CONSUMER FEELINGS AND EQUILIBRIUM PRODUCT QUALITY *

GANESH IYER  
(University of California, Berkeley)

Dmitri Kuksov  
(Washington University in St. Louis)

April, 2007

* We would like to thank Eduardo Andrade, Ben Hermalin, and Ralph Winter for comments. Address for correspondence: Haas School of Business, University of California at Berkeley, Berkeley, CA 94720-1900. E-mail addresses: giyer@haas.berkeley.edu, kuksov@wustl.edu.
ABSTRACT

This paper considers the possibility that a firm can invest not only in the true product quality, but also in activities such as merchandising and store atmosphere that create affect influencing consumer perception of the product quality. Consumers base their purchase decisions on the signal of quality they experience, where the signal is affected by both the true product quality valued by the consumer and the affect the consumer experiences at the time of the signal formation. In this situation, a firm finds it optimal to invest in both product quality and affect inducement, even though rational consumers, in equilibrium, correctly solve back for the true product quality. We uncover an asymmetry in the effects of the cost of producing quality and the cost of affect. As a firm’s cost of quality decreases, the firm will find it optimal to invest more both in the true quality and in the affect inducement, even if it does not have a lower cost of inducing affect. Conversely, if a firm’s cost of affect decreases, then the product quality decreases but affect increases.

Under competition, we find that the firm investing more in quality also invests more in affect. An implication of this is that in a competitive environment, consumers can rationally associate an uplifting store atmosphere, affect-inducing merchandising, or mood-creating communication with high quality products even when signaling motives are absent and when there is no consumption externality of the affect. We also analyze the case in which firms might have different costs and consumers are uncertain about the costs incurred by a given firm. Here again we show that the perceived quality production is positively correlated with both the true quality and the affect production.
1. Introduction

Many consumption and purchasing situations are affected by the feelings that consumers experience at the time of decision making. Furthermore, firms take an active role in inducing some of those feelings. Retail and product markets are replete with examples of firms conducting activities that create positive affect. Upscale retailers invest in store “atmospheres” including elements such as lighting, merchandising, pleasant music, attractive salespeople, and even disperse fragrance in the air in order to put consumers in a good mood during the shopping process. Firms use affect inducing ads to get consumers positively disposed towards their product. Real estate agents bake cookies, use brightly colored flowers, paintings, lighting, and fixtures to conduct open house showings. Furthermore, in the real estate context there are also companies such as Showhomes (see http://www.showhomes.com/) which specialize in the business of “staging” or dressing up homes that are up for sale.

Some interesting points may be noted regarding the above practices. First, the affect-inducing activities described above typically do not change the utility that consumers will obtain from the product. Pleasant music in a retail store should not directly affect the quality of a dress, and the smell of baked cookies in an open house showing should not affect the financial valuation of the property by a prospective buyer. Second, affect inducing activities described above involve costs. Therefore, the question arises why retailers would incur these costs rather than offer a lower price to close a sale. While retail merchandising investments are observed across a wide range of retailers, the more significant investments are present at high-end stores such as Nordstrom or Neiman Marcus.

What explains the greater supply of affect inducing activities that are unrelated to product quality by higher quality firms? How do the optimal firm decisions depend on the costs of quality and affect? Under what conditions would it be rational for consumers to expect affect to be diagnostic of the true quality of the product? We examine these questions in a model that formalizes the effect of affect on a consumer’s decision to buy product quality. Specifically, the model captures the information processing problem of a consumer who is unable to separate affect from the true valuation of product quality, but who nevertheless is rational and aware of this non-separability and takes it into account in her decision-making.

Formally, we assume that a consumer receives a signal of product quality (the quality perception that the consumer experiences), which is influenced by both the true quality as well as by the affective

\[ \text{For example, staging companies such as Showhomes may charge $2000 to $5000 upfront, plus a monthly rental fee, to dress up homes for sale (see “Secrets of an open house,” May 7, 2004 CNN Money for more details).} \]
state. The consumer at the time of decision-making cannot perfectly separate the perception of the true product quality from the perception of the affect (or any other environmental variable) which she experiences and which is unrelated to the true product quality.\(^2\) Thus, a consumer’s perception of quality is influenced by the feelings or affect that the consumer has at the time of purchase, which then gives an incentive for the seller to influence the consumer’s feelings. The paper provides an explanation for the role of affect in decision making through an equilibrium theory of a market in which consumers make rational buying decisions given the information that they have at the time of the purchase decision, while the firm acts to optimally influence consumer behavior through the marketing variables at its disposal.

Jointly considering the interaction between consumers and firms suggests an interesting question which does not arise when considering only individual level consumer behavior. While the consumer may not be able to separate the effect of feelings from her true product quality evaluation, in a market setting, the consumer knows that a firm engages in affect-inducing activities that are irrelevant for the true quality. Knowing this, a rational consumer should try to correct for the bias in quality evaluations that might be caused by the presence of affect. But if consumers indeed discount their perceptions, how would that affect firms’ incentives for quality provision? In answering the above question we are also able to identifying the type of market conditions under which it would be profitable for sellers to supply more of such affect inducing activities.

The informational problem that we consider can also be thought of as follows. There are two economic agents: a firm that is a producer of a product and a consumer. The consumer values the quality of the product, but only has an imperfect signal of the quality available for decision-making. The signal is a function of the true quality of the product and a “noise” (affect) component which is irrelevant to the true quality. The consumer is rational and tries to extract the true quality from the signal. The firm has control over the quality of the product as well as over the noise that causes the imperfection in the quality signal. The main results are as follows.

When consumers cannot separate the affect from the true quality evaluation, then firms will invest in both the product quality and the affect inducement, even though in equilibrium, rational consumers

\(^2\)One way to interpret this is to assume that a consumer’s mind automatically encodes the input signals with some loss of source information, and the information that is available for use in decision-making is a combined value of the input from both the cognitive and affective parts of the mind rather than the two separate values. Recent research provides a neurophysical basis for this construction of decision making in the presence of affect. For example, Ashby et al. (1999) propose that positive affect and the ability to perform cognitive tasks are both associated with increased levels of the neurotransmitter dopamine. Thus dopamine levels can be seen as a basis for the association of affect and cognition.
are able to fully solve back for true product quality. In comparative statics, we uncover an interesting asymmetry in the effects of the cost of quality and the cost of affect. We show that if a firm is better at producing high quality products (i.e., has a lower marginal cost for quality), it will also find it optimal to invest more both in the true quality and in the affect inducement, even if such a firm does not have a lower cost of inducing affect. Conversely, if a firm can generate affect at a lower cost, product quality decreases and affect increases. Our analysis traces this asymmetry between the effect of the cost of quality and the effect of the cost of affect to the consumer’s ability to rationally solve back for the true product quality. In other words, because consumers “think” of their feelings about quality, the market equilibrium involves a positive connection between quality and affect only when quality is easy to produce, but not when affect is easy to produce.

To see whether higher product quality can be expected along with higher affect inducement activities, we look at the equilibrium actions of firms competing in a vertically differentiated market. We show that the firm choosing higher quality also chooses to supply higher affect. An implication of this result is that in a competitive environment, consumers can rationally associate an up-lifting store atmosphere or an emotional advertisement with high quality products even in the absence of signaling motives or consumption externality for affect.

Another reason for different firms producing different levels of affect and quality is that the firms (not necessarily competing) may have different costs of producing affect and/or quality. In this case, if consumers are uncertain about a firm’s costs, the perceived quality of a good is also positively correlated with both the affect and the true quality production. Our extension of the basic model to the case when consumers are uncertain about the firm’s costs of quality or affect also allows us to investigate the possible signaling motives of the firm. We find that when consumers are uncertain about the cost of quality, then as is common in standard signaling models, a firm with low cost of quality can separate and credibly signal its cost by distorting its perceived quality production over and above the full information level which pertains to when consumers are able to observe its cost. But what is more interesting is that when consumers are uncertain about the cost of affect provision, it is the high cost of affect firm that wants now to separate. Such a firm can credibly signal its difficulty in providing more affect by distorting its perceived quality to a level which is below the full information level that it would choose if its cost were observed by the consumers.
1.1. Contribution Compared to the Existing Literature

Our approach complements the standard idea that affect inducing investments may act as signals of product quality to consumers. Costly signaling (Spence 1973) is relevant if a firm wants to communicate its private information about quality or some other characteristic. In that case by incurring costly expenditures on activities, such as advertising, which are observable to consumers a firm might signal high quality. For advertising or other expenditures to work as signals of quality, it is necessary that these signaling activities be more costly for the low quality firm than for the high quality firm (i.e., the single-crossing property must be satisfied). For example, in Milgrom and Roberts (1986) this property is satisfied through the mechanism of repeat purchases which makes it possible for the high quality firm to invest in money-burning advertising.\(^3\)

In contrast, the main point of our paper is predicated on the possibility that there is *no difference* in the costs of inducing affect for firms with different quality. Therefore, the model of this paper can be seen as explaining the provision of affect inducing activities in cases where the motivation for the firm to signal private information is either absent or when the single crossing condition cannot be satisfied, thereby making signaling impossible for the firm.\(^4\) In the real estate example, the interaction between a seller and a buyer in the sale of a house is a one-time interaction. Therefore, the seller of a low quality house would be equally inclined to incur expenditures in staging the house as a seller of a high quality house making the signaling of quality through affect investments impossible. Our model of imperfect consumer information processing contributes by explaining affect inducing investments (that are irrelevant for the true quality) in precisely these cases where either quality signaling motivations are absent, or where signaling is not possible.

This model also differs from the signaling approach in another respect: In our framework the specific amount of affect (and also quality) deployed by the firm is unobservable to the consumers, because consumers only observe the perceived quality. Consumers have to rationally infer the amount of affect supplied, and it is due to this reason that firms in equilibrium invest in affect. In contrast, it is necessary that the signaling instrument (advertising, warranties) is observable to consumers in order

\(^3\)As other examples, warranties (Grossman 1981, Gal-Or 1989, Soberman, 2003) are less costly to offer for the high quality firm than for a low quality firm, or increasing the variable fee in the contract and reducing the fixed part of the contract is optimal for a manufacturer with high demand rather than one with low demand (Tirole 1988, p. 177, Desai and Srinivasan 1995). Similarly, in Bagwell and Riordan (1991) holding high initial prices is more efficient for high quality firm and more disavantageous for a lower cost and low quality firm. Finally, in Padmanabhan, Rajiv and Srinivasan (1997) the firm signals high network externality by initially withholding quality, but by making up for it later through an upgrade, a strategy which is more costly for the low type firm.

\(^4\)The analysis is also extended in Sections 5.2 and 5.3 by considering cost uncertainty to explore the possibility and the effects of signaling.
for firms to successfully use them as quality signaling instruments. Thus our consumer information processing framework is particularly useful in explaining situations in which affect and other such investments cannot be easily separated out by consumers from what they truly value.

This signal extraction feature is one which our model shares with the models of signal jamming that have been developed in other contexts. For example, Fudenberg and Tirole (1986) consider the situation in which the information available to an entrant (who is uncertain about future profitability) is distorted by an incumbent through the use of predatory pricing. The entrant, therefore, has to infer whether adverse profit realizations were due to a low realization state drawn by the nature or because of the predatory action. Stein (1989) considers a signal-jamming model of managerial incentives in a capital market, where managers distort current earnings by borrowing at adverse rates from the future. In these models the true variables of interest (the true profitability, or the true earnings) are themselves exogenously stochastic, while distortive actions of the agent (the incumbent or the manager) create jamming because there is exogenous uncertainty to begin with. In our set-up, both the true quality and the noise (affect) are endogenous and are choice variables of the firm. Therefore, a notable difference of our paper to the above papers is that the inference problem for the consumer exists even if there is no exogenous uncertainty. Furthermore, the inference problem is due to the endogenous supply of quality and affect that are linked in equilibrium through consumer inference, which creates the positive connection between quality and affect when quality is easier to provide, but a negative connection when affect is easier to provide. Double moral hazard problems such as in Cooper and Ross (1985) have the property that the final output is dependent on two variables – the quality and the consumer effort – and it is not possible to credibly observe and verify the extent to which each variable has contributed to the output. This is somewhat similar to the unobservability of true quality and affect for the consumer in our model. However, the double moral hazard problem arises from each party in the relationship choosing one variable and the inability to credibly stipulate the individual choices of the two parties in a contract. In our model it is the firm which chooses both the variables that cannot be separated out by the consumer.

Finally, the literature in consumer psychology has established the mechanisms through which feelings interact with decision making at the individual consumer level. For example, Schwartz and

---

5 Other examples of signal jamming can be found in Holmstrom (1982) analysis of managerial incentives and Riordan’s (1985) article on dynamic conjectural variations. Note that the mixing of noise with a variable of interest is a feature that is also present in the standard principal-agent model (e.g., Ross 1973, Holmstrom 1979) in which the agent has control over the effort (quality) and the principal (the recipient of the output from the agent’s effort) has an imperfect estimate of the agent’s effort level. Unlike in the signal jamming models, the noise in the principal-agent set-up is exogenous while the agent’s effort is endogenous. In this paper, we model both variables affecting the consumer’s perception as endogenous.
Clore (1983) provide experimental evidence to show that people use their affective states as information for making judgments. However, this literature does not analyze the role of affect in markets with rational consumer decision making. Our paper contributes by jointly considering the interaction between consumers and profit maximizing firms that can take strategic actions in inducing the affect that is experienced by consumers.

The next section presents a model with full information (all parameters are known by all agents). Section 3 presents comparative statics results with respect to parameter changes and comparison between the model predictions when consumers are rational and naive. Section 4 considers competition and Section 5 discusses the robustness of our findings and also examines the effects of consumer uncertainty. Section 6 discusses the results and concludes.

2. The Model

A monopoly firm produces a single good. There is a unit mass of homogenous consumers who have the following utility for a unit of the good:

\[ U(q, p) = \gamma q - p, \]  

(1)

where \( q \) is the true quality level, \( p \) is the price, and \( \gamma \) is a parameter that represents the consumer valuation of the true quality.\(^6\)

Consumers do not directly observe \( q \), but have to make an inference about \( q \) from the perceived quality \( \tilde{q} \) which is influenced by both true product quality \( q \) and the consumer affect \( a \) at the time of evaluation, according to some perceptual mechanism \( f(q, a) \). In other words, consumer feels that the product quality is

\[ \tilde{q} = f(q, a). \]  

(2)

However, she realizes that she is also potentially being influenced by affect generated by the firm. This implies that the individual evaluation of both the true quality as well as the affect is unobservable to the consumer, in the sense that the consumer cannot separately observe how much of the perceived quality is due to the true quality and how much of it is due to the affect. Although we use the general

\(^6\)The main results are not affected if consumers are heterogenous in their valuations for true quality. In fact, in section 4 we extend the basic model to one with competition between firms and consider a market with consumer heterogeneity in quality valuations to show that all the main results are preserved.
function $f(q, a)$ to derive the equilibrium equations, we assume the additive form $f(q, a) = q + a$ to illustrate the model solution and implications.\(^7\)

Note that by the affect $a$ we mean here things *unrelated* to the product that impact consumer perception $\hat{q}$ of the product and do not directly enter the consumer utility function. The aspects that influence the product’s actual (rather than expected) value to the consumer are represented by $q$.\(^8\)

A consumer buys the product if her expected utility of the product is non-negative. The unit cost of the good of quality $q$ to the firm is assumed to be increasing and convex and given by $C(q) = c_q q^2$, and the unit cost of inducing affect $a$ is $C(a) = c_a a^2$. To summarize, the total unit cost for the firm of producing the good of quality $q$ and inducing affect $a$ is:\(^9\)

$$C(q, a) = C(q) + C(a) = c_q q^2 + c_a a^2.$$ (3)

We also assume that the parameters of the model $\gamma$, $c_q$ and $c_a$ are common knowledge (these assumptions are relaxed in Section 5). Thus, although the consumer cannot individually observe how much of the observed perceived quality is coming from the true quality or from the affect, the consumer having observed $\hat{q}$ can make a rational inference of the true quality $\hat{\hat{q}}(\hat{q})$ through inferring the profit-maximizing behavior of the firm.

The timing of actions in the game is as follows: First, the firm decides on the levels of $q$ and $a$, followed by choosing the price $p$.\(^10\) Then, consumers observe the price and the perceived quality $\hat{q}$ and make the purchase decision. We look for the perfect Bayesian Nash equilibrium of this game after elimination of strictly dominated strategies.

---

\(^7\)In Section 5.1, we discuss the robustness of the results with respect to this specification and discuss other functional forms including the multiplicative form $f(q, a) = qa$, all of which lead to similar results.

\(^8\)Note that if the affect directly enters consumer utility function, but does not interact with the utility of the product, the results will not change. However, if some part of the affect has a consumption externality value (i.e., if consumers enjoy the product consumption more if they experience a higher affect while shopping for it), then it can be viewed as a part of the quality $q$. But the affect we consider is the part which has no consumption externality, but which is induced by the firm in order to influence the consumer’s perception and evaluation of the true quality.

\(^9\)Given the inelastic consumer demand and unit mass of consumers, the results will not change at all whether either of these costs is fixed, marginal (unit), or a mix of the two. The model therefore allows both the packaging (such as container type, color, etc., which result in per-unit costs) and in-store atmospherics (such as music, fragrance, lighting, width of isles, etc., which result in mostly fixed costs) to be possible examples of the affect-inducing activities of the firm.

\(^10\)Since no other player makes a move between these three decisions, it is not consequential whether they are made sequentially, as stated above, or simultaneously. However, the sequence defined above is important when we consider, in Section 4, the extension to the competitive case.
2.1. Solution

Consumers must make their purchase decisions based on the information they have at the time of their decision-making, i.e., based on the perceived quality and price. Therefore, the profit-maximization problem of the firm can be thought of as consisting of two inter-related components:

1. Decide what values should be optimally set for the variables observed by consumers (i.e., for $\tilde{q}$ and the price), and

2. Provide these values in the most efficient way by appropriately choosing the variables under the direct control of the firm (i.e., by setting $q$, $a$ and the price).

While the optimal firm’s decision in the first component above depends on consumer behavior, the optimal choice of $q$ and $a$ in producing $\tilde{q}$ in the second component above does not depend on consumer behavior or beliefs (because beliefs can only be conditional on what consumers observe, and not on the values of $q$ and $a$ that result in the $\tilde{q}$ observed by consumers). Therefore, regardless of consumer behavior or beliefs, and no matter what $\tilde{q}$ the firm decides to produce, the firm will use the most cost efficient way of producing this value of $\tilde{q}$. This means that rational consumers know that the (rational profit maximizing) firm has used the most cost efficient way of producing the perceived quality they observe. They will, therefore, be able to infer the true values of $q$ and $a$ from the assumption of profit maximizing behavior of the firm. This results in the consumer-inferred quality function $\hat{q}$, which is a function of the (observed) perceived quality $\tilde{q}$, and which is equal to the true quality level given that the firm is profit maximizing.\textsuperscript{11}

Given the consumer inference of quality $\hat{q}(\tilde{q})$, consumers will be willing to pay (at most) $\gamma \hat{q}(\tilde{q})$ for the product. Therefore once again, regardless of the true $q$ and $a$, it is optimal for the firm to charge the price $p = \gamma \hat{q}(\tilde{q})$.\textsuperscript{12} Solving for $\tilde{q}$ that results in the highest profit given the above price, the optimal production function, and given that consumers believe the firm is following the optimal production function, determines the optimal perceived quality production of the firm.

As a result, we have the following proposition:

\textsuperscript{11}This means that even though the consumers can perfectly infer quality in equilibrium, the resulting quality under this inference may be different than the quality chosen by the firm if the consumers were able to directly observe quality $q$. As we show in section 2.3, it is, in fact, different.

\textsuperscript{12}Suppose the firm were to deviate from its optimal cost minimizing production of $q$ and $a$ as defined by the solution to the second component described above (while keeping $\tilde{q}$ observed by the consumers the same), then the consumers will still infer the same quality $\tilde{q}$. This means that the maximum price the consumer will be willing to pay still remains $p = \gamma \hat{q}(\tilde{q})$ and so it would not be optimal for the firm to deviate from its optimal production of $q$ and $a$. 


Proposition 1: In the equilibrium, the firm chooses the quality $q$ and affect $a$ as to satisfy

$$c_a a f_1(q, a) = c_q q f_2(q, a),$$

(4)

where subscripts 1 and 2 on $f$ denote the derivatives of $f$ with respect to the first and second variable, respectively, and

$$\gamma = 2c_q q + 2c_a a(q)a'(q),$$

(5)

where $a(q)$ is the function defined implicitly by Equation (4). Off-equilibrium consumer beliefs on $(q, a)$ are defined by the condition that $q$ and $a$ satisfy $\tilde{q} = f(q, a)$ and Equation (4).

Proof: See Appendix. □

We now turn to the linear specification of the function $f(q, a)$ to illustrate the model solution and its implications. Specifically, consider $f(q, a) = q + a$. In this case, equation (4) becomes $c_a a = c_q q$, implying $a(q) = (c_q/c_a) q$ and $\tilde{q} = (1 + c_q/c_a) q$. Equation (5) then becomes

$$\gamma = 2c_q q + 2c_a (c_q/c_a) q (c_q/c_a) \equiv 2c_q q + 2c_q^2 q/c_a.$$  

(6)

Therefore, the equilibrium levels of quality, affect, and perceived quality are

$$q^e = \frac{\gamma c_a}{2c_q(c_q + c_a)}, \quad a^e = \frac{\gamma}{2(c_q + c_a)}, \quad \text{and} \quad \tilde{q}^e = \frac{\gamma}{2c_q}.$$  

(7)

The costs the firm incurs on quality, inducing affect, and the total cost are, respectively,

$$C(q^e) = \frac{c_a^2 \gamma^2}{4c_q(c_q + c_a)^2}, \quad C(a^e) = \frac{c_a \gamma^2}{4(c_q + c_a)^2}, \quad \text{and} \quad C(\tilde{q}^e) = \frac{\gamma^2 c_a}{4c_q(c_q + c_a)}.$$  

(8)

Note that this implies that the firm invests in affect inducement even if the consumers are sophisticated enough to be able to infer back the true quality. The resulting equilibrium profits of the firm are

$$\pi^e = \frac{\gamma^2 c_a}{4c_q(c_q + c_a)}.$$  

(9)

13In Section 5, we show that this is equivalent to a general linear specification $f(q, a) = \rho_1 q + \rho_2 a$ by redefining cost parameters $c_a$ and $c_q$. 

9
2.2. Naive Consumers

Historically, not much attention has been paid to emotions in economic models, but there is a growing interest in modeling the effect of emotions on economic decisions. The literature on the psychology of affect generally does not consider the possibility that consumers might be sophisticated enough to account for the effect of affect and their expectations about firm behavior in evaluating products. It will be therefore useful to compare our model with the following two benchmark cases in order to better understand the results. This subsection considers the case of “naive” consumers who use perceived quality as if it were the true quality in their decision making as would be consistent with the role of affect in the psychology literature. After this, Subsection 2.3 considers the case when consumers can directly observe the true quality. Comparisons are made in Section 3.

In modification to the model above, consider what would happen if consumers are naive in the sense that they do not use rational evaluation about what they should expect the true quality to be given the perceived quality. Instead, assume they use the perceived quality as the true quality in their decision making. This means that consumers buy the product if and only if $\gamma \tilde{q} - p \geq 0$. It is still efficient for the firm to generate perceived quality through affect and true quality that are related as in equation (4); however, the optimal levels of these investments are different. Suppose the firm is considering generating a marginal increase in perceived quality. Then the marginal cost of increasing perceived quality so that the true quality would increase by $dq$ is, as before,

$$ (2c_q q + 2c_a a(q)a'(q)) dq, $$

(10)

where $a(q)$, as before, is defined by solving equation (4) for $a$ as a function of $q$. However, the marginal benefit of such an increase in the perceived quality is the full change in the perceived quality, i.e., it is

$$ \gamma f'(q, a(q)) dq = \gamma (f_1(q, a(q)) + f_2(q, a(q))a'(q)) dq $$

(11)

rather than $\gamma dq$. Therefore, if consumers are naive, the equilibrium conditions are as in Proposition 1

---

14MacLeod (1996) develops a model of heuristic decision making which is a combination between rational search and emotional judgment. Kaufman (1999) proposes that emotional arousal can enhance or degrade cognitive ability. Hermalin and Isen (2003) incorporate the effect of emotions on strategic decisions using the idea that affect at a given point in time is determined by previous affect and actions and that the current utility of the individual is a function of her current actions as well as the current affective state.
with the difference that Equation (5) is replaced by

$$\gamma(f_1(q, a(q)) + f_2(q, a(q))a'(q)) = 2c_q q + 2c_a a(q)a'(q).$$  
(12)

If \( f(q, a) = q + a \), as we have used in the solution of the model in the subsection above, Equation (12) becomes \( \gamma(1 + c_q/c_a) = 2c_q q + 2c_a(c_q/c_a)q(c_q/c_a) \), leading to the equilibrium levels of quality, affect, and perceived quality of

$$q = \frac{\gamma}{2c_q}, \quad a = \frac{\gamma}{2c_a}, \quad \text{and} \quad \bar{q} = \frac{\gamma(c_q + c_a)}{2c_q c_a}. \quad \text{(13)}$$

In this case, the costs the firm incurs on generating quality, affect, and the total costs are, respectively,

$$C(q) = \frac{\gamma^2}{4c_q}, \quad C(a) = \frac{\gamma^2}{4c_a}, \quad \text{and} \quad C(\bar{q}) = \frac{\gamma^2(c_q + c_a)}{4c_q c_a}, \quad \text{(14)}$$

whereas the profit is

$$\pi = \frac{\gamma^2(c_q + c_a)}{4c_q c_a}. \quad \text{(15)}$$

2.3. Observable Quality

Now, in another modification to the model of Section 2.1, let us consider the case when in addition to the feeling of quality \( \bar{q} \), consumers can directly observe the quality level. In this case, rational consumers will not use the perceived quality in their decision making, since the true quality affecting their utility is directly observed. Therefore, the firm would not invest in generating affect. This means that while the marginal benefit of increasing the true quality, as in the case of the base model, is \( \gamma \), the marginal cost is \( 2c_q q \). Hence, in this case, the true quality level, the total cost of the firm, and the profits are

$$q = \frac{\gamma}{2c_q}, \quad C(\bar{q}) = C(q) = \frac{\gamma^2}{4c_q}, \quad \text{and} \quad \pi = \frac{\gamma^2}{4c_q}. \quad \text{(16)}$$

Note, that in the original model, consumers correctly infer the true quality level in the equilibrium. However, the case of observable quality is different because in this case, consumers have the correct expectations of the true quality level not only given the equilibrium firm behavior, but also if the firm were to deviate. Therefore, the different payoffs given deviations in the case of observable quality result in different firm behaviors. The equilibrium true quality chosen by the firm and the firm’s profits are greater when quality is directly observed than when quality is inferred by the consumer.

Another observation is that the equilibrium quality level derived above will be the same if con-
sumers could not observe the quality directly, but the affect distorting the consumer perceptions would not be under control of the firm. Therefore, the difference between this case and the base model is due to the firm’s profit-maximization behavior.

3. Implications and Comparative Statics

From the equilibrium derived in Section 2.1, we have the following proposition that presents the comparative statics results about how the equilibrium quality levels and affect are affected by costs of quality and affect production, $c_q$ and $c_a$.

**Proposition 2:** *In the equilibrium of the model of Section 2.1*

1. If quality becomes easier to produce (i.e., if $c_q$ decreases), the perceived quality, affect, and the true quality all increase. Furthermore, profits increase as well.

2. If affect becomes easier to produce (i.e., if $c_a$ decreases), affect increases; however, the true quality decreases and the perceived quality remains the same. Furthermore, profits decrease.

**Proof:** Differentiating the equilibrium perceived quality, affect, and the true quality levels from Equation (7) and equilibrium profits from Equation (9) with respect to the costs of quality and affect production, we obtain

\[
\frac{\partial q^e}{\partial c_q} < 0, \quad \frac{\partial q^e}{\partial c_a} > 0, \quad \frac{\partial a}{\partial c_q} < 0, \quad \frac{\partial a}{\partial c_a} < 0, \quad \frac{\partial \tilde{q}^e}{\partial c_q} < 0, \quad \frac{\partial \tilde{q}^e}{\partial c_a} = 0, \quad \frac{\partial \pi^e}{\partial c_q} < 0, \quad \text{and} \quad \frac{\partial \pi^e}{\partial c_a} > 0,
\]

from which the claims of the proposition follow. □

The first statement of the proposition implies that if firms are different in the ability to produce quality, then firms producing higher quality products will also be spending more on (and achieving more of) consumer affect, even if such high quality firms do not find it easier to produce affect. In our model, affect stands for the positive consumer feelings which are over and above the feelings induced by higher product quality alone. Therefore, firms producing higher quality do not necessarily have to provide higher affect. However, the first part of Proposition 2 shows that the connection can be an equilibrium result.

In contrast, as the second result of Proposition 2 states, it is also the case that the increased ability to induce affect (lower $c_a$) makes the firm worse off when consumers are sophisticated enough to solve back the perceived quality for the true quality. If affect becomes easier to induce (i.e., $c_a$ decreases), the equilibrium level of true quality decreases, but the level of affect increases, so that the perceived
quality supplied remains the same. Therefore, if firms are different in their abilities to produce affect, it may be more difficult for consumers to infer differences in true product quality, because affect and quality differences between products would go in the opposite direction.

The asymmetry in the effect of the cost of affect versus that of the cost of quality that we see above is due to the ability of the consumers to rationally solve back for the true quality from the perceived quality. To see this, we compare the above with the case of “naive” consumers who decide on whether or not to buy based on the perceived quality \( \tilde{q} \) rather than on the inferred quality \( \hat{q}(\tilde{q}) \), and with the case when firms can not induce affect.

We will see in Proposition 3 that this association is due to sophisticated consumer behavior, whereby consumers try to infer the true quality from the perceived one, and this association does not hold if consumers are naive:

**Proposition 3:**

1. When consumers are “naive” and use only the perceived quality \( \tilde{q} \) for decision making, the equilibrium level of affect does not depend on the cost of quality, and the equilibrium level of quality does not depend on the cost of affect. However, the perceived level of quality is increasing when either cost decreases. In this case, firms profits increase when either cost decreases and the equilibrium true and perceived quality levels are higher than in the case of sophisticated consumers.

2. When consumers can directly observe true product quality, the firm does not invest in affect and relative to the unobserved true quality case, the equilibrium true quality and profits are higher.

**Proof:** Similar to the proof of Proposition 2, the claims follow directly from the equilibrium equations from the previous section. □

This proposition implies that the decrease of true quality due to lower costs of generating affect that we have observed in Proposition 2, i.e., the substitution of the investment that the firm makes from quality to affect, is due to the sophisticated consumer quality evaluations.

Note that if the firm could not change the affect \( a \) (i.e., if it is exogenous), the payoff to the firm of increasing the true quality is equal between the cases when consumers observe and do not observe the true product quality. Thus, the difference between consumer directly observing the quality and inferring it is due to the consumer expectation that the optimal firm’s investment in the affect is correlated with optimal firm’s investment in product quality.
4. Competition

In this section, we modify the model to consider a competitive environment with two firms. Given our interest in quality competition, it is natural to consider competition in a vertically differentiated market. Accordingly, consider a model of a market with a unit mass of a continuum of types of consumers who differ in their quality valuations and are uniformly distributed on the interval \( \gamma \in (0, \gamma_h) \). There are two firms (indexed by \( j = 1, 2 \)) who have the same cost of producing quality and affect and with cost functions the same as in the previous section. From the standard arguments as in Shaked and Sutton (1982) and Moorthy (1988) it follows that there is no symmetric equilibrium in quality choices in this model, and the only possible equilibrium is one in which firms differentiate in quality.

Let firm 1 be the one that produces the higher quality. Let \( \tilde{q}_1 \) and \( \tilde{q}_2 \) denote the perceived qualities offered by the two firms. As in the previous section, assume that \( f(q, a) = q + a \). Similar to the analysis in the previous section, because the perceived quality and not the true quality affects the consumers’ decision, each firm \( j \) will produce the pair \((q_j, a_j)\) to satisfy equation (4) and so \( \tilde{q}_j = (\frac{c_a}{c_q + c_a})q_j \). The consumers will therefore infer the true quality \( \hat{q}_j(q_j) = (\frac{c_a}{c_q + c_a})\tilde{q}_j \). Thus the expected utility of a type \( \gamma \) consumer from the product of firm \( j \) will be given by

\[
E_{ij} = \gamma \hat{q}_j - p_j = \left( \frac{\gamma c_a}{c_q + c_a} \right) \tilde{q}_j - p_j.
\] (18)

Since some consumers have very low valuation, not all types of consumers are served in equilibrium. The consumer type who is indifferent between the two firms is \( \gamma_x = \frac{p_1 - p_2}{q_1 - q_2} \), and the marginal consumer who is indifferent between buying and not buying from the low quality firm is \( \gamma_b = \frac{p_2}{q_2} \). This implies that the demand for the high quality firm is \( d_1 = (\gamma_h - \gamma_x) \) and that for the low quality firm is \( d_2 = (\gamma_x - \gamma_b) \). The corresponding profit functions of the firms are,

\[
\pi_1 = d_1p_1 - C(\tilde{q}_1), \quad \text{and} \quad \pi_2 = d_2p_2 - C(\tilde{q}_2).
\] (19)

The Appendix derives the equilibrium. In the equilibrium, the two firms differentiate in quality.

\[15\] Note that we could have considered consumer heterogeneity in quality valuations even for the monopoly case. However, this does not affect any of the results of that section.
equilibrium true quality, and affect of the firms are as follows:

\[ q_1^e = A_1 \frac{\gamma_2 c_a}{c_q(c_q + c_a)}, \quad a_1^e = A_1 \frac{\gamma_2}{c_q(c_q + c_a)}, \quad \text{and} \]

\[ q_2^e = A_2 \frac{\gamma_2 c_a}{c_q(c_q + c_a)}, \quad a_2^e = A_2 \frac{\gamma_2}{c_q(c_q + c_a)}, \]

where \( A_1 > A_2 > 0 \) are constants defined in the Appendix. This analysis recovers some of the comparative statics results that we presented in the monopoly case from the equilibrium actions of competing firms. The high quality firm offers higher affect and charges a higher price than the low quality firm. Therefore, in a market with competing firms, an outside observer who is able to observe affect can rationally associate higher affect provision with high quality. Thus, as opposed to the exogenous cost change rationale provided by the comparative statics results, the analysis above demonstrates the existence of the positive correlation between quality and affect in the presence of market competition.

The equilibrium profits of the two firms are

\[ \pi_1^e = B_1 \frac{\gamma_4 c_a}{c_q(c_q + c_a)}, \quad \text{and} \quad \pi_2^e = B_2 \frac{\gamma_4 c_a}{c_q(c_q + c_a)}, \]

where \( B_1 = A_1^2 \left( A_1 - A_2 \right) / (4A_1 - A_2)^2 - 1 \) and \( B_2 = A_2 \left( A_1(A_1 - A_2) / (4A_1 - A_2)^2 - A_2 \right) \) (and we have \( B_1 > B_2 > 0 \)). The equilibrium profits decrease in \( c_q \) and increase in \( c_a \). This is consistent with the previously obtained insights: When quality is harder to produce both firms produce less quality and the consumers rationally expect less quality and this reduces profits. However, when affect is harder to produce, consumers expect less affect and more quality provision for any given level of perceived quality and this increases the equilibrium firm profits.

5. Robustness of the Model Implications

We now consider several modifications to the model of Section 2 to understand better how relaxing different assumptions would change the model implications. In Section 5.1, we consider sensitivity of the results with respect to alternative specifications of the functional form of the perceived quality by considering the possibilities of the affect and true quality entering perception with different weights and also the possibility that the perceived quality is multiplicative rather than additive in the underlying factors.

Sections 5.2 and 5.3 relax the full-information assumption. Recall, that in the model of Section
2, consumers are able to figure out the firm’s strategy because they know all the parameters of the production functions of the firm. This full information assumption implies that although consumers do not observe the true quality, they can infer it for sure and without mistakes in the equilibrium. In reality, if consumers do not observe quality directly, they may be uncertain about it. This could be because they do not know some of the production function parameters, such as the costs of the firm. We now consider such possibilities. We first consider the case commonly considered in the literature: namely, uncertainty about the cost of quality (Section 5.2), and then consider the uncertainty about the cost of producing affect (Section 5.3).

Section 5.4 considers the possibility that consumers have not only a perception of quality, but also a perception of affect, but in a way that they cannot deterministically solve these two functions for the true quality. We show that the main implications of the model hold under such an extension and derive some new results.

5.1. Functional Forms of the Perceived Quality

Suppose the production of perceived quality follows the more general additively separable form

\[ f(q, a) = \rho_1 q + \rho_2 a \]  

(20)

rather than \( q + a \). One might interpret the \( \rho_1 \) and \( \rho_2 \) as the perceptual weights which the mind assigns to affect and true quality. For example, a relatively high \( \rho_1 \) compared to \( \rho_2 \) would imply that the consumers perception of quality comes primarily from the true quality. In our model this is equivalent to a re-normalization of the cost parameters \( c_q \) and \( c_a \) to \( c_q' = c_q / \rho_1 \) and \( c_a' = c_a / \rho_2 \), and hence, the implications are the same as in the base model.

It is also possible that instead of the additive functional form, the perceived quality is easier to influence in the upward direction by affect when the true quality is higher. This can be modeled by perceived quality function of the form

\[ f(q, a) = qa. \]  

(21)

The implications of this case are also similar to those of the base model. The equilibrium quality,
affect, and the perceived quality are\textsuperscript{16}

\begin{align*}
q &= \frac{\gamma}{4c_q}, \quad a = \frac{\gamma}{4(c_qc_a)^2}, \quad \text{and} \quad \bar{q} = \frac{\gamma^2}{16c_q(c_qc_a)^2}, \\
(22)
\end{align*}

which means that if the cost of quality decreases, then the quality, affect, and the perceived quality all increase (just like in the case of linear \( f(q, a) \)), and if the cost of affect decreases, the quality does not increase, but the affect increases (again, as in the case of linear \( f(q, a) \)).

### 5.2. Cost of Quality Uncertainty

We will now relax the assumption of full information about costs. Specifically, in this section, we consider the possibility that consumers are uncertain about the cost of producing quality for the firm. Accordingly, let us modify the model of Section 2 to assume that the cost of quality production \( c_q \) instead of being certain and known to consumers, is uncertain and can be either high \( c_{q2} \) or low \( c_{q1} \) with equal probability.\textsuperscript{17} Furthermore, assume that the firm knows \( c_q \), but consumers only know the prior distribution stated above. A low-cost of quality firm will therefore have the incentive to credibly signal its cost. We derive below the separating equilibrium of this signaling model.

In the full-information setting, we had that when the cost of quality is high, the firm would produce \( q, a \), and, therefore, \( \bar{q} \) at levels below those when the cost of quality is low (see equation (7)). Furthermore, consumers will discount their perception \( \bar{q} \) of quality more when the cost of quality is high. Therefore, one may expect that the high-cost firm may want to pretend to be low-cost firm, and thus, the low-cost firm may have to over-produce quality and/or affect to make sure that it is recognized as the firm with low cost of quality (and therefore, high quality). As the following proposition shows, this in fact leads to an upward distortion of the equilibrium quality and affect production levels for the firm that has a low cost of quality. In the proposition and the discussion that follows we denote the firm type (low or high cost) by 1 and 2 while the quality and affect choices (high or low levels) are denoted by \( H \) and \( L \).

**Proposition 4:** When cost of quality parameter is either \( c_{q2} \) or \( c_{q1} \) and is known to the firm but unknown to the consumers, in the unique separating equilibrium outcome that satisfies the intuitive

\textsuperscript{16}In this case, Equation (4) reduces to \( c_ua^2 = c_q\bar{q}^2 \), i.e., \( a = q\sqrt{c_q/c_a} \). Therefore, Equation (5) becomes \( \gamma = 2c_qa + 2c_a\sqrt{c_q/c_a} \cdot \bar{q}\sqrt{c_q/c_a}, \) i.e., \( q = \frac{\gamma}{4c_q} \). Thus, \( a = \frac{\gamma}{4\sqrt{c_q/c_a}} \), and \( \bar{q} = \frac{\gamma^2}{16c_q(c_qc_a)} \).

\textsuperscript{17}It turns out that the exact probabilities of states \( h \) or \( l \) are inconsequential to the equilibrium (as is common in standard signaling models), so the assumption of equal probability of the states involves no loss of generality.
criterion, we have:

1. If $c_q = c_{q2}$, the quality and affect supplied are

   \[ q_L = \frac{\gamma c_a}{2c_{q2}(c_{q2} + c_a)} \quad \text{and} \quad a_L = \frac{\gamma}{2(c_{q2} + c_a)}. \]

2. If $c_q = c_{q1}$, the quality and affect supplied are

   \[ q_H = \frac{\gamma c_a (c_{q2} + c_a + \sqrt{(c_{q2} - c_{q1})(c_{q1} + 2c_a + c_{q2})})}{2(c_{q1} + c_a)^2c_{q2}}, \quad \text{and} \quad a_H = q_H \frac{c_{q2}}{c_a}. \]

Furthermore, consumers believe that the firm has the low cost of quality if and only if they observe $\hat{q} \geq f(q_H, a_H)$.

**Proof:** See Appendix.

By comparing the equilibrium values of quality and affect to those in the full information case, one can see that the average perceived quality, true quality, and the affect will be higher than in the full information case. Furthermore, the production levels of quality and affect of the high-cost firm are at the optimal (full-information) levels, whereas the production levels of the low-cost firm are distorted upwards in both the affect and quality production levels. Thus, as in the previous comparative statics results, we observe a positive correlation between the equilibrium levels of affect and quality productions when the cost of quality production is uncertain for the consumers (i.e., affect is produced at a higher level when the quality is produced at a higher level due to lower cost of quality production).

Finally, as shown in the Appendix, the extent of distortion of perceived quality induced by the low cost firm over the full information level is given by,

\[ \text{Distortion} = \frac{\gamma (c_{q2} - c_{q1} + \sqrt{(c_{q2} - c_{q1})(c_{q1} + 2c_a + c_{q2})})}{2(c_{q1} + c_a)c_{q2}} > 0. \]  

(23)

It is interesting to note that the extent to which the low cost firm will distort its production of perceived quality decreases with $c_a$. Thus in markets where it is harder to induce affect, is easier for the low cost of quality firm to separate and credibly signal its type.
5.3. Cost of Affect Uncertainty

Suppose now that in the model of Section 2, it is the cost of affect $c_a$ is the one that is uncertain to consumers (and is known to the firm), while the cost $c_q$ of quality is certain and known to both consumers and the firm.

Let the cost of affect be either high $c_a2$ or low $c_a1$. In this set-up, the comparative statics results in Section 3 might suggest that the low-cost firm should produce the same perceived quality but lower true quality than the high-cost firm. This may suggest a pooling equilibrium which might then mean that consumers would discount the perceived quality in the same amount for the low- and high-cost firm (as would happen in a pooling equilibrium). But this would imply that the low-cost firm would want to produce a higher perceived quality than the high-cost firm. However, if it does so, consumers would identify it as a low-cost-of-affect firm. As the following proposition shows, the equilibrium satisfying the intuitive criterion is one in which the low-quality firm is producing at the optimal level given that consumers recognize that its cost of affect production is indeed low, and the high-cost firm produces both affect and quality at a lower level.

**Proposition 5:** When cost of affect parameter is either $c_a1$ or $c_a2$ and is known to the firm but unknown to the consumers, in the unique separating equilibrium outcome that satisfies the intuitive criterion, we have:

1. If $c_a = c_a1$, the quality and affect supplied are
   \[
   q_H = \frac{\gamma c_a1}{2c_q(c_q + c_a1)}, \quad \text{and} \quad a_H = \frac{\gamma}{2(c_q + c_a1)}.
   \]

2. If $c_q = c_a2$, the quality and affect supplied are
   \[
   q_L = \frac{\gamma c_a2 (c_q + c_a1) - \sqrt{(c_a2 - c_a1)c_q(c_a2c_q + 2c_a2c_a1 + c_qc_a1)}}{2c_qc_a1(c_q + c_a2)^2}, \quad \text{and} \quad a_L = q_L\frac{c_q}{c_a2}.
   \]

Furthermore, consumers believe that the firm has the low cost of creating affect if and only if they observe $\tilde{q} \geq f(q_H, a_H)$.

**Proof:** See Appendix. \qed

The above Proposition presents a counter-point to the results from signaling models involving quality. The usual result in the literature (as in the previous section) is that a low cost of quality
firm can separate and credibly signal its quality by distorting its quality level over and above its full information level (see for example Balachander and Srinivasan 1994). But unlike in the case of uncertain cost of quality, if the cost of affect is uncertain, then the distortion in affect and quality production is downwards and is done by the high-cost of affect firm. Thus, it is the high cost of affect firm here that wants to separate and credibly signal its inability to inject too much affect. It does so by providing both the level of affect and the level of quality lower than what would have provided if its costs were observed by the consumers. At the same time, just as in the cost-of-quality uncertainty and the competition cases, the result is that the affect production turns out to be positively correlated with the quality production.

5.4. Consumers Having Perceptions of Both Quality and Affect.

The main model in section 2 assumes that consumers have a perception of quality and have to infer affect on the basis of quality perception alone. In this section, we present an extension which captures two important features of the problem in the observed markets. First, consumers are likely to have not only a perception of quality, but also a perception of affect. Second, firms in reality might choose a wide range of actions which influence the feelings of consumers. This section provides a formal framework that allows one to think about both quality and affect perceptions (and the multiplicity of actions that firms might use to influence these perceptions) in a manner that still preserves the essential feature that the true quality can not be explicitly derived from the multiple perceptions. This implies that the firm’s actions in the non-quality variables would still affect the consumers’ inference about quality. As we will see from this extension, the important assumption is that consumers are not able to directly map the perceived variables into the firm’s actions, but rather they rely on their expectations of optimal firm behavior in equilibrium.

A theoretical issue that arises in introducing an affect perception defined by a function \( \tilde{a} = g(q, a) \), is that once consumers have two values they perceive, they may be able to fully solve for the two variables that the firm uses. However, in reality product quality and especially affect may be generated through a range of different inputs that are in the firm’s control. Product quality usually has different dimensions and the consumer valuation of quality might be a composite measure based on all the dimensions. Similarly, consumer feelings can also be impacted by different actions of the firm such as packaging, lighting, color theme, aroma, merchandising, music etc. Some of the decision variables of the firm may enter and enhance the true consumer utility (which we call the true quality), and some of them may affect the perceived quality and perceived affect. Even if all of them enter the true utility
(quality) in at least some amount, they may enter the quality and affect perceptions with weights
different from the weights they enter the true utility (quality) with.

To extend the model to the case of multiple perceptions of quality and affect, but still allow
consumers to have incomplete information, we consider the case when consumers have perceptions of
both quality and affect, but the number of firm’s decisions to is greater than the number of perceptions.
Specifically, let us assume that along with the quality choice, the firm chooses two instruments of affect
creation, \(a_1\) and \(a_2\), and the two variables that enter affect, enter it in a different way than the way
they enter quality perception. Formally, the quality and affect perceptions are, correspondingly,

\[
\begin{align*}
\hat{q} &= f(q, a), \\
\hat{a} &= g(q, a),
\end{align*}
\]

where \(a = (a_1, a_2)\). In other words: consumers have a perception of quality, but it is influenced
by affect, and consumers have a perception of affect, that may possibly be influenced by quality.
Furthermore, the perceptions are such that consumers are not able to explicitly derive the true quality.
For analytical tractability, assume that the functions \(f(\cdot \cdot \cdot)\) and \(g(\cdot \cdot \cdot)\) are linear. Without loss of
generality, we can then normalize the parameters in the function \(f(\cdot \cdot \cdot)\) to the identity (by rescaling
the cost parameters), to get:

\[
\hat{q} = q + a_1 + a_2, \quad (24)
\]

while keeping the specification of \(g(\cdot \cdot \cdot)\) in a general linear form:

\[
\hat{a} = \delta q + \alpha a_1 + \beta a_2. \quad (25)
\]

We further extend the cost specification to be, as in the main model, quadratic in costs of each
component: \(C(q, a_1, a_2) = c_q q^2 + c_{a_1} a_1^2 + c_{a_2} a_2^2\). Note that the interesting case for analysis is \(\alpha \neq \beta\) (in
other words, the two components influencing affect enter affect perception in a different manner than
the manner in which they enter the quality perception). Under this condition consumers will not be
able to explicitly deduce their true utility \(\gamma q\). The following proposition summarizes the equilibrium
outcome in the firm’s choices of \(q, a_1\) and \(a_2\):
Proposition 6: Denote $D = 2c_q(\alpha - \beta)^2 + 2c_{a_1}(\alpha - \delta)^2 + 2c_{a_2}(\beta - \delta)^2$. Then the firm chooses:

\[
\begin{align*}
q &= ((\beta - \delta)^2c_{a_1} + (\alpha - \delta)^2c_{a_2})\gamma/D, \\
a_1 &= (\alpha - \beta)(\alpha - \delta)\gamma/D, \\
a_2 &= (\beta - \alpha)(\beta - \delta)\gamma/D,
\end{align*}
\]

so that $\bar{q} = \gamma/(2c_q)$ and $\bar{a} = \gamma\delta/(2c_q)$. Furthermore, quality and the expenditure on quality increase when either the cost of quality decreases, or when the cost of $a_1$ or that of $a_2$ increases, while the expenditure on affect (the sum of expenditures on $a_1$ and $a_2$) and the effect of affect on quality perception increase when the cost of quality decreases or the cost of affect (either component) decreases.

Proof: See Appendix. \qed

Note that the manner in which the firm changes the allocation of resources on quality and affect depending on the costs of quality and affect, is exactly the same as in the main model (see Proposition 2 for the corresponding comparative statics). As the above proposition shows, the comparative statics on the level of quality remain the same. The comparison of comparative statics on affect components is a little bit more subtle, because there are now multiple components of affect. To establish that the comparative statics on affect are similar as well, we note the following: In the main model, consumers did not have a perception of affect, but their quality perception was influenced by the affect. In other words, the affect was defined as the amount of perceptual distortion on the true quality that the firm induces. The equivalent measure in the current setup is $\bar{q} - q = a_1 + a_2$, which in equilibrium turns out to be equal to $(\alpha - \beta)^2\gamma/D$ (see appendix). In comparative statics, this measure behaves exactly as $a$ does in the main model.

It is also the case that the consumer perception of affect in this model remains unaffected when the cost of affect (either component) changes. This can be interpreted as the firm attempting to not reveal its affect-inducing activities to consumers. The next useful result is that if the true quality does not influence the affect perception (i.e., $\delta = 0$), then in equilibrium, the firm chooses the affect components in such a way that consumers have no perception of affect ($\bar{a} = 0$). To understand this, note that it is the perception of affect $\bar{\alpha}$ that is observed by the consumer. If this perception of affect has nothing to do with the true quality, then the firm does not have the incentive to use the affect components to enhance the quality perception in a significant manner and in equilibrium this implies that the affect variables are chosen such that the induced consumer affect perception is zero. However, it must be noted that even as $\bar{a} = 0$, the affect components $a_1$ and $a_2$ are such that they together influence...
the quality perception upward. In other words, the firm sets the values of different affect-generating components so that consumers can not distinguish between this opportunistic firm behavior and the possibility that the firm does not use affect-inducing activities at all (i.e., sets \(a_1 = a_2 = 0\)).

However, if \(\delta > 0\), then the firm chooses its marketing action in a manner as if the entire consumer perception of affect was generated by the quality if the perceived quality was equal to actual quality \((\tilde{a} = \gamma \tilde{q})\). Note that since the perceived quality is higher than the actual quality, a (positive) part of perceived affect is actually induced by affect-inducing activities rather than by quality itself. In other words, when the true quality influences the affect perception, the firm does not set affect-inducing activities so as to keep their effect on perceived affect at zero as in the previous case of \(\delta = 0\), but rather the firm behaves in such a manner that consumers cannot distinguish between this behavior and choosing \(q = \tilde{q}\) together with \(a_1 = a_2 = 0\). Note that this implies that when \(\delta > 0\), the firm increases the spending on affect relative to the case when \(\delta = 0\). This is intuitive because when \(\delta > 0\), consumers view the perception of affect as diagnostic not only of the induced affect, but also of the true quality of the product.

6. Discussion and Summary

Feelings and affect seem to be present in many consumer decision making situations. But little attention has been paid to how affect might feature in decision-making by firms. Using a consumer information acquisition assumption about the non-separability of the true quality and affect, this paper provides insights into the observed strategies of firms in choosing product quality and affect inducing activities. It also points to an interesting asymmetry in a firm’s motivation to supply quality and affect. If it is easier for a firm to produce high quality products (i.e., it has a lower marginal cost for quality), the firm will also find it optimal to invest more in the true quality as well as in affect inducement, even if such a firm does not have lower cost of inducing affect. In contrast, if a firm can generate affect at a lower cost, the product quality supplied decreases and affect increases.

The above insight seems to have empirical validity as evidenced by the frequent incidence of high quality along with affect creating activities. Store atmosphere, emotional advertising, classy merchandising, and sophisticated salespeople are often associated with high quality products even in the absence of signaling motives. Our analysis traces this association to two specific aspects of markets: i) markets where quality rather than affect is easy to produce and ii) markets where consumers are sophisticated enough to attempt to solve back for the true quality.
Given the above, our model adds to the understanding of the supply of affect in markets in comparison to some obvious alternative motives. One alternative explanation for why sellers invest in affect is that consumers value affect directly. In our model if consumers have direct utility for some aspect, then that aspect would act exactly like the quality variable in the model. Our point is that even if there are aspects of affect that have no direct utility for the consumer, they can still be offered by the firm in equilibrium. Second, the point of our paper is also that the positive correlation between affect and quality will be observed even in the absence of correlation in the valuations for quality and affect. This distinction has practical importance, because it helps firms to decide on the attributes of store ambience consumers may not directly care about (such as specific variations of the packaging, music, lighting), but which still ends up influencing their purchase through the rational consumer inference.\textsuperscript{18}

It might be interesting to consider an extension to the case where consumers are heterogeneous in the information they acquire – if some consumers are only able to observe the affect, others, perhaps, are able to observe the true quality, and still others are able to observe a summary measure of the two – then the consumers who observe only the affect may find it rational to use it to infer the true quality in a market equilibrium. This is because the analysis in this paper suggests that the existence of consumers who observe the summary measure and try to rationally solve back for the true quality results in the positive correlation between the quality and affect in the marketplace. Therefore, these consumers place a positive informational externality on the consumers who only observe the affect.\textsuperscript{18}

\textsuperscript{18}For example, in the “Why You Buy” feature run by ABC News “20/20” on 3/29/1996 one can see that real life examples of how retailers supply music, lighting, packaging colors and other ambience elements which while not necessarily entering the consumer’s utility directly, may induce perceptions of product attributes which in turn influences purchase.
Appendix

Proof of Proposition 1

We begin the proof by solving for the optimal inter-relationship between \( q \) and \( a \) that the firm should use to create the consumer perceived quality \( \tilde{q} \). Note that this function comes from the problem of cost minimization subject to the constraint that a given perceived quality \( \tilde{q} \) must be generated and is therefore derived independent of consumer behavior (and/or beliefs). We can solve this cost minimization problem using the Lagrangian

\[
L(q, a, \lambda) = c_q q^2 + c_a a^2 - \lambda(\tilde{q} - f(q, a)).
\]

By equating the derivatives of this function with respect to \( q \) and \( a \) to zero, we obtain a system of two equations. Then by eliminating \( \lambda \) from this system we get Equation (4). This equation links \( q \) and \( a \) and it represents the manner in which these two variables must be chosen by a profit-maximizing firm regardless of its decisions on the price, perceived quality production, and consumer behavior (and/or beliefs). Therefore, rational consumers must believe that, no matter what price and \( \tilde{q} \) they observe, the true \( q \) and \( a \) are linked by Equation (4) because any other values of \( q \) and \( a \) that result in the same perceived quality observed by the consumer would involve strictly higher cost for the firm.

Equation (4) can be also thought of as defining the function \( a(q) \) determining how the firm’s choice of the affect \( a \) is connected with the firm’s choice of quality \( q \). Thus, consumers observing \( \tilde{q} \) should infer that it must be coming from the firm’s choice of some \( q \) and \( a = a(q) \) such that \( \tilde{q} = f(q, a(q)) \).

Let \( \hat{q} = \hat{q}(\tilde{q}) \) be the solution of the above equation for \( q \) given \( \tilde{q} \). Then, consumers observing \( \tilde{q} \) and any price expect the true quality to be \( \hat{q}(\tilde{q}) \), they value the product at \( \gamma \hat{q}(\tilde{q}) \). Therefore, consumers buy if and only if the price is at or below \( \gamma \hat{q}(\tilde{q}) \). Therefore, the firm’s profit is \( \pi = \gamma \hat{q}(\tilde{q}) - C(q, a(q)) \). The firm’s needs to maximizing the above profit as a function of quality choice \( q \), where the affect \( a \) has to be set by the firm according to the function \( a(q) \) defined by the implicit Equation (4). The first order condition for the optimality of the firm’s behavior is

\[
\gamma = C'(q) + C'(a(q))a'(q) = 2c_q q + 2c_a a(q) a'(q),
\]

where the left hand side is the marginal benefit of increasing the quality for the firm (increase in revenue), and the right hand side is the derivative of the cost function \( C(q, a(q)) \) with respect to \( q \). This is the second equation (Equation (5)) stated in the proposition. This proves the proposition.

Solution of the Model with Competition (Defined in Section 4).

Remind that the profits as functions of the price and quality choices of the firms are defined in Equation (19), where the consumer demands are defined right above Equation (19). Also not that since \( a = \frac{c_q}{c_a} q \), the cost functions of the firms can be written as

\[
C(q_j) = c_q q_j^2 + \frac{c_a}{c_q} q_j^2.
\]

We first solve for the pricing choices of the firms and then for the quality choices given the prices. Simultaneously
solving the first order conditions for the prices yields \( p_1(q_1, q_2) = 2\gamma_h q_1(q_1 - q_2)/(4q_1 - q_2) \) and \( p_2(q_1, q_2) = \gamma_h (q_1 - q_2) q_2/(4q_1 - q_2) \). The second order conditions on profit maximum with respect to the price choice are always satisfied. Substituting the above prices in the profit functions and then solving first order conditions for the optimal choice of \( q_1 \) and \( q_2 \), we obtain

\[
q_1^e = A_1 \frac{\gamma_h^2 c_a}{\gamma_h c_q (c_q + c_a)}, \quad q_2^e = A_2 \frac{\gamma_h^2 c_a}{c_q (c_q + c_a)},
\]

where \( A_2 \approx 0.024119 \) is the (unique) solution of \( 58956A_2^2 - 13057A_2^2 + 944A_2 - 16 = 0 \), and \( A_1 = (24080 - 949072A_2 - 2162935A_2^2)/[235824(5 - 256A_2 + 2015A_2^2)] \approx 0.12783 \). Substituting \( q_1^e \) and \( q_2^e \) in the profit functions lead to

\[
\pi_1^e = A_1 \left( \frac{4(A_1 - A_2)}{(4A_1 - A_2)^2 - 1} \right) \frac{\gamma_h^2 c_a}{c_q (c_q + c_a)}, \quad \text{and} \quad \pi_2^e = A_2 \left( \frac{A_1(A_1 - A_2)}{(4A_1 - A_2)^2} - A_2 \right) \frac{\gamma_h^2 c_a}{c_q (c_q + c_a)},
\]

where the constants in front of the last fraction of each profit function are approximately equal to \( 1.2218 \times 10^{-2} \) and \( 7.6538 \times 10^{-4} \), correspondingly.

To check the second order conditions, we derive \( \frac{\partial^2 \pi_1}{\partial q_1^2} = -2.054\frac{c_q (c_q + c_a)}{c_a} < 0 \) and \( \frac{\partial^2 \pi_2}{\partial q_2^2} = -2.6911\frac{c_q (c_q + c_a)}{c_a} < 0 \). Thus \( q_1^e \) and \( q_2^e \) defined above satisfy the second order conditions, and therefore, they lead to the profit maximum for both firms given that \( q_1^e > q_2^e \). One can also easily check that deviations by either firm to make \( q_1 \leq q_2 \) result in a lower profit to the deviating firm. This proves that \( q_1^e \) and \( q_2^e \) are the equilibrium quality choices. It remains to be noted that the equilibrium affect choices are \( a_j^e = \frac{c_q}{c_a} q_j^e \) for \( j = 1, 2 \).

**Proof of Proposition 4.**

If consumers believe that a firm has some cost \( c_q \) then they will discount the perceived quality by \( \frac{c_q}{c_q + c_a} \). Thus if consumers believe that the firm has the lower cost of quality \( (c_{q1}) \), then they will discount the perceived quality by a lower amount. Therefore, the firm’s profits are higher if it is able to convince consumers that it has the lower cost of quality production. Given this and because increasing the perceived quality is less costly for the firm with lower cost of quality, this firm can credibly signal its low cost by increasing the perceived quality.

Therefore, in a separating PBE the following properties must hold:

i. There are two equilibrium levels of perceived-quality production, the lower one of which we denote by \( \tilde{q}_L \) and the other by \( \tilde{q}_H \). Each output is produced by one of the types, and the incentive compatibility constraints are satisfied. In particular:
ii. The firm of the high-cost type is producing at the level optimal given that consumers correctly recognize the firm’s type (since consumers already have the worst possible belief about this production), and:

iii. The firm of the low-cost type is producing at a level such that the high-cost firm does not prefer to deviate to this level and be perceived as a low-cost type rather than produce at the other level and be perceived as the high-cost type.

Furthermore, according to the Intuitive Criterion, if any firm produces at an off-equilibrium level and if only the low-cost firm could possibly benefit given any off-equilibrium beliefs, then the off-equilibrium beliefs for this action should be that the firm is the low-cost type. In other words, any perceived-quality choice at a level higher than or equal to \( q_{Hic} \) which the high-cost firm has no incentive to mimic (instead of \( q_L \)) even if consumers were to believe it to be the low-cost firm under such production (i.e., such that the incentive compatibility constraint is satisfied for the high-cost firm), has to be perceived as having been made by the low-cost firm.

In other words, for a separating equilibrium, the perceived quality choice \( \tilde{q}_H \) of the low-cost firm is at least as high as the maximum of (a) \( q_{Ho} \) which is the full information level that would be optimal for the firm given that its cost is \( c_{q_1} \) and that consumers believe that it has low cost and (b) \( q_{Hic} \) which is what would be necessary such that the high-cost firm does not want to pretend to be low-cost.

The Intuitive Criterion implies that the the perceived quality choice \( \tilde{q}_H \) of the low-cost firm is exactly equal to the maximum of the above (see Cho and Kreps, 1987, for this characterization of the Intuitive Criterion for games with only two types of the sender).

In formal notation, the Intuitive Criterion for deviations from the equilibrium of the low cost firm is as follows: Suppose the low cost firm deviates to some \( \tilde{q}_{Hdev} \neq \tilde{q}_H \), then the firm should get lower profits in this deviation than in the equilibrium no matter what the beliefs are that consumers assign for the information set pertaining to the deviation. The choice of the low-cost firm is \( \tilde{q}_H = \max(\tilde{q}_{Ho}, \tilde{q}_{Hic}) \).

Now if \( \tilde{q}_{Ho} > \tilde{q}_{Hic} \), then any deviation \( \tilde{q}_{Hdev} \) will satisfy the intuitive criterion because any \( \tilde{q}_{Hdev} \) will imply higher costs for the low cost firm irrespective of consumer beliefs. Therefore, all such deviations are ruled out. Now suppose \( \tilde{q}_{Hic} > \tilde{q}_{Ho} \), then any \( \tilde{q}_{Hdev} > \tilde{q}_{Hic} \) is ruled out because it means an even higher perceived quality choice than what is dictated by cost minimization for the firm. To rule out any deviation \( \tilde{q}_{Ho} < \tilde{q}_H < \tilde{q}_{Hic} \) we specify the off-equilibrium belief that consumers believe the firm to be a high cost firm if they see a perceived quality in this range.

Thus, according to Equation (7), the quality and affect production levels of the high-cost firm are
as in part 1 of the Proposition, and the production levels of the low-cost firm are at least as high as
to make the high-cost firm indifferent between its equilibrium production levels with the associated
consumer belief that the firm is high-cost, and the deviation to the low-cost production levels with
the associated consumer beliefs that it is a low-cost firm. This indifference is given by the following
equation:

\[
\frac{\gamma^2 c_a}{4c_{q2}(c_{q2} + c_a)} = \frac{\gamma \bar{q}_H c_a}{c_{q1} + c_a} - C_h(\bar{q}_H),
\]

(27)

where the left hand side is the profits of the high-cost firm given the equilibrium level of its quality
and affect (see Equation (9)), \(\bar{q}_H\) is the perceived quality under which consumers start assuming that
this firm is one with low-cost and start discounting the perceived quality by \(c_a/(c_{q1} + c_a)\) instead of
by \(c_a/(c_{q2} + c_a)\), and \(C_h(\bar{q}_H)\) is the cost of inducing the quality perception \(\bar{q}_H\) by the high-cost firm.
Since for a production of any perceived quality \(\bar{q}\), the optimal choice of affect for the high-cost firm is
\(qc_{q2}/c_a\) where \(q\) is the choice of quality, the cost for high-cost firm to produce the perceived quality \(\bar{q}_H\)
is equal to \(c_{q2}c_a\bar{q}_H^2/(c_{q2} + c_a)\). Substituting this in the above equation and solving for \(\bar{q}_H\), we obtain:

\[
\bar{q}_H = \frac{\gamma (c_{q2} + c_a \pm \sqrt{(c_{q2} - c_{q1})(c_{q1} + 2c_a + c_{q2})})}{2(c_{q1} + c_a)c_{q2}}.
\]

(28)

The higher value (positive sign in front of the square root) is the one pertaining to the incentive
compatibility constraint, since it is the one corresponding to the deviation above rather than below
the equilibrium value for the high-cost firm production.

We have to check whether or not the incentive compatibility constraint is binding for the low-cost
firm. For this note that the optimal production level of perceived quality for the low cost firm if
consumers correctly recognized it as a low-cost firm would be \(\bar{q}_{Ho} = \gamma/(2c_{q1})\), which is indeed smaller
than the \(\bar{q}_H = \bar{q}_{Hic}\) determined by the incentive compatibility constraint. Because the incentive
compatibility constraint is binding, the intuitive criterion described above is satisfied given the cost
minimization incentive of the firm and the off-equilibrium consumer belief specification in the event
a deviation to a perceived quality which is lower than that implied by the incentive compatibility
constraint is observed. The difference \(\bar{q}_{Hic} - \bar{q}_{Ho}\) is the extent of distortion in the perceived quality
choice of the low cost firm over its full information level.

\[
\text{Distortion} = \frac{\gamma (c_{q2} - c_{q1} + \sqrt{(c_{q2} - c_{q1})(c_{q1} + 2c_a + c_{q2})})}{2(c_{q1} + c_a)c_{q2}} > 0.
\]

(29)

The optimal quality and affect production levels for the low-quality firm now follow immediately from
the cost-minimization conditions given the perceived quality (see Section 2.1). □
Proof of Proposition 5.

The method of proof is similar to the one of the previous proposition. Recall that consumers discount the perceived quality by $\frac{c_q}{c_q + c_a}$. So the difference now is that, since consumers will discount the perceived quality less if they think the firm is the high-cost of affect one, the high-cost firm would like to convince consumers that it is indeed high-cost. It will be able to do this by reducing the perceived quality level, because such a reduction, will have the same negative effect on consumer willingness to pay, but will save more in costs for the high-cost firm than for the low-cost firm.

The incentive compatibility constraint now determines the value of the perceived quality produced by the high-quality firm (or the low-cost of affect firm) and is the solution $\tilde{q}_L$ of the following equation representing the low-cost firm indifference.

\[
\frac{\gamma^2 c_{a1}}{4c_q(c_q + c_{a1})} = \frac{c_q c_{a1} \tilde{q}_L}{c_q + c_{a1}} - \frac{c_q c_{a1} \tilde{q}_L}{c_q + c_{a1}}
\]

Solving the above equation for $\tilde{q}_L$, we obtain

\[
\tilde{q}_L = \frac{\gamma (c_{a2}(c_q + c_{a1}) - \sqrt{(c_{a2} - c_{a1})c_q(c_{a2}c_q + 2c_{a2}c_{a1} + c_q c_{a1}))}}{2c_{a1}c_q(c_q + c_{a2})}.
\]

As in the last proposition, this incentive compatibility constraint is binding and the reduction in perceived quality of the high-cost of affect firm, relative to what it would choose if its costs were observed is

\[
\text{Distortion} = \frac{\gamma \left( \sqrt{(c_{a2} - c_{a1})c_q(c_{a2}c_q + 2c_{a2}c_{a1} + c_q c_{a1})) - c_q(c_{a2} - c_{a1})} \right)}{c_{a1}c_q(c_q + c_{a2})} > 0.
\]

In this case the distortion represents the amount of the downward distortion in perceived quality by high-cost of affect firm. The optimal levels of $q$ and $a$ production for the high-cost of affect firm now follow from the perceived quality value $\tilde{q}_L$ defined above.

□

Proof of Proposition 6.

Similarly to the proof of Proposition 1, since consumer behavior, and therefore, revenue, only depends on $\tilde{a}$ and $\tilde{q}$, if the firm decides on certain values of these variables, it should optimally choose $q$, $a_1$, and $a_2$ as to minimize it’s cost. Hence, the optimal behavior by the firm must satisfy the following conditional cost minimization property: $q$, $a_1$, and $a_2$ are such that $C(q, a_1, a_2)$ is minimized subject to the constraint that $\tilde{q}$ and $\tilde{a}$ are at the given level. Using Lagrangian multipliers to solve this condition, we write $L(q, a_1, a_2, \lambda, \mu, \tilde{a}, \tilde{q}) = C(q, a_1, a_2) + \lambda(f(q, a_1, a_2) - \tilde{q}) + \mu(g(q, a_1, a_2) - \tilde{a})$. Differentiating
this function with respect to $q$, $a_1$, and $a_2$ (and using the functional forms of $f(\cdot)$, $g(\cdot)$ and $C(\cdot)$), we obtain the following system of equations

$$
\begin{align*}
2c_q q + \lambda + \mu \delta &= 0, \\
2c_{a_1} a_1 + \lambda + \mu \alpha &= 0, \\
2c_{a_2} a_2 + \lambda + \mu \beta &= 0,
\end{align*}
$$

(33)

Solving this system for $\lambda$, $\mu$, and $a_2$, we obtain that a necessary condition for the firm’s profit maximization is

$$
a_2 = \frac{c_q q \alpha - \delta c_{a_1} a_1 + \beta c_{a_1} a_1 - \beta c_q q}{(\alpha - \delta)c_{a_2}}.
$$

(34)

Since the above equation must be satisfied regardless of consumer behavior (and beliefs, if present), the firm always follows it, and hence, rational consumers know that it holds. Furthermore, as far as the above equation is satisfied, firm’s decisions on $a_1$, $a_2$, and $q$ are uniquely determined by $\tilde{q}$ and $\tilde{a}$. Therefore, consumers can infer the true value of $q$ from $\tilde{q}$ and $\tilde{a}$ without error as far as the firm follows the above equation. This means that the firm’s profit under any $q$, $a_1$, and $a_2$ satisfying the above equation, is $\pi = \gamma q - C(q, a_1, a_2)$. Differentiating this profit function with respect to $a_1$ and $q$, after substituting $a_2 = a_2(q, a_1)$ determined by Equation (34), results in the system of two linear equations on $a_1$ and $q$ with two unknowns. The solution is unique, thereby giving the point of profit maximum. The solution values are reported in Equation (26). The comparative statics reported in the statement of the proposition immediately follow from these values. The proposition is proven.
REFERENCES


