate needs a mechanism to convey its information (about \( L_m \) or \( L_r \)) to the other firm when it is profitable to do so. The transmission of this information can only be successful if the other firm finds itself better off, or at least no worse off, to use such information. Throughout this paper, we say that information sharing occurs when such transmission is successful, that is, when the firms share their private information about demand volatility (\( L_m \) or \( L_r \)). An important observation explained in Sections 2 is that belief updating is moot for the situation we consider.

To better understand the incentive and credibility problems in information sharing, we outline the taxonomy of alternative information structures, summarized in Table 1 (on p. 10). We discuss each of these scenarios in turn.

In Scenario 1, if the manufacturer’s estimate is superior (Scenario 1a), he needs to credibly convey this information to the retailer when doing so improves his profit. The incentive problem arises since the manufacturer only wants to share his better information if the retailer’s order quantity is too low and hence hurts his profit. However, the manufacturer may also have an incentive to induce the retailer to order too much. Thus, the credibility problem also arises as the retailer does not know whose estimate is superior and would suffer from a lower profit if he orders too much. If the retailer’s estimate is superior (Scenario 1b), no action is necessary. The manufacturer simply complies with the retailer’s order and the retailer’s information is transferred to the manufacturer costlessly. Neither credibility nor incentive problems are present since the retailer has no incentive to manipulate the order quantity given his lack of knowledge on whose demand estimate is superior. In Scenario 2, credibility and incentive problems arise when the manufacturer possesses a better demand estimate (Scenario 2a), but no action is necessary if the retailer’s demand estimate is superior (Scenario 2b). The underlying logic is similar to that discussed in Scenario 1. In Scenario 3, it is not clear that information sharing is at all feasible. At least one firm should be able to evaluate the expected improvement in profits when
information sharing occurs, but such judgment cannot be made if neither firm knows whose demand estimate is superior. In sum, information sharing is feasible when at least one firm knows the quality of the demand estimates and when the firms can overcome the incentive and credibility problems.

In a coordinated channel, the two firms act as if they were a single entity, which implies the absence of double marginalization and a retail price and order quantity that maximize the channel profits. As we demonstrate in Section 3, the wholesale price by itself neither coordinates the channel nor achieves information sharing. Iyer and Villas-Boas (2003) show that, under deterministic demand and complete information, a wholesale price only contract coordinates the channel when the retailer’s bargaining power is sufficiently large. A bargaining framework, however, is beyond the scope of this paper. An alternative pricing scheme, the two-part tariff consisting of a fixed fee and a variable fee, coordinates the channel under symmetric information. However, it is not clear whether a two-part tariff achieves information sharing in the presence of information asymmetry. In this section, we examine the potential value of information sharing under a wholesale price regime, and explore how information can be shared in a mutually beneficial manner using profit sharing and buy back contracts. A parallel analysis under a two-part tariff regime is provided in Section 5.

4.1 The Value of Information Sharing

In Section 3 we analyze the impact of the manufacturer and retailer having asymmetric estimates of the demand volatility under a wholesale price only regime without information sharing. In this section we examine the potential value of sharing such information in terms of increased channel profits. We also consider the consequences for the profit maximizing strategies in terms of pricing and order quantities.

The fundamental question is whether information sharing always has a potential to improve channel profits and how information sharing impacts each firm’s expected profit. To simplify the analysis without sacrificing any insights, we henceforth consider a situation where the manufacturer’s estimate of demand volatility is perfect (i.e., \( L_m = L \)), but the retailer’s estimate deviates from the true
state. The potential value of information sharing is obtained by comparing the situation of no information sharing (analyzed in Section 3) with the situation of perfect information sharing. The latter referring to the situation where both firms use the best available estimate $L_m$. (Recall that whenever $L_r$ is superior, information sharing takes care of itself and is of little interest to analyze further.)

**Proposition 3:** Suppose $L_m = L$ and let $L_r = L_m + \delta$, where $\delta$ denotes the deviation from the true state, $L_m$, and satisfies the regularity condition $0 \leq L_m + \delta \leq 1$. Under a wholesale price only regime, information sharing improves expected channel profits when $\delta > 0$, but may either increase or decrease channel profits when $\delta < 0$. More precisely, when $\delta < 0$ information sharing increases expected channel profits if and only if the information sharing condition (ISC) is satisfied:

$$\Omega(L_m, \delta, b) + \frac{\hat{H}(b, L_m, \delta)}{b-1} \left(1 - \frac{b\hat{H}(b, L_m, \delta)}{4L_m}\right) \left(\frac{H(b, L_m)}{H(b, L_m + \delta)}\right)^{b+1} \leq 1$$

where $\hat{H}(b, L_m, \delta)$ and $\Omega(b, L_m, \delta)$ only depend on $b, L_m$ and $\delta$.

To appreciate the above proposition, we identify the sources of distortion in the channel. Compared to a coordinated channel with complete information, the current channel suffers from three types of distortion: double marginalization, demand uncertainty, and miscalculation of demand uncertainty. While the first two types of distortion is detrimental to both firms (the effect of demand uncertainty on firms’ profits is delineated in Proposition 2), the effect of the third type of distortion on the two firms can be asymmetric. As asserted in Proposition 1, both the profit maximizing retail price, $p_r^*(w_m^*)$, and order quantity, $q_r^*(w_m^*)$, are distorted by uncertainty. By contrast, the wholesale price is unaffected by the demand volatility. In the absence of information sharing, the manufacturer takes the retailer’s order quantity as given and produces exactly that amount. Hence, the manufacturer does not bear any risk associated with demand volatility. When $\delta > 0$, which means that the retailer overestimates the demand uncertainty, it leads to a higher retail price and a lower order quantity than what would otherwise be
the case (see discussion in Section 3). The opposite is true when $\delta < 0$. An interesting conclusion from Proposition 3 is that whenever the retailer underestimates the demand volatility (i.e., $\delta < 0$) it is only the price elasticity, $b$, and the manufacturers estimate, $L_m$, that determines whether information sharing causes channel profits to increase or decrease (see (ISC)). Hence, using Proposition 3, the manufacturer can assess for which $\delta$ (or equivalently $L_r$ estimates) sharing its superior information about the demand volatility would increase channel profits and when it would reduce them. Furthermore, ISC can help determine when information sharing is beneficial to the manufacturer.

To illustrate the above points, Figure 3 shows the difference in the manufacturer’s and the retailer’s expected profits when perfect sharing takes place ($L_r = L_m = L$) compared to when no information sharing occurs. We use the notation:

$$\Delta \pi_m = \pi_m \left( z_m^*(w_m^*), p_m^*(w_m^*), w_m^* \right) - \pi_m \left( z_r^*(w_m^*), p_r^*(w_m^*), w_m^* \right),$$

and

$$\Delta \pi_r = E_{z_r} \left\{ \pi_r \left( z_r^*(w_m^*), p_r^*(w_m^*), w_m^* \right) \right\} - E_{z_r} \left\{ \pi_r \left( z_r^*(w_m^*), p_r^*(w_m^*), w_m^* \right) \right\},$$

where $E_{z_r} \left\{ \pi_r \left( z_r^*(w_m^*), p_r^*(w_m^*), w_m^* \right) \right\}$ is the retailer’s actual expected profit when the optimal strategies are based on the superior estimate $L_m$ (which in our case happens to coincide with the true state, $L$).
\[ E_x \left\{ \pi_r(z^*_r(w^*_m), p^*_r(w^*_m), w^*_m) \right\} \] is the actual expected retailer profit when no information sharing takes place. A positive \( \Delta \pi_r \) (or \( \Delta \pi_m \)) implies that the firm’s expected profit increases when information is shared, while a negative value means it decreases.

From Figure 3 we can see that information sharing increases both the retailer’s and the manufacturer’s profits when the retailer overestimates the demand uncertainty (\( \Delta \pi_m > 0 \) when \( \delta > 0 \)), a result that follows from Proposition 2. In this case, information sharing is incentive compatible and improves channel profits (Proposition 3). That is, both parties are better off with respect to their expected profits when using \( L_m \). On the other hand, when the retailer underestimates the demand uncertainty (\( \delta < 0 \)), we can see that information sharing decreases the manufacturer’s profit while it still benefits the retailer (\( \Delta \pi_m < 0 \) and \( \Delta \pi_r > 0 \) when \( \delta < 0 \)). This result also follows from Proposition 2. Moreover, as asserted in Proposition 3, Figure 3 indicates that the loss in the manufacturer’s profit of sharing the information on demand volatility can exceed the gain in the retailer’s profit (\( \Delta \pi_m > \Delta \pi_r \) when \( \delta < 0 \)), i.e., condition (ISC) is not satisfied. This causes a conflict of interest and information sharing reduces channel profits. Thus, we conclude that information sharing may or may not be valuable in an uncoordinated channel, and it depends in a complicated way on the price elasticity \( b \), and the demand estimates \( L_m \) and \( L_r \).

An important insight is that before the firms actually share their information, they do not know with certainty whether they are facing a situation where it is beneficial to do so. This implies incentive issues for the manufacturer to share its superior information. To explain this further, consider the manufacturer before information sharing. Given that it has no prior knowledge of the retailer’s estimate \( L_r \) (beyond the fact that it deviates from the true volatility \( L_m \)), one can argue that from the manufacturer’s perspective, \( L_r \) is a stochastic variable with equal (ex ante) probability for \( 0 \leq L_r \leq 1 \) (or equivalently \( 0 \leq L_m + \delta \leq 1 \)). Using Proposition 3 and (ISC), the manufacturer can easily determine the
range of $\delta$ where information sharing increases and decreases expected channel profits, respectively, and thus the probability for information sharing to be beneficial. For example, if $L_m < 0.5$ we know without using condition (ISC) that there is at least a 50% chance information sharing will increase the expected channel profits (the probability that $\delta>0$ is $1-L_m$). One can therefore argue that in this case it is rational for the manufacturer to share its information. (Note that this argument is not contingent on $L_m = L$, since the $L$ is unknown and $L_m$ is the best estimate available when the manufacturer makes its decision.) However, for certain values of $L_m$ and $b$ sharing is bound to not take place because the manufacturer concludes that there is a larger chance profits will decrease than increase.

As an alternative, before sharing the manufacturer can also compute the expected gain in channel profits across the entire range of feasible $\delta$ values (for example using the expressions specified in the proof of Proposition 3 and integrate numerically). If this gain is positive it would be rational for the manufacturer to share information, otherwise not.

To conclude, it is not obvious that the manufacturer is always willing to share its superior information since doing so may improve the retailer’s profit at the expense of reducing its own profit. Although information sharing reduce/eliminate the distortion caused by the miscalculation of demand uncertainty, its impact on the profits of the manufacturer and the retailer is asymmetric. In particular, information sharing is not incentive compatible when the retailer underestimates demand volatility. In sum, information sharing is feasible only when it benefits both firms, i.e., when the information sharing condition illustrated in Proposition 3 holds.

4.2 Information Sharing Contracts

As discussed earlier, firms need to overcome incentive and credibility problems to achieve information sharing. In what follows, we investigate two alternative contractual arrangements that help achieve information sharing. In particular, these contracts are relatively easy to implement. We first analyze the profit sharing contract when the quality of the demand estimates is common knowledge;
we then examine the buy back contract when only one firm knows whose demand estimate is superior.

In addition, we study the effect of information sharing on pricing, firms’ profits, and channel profits.

4.2.1 The profit sharing contract

When both the manufacturer and the retailer know which demand estimate is superior, credibility is not an issue but the incentive problem remains. A profit sharing contract is a simple mechanism to facilitate information sharing under such circumstances. Since information sharing takes care of itself when the retailer’s estimate is superior, we only consider the case when the manufacturer’s estimate is superior. Our analysis presumes that the manufacturer has an interest in sharing. (As explained in Section 3.1., this is not always the case since the manufacturer may conclude before the fact that the risk of reducing the expected channel profit is too high.) The sequence of actions is as follows. First, the manufacturer and the retailer negotiate for a division of the combined gain in profits when information sharing occurs. The gain (loss) in profit for each firm is relative to the no information sharing case, where the firms use their own demand estimates. Although the division can be arbitrary depending on each firm’s bargaining power, we assume without loss of generality that both firms split the gain equally. If the negotiation is successful, both firms submit their demand estimates simultaneously. Otherwise, the firms do not reveal their demand estimates and the game proceeds as in the no-information-sharing case described in Section 3. The solution concept in the profit sharing contract is one of Nash bargaining. Without information sharing, the equilibrium strategy \((z^*_r(w^*_m), p^*_r(w^*_m), w^*_m)\) is based on the retailer’s demand estimate. With information sharing, both firms use the superior estimate \(L_m\) when determining the optimal strategy \((z^*_m(w^*_m), p^*_m(w^*_m), w^*_m)\). After the demand estimates are revealed both parties agree that \(L_m\) is the best available estimate of the true demand volatility. Under this estimate the retailer’s expected profit with and without information sharing are \(E_{z_m}(\pi_r(z^*_m(w^*_m), p^*_m(w^*_m), w^*_m))\) and \(E_{z_r}(\pi_r(z^*_r(w^*_m), p^*_r(w^*_m), w^*_m))\) respectively. Similarly the manufacturer’s profit with and without information sharing are \(\pi_m(z^*_m(w^*_m), p^*_m(w^*_m), w^*_m)\) and
\[ \pi_m(z_r^*(w_m^*), p_r^*(w_m^*), w_m^*) \]. Hence, after the contract is signed (in our case after the fifty-fifty split is agreed on), the firms’ perceived change in the expected total channel profits is

\[
\Delta \Pi = \left( E_{z_m} \{ \pi_r(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*) \} - E_{z_m} \{ \pi_r(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*) \} \right)
+ \left( \pi_m(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*) - \pi_m(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*) \right)
\]

and the profit sharing contract dictates that the manufacturer and the retailer each receives \( \frac{1}{2} \Delta \Pi \) in equilibrium.

A prominent feature of the profit sharing contract is that although the agreement on how to split the gain in channel profits is reached \textit{ex ante}, before the demand is realized, the actual profit sharing occurs \textit{ex post}, after the demand realization. Note that \( \Delta \Pi \) in (15) represents the expected \textit{actual} gain in profits if \( L_m = L \). Given the intrinsic stochasticity of the demand, a profit sharing contract can lead to either an increase or a decrease of the profits of both firms in realization. This means that under this contract the manufacturer and the retailer share the risk associated with the demand uncertainty. (Under a wholesale price only contract the retailer bears this risk on his own.). As outlined in Proposition 1, the optimal retail price and order quantity can either increase or decrease depending on whether information sharing reduces the retailer’s overestimation or underestimation. Furthermore, from Proposition 3 and Figure 3, we can conclude that the sign of \( \Delta \Pi \) as defined in (15) is ambiguous (note that \( \Delta \Pi \) is equivalent to \( \Delta \pi_r + \Delta \pi_m \) under the presumption that the manufacturer’s estimate is correct). This suggests that after signing the contract the firms may realize that the perceived change in expected profits due to information sharing, \( \Delta \Pi \), is negative. Profit sharing in this case means that the retailer has to give up profits to the manufacturer and both parties end up being worse off. Consequently, the retailer has incentives to break the contract, to use the manufacturer’s estimate, which it now knows, and leave the manufacturer to take the entire profit loss. Hence, some kind of enforcing mechanisms may be required in order to secure the contract.
4.2.2 The buy back contract

When only the manufacturer knows the quality of the demand estimates and the retailer overestimates the demand volatility, the manufacturer can offer a buy back contract to induce the retailer to order more and credibly convey his information to the retailer in the process.

When the manufacturer offers a return policy, $s$, the retailer’s profit function is given by

$$\pi_r(z, p, w, s) = \begin{cases} y(p) \left[ (p-w)\varepsilon_r - (w-s)(z-\varepsilon_r) \right], & \varepsilon_r \leq z \\ y(p) \left[ (p-w)\varepsilon_r - (p-w)(\varepsilon_r - z) \right], & \varepsilon_r > z \end{cases},$$

(16)

the retailer’s profit maximization problem, given $w$ and $s$, can be written as

$$\max_{z, p} E_{\varepsilon_r} \left\{ \pi_r(z, p, w, s) \right\} = y(p) \begin{cases} \int_{1-L_r}^{z} (p-w)x f_{\varepsilon_r}(x)dx - \int_{1-L_r}^{z} (w-s)(z-x)f_{\varepsilon_r}(x)dx \\ + \int_{z}^{1+L_r} (p-w)x f_{\varepsilon_r}(x)dx - \int_{z}^{1+L_r} (p-w)(x-z)f_{\varepsilon_r}(x)dx \end{cases}$$

(17)

$$= y(p) \left\{ (p-w) - \int_{1-L_r}^{z} (w-s)(z-x)f_{\varepsilon_r}(x)dx - \int_{z}^{1+L_r} (p-w)(x-z)f_{\varepsilon_r}(x)dx \right\}$$

Noting that a return policy is only activated when the mean adjusted demand falls below $z_r^* = z_r^*(w, s)$, we can write the manufacturer’s profit maximization problem as

$$\max_{w, s} E_{\varepsilon_r} \left\{ \pi_m(w, s, q_r^*) \right\} = y(p_r^*) \begin{cases} \int_{1-L_m}^{G(t_w, z)} (w-c)z_r^*-s (z_r^*-x)f_{\varepsilon_r}(x)dx \\ \left( (w-c)z_r^*, \right) \text{ if } z_r^* < 1-L_m \\ \left( (w-c)z_r^*-s \left[ z_r^* - (1-L_m) \right]^2 \right) / 4L_m \text{ if } 1-L_m \leq z_r^* \leq 1+L_m \\ \left( (w-c)z_r^*-s \left( z_r^*-1 \right) \right) \text{ if } z_r^* > 1+L_m \end{cases}$$

(18)

The optimal wholesale price, return policy, and retail price can be obtained through a standard backward induction approach. However, because of the complexity of the profit function, closed form expressions for these parameters do not exist in the general case. To attain an understanding of the
behavior of $p^*_r(w,s)$ and $q^*_r(w,s)$ we therefore focus on the special case where $L_r=1$ and we can derive analytical results.

**Proposition 4:** Given $w$, and $L_r=1$, the optimal retail price $p^*_r(w,s)$ is decreasing in the return policy $s$ and the optimal order quantity $q^*_r(w,s)$ is increasing in the return policy $s$.

Proposition 4 asserts that for any given wholesale price, the manufacturer can offer a return policy to induce the retailer to charge a lower retail price and order more. Hence, when pricing is endogenous a return policy will stimulate larger retailer orders. The manufacturer will only offer a return policy that improves its expected profit (18). Similarly, the retailer will only accept a policy that improves its expected profit (17). This implies that by offering a return policy, the retailer’s overestimation of demand volatility is reduced, and the expected profits for both firms are improved. Although we are able to demonstrate these relationships analytically only for $L_r = 1$, our numerical studies suggest that Proposition 3 holds for any $L_r > L_m$. Padmanabhan and Png (1997) and Emmons and Gilbert (1998) show similar relationships under symmetric demand information. Consequently, our results demonstrate that a return policy helps to achieve information sharing under asymmetric information, and continues to be an effective tool for channel coordination. We note, however, that a return policy is generally not as efficient as other types of contracts such as the profit sharing contract and the two-part tariff (discussed in Section 5). The intuition is as follows. A return policy impacts the optimal wholesale price and retail price, thereby inducing the retailer to place a larger order. The distortion of the pricing decisions results in deadweight loss. By contrast, a profit sharing contract achieves information sharing through a fixed fee, which is free from distortion and deadweight loss. However, the key advantage of the return policy is the ease of implementation, which probably explains the popularity of this type of policy in real world practice.
5. A Two-part Tariff Regime

In Section 4, we examine the value of information sharing, and some contracts to facilitate sharing in an uncoordinated channel operating under a wholesale price only regime. We now turn to an alternative two-part tariff regime. Under a bilateral monopoly and symmetric information, it is well known (e.g., Locay and Rodriguez (1992), Raju and Zhang (2005)) that the optimal two-part tariff is in the form of the manufacturer charging the retailer a fixed fee and a wholesale price at the marginal cost. A wholesale price at the marginal cost eliminates double marginalization and ensures maximum channel profits. The fixed fee serves to split the channel profits between the manufacturer and the retailer. It is worth noting that under asymmetric information about the demand volatility the firms have different perceptions of the expected channel profits. From the manufacturer’s perspective the expected channel profit before information sharing is \( E_{\pi_m} \{ \pi_m(z_m(c), p_m^*(c), c) \} \), and the fixed fee can be expressed as \( \theta_m E_{\pi_m} \{ \pi_m(z_m(c), p_m^*(c), c) \} \), where \( \theta_m \in (0,1) \). Because the wholesale price is \( c \), without the fixed fee the manufacturer’s profit is zero. From the retailers perspective the expected channel profit is \( E_{\pi_r} \{ \pi_r(z_r(c), p_r^*(c), c) \} \) and the fixed fee is \( \theta_r E_{\pi_r} \{ \pi_r(z_r(c), p_r^*(c), c) \} \), with \( \theta_r \in (0,1) \). The actual expected channel profit, unknown to the firms, is \( E_{\pi} \{ \pi(z(c), p^*(c), c) \} \). In order to do business, the firms must agree on the fixed fee, i.e., \( \theta_m E_{\pi_m} \{ \pi_m(z_m(c), p_m^*(c), c) \} = \theta_r E_{\pi_r} \{ \pi_r(z_r(c), p_r^*(c), c) \} \). Note that unless \( L_r = L_m \), the negotiated fee constitutes different portions of the firms’ perceived channel profits, i.e., \( \theta_m \neq \theta_r \). The size of the fee clearly depends on the firms’ bargaining power, which may be influenced by the quality of the demand information and knowledge thereof, i.e., which firm has the superior demand estimate and which knows about this. However, as we asserted in Section 4, a detailed treatment of bargaining is beyond the scope of this paper.
Clearly, a two-part tariff coordinates the channel when the manufacturer and the retailer have symmetric information (i.e., $L_r = L_m$ and $\theta_m = \theta_r$). This is because the wholesale price is set to the marginal cost and the manufacturer derives his profit solely from the fixed fee, which is a portion of the retailer’s expected profit. As a result, maximizing the retailer’s expected profit amounts to maximizing the expected channel profits. In the absence of information asymmetry, the manufacturer concurs with the retailer’s perceived expected profit. Thus, there is no disagreement regarding the portion of the channel profits that constitute the fixed fee.

As under the wholesale price regime in Section 4, a fundamental question under a two-part tariff regime is the potential for information sharing to improve expected channel profits, and how that impacts the firms’ individual profits. Noting that the expected channel profit equals the retailer’s expected profit, it follows that information sharing as defined in this paper only affects the channel profit when the manufacturer’s estimate $L_m$ is superior. To assess the value of information sharing we therefore follow the approach in Section 4 and compare the situation of no information sharing with that of perfect information sharing where both firms use the superior estimate $L_m=L$. 

![Figure 4: The value of information sharing under a two-part tariff regime.](image-url)
Lemma 4: Under a two-part tariff, perfect information sharing always improves the expected channel profits.

Without information sharing, the expected channel profit is equivalent to the retailer’s actual expected profit \[ E_c \{ \pi_r^*(z_r^*(c), p_r^*(c), c) \} \] where the pricing and order decisions are based on the retailer’s demand estimate, \( L_r \). Consistent with Proposition 2, both overestimation and underestimation of the demand volatility are detrimental to the retailer’s actual expected profit and therefore also to the channel (Figure 4). When the manufacturer shares its superior demand information \( L_m \) with the retailer the expected channel profit increases and creates an opportunity for both firms to be better off than before. It is not clear, however, whether a two-part tariff suffices to achieve information sharing when the manufacturer and the retailer have different demand estimates, which is the case in our model. We therefore, need to further examine when (if ever) a two-part tariff coordinates the channel under asymmetric demand information. We address these issues next, for each option in our information sharing taxonomy outlined in Table 1.

Proposition 5:

(i) If the quality of the demand volatility estimates is private information, a two-part tariff achieves information sharing and coordinates the channel in Scenario 1b and Scenario 2a; in Scenario 1a, Scenario 2b, and Scenario 3, it does not achieve information sharing but it may coordinate the channel if there is no dispute about the fixed fee.

(ii) If the quality of the estimates of demand volatility is common knowledge, a two-part tariff achieves information sharing and coordinates the channel through a profit sharing contract.

When firms have asymmetric demand estimates, a two-part tariff does not always achieve information sharing and fails to coordinate the channel under certain conditions. This is in contrast to the situation under symmetric information, where a two-part tariff always coordinates the channel. The intuitions are as follows. When each firm has its own demand forecast, it also has an asymmetric
forecast of the total channel profits, and this in turn may lead to disagreement on the amount of fixed fee. When the quality of the demand estimates is private information, a two-part tariff by itself cannot resolve such dispute.

Under a two-part tariff, the incentive problem in a profit sharing contract is relatively simple. Since information sharing always improves the expected channel profits (Figure 4), the firms only need to negotiate an agreement on how to split the benefits from information sharing. However, a profit sharing contract is feasible only when the quality of demand estimates is common knowledge. It is interesting to compare the properties of a two-part tariff with those of a profit sharing contract. A two-part tariff is strictly an *ex ante* contract, both the wholesale price and the fixed fee are set before the demand realization. By contrast, a profit sharing contract has both *ex ante* and *ex post* components. While the terms of the contracts are determined before the demand realization, the execution of the contract occurs after the demand realization. Therefore, a two-part tariff is risk free to the manufacturer, whereas a profit sharing contract leads to risk sharing between the manufacturer and the retailer.

### 6. Conclusion

As Padmanabhan and Png (1997) eloquently put it, “one of the few certainties about the demand for products such as new books, CDs, software, fashion wear, and winter clothing is that it is uncertain.” Fortunately, many of the results from the extant channel literature remain robust with or without demand uncertainty. However, important questions arise when firms in a distribution channel face asymmetric demand uncertainty: should this information be shared, what is the value of sharing, and how can sharing be accomplished? In this paper, we provide insights into these issues. Rather than looking at uncertainty about demand trend, we focus on firms’ asymmetric information on demand volatility. Specifically, we consider a situation with price dependent multiplicative demand uncertainty where firms are correct in estimating the average demand trend, but they may miscalculate the extent of demand volatility. The value of information sharing stems from the fact that one member of the
distribution channel may possess a superior estimate of the demand volatility compared to the other member. However, the firms may not possess the same information on whose demand estimate is superior. This asymmetric information on the quality of the demand estimates further complicates information sharing. In the context of bilateral monopoly and short-term relationships, we demonstrate through our model that demand volatility can have substantial impact on retail price, order quantity, and the firms’ profits. Although information sharing always improves channel profits in a coordinated channel, it is not necessarily so if the channel is uncoordinated because of asymmetric demand information. Depending on the specific information structure, there are different strategies and contracts that firms can use to facilitate information sharing. Two prominent examples that we consider in this work are profit sharing contracts and return policies.

There are several directions to extend our model. In this paper, we consider a channel of bilateral monopoly in a single period setting, it is interesting to investigate how upstream and downstream competition moderate information sharing; it is also interesting to use a signaling framework to examine information sharing in a multi-period setting. Although pricing is a decision variable in our model, we do not consider retail promotion. An important form of uncertainty not covered in the paper is the uncertainty stemming from lead-time and capacity constraint. It would also be interesting to investigate the impact of using other types of demand models, additive demand uncertainty, alternative distributions etc. Finally, many alternative contracts, such as quantity discount contracts, are good candidates for achieving information sharing. We leave these issues to future research.
Appendix: Proofs

Proof of Claim 1:

Suppose a firm’s estimate of $L$ follows a distribution $g(L)$ with positive support over the interval $[L, \bar{L}]$, with $L \geq 0$ and $L < \bar{L}$. Since demand is uniformly distributed over the interval $[y(p)(1-L), y(p)(1+L)]$, it follows that the firm’s estimated distribution of $\varepsilon$ equals

$$f(x) = \int_{L}^{\bar{L}} \tilde{f}(x | L) g(L) dL,$$

where

$$\tilde{f}(x | L) = \begin{cases} 
     \frac{1}{2L} & \text{if } 1-L \leq x \leq 1+L \\
     0 & \text{otherwise}
\end{cases}$$

represents the pdf given a bound $L$ on the uniform distribution. As a result, for any $g(L)$ the implied distribution from the above expression is not uniform, since $f(L) \neq f(L+\ell)$ for any $\ell$ such that $0 < \ell \leq \bar{L} - L$. Given that we assume that it is common knowledge that $\varepsilon$ is uniformly distributed, any estimate that follows some distribution with positive support over some interval will therefore yield a contradiction. $Q.E.D.$

Proof of Claim 2:

Refer to Section 3.2.

Proof of Claim 3:

Without information sharing, the retailer cannot extract any additional information from the manufacturer since the wholesale price $w$ is uninformative. Because $L_r$ can be either greater than or less than the true state $L$, the retailer has no way of knowing which way she should adjust $L_r$ even when $\sigma_r$ dictates that the manufacturer is of type $h$. In a long-term relationship, an update is possible by using the correlation between $\varepsilon_m$ and $\varepsilon_r$ based on historical data. However, such correlation is unknown in the one-shot setting we consider. Therefore, it is rational for the retailer to base her decisions on her own estimate $L_r$. 

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By the same token, the decision rule for the retailer as to whether to base her decisions on $L_m$ or $L_r$ when information sharing occurs is as follows. She would use $L_m$ if $E\{\pi_r(L_m)|L_r,\sigma_r\} \geq E\{\pi_r(L_r)|L_r,\sigma_r\}$. Otherwise she would continue to base her decisions on $L_r$.

Q.E.D.

Proof of Lemma 1:

After substituting $z^*_r(p,w)$ into expression (4), we take the first derivative of the retailer’s expected profit function with respect to $p$, yielding

$$\frac{\partial E_r\{\pi_r(z^*_r(p,w),p)\}}{\partial p} = \partial \left[ y(p)\left( (p-w) - Lw\frac{p-w}{p} \right) \right]$$

$$= \frac{\partial y(p)}{\partial p} \left( (p-w) - Lw\frac{p-w}{p} \right) + y(p) \frac{\partial \left( (p-w) - Lw\frac{p-w}{p} \right)}{\partial p}$$

$$= y(p) \frac{-b}{p} \left( (p-w) - Lw\frac{p-w}{p} \right) + y(p) \left( 1 - \frac{Lw^2}{p^2} \right)$$

$$= y(p) \left( \frac{1}{p^2} \left( (1-b)p^2 + bw(L+1)p + (b+1)w^2L \right) \right),$$

and use the first-order optimality conditions to obtain

$$\frac{\partial E_r\{\pi_r(z^*_r(p,w),p)\}}{\partial p} = 0 \quad \Rightarrow \quad (1-b)p^2 + bw(L+1)p + (b+1)w^2L = 0 .$$

Solving for the resulting equality for $p$ yields

$$p^*_r(w) = \frac{b}{b-1}w \left( \frac{1+L}{2} + \sqrt{\frac{(1-L)^2}{2} + \frac{L}{b^2}} \right) \quad \text{or} \quad p^*_r(w) = \frac{b}{b-1}w \left( \frac{1+L}{2} - \sqrt{\frac{(1-L)^2}{2} + \frac{L}{b^2}} \right)$$

The additional constraint $p^*_r(w) \geq w$, however, ensures that only the first of these expressions is valid. To show this, we first note that $p^* \geq w$ holds for the first solution, since
\[
\left(\frac{1-L}{2}\right)^2 + \frac{L}{b^2} > \frac{1-L}{2} \quad \text{and therefore} \quad \frac{1+L}{2} + \sqrt{\left(\frac{1-L}{2}\right)^2 + \frac{L}{b^2}} > \frac{1+L}{2} + \frac{1-L}{2} = 1
\]

On the other hand, using the same logic for the second expression yields
\[
\frac{1+L}{2} - \sqrt{\left(\frac{1-L}{2}\right)^2 + \frac{L}{b^2}} < \frac{1+L}{2} - \frac{1-L}{2} = L,
\]
and therefore this solution is invalid if \( L \leq \frac{b-1}{b} \). Suppose therefore that \( L > \frac{b-1}{b} \). Then,
\[
\frac{1+L}{2} - \sqrt{\left(\frac{1-L}{2}\right)^2 + \frac{L}{b^2}} \geq \frac{b-1}{b}
\]
implies
\[
\sqrt{\left(\frac{1-L}{2}\right)^2 + \frac{L}{b^2}} \leq \frac{1+L}{2} - \frac{b-1}{b} = \frac{2-(1-L)b}{2b},
\]
and therefore (since \( L > \frac{b-1}{b} \) implies \( 2-(1-L)b > 0 \)),
\[
\frac{b^2(1-L)^2 + 4L'}{4b^2} \leq \frac{4 - 4(1-L)b + (1-L)^2b^2}{4b^2},
\]
or
\[
4(1-L)b \leq 4(1-L),
\]
which contradicts our assumption that \( b > 1 \). Thus, the second expression for the optimal price is never valid.  \( Q.E.D. \)

**Proof of Lemma 2:**

\[
\iff z(p^*(w),w) > 1 \\
L\left(1 - \frac{2w}{p^*(w)}\right) > 0 \\
\iff \frac{p^*(w)}{2w} > 2w \\
H(b,L) > 2 \frac{b-1}{b}
\]
Observe that there is a single value of $0 < L_r < 1$ for which $z(p^*_r(w), w) = 1$, since $H(b, L)$ is increasing in $L_r$, as was shown in the proof of proposition 1. \textit{Q.E.D.}

**Proof of Proposition 1:**

To show that $p^*_r(w) = w \frac{b}{b-1} H(b, L)$ is strictly increasing in $L_r$, we consider the derivative $\frac{\partial H(b, L)}{\partial L_r}$

$$\frac{\partial H(b, L)}{\partial L_r} = \frac{\partial}{\partial L_r} \left( \frac{1 + L}{2} + \sqrt{\left(\frac{1 - L}{2}\right)^2 + \frac{L}{b^2}} \right)$$

$$= \frac{1}{2} + \frac{1}{2b} \left( \frac{b^2(L-1) + 2}{\sqrt{b^2(1-L)^2 + 4L}} \right)$$

$$= \frac{1}{b\sqrt{b^2(1-L)^2 + 4L}} + \frac{1}{2} \left( 1 - \frac{b(1-L)}{\sqrt{b^2(1-L)^2 + 4L}} \right) > 0,$$

since $\frac{b(1-L)}{\sqrt{b^2(1-L)^2 + 4L}} < 1$. \textit{Q.E.D.}

**Proof of Lemma 3:**

Lemma 3 follows by taking the first derivative of the order quantity $q^*_r(p, w)$ with respect to $p$, yielding

$$\frac{\partial q^*_r(p, w)}{\partial p} = \frac{\partial}{\partial p} \left[ y(p) \left( 1 + L - 2L \frac{w}{p} \right) \right]$$

$$= \frac{\partial y(p)}{\partial p} \left( 1 + L - 2L \frac{w}{p} \right) + y(p) \frac{\partial}{\partial p} \left( 1 + L - 2L \frac{w}{p} \right)$$

$$= y(p) \frac{-b}{p} \left( 1 + L - 2L \frac{w}{p} \right) + y(p) \left( 2Lw \frac{p}{p^2} \right)$$

$$= y(p) \left( \frac{2Lw(b+1)}{p^2} - \frac{(1+L)b}{p} \right).$$

Using the first-order optimality conditions, we obtain
\[
\frac{\partial q^*_r(p, w)}{\partial p} = 0 \Rightarrow \left( \frac{2Lw(b+1)}{p^2} - \frac{(1+L)b}{p} \right) = 0
\]
\[
\Rightarrow p = w\left( \frac{b+1}{b} \right)\left( \frac{2L}{L+1} \right)
\]

\(q^*_r(p, w)\) strictly increases with respect to \(p\) when \(\frac{\partial q^*_r(p, w)}{\partial p} > 0\), or alternatively

\(p < w\left( \frac{b}{b+1} \right)\). \(q^*_r(p, w)\) strictly decreases with respect to \(p\) when \(\frac{\partial q^*_r(p, w)}{\partial p} < 0\), that is,

when \(p > w\left( \frac{b}{b+1} \right)\). \(Q.E.D.\)

**Proof of Proposition 2:**

(i) When the manufacturer only charges a wholesale price, the manufacturer’s expected profit is given by \(\pi_M\left(z^*_m, p^*_m, w\right) = (w-c)q^*_r\), hence the sign of \(\frac{\partial \pi_M(z^*_m, p^*_m, w)}{\partial L_r}\) is determined by \(\frac{\partial q^*_r(w)}{\partial L_r}\). It follows from Proposition 1 that the manufacturer’s expected profit is strictly decreasing for all \(L_r\).

To show that the retailer’s perceived expected profit, \(E_r\left\{\pi_r(z^*_r(w^*_m), p^*_r(w^*_m), w^*_m)\right\}\), is decreasing in \(L_r\), we consider its derivative with respect to \(L_r\). For notational convenience let

\[
H = H(b, L_r), \quad H' = \frac{\partial H(b, L_r)}{\partial L_r}, \quad p^*_r = p^*_r(w^*_m), \quad y = y\left(p^*_r(w^*_m)\right), \quad y' = \frac{\partial y\left(p^*_r(w^*_m)\right)}{\partial L_r}.
\]

\[
\frac{\partial E_r}{\partial L_r} = \frac{\partial y\left(p^*_r(w^*_m)\right)}{\partial L_r} \left\{ \left( p^*_r(w^*_m) - w^*_m \right) \left[ 1 - L_r \frac{w^*_m}{p^*_r(w^*_m)} \right] \right\}
\]

\[
= y' \left\{ \left( p^*_r - w^*_m \right) \left[ 1 - L_r \frac{w^*_m}{p^*_r} \right] \right\} + y \left[ 1 - L_r \frac{w^*_m}{p^*_r} \right] p' - \left( p^*_r - w^*_m \right) \left[ \frac{w^*_m}{p^*_r} + L_r \frac{\partial}{\partial L_r} \left( \frac{w^*_m}{p^*_r} \right) \right]
\]

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\[
\begin{align*}
= y' \left[ w_m^* (H - 1) \left(1 - \frac{L_m}{H}\right) \right] + y' \left[ w_m^* H' \left(1 - \frac{L_m}{H}\right) - w_m^* (H - 1) \left[ \frac{1}{H} + L_r \frac{\partial}{\partial L_r} \left( \frac{1}{H} \right) \right] \right] \\
= w_m^* \left[ \frac{-bH' (H - 1) [H - L_r] + HH' [H - L_r] - (H - 1) [H - L_r, H']} {H^2} \right] \\
= w_m^* \left[ \frac{-bH^2 + bHL_r + bH - bL_r - H^2 + H - H^2 + H} {H^2} \right] = -w_m^* \left( \frac{H - 1}{H} \right) y
\end{align*}
\]

Clearly, \( E_E \{ \pi_r(z_m^*(w_m^*), p_r^*(w_m^*), w_m^*) \} \) is decreasing in \( L_r \) since from Lemma 2 we know that \( H > 1 \).

(ii) Let \( p^*(w_m^*) \) and \( z^*(w_m^*) \) denote the retail price and mean adjusted order quantity, which are optimal under the true demand uncertainty \( \varepsilon \), and suppose that \( E_E \{ \pi_r(z_m^*(w_m^*), p_r^*(w_m^*), w_m^*) \} \geq E_E \{ \pi_r(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*) \} \) for some \( L_r \neq L \). This implies, however, that \( p^*(w_m^*) \) and \( z^*(w_m^*) \) are not optimal for the true state \( L \), which is a contradiction since optimality dictates that \( E_E \{ \pi_r(z_m^*(w_m^*), p_r^*(w_m^*), w_m^*) \} \) is maximized when the retailer’s demand estimate is correct (\( L_r = L \)) and 
\[ p_r^*(w_m^*) = p^*(w_m^*), z_r^*(w_m^*) = z^*(w_m^*). \]

(iii) This result follows directly from (i) when \( L_r = L \) and the perceived expected profit coincides with the actual expected profit. \( \text{Q.E.D.} \)

**Proof of Proposition 3:**

Denote the gain in expected manufacturer, retailer and channel profits due to perfect information sharing \( \Delta \pi_m = \pi_m(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*) - \pi_m(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*), \)
\[ \Delta \pi_r = E_{E_m} \{ \pi_r(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*) \} - E_{E_m} \{ \pi_r(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*) \}, \]
and \( \Delta \Pi = \Delta \pi_m + \Delta \pi_r, \) respectively. It follows from Proposition 2 (i) that \( \Delta \pi_m \geq 0 \) for \( \delta \geq 0 \) and \( \Delta \pi_m < 0 \) for \( \delta < 0 \).

Furthermore, since \( L_m = L \) Proposition 2 (ii) asserts that \( \Delta \pi_r \geq 0 \) for all \( \delta \) such that \( 0 \leq L_m + \delta \leq 1 \).

Hence, it is clear that information sharing always improves channel profits when the retailer
overestimates the demand variability, i.e., $\Delta \Pi > 0$ for $\delta \geq 0$. To prove that information sharing may either decrease or increase channel profits when $\delta < 0$, we show that the sign of $\Delta \Pi$ is ambiguous.

For notational simplicity we use $z_i^* = z_i^*(w_i^*)$ and $p_i^* = p_i^*(w_i^*)$ for $i=m,r$.

When $\delta < 0$ it follows that $1 - L_m \leq z_r^* \leq 1 + L_m$, and by definition $1 - L_m \leq z_m^* \leq 1 + L_m$. We get

$$E_{e_m} \{\pi_r(z_r^*, p_r^*, w_m^*)\} = y(p_r^*) \left\{ (p_r^* - w_m^*) - \int_{1-L_m}^{1+L_m} (z_r^*-x)f_m(x)dx - (p_r^* - w_m^*) \int_{z_r^*}^{1-L_m} (x-z_r^*)f_m(x)dx \right\}$$

$$= c^{-b+1} y \left( \frac{b^2 H(b, L_m + \delta)}{(b-1)^2} \right) \left( \frac{b}{b-1} \right) \left( \frac{bH(b, L_m)}{b-1} - 1 \right) \left( \frac{1 - (1 + L_m - z_r^*)^2}{4L_m} \right) - \left( \frac{z_r^* - 1 + L_m}{4L_m} \right)^2 \right\}$$

$$E_{e_m} \{\pi_m(z_m^*, p_m^*, w_m^*)\} = c^{-b+1} y \left( \frac{b^2 H(b, L_m)}{(b-1)^2} \right) \left( \frac{b}{b-1} \right) \left( \frac{bH(b, L_m)}{b-1} - 1 \right) \left( \frac{1 - (1 + L_m - z_m^*)^2}{4L_m} \right) - \left( \frac{z_m^* - 1 + L_m}{4L_m} \right)^2 \right\}.$$

Recall that $\varepsilon_m \in U[1-L_m, 1+L_m]$, $y(p) = ap^{-b}$, $w_m^* = \frac{b}{b-1} - \frac{b}{b-1} H(b, L_m + \delta)$ and $p_m^* = \frac{b}{b-1} H(b, L_m)$. Similarly, $\pi_m(z_r^*, p_r^*, w_m^*) = c^{-b+1} y \left( \frac{b^2 H(b, L_m + \delta)}{(b-1)^2} \right) \left( \frac{1}{b-1} \right) z_r^*$

$$\pi_m(z_m^*, p_m^*, w_m^*) = c^{-b+1} y \left( \frac{b^2 H(b, L_m + \delta)}{(b-1)^2} \right) \left( \frac{1}{b-1} \right) z_m^*.$$

Let the expected channel profit with and without perfect information sharing be

$$\Pi(z_m^*, p_m^*, w_m^*)$$

and $\Pi(z_r^*, p_r^*, w_r^*)$, where $\Pi(z_i^*, p_i^*, w_i^*) = \pi_m(z_i^*, p_i^*, w_i^*) + E_{e_m} \{\pi_r(z_i^*, p_i^*, w_i^*)\}.$

Staring from the expressions for the expected manufacturer’s and retailer’s profits above, simplified expressions for the channel profits can be derived, omitting the algebraic details we get (note that $z_r^*$ and $z_m^*$ are given by (9))

$$\Pi(z_m^*, p_m^*, w_m^*) = c^{-b+1} y \left( \frac{b^2 H(b, L_m)}{(b-1)^2} \right) \left( \frac{b-1}{bH(b, L_m)} \right) \Omega(L_m, 0, b) \right\}$$
\[ \Pi(z^*_r, p^*_r, w^*_m) = c^{-b+1} \cdot \gamma \left( \frac{b^2 H(b, L_m + \delta)}{(b - 1)^2} \right) \left( \frac{b - 1}{bH(b, L_m + \delta)} \right) \left( \Omega(L_m, \delta, b) + \frac{\hat{H}(L_m, \delta, b)}{b - 1} \left( 1 - \frac{b\hat{H}(L_m, \delta, b)}{4L_m} \right) \right) \]

where

\[ \hat{H}(b, L_m, \delta) = \delta \left( \frac{bH(b, L_m + \delta)}{b - 1} - 2 \right) \]

\[ \Omega(b, L_m, \delta) = \frac{b}{b - 1} \left( \frac{b^2 H(b, L_m + \delta)^2}{(b - 1)^2} - L_m \right) - \left( \frac{b(1 + L_m)H(b, L_m + \delta)}{b - 1} - 2L_m \right). \]

The condition \( \Delta \Pi = \Pi(z^*_m, p^*_m, w^*_m) - \Pi(z^*_r, p^*_r, w^*_r) \geq 0 \) can now be simplified into (ISC) which holds if and only if \( \Delta \Pi \geq 0 \). (Note that by definition \( \Pi(z^*_m, p^*_m, w^*_m) \geq 0 \))

\[ \frac{\Omega(L_m, \delta, b) + \frac{\hat{H}(b, L_m, \delta)}{b - 1} \left( 1 - \frac{b\hat{H}(b, L_m, \delta)}{4L_m} \right)}{\Omega(L_m, 0, b)} \left( \frac{H(b, L_m)}{H(b, L_m + \delta)} \right)^{b+1} \leq 1 \quad (\text{ISC}) \]

It follows that the sign of the expected gain in channel profits \( \Delta \Pi \) is a function of \( b, L_m \) and \( \delta \), but independent of \( c \) and \( a \). By example, it is easy to show that condition (ISC) may or may not be satisfied for \( \delta < 0 \). For example if \( b = 2 \) and \( L_m = 1 \), (ISC) is satisfied (i.e., \( \Delta \Pi \geq 0 \)) for all possible \( \delta \) \((-1 < \delta < 0)\). On the other hand for \( b=1.05, \ L_m=1 \), (ISC) is not satisfied for any relevant \( \delta \) \((-1 < \delta < 0)\), i.e., \( \Delta \Pi < 0 \). Finally, if \( b=1.5 \) and \( L_m=1 \), (ISC) is satisfied for \(-0.48 \leq \delta < 0 \), but not satisfied for \(-1 < \delta \leq -0.49 \). \( \square \)

**Proof of Proposition 4:**

Given \( w, s \) and \( L_r = 1 \), we determine a solution to the maximization problem (17) using the first order optimality conditions. First, the partial derivative of \( E_{\pi_r} \{ \pi_r(z, p, w, s) \} \) with respect to \( z \) renders the optimal mean adjusted order quantity for any given retail price \( p \geq w \):

\[ z^*_r(p, w, s) = 1 + \frac{p + s}{p - s} L_r - \frac{2wL_r}{p - s} \]
Substituting this expression back into the retailer’s profit function and solving for \( p \) using the first order optimality condition we obtain the optimal retail price as a function of \( w \) and \( s \):

\[
p^*_r(w, s) = \frac{1}{2(b-1)} \left[ (b+1)w + (b-2)s + A \right], \text{ where } A = \sqrt{(b+1)^2 w^2 + (b-2)^2 s^2 - (2b^2 - 2b + 4)ws}.
\]

Under the necessary condition \( p^*_r(w, s) \geq w \geq s \) it is possible to show that \( A \) is real for all \( b > 1 \) and that the solution is unique.

Omitting the algebraic details, it is straightforward to show that

\[
\frac{\partial p^*_r(w, s)}{\partial s} = \frac{(b-2)A + (b-2)^2 s - (b^2 - b + 2)w}{2A(b-1)} \leq 0 \text{ for all } b > 1 \text{ and } w \geq s.
\]

Similarly, for the mean adjusted order quantity \( z^*_r(w, s) \) (obtained by substituting \( p^*_r(w, s) \) into the expression for \( z^*_r(p, w, s) \)), it can be shown that after defining \( g = (b+1)w - bs + A \),

\[
\frac{\partial z^*_r(w, s)}{\partial s} = \frac{4(b-1)}{g^2 A} \left[ \frac{A + (b+1)w - (3b-2)s}{3g^2 A} \right] \geq 0 \text{ for all } b \geq 1 \text{ and } w \geq s.
\]

We also have

\[
\frac{\partial y(p^*_r(w, s))}{\partial s} = y(p^*_r(w, s)) \left( -\frac{b}{p^*_r(w, s)} \right) \frac{\partial p^*_r(w, s)}{\partial s} \geq 0,
\]

where the last inequality is a consequence of the result that \( \frac{\partial p^*_r(w, s)}{\partial s} \leq 0 \). Since \( z^*_r(w, s) \geq 0 \) and \( p^*_r(w, s) \geq 0 \), it follows that

\[
\frac{\partial q^*_r(w, s)}{\partial s} = \frac{\partial}{\partial s} \left[ z^*_r(w, s) y(p^*_r(w, s)) \right] = y(p^*_r(w, s)) \left[ \frac{\partial z^*_r(w, s)}{\partial s} - \frac{b z^*_r(w, s)}{p^*_r(w, s)} \frac{\partial p^*_r(w, s)}{\partial s} \right] \geq 0 \text{ for } b \geq 1 \text{ and } w \geq s.
\]

Q.E.D.

**Proof of Lemma 4:**

First note that under the optimal two-part tariff, the channel profit is equal to the retailer’s expected profit. Moreover, under perfect information sharing the manufacturer shares its superior estimate \( L_m = \)

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$L$ with the retailer who then uses this estimate to optimize its order and pricing decisions. Hence the gain in expected channel profits compared to a situation with no information sharing and $L_r \neq L$ is

$$E_{e_m} \{\pi_r(z_m^*(w_m^*), p_m^*(w_m^*), w_m^*)\} - E_{e_m} \{\pi_r(z_r^*(w_m^*), p_r^*(w_m^*), w_m^*)\} > 0$$

The inequality follows from Proposition 2, which asserts that the retailer’s actual expected profit is maximized when the retailers demand estimate is correct. \textit{Q.E.D.}

\textbf{Proof of Proposition 5:}

Under a two-part tariff and symmetric demand estimates (i.e., $\varepsilon_r = \varepsilon_m$), the manufacturer’s profit is given by

$$\theta E_{e_r} \{\pi_r(\varepsilon_r(c), p_r^*(c), c)\} \text{, where } \theta \in (0,1)$$

is determined by the bargaining power of the manufacturer. We first consider the situation where the quality of the demand estimates is private information.

1). When the manufacturer knows whose estimate is superior but the retailer does not:

a.) If the manufacturer’s estimate is superior, it would expect a higher fixed fee if it reveals it to the retailer, since by using it the retailer can increase the expected channel profits. However, the retailer has no knowledge of the quality of the demand estimates, and therefore has no reason to abandon its own estimate and use the manufacturer’s instead. Since the manufacturer has no means to convince the manufacturer to use its superior estimate without reducing the fixed fee, and thereby lower its own profit a two-part tariff by itself does not achieve information sharing. Using their own demand estimates, if the smallest fee that the manufacturer is willing to accept is

$$\theta E_{e_m} \{\pi_r(z_m^*(c), p_m^*(c), c)\}$$

and this amount is lower than

$$\theta \cdot E_{e_r} \{\pi_r(z_r^*(c), p_r^*(c), c)\}$$

an agreement can be reached and a two-part tariff coordinates the chain.

b.) If the retailer’s estimate is superior, the manufacturer simply accepts that, and both firms negotiate a fixed fee of

$$\theta \cdot E_{e_r} \{\pi_r(z_r^*(c), p_r^*(c), c)\}$$

based on the best available estimate of the expected
channel profits. In doing so, information sharing occurs and the two firms share the benefits of the superior information.

2). When the retailer knows whose information is superior but the manufacturer does not:

   a.) If the manufacturer’s estimate is superior, the retailer simply agrees to use that estimate and negotiate a fixed fee of $\theta \cdot E_{z_m} \{\pi_r (z_m^*(c), p^*_m(c), c)\}$ based on the best available estimate of the expected channel profits. In doing so, information sharing occurs, and the two firms share the benefits of the superior information.

   b. If the retailer’s estimate is superior, the retailer agrees to pay a fixed fee of no more than $\theta \cdot E_{z_r} \{\pi_r (z_r^*(c), p^*_r(c), c)\}$ but it has no means to convince the manufacturer that its information renders the best available estimate of the channel profits. The manufacturer therefore uses its own demand estimate to determine the expected channel profit as a basis for negotiating the fixed fee. An agreement can be reached if the smallest fee that the manufacturer is willing to accept is $\theta \cdot E_{z_m} \{\pi_r (z_m^*(c), p^*_m(c), c)\}$ and this amount is lower than $\theta \cdot E_{z_r} \{\pi_r (z_r^*(c), p^*_r(c), c)\}$, but no information sharing occurs.

3). When neither firm knows whose information is superior, the firms can reach an agreement on the fixed fee if $\theta \cdot E_{z_m} \{\pi_r (z_m^*(c), p^*_m(c), c)\} \leq \theta \cdot E_{z_r} \{\pi_r (z_r^*(c), p^*_r(c), c)\}$ but not otherwise as each firm has its own estimate and there is no incentive for either firm to accept the other firm’s proposal that is perceived to make itself worse off. In either case information sharing does not occur since neither firm has any incentive to abandon its own estimate.

When the quality of the demand estimates is common knowledge, the two firms can negotiate a profit sharing contract to achieve information sharing, as outlined in Section 3.2.1. In doing so, the channel is coordinated. 

Q.E.D.
References


