The Selective Reporting of Factual Content by Commercial Media

Yi Zhu
Doctoral Student
Department of Marketing
Marshall School of Business
University of Southern California
zhuy@usc.edu
http://www-scf.usc.edu/~zhuy/

Anthony Dukes
Associate Professor of Marketing
Department of Marketing
Marshall School of Business
University of Southern California
dukes@marshall.usc.edu
http://www-bcf.usc.edu/~dukes/

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Abstract

We study the market for factual content and ask whether competition increases or decreases its provision. Factual content is supplied by commercial media who observe a set of facts depicting the true state of the world and selectively decide how to report them. Consumers value content that matches their opinion, which gives media an incentive to slant their reports by omitting certain facts. The novel feature in our model is that consumers anticipate media’s incentives for slant and all stances taken by media must be supported by facts. Furthermore, reports with more facts are more convincing. Despite consumers ability to detect slant and demand for factual support, our results show that competition results in consumers reading fewer facts and unable to update their priors about the state. We also find that a monopoly medium may be more polarizing than competitive media, and a polarized reporting can be less biased.

Keywords: Cheap-Talk, Factual Content, Information Product, Media Competition

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1. Introduction
Consumers demand useful information to improve their understanding of the world, gain access to truth, and satisfy their curiosity. This information, referred to hereinafter as factual content, is typically provided by commercial media, who earn profit by collecting payments (money or attention) from readers or advertisers. In this paper, we study the marketing of factual content by such media and examine how the market structure affects medias’ content strategies.

The internet and mobile technology have made it easier and cheaper to distribute factual content. Newspapers, magazines, radio stations, and television, distribute factual content through the internet and mobile devices. In addition, there are thousands of online-only distributors of content as well, not to mention the countless number of bloggers who provide factual content for commercial gain. The emergence of these media technologies has, consequently, increased competition for readers’ attention. The central question we ask in this research is whether this increase in competition corresponds to an increase in the provision of factual content?

A special feature of factual content is that it is made up of two components: the facts themselves and the overall picture told by the collection of presented facts. Facts themselves are objective and, while novel and interesting in their own right, they do not provide a comprehensive sense of the true state of the world (or simply the state). Viewed collectively, however, facts offer such a perspective. And, the more facts one is presented about a certain issue, the better her perspective becomes and the closer she is to having an accurate understanding of the true state of the world. For many important issues, we rely on commercial media to present a collection of facts of their choosing.

Despite consumers’ common desire to learn the true state, media may be unable to simply make a claim without providing any supporting evidence. For instance, for crucial issues, consumers may not be convinced about a medium’s claim about the state of the world (e.g. “global warming is real.”) without facts to support it. In other words, claims with no evidence are merely allegations. By using factual content to support a claim, consumers are more convinced and improve their understanding of the state. Media, therefore, have a commercial incentive to provide content with convincing perspectives supported by a lot of factual evidence.

Confounding this incentive, however, is that people prefer that the perspective offered by the collection of facts be consistent with their opinions, all else equal (Klayman 1995, Rabin & Schrag 1999). And, since consumers hold varied opinions on most issues, a commercial medium
is forced to balance consumers’ desire for credibility with their preference for content that matches their opinion. This trade-off, the fundamental aspect of our research, implies that media must selectively omit some facts so that the overall collection of presented facts fits the desired perspective—a notion we call media slant.

In contrast to previous work on media slant, our study is based on the micro-foundation that consumers’ desire for knowing the truth is reflected in their demand for convincing evidence in the form of objective facts and asks how slant is constructed via the careful selection of objective facts. To illustrate the selection process in practice, consider the issue of global warming, which was among the top 10 subjects of interest among Americans during the period 1986–2006 (Robinson 2007). The media reported on the subject differently. For example, a New York Times report had the title “Past Decade Warmest on Record, NASA Data Shows”, and it contained the following two scientific facts: (1) “2009 was the second warmest year since 1880, when modern temperature measurement began,” and (2) “An upward temperature trend of about 0.36 degrees Fahrenheit (0.2 degrees Celsius) per decade over the past 30 years.” In contrast, the Wall Street Journal reported an article with the title: “Global Warming Models Are Wrong Again.” In this article, they referred to two facts: (1) “(February 2012) monthly global temperature …was minus 0.12 degrees Celsius, slightly less than the average since …1979,” and (2) “Weather conditions similar to 2012 occurred in the winter of 1942.” The cited sentences from New York Times and Wall Street Journal are separate facts generated from the same state of the world. As we can see, the first two give readers the impression that global warming is occurring, but the last two provoke consumers’ skepticism about its occurrence. Furthermore, the New York Times report does not mention any facts that support global warming skepticism and the Wall Street Journal report does not mention any facts that support global warming. It suggests that the facts embedded in these reports were carefully chosen so that they support the respective stances as indicated in the titles of the reports.

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3 An alternative interpretation is that these media possess more facts than they can possibly report due space or bandwidth constraints (Bhardwaj, Chen and Godes 2007, Mayzlin and Shin 2011). In our setting, media must have the freedom to provide as many facts as they like as long as the collection of facts supports the chosen stance. Hence, we consider settings in which bandwidth constraints are not binding. We have in mind the case when media provide a series of reports on a single and controversial topic. For instance, NYT has reported repeatedly on important topics like global warming and maintained similar stances over time. In those cases, an alternative explanation of media stance is the general position media take on an important topic. In contrast, our model would have limited application to cases in which media must choose which facts to omit in order to meet a physical or bandwidth
Arguably, the idea that media may slant their reports is suspected by most consumers. Popular political commentators (Goldberg 2002 and Franken 2003) as well as academics (Groseclose & Milyo 2005 and Gentzkow & Shapiro 2010) have written about the US media’s attempt to appeal to consumers’ opinion though the slanting of reports. Anticipation of a medium’s incentive to slant its content implies that a consumer can infer something about the quality of the reporting (the number of facts) by observing the media’s stance on an issue. For example, even before reading the body of the reports mentioned above, readers can anticipate from the titles that few facts of global warming skepticism will be presented in the first report and that little content supporting global warming’s occurrence will be found in the second one. Therefore, the media stance indicates the type and quality of information consumers expect before they decide which medium to read. If consumers update their beliefs based on their observations of media stances, even if slanted, they can improve their understanding about the state of the world. This is an important distinction from the earlier work on media slant (Mullainathan & Shleifer 2005, hereinafter MS; and Xiang & Sarvary 2007, hereinafter XS), which assumes consumers have no ability to learn something other than what they are explicitly told by the media, and is the key novelty of this research. Furthermore, by endowing consumers with the ability to anticipate the incentives for slant implies that strategic media must account for consumer’s anticipation of slant when choosing what content to report. By including this additional consideration, we provoke extant intuitions about media bias, polarization, and journalistic balance.

We compare various media market structures and assess the degree to which consumers can meaningfully update beliefs about the state of the world from the available media options, a measure which we call “media informativeness”. To illustrate this measure consider two scenarios. First, suppose the New York Times is the only medium to report on global warming. After knowing the stance, consumers update their beliefs (possibly imperfectly) in favor of global warming’s occurrence. However, when both New York Times and Wall Street Journal cover the same subject matter, consumers are unable to update their beliefs since their stances are equally opposing. In this example, media informativeness is higher in the first scenario in which there is only one medium.

constraint. When media face a binding bandwidth constraint, our framework degenerates into a pure location model which has similar predictions as Mullainathan & Shleifer (2005).
The idea that media can slant their content by taking a stance different than how they actually observe the state of the world gives rise to the notion of media bias. It is commonly believed that stronger bias in the media is associated with consumers being less informed. However, in our setting consumers understand the incentive for biased reporting and may, therefore, infer something about the state of the world from media stances, even if slanted. This raises the question of whether media bias is a good measure of consumer ignorance. We find, in fact, that more informative reporting is not necessarily associated with less media bias.

We highlight the three most important findings from this research. The first regards the impact of competition on the content provision of media. We find that competition does not increase media content provision, and, when the value consumers place on facts is large, competition strictly reduces it. In a monopoly, if consumers place a large value on facts, then the medium may be able to appropriate some of this value if consumers believe its reports will contain a lot of facts. Because consumers recognize this incentive, they perceive the monopolist’s stance as a credible signal of the true state. Consumers also recognize that media have an incentive to cater to readers’ opinions. Consequently, the medium’s stance cannot be perfectly informative about the state of the world and must carefully omit some facts. Even though the monopolist’s reports selectively omit facts in their reports, we find it still outperforms competitive media. The pressure of competition prevents either medium from being providing credible stances. We show that if one medium attempts to provide an informative stance, which is close the state, then a competitor’s best response is to jam its rival’s stance by choosing an opposing stance. By providing conflicting state of the world, consumers cannot update their prior beliefs about the state of the world. It is important to note that opposing media stances are not simply product differentiation strategies. Rather, a medium chooses an opposing stance in order to disable consumers’ ability to determine which medium is more informative and eliminate its rival’s potential competitive advantage in content quality. Hence, no medium can be informative in equilibrium, and as a result we find that each medium produces content with fewer facts than a monopolist. This outcome, in fact, may be connected to the growth of micro-blogging platforms such as Facebook, Twitter and Tumblr in reaction to the intensified competition of internet content, in order to conceal the reduction of facts provision.

Our second result regards the impact of competition on polarization and media extremism. In contrast to earlier work on media slant (MS & XS), we find that a monopoly medium may take
more extreme positions than duopolistic media. Stances in a duopoly are independent of the state of the world because their stances are motivated by the desire to confuse consumers about which medium has more facts. As noted above, they do this by choosing opposite stances rather than by maximally differentiating their product (MS, XS). The rival takes a mirror (though not necessarily extreme) stance to eradicate the informative medium’s competitive advantage and thereby focus readers’ choices solely on their opinions. Even though competitive media may take conflicting positions, they do so only to the extent of heterogeneity in consumer opinion. If consumers’ opinions are relatively consensual, the competitive media need not choose extreme stances to prevent consumers from updating their beliefs. The monopolist, on the other hand, may want to take an extreme stance to communicate that its report is of high quality in that it contains a lot of facts.

Finally, our third finding challenges the use of media bias as a measure of consumers’ ability to be informed. The term “media bias” has been well discussed in the literature and is typically defined as the expected relative difference between the media report and the underlying state. We find that media bias is an imperfect indicator of the quality of communication between media and consumers because consumers believe more than what they are explicitly told and understand media’s incentive to slant. If consumers sufficiently value facts, a monopoly medium can bias its stance in order to signal that its report contains a lot of factual content. In this way, more-informative reporting does not necessarily correspond to less media bias. Additionally, because competition (weakly) reduces media informativeness, we find that competition may actually shrink media bias. This result is sharply different from previous findings that indicate competition (weakly) increases media bias.

This paper relates to the literature on media strategies and the influence of commercial incentives on content provision. Gal-Or and Dukes (2003), Anderson and Coate (2005), and Godes et al. (2009), among others, consider media strategies in which the content is non-informational (e.g., entertainment). In the current paper, we focus on media strategies when the content is factual and when consumers desire objective information. While more recent work has explored media competition in markets for informational products, their focus has been on identifying factors leading to media bias. Most notably, MS, Anand et al. (2007), and XS identify the incentives of duopolists to slant news to extreme positions in order to differentiate their content from that of competitors, which results in greater media bias in news reporting. Ellman
and Germano (2009), Gal-Or et al. (2012), and Yildirim et al. (2013) explore the relationship between the media’s desire to appeal to advertisers’ interests and media bias. Our focus, in contrast, is on how media choose to present or conceal factual content and the extent to which consumers become informed. Also, unlike the previously mentioned research, we focus on consumers’ desire for objective facts and their ability to infer information from the media’s marketing strategies.

This distinction is important for two reasons. First, if consumers have a common desire for content with more factual support, then media create value for customers by being more informative. With this consideration, it is not entirely clear whether commercial incentives or competition will exacerbate or mitigate media bias. Second, if consumers are able to anticipate incentives for biased content, consumers may be able to make inferences from media strategies and update their understanding about the state of the world. Consequently, our work introduces the notion that media bias may not be the best measure of reader ignorance. This is a relevant distinction in light of the attention on identifying and empirically measuring media bias in the news media (Lichter et al. 1986, Groseclose & Milyo 2005, and Gentzkow & Shapiro 2010).

Similar to our work, Strömberg (2004), Anand et al. (2007), and XS study the incentives of media to provide objective factual content. Strömberg (2004) focuses on information that is related to popular elections, and Anand et al. (2007) are concerned with the verifiability of facts. XS introduce the notion of the conscientious consumer who has no personal opinions. Despite having “unbiased” consumers, XS show that extreme positions arise in equilibrium. The microfoundation in our work is closest to XS, with three important distinctions. First, we concentrate on the notion that consumers enjoy reading factual content, and therefore more facts are better for consumers. Second, we permit consumers to make inferences based on the stances media take. Because consumers make such inferences, they can be informed about the state of the world despite media slanting. Third, while XS assume that the media’s ability to slant their reporting is bounded, we assume that the media can always generate a report to support the stance. From the media’s perspective, if facts are assumed to be truthful, then a stance binds them to reporting a set of facts that are consistent with that stance. It is not entirely clear how these forces affect the incentives for media slant as a differentiation strategy, as found in earlier work.

We consider a setting in which the media declare their stances publically so that they are known to consumers before they read or purchase a report. Such is the case when consumers can
read headlines before choosing which medium to consume or when media declare their general positioning. It is through this declaration that consumers may update their beliefs about the state of the world and estimate the number of facts a medium will provide in its report. In this sense, stances resemble “cheap-talk” (Crawford & Sobel 1982, hereinafter CS). As in CS, our setting exhibits a “less-than-fully informative” equilibrium in which the support of the information variable is partitioned in a countable set of intervals and the sender (the medium) reports a message (a stance) only when the observed information lies in the associated interval. But, in contrast to the cheap-talk literature, the sender’s message serves two roles. In addition to serving as a signaling mechanism to (partially) inform consumers about the state of the world, the message also serves a positioning function, in which the stance enters consumers’ utility, through appeals to their opinions. These two features imply that the communication process between the information sender (media) and the receiver (consumers) in our framework is a non-trivial extension of a cheap-talk game. Nevertheless, the cheap-talk approach is helpful in understanding how commercial incentives confound the media’s ability to be fully informative and why the informativeness of media reporting and content provision are affected by competition.5

2. The Model

In this section, we first discuss the fundamental connection between the state of the world and the generation of facts. We then illustrate how the choice of stance binds the medium’s maximum number of facts in the report and then derive consumers’ demands for media’s reports. For consistency, we use the term media to represent the commercial information providers, websites, news outlets, and UGC platforms, which observe and (selectively) report factual content in exchange for a payoff. The term report refers to the mixture of facts that the media choose to present to support their stance. The term consumer represents the reader or viewer.

4 While our monopoly model involves a single sender (as in CS), the duopoly media model we analyze has multiple senders. Cheap-talk with multiple senders has been studied in a variety of settings (Gilligan & Krehbiel 1989, Krishna & Morgan 2001) and more generally in Battaglini (2002). This earlier work takes the receivers’ preferences among senders as exogenous. In our duopolistic media setting, however, we assume consumers’ preferences for senders is determined endogenously by the senders’ choice of stance.

5 Similarly, Li (2005), Chakraborty and Harbaugh (2010, 2013) and Gardete (2013) studied diverse marketing related questions under a cheap-talk game setting.
2.1 The State of the World and the Source of Facts

The state of the world, or simply the state, represents the underlying truth about some focal event or subject. We assume that the state is a random variable, $t$, that is uniformly distributed between [0,1]. Relating to the opening example, a value of $t = 0.4$ means that there is a 40% chance that global warming is occurring. The value of the state $t$ dictates the overall composition of the set of facts. Each fact has incremental value to consumers in the form of novel information. In addition, each fact contains a binary signal $Y \in \{0,1\}$ about $t$. In the global-warming example, “2009 was the second warmest year since 1880,” and “An upward temperature trend of about 0.36 degrees Fahrenheit … per decade over the past 30 years” are two facts that contain different informational content but that are both associated with an affirmative signal, $Y = 1$, or that global warming is occurring. However, “Weather conditions similar to 2012 occurred in the winter of 1942” is an alternative fact, that contains an opposite signal, $Y = 0$, about the occurrence of global warming. We further assume that every signal, $Y$, is an independently and identically distributed (i.i.d.) random draw from a Bernoulli process with $\Pr(Y = 1) = t$, and $\Pr(Y = 0) = 1 - t$. In this way, a fact is a noisy signal about the state, but there is no ambiguity about which “side” of the state the fact is located.

Let $\Omega$ denote the total number of observable facts, and assume that every medium observes all of the facts. Among them, $\Omega_0$ we have facts with signal 0 and $\Omega_1$ facts with signal 1, and $\Omega_0 + \Omega_1 = \Omega$.

2.2 Media Stances and Reports

Each medium chooses a stance $s$, which represents an announcement about the state of the world. A stance is supported by a report – a combination of facts that serve as evidence for the chosen stance. We assume that the media can only report “true” facts and cannot fabricate non-existing facts.

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6 The fact-generation process in previous literature (Hayakawa 1990, MS, XS) assumes that the facts are only a string of data consisting of 0’s and 1’s. In our setting, there is additional value from facts – each contains a novel piece of information that readers consume. We specify this in more detail section 2.3.

7 It is reasonable to suggest that each fact contains additional noise, making it harder to interpret which side of the state the fact lays. Incorporating this aspect implies that consumers value facts less than what we assume in equation (1) below: $M_{noise} < M$.

8 We assume media acquire all facts at no cost. This assumption allows us to abstract away the fact collection process and focus on the selective reporting problem. A considerable amount of the previous literature has made the same assumption. For example: MS, XS and Gal-or et al. (2012). If the cost of collecting facts is not zero, as is realistically plausible, then media will have lower incentives to provide facts, which reduces the total number of observable facts.
facts. This is a central assumption of our model because it requires media to form their positions through the careful selection of objectively interpretable facts. As an alternative to this assumption, media could slant reports without factual support simply by the choice of words or framing. Incorporating such an assumption in our model enhances media’s ability to differentiate without providing additional factual content. Given our objective to understand factual content provision when consumers anticipate media’s commercial incentive, we bind the media in a way to give them a stronger incentive to be informative. As we show, even with the strongest possible incentives to be informative, media cannot be fully informative in equilibrium, and even less so under competition. Under the no fabricating and no framing assumption, the maximum number of facts in a report can never exceed total amount facts $\Omega$. We also require media reports to be consistent with the chosen stance. This requirement implies that a medium’s stance affects the number of facts it can report. To illustrate this key implication, let $n \leq \Omega$ be the number of reported facts and $n_0, n_1 \leq n$ be the number of facts with signal 0 and 1, respectively. By announcing $s$, the total number of facts must satisfy $s = \frac{n_0}{n}$. The consistency between stance and reports therefore implies the following inequalities:

$$n_0 = n \times (1 - s) \leq \Omega_0 \quad \text{and} \quad n_1 = n \times s \leq \Omega_1.$$ 

Hence, the total number of observable facts and the choice of media stance $s$ bounds the maximum number of facts in a report. Because a stance is more credible with the more factual evidence used to support it, consumers always prefer more facts to less. So we can restrict attention to maximum number of facts $N$ (and $N_0, N_1 \leq N$ be the maximum number of facts with signal 0 and 1) a medium can report for a given stance $s$. We can see that

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9 Incorporating framing in our model may have different impacts on the provision of factual content depending on whether consumers can understand the commercial incentive of framing. If consumers examine the framed stances, anticipate the motives for framing, and try to “back-out” the original unframed stances, consumers can de-differentiate the stance. Therefore, framing may not impede consumers’ ability to make inferences nor affect the equilibrium level of content provision. On the other hand, if consumers cannot anticipate the commercial incentive to frame stances without additional factual support, then media have means to differentiate their stances without regard to the selection of facts. In this case, our framework degenerates to a location model in which differentiation does not involve a trade-off with the number of facts. Such is the assumption made in earlier work (e.g. MS), from which we depart.

10 As we show later in the game timing, consumers make a purchase decision before reading the report. Since there is no cost associated with the selection of “stances,” the media do not have incentive to “bait and switch” from the claimed stance.
\[ N = \frac{\omega}{t}, \text{ if and only if } s \leq t, \text{ and otherwise } N = \frac{\Omega}{s} \text{ or } N = \min\{\frac{\omega}{1-t}, \frac{\Omega}{s}\}. \]

In order to avoid the rounding problem, we also assume that \( N \) is continuous. To help the reader to better understand the trade-off between the media stance and the number of facts, we use a simple numerical example to illustrate the idea.

**Table 1 A Numerical Example of \( t = 0.4 \) and \( \Omega = 10 \)**

<table>
<thead>
<tr>
<th>Facts: Content</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facts: Signals</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1 shows a case in which \( \Omega = 10 \) and \( t = 0.4 \). As we can see, among a total of 10 facts, the facts A, C, D, G, I, and J contain signal 0, and facts B, E, F, and H contain signal 1. Now, suppose that the medium chooses to produce a report with media stance \( s = 0.5 \). To support this stance, factual content must satisfy \( s = \frac{N - N_0}{N} \), or simply \( N_0 = N_1 \). However, there are 6 facts with signal 0 but only 4 facts with signal 1 in the total data set. The best the medium can do is to produce a report with all 4 facts with signal 1 and randomly pick another 4 facts with signal 0 to fulfill the media stance \( s = 0.5 \). In this situation, \( N \) is 8. Now, suppose that the medium wants to produce a report with media stance \( s = 0.8 \). We can clearly see this media report has been slanted farther away from the state of the world, \( t = 0.4 \). To support this media stance, we need \( \frac{N_1}{N_0} = 4 \), or for every fact with signal 0 we need 4 facts with signal 1. Because we only have 4 facts (B, E, F, and H) that contain signal 1, the best the medium can do is to combine these 4 facts with a fact of signal 0. Therefore, \( N \) is 5 under the second stance, which is fewer than the maximum number of facts the medium can report when \( s = 0.5 \). As these examples demonstrate, the further a medium’s stance is from the state, the fewer facts it can use in its report to support that stance.

### 2.3 Consumer Demand

We model the consumer’s utility function in the following manner. Recall that each fact not only represents a signal, 0 or 1, but also contains unique novel information and a piece of supporting evidence for a chosen stance. Consumers obtain positive utility from reading an additional fact in a report because it provides a more convincing support of the stance. Therefore, more facts
always increase a consumer’s willingness-to-pay for the report. This preference structure embeds consumers’ inherent desire to learn the true state of the world.\textsuperscript{11}

Consumers also have opinions and, all else equal, prefer reports to be consistent with their opinions. Thus, we assume consumers will pay more for a report when the media stance is close to the consumers’ opinions. Denote a consumer by her opinion, \( b \), in \([z,1-z]\), where \( 0 \leq z < \frac{1}{2} \). We assume \( b \sim U[z,1-z] \), where \( z \) captures the divergence of the consumer opinion space.\textsuperscript{12} A higher \( z \) means that consumers are more homogenous in their opinions about the state, and a lower \( z \) means that opinions are more diversified. Consumer \( b \)’s expected utility function from reading the report on media outlet \( A \) is given by:

\[
E \left[ u_b (N, s, p \mid s) \right] = E[V + MN - d(s-b)^2 - p \mid s],
\]

where \( V \) is the intrinsic value of consuming the report, and \( M \) and \( d \) represent the consumer’s preference about the number of facts and about reading a report that differs from her opinion, respectively. Hence, \( M \) and \( d \) are assumed to be non-negative.\textsuperscript{13}

Even though consumers observe a medium’s stance, \( s \), before deciding whether to buy the report, they do not know exactly how many facts the report will contain. Consumers must, therefore, formalize their beliefs about the state of the world based on the medium’s stance in order to formalize an expectation about the number of facts and make a purchase decision. The consumer’s prior belief about the state is \( t \sim U[0,1] \), and therefore the updated belief \( \mu[t \mid s] \), is also a distribution on \( t \).

Consumers also observe a medium’s pricing decision before deciding whether to purchase, so a natural question to ask is whether consumers can utilize \( p \) to update their beliefs about the state \( t \). Even though we make no restriction on consumer’s ability to do so, it is impossible for price to ever serve as a signal in our model. A necessary condition for a separating equilibrium involving prices is the “single-crossing property” of firm types’ profit functions.

\textsuperscript{11} A formal proof and detailed discussion on this point is provided in a Supplemental Appendix, which is available upon request.

\textsuperscript{12} The scope of consumer opinion when \( z > 0 \) is smaller than the scope for the state of the world. This permits the possibility of states that exceed relatively narrow opinion (e.g. 14\textsuperscript{th} Century opinions that the world was flat.) Media also have the additional flexibility to take stances even more extreme than consumer opinion to signal that their stances are supported with many facts. Restricting \( z = 0 \) aligns the set of consumer opinion with the state of the world, but does not alter the basic findings.

\textsuperscript{13} A simplification embedded in (1) is that every fact has equal value to consumers. While in reality different facts might have different effects on consumer preferences, we assume that consumers have constant marginal utility from reading extra facts. However, our results hold as long as consumers’ valuation for facts is increasing and weakly concave.
This condition is absent in our model because all firm types (as defined by the expected number of facts in their report) have the same full-information optimal price. Therefore, the only firm decision, which may possibly facilitate updated beliefs about \( t \) is the medium’s stance.

A consumer receives zero utility if she does not read any report and will purchase the report as long as it gives them non-negative utility. We also assume that \( V \) is large enough so that it is always optimal for the media to serve all consumers. If the media stance reflects the true state of the world consumers believe it, then a consumer’s utility from consuming the report is 

\[
u_b(N, s, p) = V + M\Omega - d(s - b)^2 - p.
\]

However, as we will show in the next section, such an outcome is impossible in equilibrium as long as consumers value reports consistent with their opinion (\( d > 0 \)).

In order to derive equilibria in this game, we treat media stances as a form of “cheap talk” (CS). As in cheap talk settings, the only feasible pure-strategy equilibria take the form of an interval equilibrium. An interval equilibrium is characterized by a partition \( \{a_i\}_{i=0}^\infty \), some integer \( x > 0 \), with \( 0 = a_0 \leq \cdots \leq a_x = 1 \) in which the media stance is \( s_i \in [a_{i-1}, a_i] \), for any \( t \in [a_{i-1}, a_i] \), where \( i \) represents the index of interval. Under this situation, the number of facts that the medium will produce is

\[
N = \begin{cases} \frac{\Omega_t}{s_i} = \frac{\Omega \times t}{s_i}; & \text{if } t < s_i \\ \frac{\Omega_0}{1-s_i} = \frac{\Omega \times (1-t)}{1-s_i}; & \text{otherwise} \end{cases}
\]

Therefore, the expected utility under this situation is

\[
E[u_b(N, s_i, p \mid a_{i-1} \leq s_i \leq a_i)] = E[u_b(N, s_i, p \mid a_{i-1} \leq t \leq s_i)] \Pr(a_{i-1} \leq t \leq s_i) + E[u_b(N, s_i, p \mid s_i < t \leq a_i)] \Pr(s_i < t \leq a_i)
\]

\[
= V + \frac{1}{a_i - a_{i-1}} \int_{a_{i-1}}^{a_i} f(t)dt + \frac{1}{1-s_i} \int_{s_i}^{a_i} f(t)dt - d(s_i - b)^2 - p
\]

\[
= V + \frac{M\Omega}{a_i - a_{i-1}} \left[ \frac{1}{2} - \frac{a_i^2}{2s_i} - \frac{(1-a_i)^2}{2(1-s_i)} \right] - d(s_i - b)^2 - p.
\]

Given the belief that \( t \in [a_{i-1}, a_i] \), the consumer with opinion \( b \) will purchase a report if and only if

\[14\] Additional details of this argument are provided in a Supplemental Appendix available from the authors.
\[ E[u_b(N, s_i, p | s_i)] \geq 0. \]  
\[ (4) \]

Because \( M \Omega \) always appear together, we normalize \( \Omega = 1 \) without loss of generality. In this paper, we consider the case that consumers’ opinions are not influenced by the updated belief.

2.4 Game Timing

The timing of the game is as follows.

\begin{itemize}
  \item **Step 1:** The media outlet observes all available facts, announces a stance, \( s \), and generates a report. If there are two media outlets, they announce their stances simultaneously.
  \item **Step 2:** The media outlet announces the price, \( p \), for its report. If there are two media outlets, they announce their prices simultaneously.
  \item **Step 3:** Consumers update their beliefs about the state of the world, \( t \), formalize their expectations about the reading utility associated with the media based on the media stance and the price, and then make their purchasing decisions.
\end{itemize}

Next, we investigate a model with a monopoly medium; then we examine the effects of competition on informative reporting and factual content provision by analyzing a duopolistic model and comparing it with the monopoly model.

3. The Monopoly Medium

We begin our analysis by studying the equilibrium with a monopoly medium, whose objective is to maximize the profit. Inherent in our model is that media have no incentive to misrepresent the message unless it is strictly profitable. The results show that, even though consumers value facts, the commercial incentives induce some degree of misrepresentation. To show this, we first show that an equilibrium in which consumers are fully informed about the state can never exist.

**Definition 1:** A fully informative equilibrium is a pure-strategy equilibrium such that if \( \forall t_1, t_2 \in [0,1] \) with \( t_1 \neq t_2 \), and corresponding stances \( s_1 \) & \( s_2 \) with \( s_1 \neq s_2 \) and \( \mu[t_1 | s_i] = t_i \), for \( i = 1,2 \).

**Lemma 1** A fully informative equilibrium does not exist unless \( d = 0 \).

Lemma 1 shows that the medium cannot communicate the state fully informatively unless the consumers do not have divergent opinions about the state. The intuition behind this result is
as follows. Because consumers value a report that appeals to their own opinions \( (d > 0) \), the medium has an incentive to fit the report to the consumers’ opinions and charge a higher price. If consumers believe the medium is fully informative, then it is always optimal for the medium to report the state with \( s = \frac{1}{2} \), regardless of what \( t \) is, in order to convince consumers that the state is \( \frac{1}{2} \). Doing so not only increases consumers’ expectations about the number of facts but also best fits consumers’ opinions, which enables the medium to charge a higher price. Even though consumers desire to know the true state of the world, they anticipate the medium’s commercial incentives and therefore would not trust the stance to be fully informative. From now on, we focus on the more interesting cases by assuming that \( d > 0 \).

It is reasonable to wonder whether the non-existence of a fully-informative equilibrium is a direct implication of the assumption that consumers have no recourse if they catch a medium slanting their reports. But this is not the case: consumers, in our model are never misled. In fact, as long as \( d > 0 \), consumers prefer some degree of slant over a fully accurate reports and would not take recourse. Although fully informative equilibria are never possible, room still exists for consumers and the medium to construct a less-than-fully informative reporting equilibrium. We find that, more informative equilibria exist in which the space \([0,1]\) is divided into several (smaller) intervals, and for different intervals the medium reports dissimilar media stances.

**Definition 2:** A less-than-fully informative equilibrium in a monopoly is a Perfect Bayesian equilibrium (PBE) characterized by:

- Reporting rules: There must be a media stance reporting profile vector with
  \[ S : [0,1] \rightarrow \{s_1, s_2, \ldots, s_x\}, \] where \( x \) is the number of intervals, for each medium, and a set of dividing points \( (a_i)_{i=0,\ldots,x} \) with \( a_{i-1} \leq a_i \), such that \( s = s_i \), for any \( t \in [a_{i-1}, a_i] \).
- Belief functions: These are consumers’ updated beliefs about the state of the world from the media stance, \( \mu(t \mid s_i) \).
- Purchasing rules: Consumers will purchase reports from the media if the expected payoff is nonnegative. Hence the following purchasing rule applies:
  \[ \Psi_b(N, s_i, p \mid s_i) = \begin{cases} 1, & \text{if } E[N, s_i, p \mid s_i] \geq 0 \\ 0, & \text{otherwise} \end{cases} \] (5)
• Furthermore, (1) the reporting rules are optimal for the medium given the belief functions and purchasing rules, (2) the purchasing rules are optimal for consumers given the belief functions, and (3) the belief functions are derived from the reporting rules using Bayes’ rule whenever possible.

The following proposition establishes the existence of a partially informative reporting equilibrium and fully characterizes it. We show that, as in CS, all reporting equilibria are interval equilibria in which the medium only reveals the state of the world to which the interval belongs.

**Proposition 1** A less-than-fully informative equilibrium is characterized by a partition of $[0,1]$ defined by a set of dividing points with $a_0 = 0 < a_1 < \cdots < a_{x-1} < a_x = 1$ with $x \geq 1$, and a set of media stances $(s_i)_{i=1,\ldots,x}$, where $s_i \neq s_i', \forall i,i' \in \{1,\ldots,x\}$ such that:

1. $(a_i)_{i=0,\ldots,x}$ and $(s_i)_{i=1,\ldots,x}$ satisfies $\pi(s_i, a_{i-1}, a_i) = \pi(s_{i+1}, a_i, a_{i+1})$, for $i = 1,\ldots,x-1$

   where:

   $$\pi(s_i, a_{i-1}, a_i) = V + \frac{1}{(a_i - a_{i-1})} \left[ \frac{M}{2} - \frac{M a_i^2}{2 s_i} - \frac{M(1-a_i)^2}{2(1-s_i)} \right] - \frac{d(s_i - 1 + z)^2, \text{if } a_i \leq \frac{1}{2}}{d(s_i - z)^2, \text{if } a_{i+1} \geq \frac{1}{2}}$$

2. $s_i = \arg \max_s \pi(s, a_{i-1}, a_i)$ with $s \in [a_{i-1}, a_i]$ for any $t \in [a_i, a_{i+1}]$;

3. There is symmetry around the middle point: $a_i = 1 - a_{i-1}$, $s_i = 1 - s_{i+1}$ for all $i = 1,\ldots,x$; and

4. $\mu(t \mid s_i)$ is uniformly supported on $[a_{i-1}, a_i]$ if $s_i \in [a_{i-1}, a_i]$.

Proposition 1 establishes necessary and sufficient conditions for the existence of a partition equilibrium in this setting. Although this equilibrium characterization closely resembles the one in the CS’s classic cheap-talk game, a significant difference exists. In our equilibrium, the choice of the media stance is NOT randomized within the interval. The reason for this distinction is due to the fact that the medium’s stance not only serves the function of affecting consumers’ expectations about the state but also directly affects their utility and corresponding WTP. Therefore, the medium is not indifferent to messages within an interval, as in CS. The optimal stance is unique within the interval and is affected by consumer preference parameters $M$ and $d$. 
Figure 1 shows an example of the less-than-fully informative equilibrium with $x$ intervals. For any possible $t$ within a given interval, this stance may alter between intervals so that the media stances can signal to consumers about the interval of the state to which it belongs. Thus, unlike the uninformative equilibrium, the medium’s stance in the partially informative equilibrium depends on the state, $t$.

![Diagram showing intervals and media stances](image)

**Figure 1: Less-Than-Fully Informative Equilibrium with $x$ Intervals**

Next, we discuss two cases of Proposition 1. The first case is when there is only one interval ($x = 1$) so that the medium reports the same stance for any $t \in [0,1]$. As we can see, consumers cannot update their beliefs about the state of the world by observing the media stance. Therefore, this special case of a less-than-fully informative equilibrium is called the *uninformative equilibrium*, as shown in Fig. 2. Lemma 2 describes the medium’s profit and optimal choice of media stance when $x = 1$, and it also shows that when $M$ is small enough we can expect the medium to report uninformatively.

![Diagram showing uninformative equilibrium](image)

**Figure 2: The Uninformed Equilibrium**

**Lemma 2** There always exists an uninformative equilibrium, and the optimal stance for the monopoly is to report the state at $s = \frac{1}{2}$, with the expected profit $\Pi = V + \frac{2}{3} - d(\frac{1}{2} - z)^2$.

Furthermore, there is a cutoff point $M_1$ such that when $0 < M < M_1$ the uninformative equilibrium is unique.

Lemma 2 shows that in the uninformative equilibrium, the medium’s best choice of stance is in the middle. That is because consumers cannot update the expected number of facts regardless of $s$, and the best the medium can do is to cater its stance as well as possible to consumers’ opinions by locating at the center of the $[z, 1 - z]$. It also shows that although an
uninformative equilibrium always exists, when $M$ is small enough, the monopoly reports are uninformative. Smaller intervals would indeed enable the medium to raise consumers’ expectations about the number of facts in the report. But if consumers do not value the facts ($M < M_1$) then the medium is unable to provide value by appealing to consumer opinion. Consumers are less inclined to interpret stances as meaningful for updating their expectations.

The second case is when $x \geq 2$, which we refer to as the *partially informative equilibrium*. A specific example with $x = 2$ is illustrated in Fig. 3. Under this example, the medium chooses a different media stance when the observed state falls into different intervals. More precisely, the medium will announce $s_1 \in [0,a]$ if $t \in [0,a]$ and $s_2 \in [a,1]$ if $t \in [a,1]$, where $s_1 \neq s_2$. Proposition 2 compares these two cases and shows when the medium can be rewarded for being more informative.

\[
\begin{array}{cc}
  & s_1 & \\ 0 & z & 1-z & 1 \\
 s_2 & & &
\end{array}
\]

**Figure 3**: A Partially Informative Equilibrium ($x = 2$)

**Proposition 2** There exists a cutoff point $M_2$ such that when $M > M_2$ there exists a partially informative equilibrium with $x \geq 2$ as well as the uninformative equilibrium. The medium is better off in the partially informative equilibrium with $x \geq 2$.

Proposition 2 shows that as consumers value facts more, it is possible for medium to be partially informative in equilibrium. It also says that the monopoly does not always benefit from producing partially informative reports even when it is possible. Only when $M$ is large enough is it in the medium’s interest to increase communication efficiency and deliver a more informative report about the state to consumers.

To illustrate the intuition, consider any interval $[a_{i-1}, a_i]$ containing the state $t$ and to left of 0.5. The stance that maximizes the number of facts is always left of the middle point of the interval $\frac{a_{i-1}+a_i}{2}$. Recall that since $t < 0.5$, there are more facts with signal 0’s than with 1’s. So, while a stance of $a_i$ best appeals to consumer opinion, the medium can report more facts by shifting left toward the point that maximizes the number of facts. The optimal balance between
these two effects is influenced by two parameters: $M$ and $d$. When $M$ increases, the optimal stance shifts left and closer to the position that maximizes the expected number of facts. This effect grows for $t$ closer to 0. In fact, for the extreme case of the left most interval, when $a_{i-1} = 0$, it is always profitable to reduce the chance $t < s_i$ since the medium might have nothing to report when $t = 0$. Hence the stance that maximizes the expected number of facts is actually a corner solution of 0 when $t \in [0, a_i]$. We can show when $M > 8d(1 - z)$, the optimal stance for the medium in interval $[0, a_i]$ is 0.

We now define a measure of media informativeness as the extent to which the audience can infer the state of the world from seeing the stance. It also represents communication efficiency (Alonso et al. 2008, XS) between the media and consumers.

**Definition 3:** Media informativeness is the residual variance of the state of the world under consumers’ updated beliefs: $MI = -E_i[(t - E[t|s_i])^2]$ if the media report with $s_i$ when $t \in [a_{i-1}, a_i]$.

This definition represents the residual uncertainty a consumer experiences after reading a report. We can see that as the medium reports become more informative (e.g. $x = 1$ vs. $x = 2$), the interval shrinks and $MI$ increases. While there are multiple equilibrium when $M > M_2$, Proposition 2 shows partially informative equilibria benefit both medium and consumers. Therefore from now on we focus on the Pareto efficient equilibrium as in CS, which is the most informative equilibrium. Following this mild refinement, consumers will have a better understanding about the state of the world and obtain more factual content when $M > M_2$.\(^{15}\)

**4 Competitive Media**

To understand the effect of media competition on media informativeness, we investigate a duopoly model and compare it with the previous monopoly case. One may first wonder whether competition forces media to be fully informative.\(^{16}\) Proposition 3 tells us that the answer is no.

\(^{15}\) There are no explicit expressions for the partition of intervals for $x > 2$. Nevertheless, general properties of equilibrium can be implicitly derived. Therefore most of the results hold without any restrictions on $x$. An exception occurs in section 5.3, where we restrict attention to the cases of $x \leq 2$.

\(^{16}\) Our duopoly model is similar to a cheap-talk game with multiple senders. Battaglini (2002) showed that the fully revealing equilibrium is not stable when the information space is one-dimensional. Although others (Ambrus and Lu
Proposition 3 There does not exist a fully informative equilibrium with two competitive media.

The intuition for this result is immediate. To see the intuition, suppose there is a fully informative equilibrium. Then, both media have identical stances located both at the state, with $s^A = s^B = t$. In this case, media are undifferentiated substitutes and will engage into fierce Bertrand price competition and earn zero profit. A positive profit is possible by deviating to a different stance. It is useful to compare the intuition of this result to that of Lemma 1. In the monopoly case, the medium’s incentive to deviate from a fully informative stance arose purely from consumers’ opinions. With duopoly media taking fully informative stances, there are even stronger incentives to deviate due to competitive pressure.

Despite the result that media are never fully informative in competition, the question remains as to whether there exists partially informative equilibria. Before addressing this question we must define consumers belief-updating function after observing $s^A$ and $s^*$, as well as what the out-of-equilibrium belief is with regard to the equilibrium. We first specify the equilibrium concept and the associated equilibrium belief.

Because we have two media outlets, consumers’ purchasing rule can be affected by the media stance choices of the two media. We therefore need to provide a new definition of PBE under duopoly. With two media, A and B, a pure-strategy PBE is characterized by:

- **Reporting rules:** A media stance profile vector for each medium $l \in \{A, B\}$ with $S^l : [0,1] \to \{s^l_1, s^l_2, \ldots \}$, and a set of dividing points $(a^l_i)_{i=0,\ldots,x}$ with $a^l_i \leq a^l_{i+1}$, such that $s(t) = s^l_i$ for any $t \in [a^l_{i-1}, a^l_i]$.

- **Belief functions:** Consumers’ updating of beliefs about the state of the world from the media stance $\mu(t \mid s^A, s^B)$.

- **Purchasing rules:**

2010, Lu 2011) have established the robustness of fully informative equilibria with multiple senders, our paper provides more insights on this important topic. Different from previous findings in standard cheap-talk game settings, we found first that a fully informative equilibrium can never exist in our framework. In what follows, we show that there is no equilibrium in which one or both media outlets are partially informative either.
\[ \psi^A_b(\cdot) = \begin{cases} 1, & \text{if } E[u_b(N^A, s^A, p^A|s^A)] \geq 0 \text{, and } E[u_b(N^A, s^A, p^A|s^A)] > E[u_b(N^B, s^B, p^B|s^B)] \\ 0, & \text{otherwise} \end{cases} \tag{7} \]

In words, a PBE requires that (1) reporting rules must be optimal for the media given the belief functions and purchasing rules, (2) purchasing rules must be optimal for consumers given the belief functions, and (3) the belief functions must be derived from the reporting rules using Bayes’ rule whenever possible.

**Definition 4:** A partially informative equilibrium with two competitive media is a pure-strategy PBE if there exists a stance profile vector for each medium with

\[ S^l : [0,1] \to \{ s^l_1, s^l_2, \ldots \}, \ l \in \{ A, B \} \] for each medium and a set of dividing points

\[ (a_i)_{i=0,1,\ldots} \text{ with } x \geq 2, \text{ so that } s^l(t) = s^l_i \text{ for any } t \in [a_{i-1}, a_i] \text{, that the media choice is optimal for both media and that } \mu(t|s^A, s^B) \text{ is uniformly supported on } [a_{i-1}, a_i] \text{ if } s^A, s^B \in [a_{i-1}, a_i]. \]

If a partially informative equilibrium with two media exists, then the definition implies that consumers can partially infer the state of the world from the media reporting. However, our next proposition shows that such an outcome is impossible.

**Proposition 4** There does not exist any partially informative equilibrium with two competitive media.

To see the intuition behind this result, suppose both media are partially informative. Either medium has a profitable deviation, which is to “jam” the stance of its rival by choosing a media stance from another interval. Consumers see stances from different intervals and are therefore unable to update their beliefs. Jamming implies that consumers consider both media uninformative.\(^{17}\)

Because jamming is a crucial and effective strategy in the duopoly case, we provide an elaboration of it. Assume consumers believe that both media are partially informative. Based on

\(^{17}\) If each medium can frame any given fact differently, it is natural to ask whether media can accentuate differentiation without affecting the number of facts each report. As long as consumers can anticipate such incentives to frame, our framework suggests that they would adjust their beliefs to account for this distortion. However, more complete research on the consequences of framing is necessary.
consumers’ beliefs, whenever medium A chooses \( s^A \) from \([a_{i-1}, a_i]\) and medium B chooses \( s^B \) from \([a_{i-1}, a_i]\), with \([a_{i-1}, a_i]\cap[a_{i-1}, a_i]=\emptyset\), these two media stances are conflicting since it is impossible to have a state of the world from both \([a_{i-1}, a_i]\) and \([a_{i-1}, a_i]\). Therefore, consumers cannot infer which medium is actually informative about the state of the world under conflicting media reporting. As a result, consumers have no means to update their beliefs about the state.

Given Proposition 4, we know it is impossible to find an equilibrium in which both media are informative, even partially. Thus, the only possible cases are when one medium is partially informative and the other is uninformative or when both media are uninformative. We turn to the possibility of an asymmetric equilibrium after Proposition 5, which confirms that both media are uninformative in equilibrium when consumers really value facts.

**Proposition 5** There exists a cutoff point \( M_3 \) such that, when \( M > M_3 \), there exists only the uninformative equilibrium. The equilibrium choices of media stances (assuming \( s^A \leq s^B \)) are given by \( \bar{s}^A = \max\{\frac{1}{2} - \frac{1}{2}(1-2z),0\} \) and \( \bar{s}^B = \min\{\frac{1}{2} + \frac{1}{2}(1-2z),1\} \).

This proposition shows that when consumers sufficiently value facts, there only exists uninformative media in equilibrium. The intuition is the following: suppose one medium is uninformative and the other is partially informative. The competitive disadvantage of remaining uninformative is so big when consumers really value facts that the uninformative medium cannot obtain positive profit. Therefore, the asymmetric equilibrium cannot exist because one medium will jam the other’s stance. In this equilibrium, consumers believe that media stances are uninformative and therefore, any attempt of one media to deviate by choosing a stance closer to the state, which convinces consumers of a more informative position, is impossible. From Proposition 2 we know that when \( M > M_3 \) (\( M_3 > M_2 \) by comparison), the monopolist’s reports are partially informative. Therefore, even with the possibility that one medium can be informative in equilibrium, Propositions 4 and 5 together imply the basic insights of this research: 1) at least one competitive media is (weakly) less informative than monopoly; 2) competition strictly decreases the informativeness of media when consumers value facts.

We now turn to the question of whether there exist equilibria in which one medium is partially informative. In almost all signaling models, including cheap-talk games, our definition of PBE does not restrict out-of-equilibrium beliefs. In general, therefore, an infinite number of
equilibria are sustainable, including the asymmetric equilibrium with exactly one partially informative medium, depending on how one concocts consumers’ out-of-equilibrium beliefs. To identify which of these equilibria are most plausible, we must specify an equilibrium selection criterion.\footnote{In the game-theory literature, there are several established equilibrium-selection criteria. For example, the intuitive criterion has been shown to be an effective refinement in signaling games with two types. In the cheap-talk literature with multiple senders there is unfortunately no commonly accepted criterion on equilibrium refinement (Battaglini 2002). In addition, given that our setting is actually a combination of cheap-talk and location model, there is no existing criteria that can be effectively implemented into our new setting.}

The refinement we introduce, the “favorable criterion,” provides a means to select the most intuitive equilibrium. Like refinements in other signaling games, the favorable criterion filters out equilibria supported by implausible out-of-equilibrium beliefs. The favorable criterion relies on the following characterization of beliefs.

**Definition 5:** A favorable belief is an out-of-equilibrium belief such that, for any equilibrium deviation by a player, other players believe this player is (weakly) more informative with this belief, all else equal.

For example, if in equilibrium the optimal choice of media stance is $\tilde{s}^A$, then given the equilibrium belief, consumers expect that medium $A$ has $\tilde{N}^A$ facts. However, if medium $A$ chooses a media stance $\tilde{s}^A$ different from $\tilde{s}^A$, then a favorable belief is any belief such that the medium’s expected number of facts under the belief is $\tilde{N}^A \geq \tilde{N}^A$.

**Definition 6:** An equilibrium satisfies the favorable criterion if for any deviations from the equilibrium path, the player’s payoff is strictly decreasing under some favorable out-of-equilibrium belief from other players, all else being equal.

From this definition, we can see that if an equilibrium fails the favorable criterion it means that any favorable out-of-equilibrium belief rewards the deviation. Then the medium can make a statement like the following to consumers:

“If I choose a media stance $\tilde{s}^A$, you should believe I generate at least as many facts as $\tilde{s}^A$ because as long as you believe the number of facts of the new report is (weakly) higher, my payoff is no less than what I obtained before. In this way, I will deviate from the equilibrium to provide a report with (weakly) more facts, and thus you should believe me.”
As we can see, this criterion is essentially asking consumers to use forward induction (Cho and Kreps 1987, Gibbons 1992) when interpreting a stance choice. When consumers form the updated belief, they ask whether the medium’s choice is rational, which means a deviation from the equilibrium path should at least not decrease the medium’s payoff if consumers believe that this deviation brings more facts. Therefore, the statement of deviation is credible.\(^{19}\)

If the equilibrium satisfies this criterion, then there exists at least one favorable out-of-equilibrium belief that could reduce the medium’s payoff if the medium deviates from the equilibrium. Hence, this above statement is not “credible” any more since not all favorable out-of-equilibrium beliefs can benefit from the deviation. As we can see, this equilibrium refinement can help us to find the most stable and credible equilibrium.

To see how this criterion works, we examine an example. Suppose an equilibrium belief indicates that consumers always believe that media stance \(s = \frac{1}{3}\) represents \(t \in [0, \frac{1}{3}]\) and that \(s = \frac{2}{3}\) represents \(t \in \left[\frac{1}{3}, 1\right]\)—in other words, consumers believe that any other choices of media stances are not informative. We can see that there exist at least two types pure-strategy PBEs under this set of equilibrium beliefs. Let’s further assume that \(s_A \leq s_B\). The first one is \(\bar{s}_A = \frac{1}{3}\) and \(\bar{s}_B = \frac{2}{3}\). Under this case, consumers do not know which medium is actually partially informative because stances are exactly mirror opposites from different intervals. Hence, \(\bar{\Delta} \equiv (\bar{N}_A - \bar{N}_B) = 0\). Now, if medium \(A\) deviates from the equilibrium by choosing \(\bar{s}_A = \arg \max_{s_A} \pi_A(s^A_A, \bar{s}_B, \bar{\Delta})\), then consumers’ favorable out-of-equilibrium belief is to believe that medium \(A\) is either still uninformative or partially informative. If the belief is the former, then we know that \(\pi_A(\bar{s}_A^A, \bar{s}_B^A, \bar{\Delta}) \geq \pi_A(\tilde{s}_A^A, \bar{s}_B^A, \bar{\Delta})\). Otherwise, if consumers believe medium \(A\) is

\[^{19}\text{In a separate note, which is available upon request, we show that for any equilibrium that satisfies the favorable criterion, the stances are (i) mutually optimal (in a Nash sense) given consumers’ beliefs about the state-of-the-world and (ii) “stable” to small mistakes in beliefs about stances (trembling-hand perfect). To see why this criterion is desirable and filters out unintuitive equilibria, assume we are in an equilibrium that doesn’t satisfy the criterion. Then there must exist a set of stances for the partially informative medium \(A\) such that for stance \(s_i^A\), consumers believe the state \(t \in [a_i^A, b_i^A]\) and formalize the expected number of facts to be \(N^A_i(s_i^A)\). Because this equilibrium fails the favorable criterion, it does not have the mutually optimal property, (i) above. Therefore the equilibrium stance \(s_i^A \neq \arg \max, \pi_A[s, N^A_i(s_i^A), s_B^A, N^B]\). Essentially this means that conditional on the expected number of facts \(N^A_i(s_i^A)\), and the rival medium’s stance, the equilibrium stance \(s_i^A\) does not maximize medium \(A\)’s profit. We can clearly see this equilibrium belief is not intuitive.}\]
partially informative, $\tilde{\Delta} > \bar{\Delta}$. Therefore, $\pi^A(\tilde{s}^A, \tilde{s}^B, \tilde{\Delta}) > \pi^A(\tilde{s}^A, \bar{s}^B, \Bar{\Delta}) \geq \pi^A(\tilde{s}^A, \bar{s}^B, \bar{\Delta})$, and hence this equilibrium does not pass the favorable criterion unless $\tilde{s}^A = \bar{s}^A = \frac{1}{3}$.

Another possible equilibrium is one in which a medium reports based on the equilibrium belief and another medium reports un informatively. Let us assume that $A$ is informative and chooses $\bar{s}^A = \frac{1}{2}$ when $t \in [0, \frac{1}{2}]$. Since media $B$ is uninformative, we know that $\bar{\Delta} > 0$. Medium $B$ will choose $\bar{s}^B = \arg \max_{s^B} \pi^B(\tilde{s}^A, s^B, \bar{\Delta})$. Now, if medium $A$ deviates from $\tilde{s}^A$, then it can be profitable by choosing $\tilde{s}^A = \arg \max_{s^A} \pi^A(s^A, \tilde{s}^B, \tilde{\Delta})$. A favorable out-of-equilibrium belief is that $A$ reports at least the same number of facts as before, so that $\tilde{\Delta} \geq \bar{\Delta}$. Then, $\pi^A(\tilde{s}^A, \tilde{s}^B, \tilde{\Delta}) \geq \pi^A(\tilde{s}^A, \bar{s}^B, \bar{\Delta}) \geq \pi^A(\tilde{s}^A, \tilde{s}^B, \bar{\Delta})$. Hence, we can always find a deviation that could benefit medium $A$ given the favorable out-of-equilibrium belief unless $\tilde{s}^A = \bar{s}^A = \frac{1}{3}$. Similarly, we can show that there exists a deviation for medium $B$ given the favorable out-of-equilibrium belief unless $\tilde{s}^B = \bar{s}^B = \frac{2}{3}$. Therefore, we can see that the favorable criterion can effectively filter out any equilibrium in which one medium’s choice of stance is not an optimal response to the other given the equilibrium belief. Otherwise, a medium can always benefit from deviation. Hence, by applying the favorable criterion, we are left with exactly one type of equilibrium under competition.

**Proposition 6** Under the favorable criterion, there does not exist any asymmetric equilibrium in which one medium is uninformative and the other medium is partially informative; implying that the uninformative equilibrium characterized in Proposition 5 is unique.

The intuition for this proposition is the following: if there existed an equilibrium in which only one medium is partially informative, then consumers would believe the uninformative medium provides fewer facts than the other medium. The uninformative medium suffers a profit loss from the lower expectation of the number of facts that will be in its report. Therefore, it has an incentive to jam the other medium by taking the “mirror” stance, so that consumers do not

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20 Note here the criterion requires the strictly decreasing in payoff. Therefore, if a deviation can weakly increase the payoff under any favorable belief, it fails the criterion.
know which medium is actually partially informative. Our result follows from the fact that the uninformative medium always benefits from jamming, and that this incentive for deviation becomes stronger as $M$ enlarges. Therefore, consumers should not believe any medium is partially informative in equilibrium implying that an asymmetric equilibrium is impossible.  

Proposition 6 establishes the existence and uniqueness of an equilibrium with two competitive media. Furthermore, it characterizes the stances in an uninformative equilibrium for both media. Interestingly, since both media are uninformative and have an equal expected number of facts, the equilibrium stance in this case is independent of consumers’ valuation of facts ($M$) and instead is only decided by the heterogeneity of consumer opinions ($z$). A graphical example is shown in Fig 4.

\[0 \leq b \leq 0.5 \leq 1-z\]

**Figure 4:** Uninformative Equilibrium with Two Media ($s^A \leq s^B$)

It is worth mentioning that the uniqueness result of Proposition 6 is a direct application of the favorable criterion, which rules out asymmetric equilibria supported by unintuitive beliefs. However, the existence aspect of the uninformative equilibrium with competitive media is independent of the equilibrium selection criterion used. Therefore, regardless of which equilibrium selection criterion used in our model, one can be sure that there is no more than one informative media in any equilibrium. In the comparisons that follow, we focus on the most intuitive equilibria and hence consider only those that survive the favorable criterion.

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21 Although our model is a one-shot game, we can consider a dynamic model with reputation for a certain position, perhaps influenced by its previous choices of slant. Incorporating such dynamics reinforces the results from this static model if media establish reputations for a consistent position (e.g. left-wing or right-wing stances). On the other hand, if consumers have the ability to verify all facts, including the unreported ones, and punish media (e.g. by not buying in the future), then media would have a greater incentive to choose stances closer to the state-of-the-world. As these two scenarios suggest, the implication of dynamics could go either way depending on which conditions hold. Which scenario is more plausible is subject to debate. If verification costs were always sufficiently low for consumers to check unreported facts, then there would be no role for media in the first place. Thus, we feel that our setting is realistic in many plausible scenarios.
5. Competition vs. Monopoly

In this section, we compare the equilibrium results of the monopoly and duopoly cases by first examining whether competition improves media informativeness and factual content provision. We then examine whether competition causes media reporting to be more polarized. Finally, we extend the scope of the analysis to incorporate the concept of media bias into our model and examine how competition affects it.

5.1 Media Informativeness and Factual Content Provision

In this section, we focus on the comparison of media informativeness and content provision under the monopolistic and duopolistic cases. Propositions 2 & 4-6 indicate that competition deteriorates media informativeness, especially when consumers value facts more.

**Proposition 7** *Competition does not increase the informativeness of media reporting when* \( M < M_1 \), *but it decreases the informativeness when* \( M > M_2 \).*

Proposition 7 says that competitive commercial media leave consumers less informed. Since duopolistic media are always uninformative, the informativeness is (weakly) lower under competition. The competitive environment prevents each medium from being even partially informative. Even if one medium tries to report informatively, competition encourages the other medium to “jam” its stance by taking an equal but opposing stance in order to neutralize the rival’s advantages of being believed to have more facts. This is a counter example to Shaked and Sutton (1982, 1983)’s vertical differentiation case, in which they showed that competing firms can have an incentive to differentiate themselves by quality. In our framework, even though we allow that the product quality (the number of facts) to be endogenously chosen, any efforts to build up the (vertical) advantage in content reporting through staying partially informative will be offset by the rival’s jamming behavior. While the single medium benefits from being informative when consumers value the facts, this benefit disappears with competition. Hence, competition reduces the communication efficiency—the additional voice actually confuses consumers.\(^{22}\)

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\(^{22}\) Another source of confusion, which we do not consider here, could be due to an excessive number of facts or a form of information overload. In our model setting, this would be captured by a decreasing marginal utility of facts in equation (1) and imply that media stances are more responsive to differentiated opinions and less to factual content provision.
Next, we study the impact of competition on the factual content provision. The following proposition indicates that competition decreases the average provision of factual content per medium.

**Proposition 8** Relative to a monopoly medium, a competitive medium reports strictly fewer facts when \( M > M_2 \).

Our results indicate that a medium becomes less informative and reports fewer facts under competition. This last finding may provide an alternative explanation of why micro-blogging platforms such as Facebook, Twitter, and Tumbler have become so popular. A competitive environment drives the media to reduce product quality in the form of the factual content provision. The reduction of factual content leads to an increased need for micro-blogging platforms, which restrict the amount of factual information that each medium can provide, in order to conceal the reduction in information provided. This competitive reason for the booming of the micro-blogging industry has never been examined in the previous literature.

### 5.2. Media Polarization

We call a media stance more polarized if the stance significantly diverges from the middle point of the state. Specifically, for any equilibrium stance in monopoly \( \bar{s} \), define the polarization measure \( MP_{Mon} = |\bar{s} - \frac{1}{2}| \). And for any pair of equilibrium stances in duopoly, \( \bar{s}^A, \bar{s}^B \) which are symmetric around the middle point by Proposition 6, the corresponding measure is \( MP_{Duo} = \frac{1}{2}(|\bar{s}^A - \frac{1}{2}| + |\bar{s}^B - \frac{1}{2}|) \), which is the average distance of media stances from \( \frac{1}{2} \). Both MS and XS find that competition pushes media stances toward more extreme positions. Our result is consistent with their finding, but only under certain conditions. If those conditions do not hold, competition does not necessarily drive the media to be more polarized.

**Proposition 9** Competition does not necessarily imply that media stances are more polarized.

1. **When** \( z \) **and** \( M \) **are large enough, monopoly is more polarized than duopoly**

   \[
   MP_{Mon} > MP_{Duo}.
   \]

2. **When** \( z \) **and** \( M \) **are small enough, duopoly is more polarized than monopoly**

   \[
   MP_{Mon} < MP_{Duo}.
   \]
The intuition of the proposition is based on the different driving forces for the choice of stance under different competitive environments. Under monopoly, the medium’s reporting depends on consumers’ preference for facts ($M$). When $M$ is large enough, the monopolist’s reports are partially informative reports, and therefore its stance is in the same interval as the state. A higher $M$ drives the stance further away from the middle point. However, the duopolistic media are always uninformative. The stances are only decided by the degree of consumer heterogeneity ($z$). When $z$ is large, the stance by each medium is closer to the middle point. Therefore, when $z$ and $M$ are large enough, monopoly is more polarized than duopoly. But if $z$ and $M$ are small enough, the stance of the monopolistic medium is positioned at the middle point. At the same time, duopolistic media position themselves toward the endpoints to accommodate diverse consumer opinions. Therefore, monopoly is less polarized than duopoly in this case.

One thing we want to emphasize here is that this result is not simply a maximum differentiation result (MS, XS). Although the competitive equilibrium outcome is quite similar, the underlying reason is very different. The rival takes a mirror stance to eradicate the informative medium’s competitive advantage and to focus readers’ choices solely on opinions. Even though competitive media may take opposite positions, they do so only to the extent of heterogeneity in consumer opinion. Also, our result, which is different from previous findings, indicates that a monopoly medium’s stance is more polarized than a duopolist’s under certain conditions.

5.3 Media Bias and Media Competition
Relative to previous research, the results of our analysis provide new insights regarding the relationship between media bias and the degree of competition. Earlier work, notably MS and XS, define media bias as a relative measure of the distance between the state and the media stance, which captures the distortion of a media position. Because those works do not permit consumers to update beliefs about the state based on media strategies, they are unable to tell whether consumers obtain better understanding from reading a media report, nor how consumers make inferences about the state from the distorted information. To understand how these distinctions matter, we must redefine media bias to account for consumers’ inference. Media bias in our model is measured by the weighted distance between the media stance and the state of the world.
**Definition 7:** Media bias is the expectation of weighted distance by which the media stance deviates from the state of the world: 

\[ MB_{\text{Mon}} = E_t\left(\frac{(s_t-i)^2}{a_i-a_{i+1}}\right), \]

if the media reports with \( s_t \) when \( t \in [a_{i-1}, a_i] \), while in duopoly 

\[ MB_{\text{Duo}} = \frac{1}{2}(E_t[(\bar{s}_A-i)^2] + E_t[(\bar{s}_B-i)^2]) \]

An important aspect about this definition of media bias is the denominator of \( (a_i-a_{i+1}) \). This enables us to compare media bias under different levels of informativeness. The measure of media bias used by MS or XS is not suitable in our setting because it does not adjust for consumers’ ability to identify the interval from which the state lies. Therefore, we normalize the expected difference by the size of the updated interval. Also, in order to compare the media bias in different informative levels, a full characterization of the interval structure is necessary. However, it is impossible to obtain tractable solutions for the intervals when \( x > 2 \). Hence we restrict our attention in the following examples to \( x \leq 2 \) in this section. Using this measure of media bias, we find two results: (i) a higher informative reporting does not necessarily lead to lower media bias, and (ii) competition can actually reduce it.

**Proposition 10** When \( M > 8d(1-z) \), the partially informative reporting with \( x=2 \) in monopoly has higher media bias than uninformative reporting.

![Figure 5: Media Bias in Monopoly under Different Informative Level](image)

Normally we would think a more informative report should be less biased. Proposition 10 shows this is not always the case. The fundamental conflict in consumers’ preferences drives this
result: consumers want to read more facts, but at the same time like to read a report appealing to their opinions. A more informative reporting strategy certainly raises consumer expectation about the facts, but at the same time, the medium still has an incentive to appeal to consumer opinion, which forces it to report the state of the world towards the expectation of consumer opinions. The dispersion between the updated expectation of the state of the world and the expectation of consumer’s opinion will be higher as consumers’ preference for facts increases. As a result, the polarization becomes stronger when $M$ is large enough, which increases the media bias.

Figure 5 shows the disparity of media informativeness and media bias under different reporting equilibria. The first picture is when the reporting is uninformative. From Lemma 2 we know the optimal media stance is 0.5, which minimizes the media bias of the uninformative equilibrium. On the other hand, following the discussion about the choice of the media stance in section 3, in a partially informative equilibrium with two intervals, when $M > 8d(1 - z)$, the medium chooses a media stance $s_i = 0$ when $t$ belongs to $[0, 0.5)$. We can clearly see that $s_i$ is very polarized relative to either the old expectation of the state (0.5) or the new expectation (0.25), which demonstrates the increase in media bias with more informative reporting.

Proposition 10 illustrates the fundamental difference between the media informativeness and media bias, and shows how a report with higher media bias can still be more informative. Proposition 10 also shows that the media bias concept is not an ideal measure of the quality of communication between media and consumers. The medium can be “biased but informative” at the same time, which challenges the use of media bias as a measure of consumers’ ability to be informed.

The last proposition summarizes the findings regarding media bias and competition.

**Proposition 11** Competition does not necessary increase media bias.

i) When $z$ and $M$ are large enough, the partially informative equilibrium with $x = 2$

under monopoly has higher media bias than duopoly: $MB_{Mon} > MB_{Duo}$.

ii) When $z$ and $M$ is small enough, monopoly has lower media bias than duopoly

$MB_{Mon} < MB_{Duo}$. 

Starting from MS, a common result in the literature is that competition increases the media bias by the principle of maximum horizontal differentiation. Our framework echoes the previous finding, but only under certain boundary conditions. The fundamental reason for this result is again because consumers understand the incentives for bias. Proposition 10 establishes the uninformative equilibrium in monopoly might have lower media bias than the more informative equilibrium. Therefore, if \( z \) is high enough, the media bias of a duopoly is closer to the media bias of the uninformative equilibrium in monopoly, as indicated in Figure 6. However, if \( M \) is large, the monopoly medium’s report is partially informatively and can have higher media bias than in the uninformative equilibrium. That explains why the media bias of monopoly is higher than the duopoly when \( z \) and \( M \) are large enough. The intuition of the second result is similar but opposite.

![Diagram of Media Bias Comparison](image)

(a) Monopoly (x=2)

(b) Duopoly

**Figure 6**: Media Bias Comparison under Competition (\( z \) and \( M \) are large)

6. Implications and Discussions

Our results suggest that encouraging competition in the commercial media market does not necessarily make consumers better informed. This result is consistent with recent survey evidence about how informed consumers are depending on their news source. For example, one study found that Fox News and MSNBC viewers are the least informed about current events compared with those who use other news sources.\(^{23}\)

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The result from Section 5.1—that competitive forces encourage media to take stances that compromise their ability to produce factual content—may have an interesting connection to the growth of micro-blogging sites such as Facebook, Twitter and Tumblr, which limit the amount of content users can provide. These forms of media may have gained popularity because of heavy competition for consumer attention, which is arguably due to the internet lowering the barriers to content provision. While publishers on those platforms do not typically collect monetary transfer from consumers, they do benefit from additional consumer attention, which is obviously a scarce resource (e.g. a tweeter is better off with more followers). In light of our results, heavier competition encourages media to switch to microblogging platforms since they can deliberately obscure the reduction of content provision.

Another interesting finding in this paper is that a balanced report (for example: an article claims global warming is inconclusive and put equal amount of facts from both sides) does not necessarily facilitate viewers’ understanding about the state of the world. Some research attributes balanced reporting as implied by journalistic norms and values (Boykoff & Boykoff 2004), but our paper points to a different explanation. When readers value balanced reports more highly than unbalanced reports (as is the case in our model when $M \rightarrow 0$, the media have an incentive to align their content to consumer opinions). In this way, a medium’s incentive to appeal to balanced reporting causes a divergence between the general scientific discourse and viewer attitudes toward it. That is, the media could simultaneously be balanced, yet uninformative. In the case of global warming, for example, if scientists generally accept that global warming is occurring ($t \sim 1$) but consumers prefer balanced reporting over factual content, then a medium has a tendency to slant content toward a balanced angle indicating that global warming’s occurrence is inconclusive.

On the other hand, as this paper shows, an unbalanced report under monopoly can be more informative than a balanced report, which implies that biased reporting does not leave viewers with less knowledge. The important thing to keep in mind is that a media stance that is polarized is not always meaningless, as long as there are no jamming voices opposing it. A provocative report, even though it is biased, could tell consumers more about current events than a plain and “politically neutral” one, when competition among commercial media is not high.

Finally, our model can be used to understand the implication of “cross-checking” – the notion that consumers read multiple media to obtain different perspectives. XS suggests that, by reading both media, “a conscientious reader gets all the facts, as if she were able to read an unslanted newspaper.” Our paper indicates that conscientious readers do not necessarily learn all the facts by cross checking different media. The reason is that when the state lies on one side of both media stances, the media will use either all the facts with signal “1”s or “0”s, but not both. Hence reading both media does NOT guarantee that readers read all the facts. It is only when the state is located between both media stances that conscientious readers obtain all the facts. Since media stances are more polarized as $z$ decreases, consumers’ heterogeneous opinions help conscientious readers to be more informed. Also, the average media bias increases as media stances are more polarized, which means that conscientious readers are more informed when the media are more biased (this intuition is very similar to XS). This finding again echoes the counterintuitive relationship between media informativeness and media bias.

7 Conclusions & Limitations

This paper has presented the first analysis of selective factual content provision by commercial media. Our model differs from previous research by incorporating the following elements (1) Factual content provision is bounded by the choice of media stance. (2) Consumers appreciate more facts. (3) Consumers can (partially) infer the state from the media stance. We find that while the commercial incentives of media prevent the monopoly from being fully informative, it can help consumers to understand the state of the world better by providing informative reporting. In contrast, readers facing competitive media end up learning little or nothing and read fewer facts. We also find several other counterintuitive results: competition does not cause the media stance to be more extreme when consumers value facts and are less diverse, a more-informative report does not necessarily lead to a lower media bias, and competition can reduce media bias.

We related our results to anecdotal evidence and shed light on media regulation, the booming of micro-blogging, and other important subjects related to selective factual content provision by commercial media.

One important assumption we made is that the state of the world is unidimensional. However, it is natural to consider a multidimensional state space. For example, the controversial

Mathematically: $s^A < t < s^B$. 

24
issue of global warming involves issues related not only to climate change but also to the development of alternative energy sources. Battaglini’s (2002) analysis of standard “cheap-talk” in a multidimensional state space may provide some guidance. He shows, in fact, that multidimensionality can improve communication efficiency with multiple senders. Intuitively, multidimensionality can soften the conflicts among information senders and, therefore, leave room for senders to coordinate. Future work can extend our framework by considering the case in which the state is conceived as having more than one dimension.

In this paper, we considered a setting in which the media do not have any preference about the stance. This allows us to isolate the impact of competition on factual content from other factors. However, it is not hard to imagine that the media itself is not perfectly neutral about the stance. Several studies (Balan et al. 2004, Baron 2004, Anderson & McLaren 2009) provide insight on the supply side analyses of media reporting with media owners having preferences about what consumers should read. We do not anticipate that the introduction of media preference will alter the main driving force of the main result: media optimally balance consumers’ desire for facts and their taste for slant. It would still be interesting to investigate, however, whether the media would be less or more informative when the goal of reporting is considered to be not exclusively profits.

References


Appendix

Proof of Lemma 1

Assume there exists a fully informative equilibrium with a one-to-one stance mapping 
\[ s^* : [0,1] \rightarrow [0,1] \] between \( s \) and \( t \). We establish a contradiction in two steps: we first show that if a fully informative equilibrium exists, then for \( t \in [0, \frac{1}{2}] \), the equilibrium media stance \( s^*(t) \in [t, \frac{1}{2}] \). (Similarly, if \( t \in [\frac{1}{2}, 1] \), \( s^*(t) \in [\frac{1}{2}, t] \)). Second we show these conditions provide a unique optimal choice of media stance regardless the value of \( t \), which means that consumers cannot update their beliefs fully informatively.

Step 1. Fix \( t_i \in [0, \frac{1}{2}] \) and denote \( s_i = s^*(t_i) \). We first establish the following fact. For any \( t_2 \neq t_1 \), we must have \( \pi(t_1 \mid s_i) \neq \pi(t_2 \mid s_i) \). Suppose otherwise. Then the media could report either \( s_i \) or \( s^*(t_2) \neq s_i \) and earn equilibrium payoffs. Consumers could not, therefore, update their beliefs fully informatively. Using this condition, we show that \( s_i \) lies in \([t_1, \frac{1}{2}]\). First observe that if \( s_i \geq t_1 \) then \( s_i \leq \frac{1}{2} \). To see this, take some state of the world \( t_2 \geq s_i \). Profits at these states of the world with a stance \( s_i \) are expressed:
\[ \pi(t_i | s_i) = V + M \frac{t_i}{s_i} - d \max \{ (s_i - 1 + z)^2, (s_i - z)^2 \} ; \text{ and} \]
\[ \pi(t_2 | s_i) = V + M \frac{1-t_2}{1-s_i} - d \max \{ (s_i - 1 + z)^2, (s_i - z)^2 \} \]

The condition \( \pi(t_i | s_i) \neq \pi(t_2 | s_i) \) implies that \( \frac{t_i}{s_i} \neq \frac{1-t_2}{1-s_i} \), or \( t_2 \neq 1 - \frac{1-s_i}{s_i} t_1 \). Given that \( t_2 \in [s_i,1] \) we have \( \frac{1-s_i}{s_i} \geq 1 \), or \( s_i \leq \frac{1}{2} \). We now show that \( s_i \geq t_i \). For some \( t_2 \leq s_i \), the profit expressions above and the condition \( \pi(t_i | s_i) \neq \pi(t_2 | s_i) \) imply that \( \frac{t_i}{s_i} \neq \frac{1-t_2}{1-s_i} \). But \( t_2 \in [0,s_i] \) implies \( \frac{1-t_2}{1-s_i} \geq 1 \) or \( s_i \geq t_i \). A similar argument establishes that \( s^*(t) \in [\frac{1}{2},1] \) whenever \( t \in [\frac{1}{2},1] \).

Step 2: We show that it is impossible to have a one to one mapping that can satisfy the necessary conditions we specified above. Suppose there is a \( s_i(\varepsilon) = s^*(t_i = \frac{1}{2} - \varepsilon) \) which is associated with \( t_i = \frac{1}{2} - \varepsilon \) with \( \varepsilon \geq 0 \). From Step 1 we know \( s_i(\varepsilon) \in [t_i, \frac{1}{2}] \). We can see \( \lim_{\varepsilon \to 0} t_i = \frac{1}{2} \), therefore \( \lim_{\varepsilon \to 0} s_i(\varepsilon) = \frac{1}{2} \). That means when \( t = \frac{1}{2} \) the associated media stance \( s^*(\frac{1}{2}) = \frac{1}{2} \) under fully informative equilibrium.

However, we can see \( \pi(t = \frac{1}{2} | s = \frac{1}{2}) = V + M - d(\frac{1}{2} - z) > \pi(t' | s') \) for all \( t' \) and for all \( s' \neq \frac{1}{2} \). That is, for any state of the world, the media’s profit is strictly highest when announcing \( s' = \frac{1}{2} \). The monopoly can charge a higher price and earn more profit by not telling the truth of the state. This contradicts the requirement of a fully informative equilibrium within \([0,1]\).

Finally, it follows directly that when \( d = 0 \), the medium has no incentive to distort the reporting and the only equilibrium is to report \( s^*(t) = t \), which is fully informative. 

Q.E.D.

**Proof of Proposition 1**

We show that when \( x \geq 1 \), for any set of dividing points \((a_i)_{i=0,\ldots,x}\) and media stances \((s_i)_{i=1,\ldots,x}\) that satisfies (1), (2), and (3), the reporting rule defined in (2) is optimal for the medium’s profit maximization given purchasing rules under (4), and the purchasing rules are optimal for consumers under the reporting rule defined by (1) and (2).

For any PBE, we must specify consumers’ out-of-equilibrium beliefs. The most intuitive belief system specifies that when consumers observe a media stance \( s' \neq s_i \), all \( i \), consumers do
not update their knowledge about the state of the world and believe the medium’s reporting is uninformative. Under this out-of-equilibrium belief, the expected number of facts is \( \frac{1}{2} \).

We show that when \( x \geq 1 \), if a set of dividing points \( (a_i)_{i=0,\ldots,x} \) and media stances \( (s_i)_{i=1,\ldots,x} \) satisfies (1), then the reporting rule defined in (2) is optimal for the medium’s profit given purchasing rules under (4). Suppose \( t \in [a_{i-1}, a_i] \) and let \( s \) be any stance in \([a_{i-1}, a_i]\).

Since
\[
\frac{\partial^2 \pi_i(s, a_{i-1}, a_i)}{\partial s^2} = -\frac{Ma_i^2}{(a_i - a_{i-1})(s)^3} - \frac{M(1-a_i)^2}{(a_i - a_{i-1})(1-s)^3} - 2d < 0,
\]
we know that \( \pi_i(s, a_{i-1}, a_i) \) is concave in \( s \) on the compact set \([a_{i-1}, a_i]\) which guarantees the maximizer in (2) is unique. What’s left is to show consumers decision rule \( \Psi_i(\cdot) = 1 \) is optimal for all consumers when reporting rules and beliefs are specified as (1) to (4). We can see that not buying cannot increase a consumer’s payoff since \( E[u_b(n, s_i, p \mid s_i)] \geq 0 \) for all consumers.

Last we show if a set of dividing points \( (a_i)_{i=0,\ldots,x} \) and media stances \( (s_i)_{i=1,\ldots,x} \) satisfies (1) and (2), then (3) is satisfied. We know that when \( i = 1 \), and \( x \):

\[
\pi(s, 0, a_i) = V + \left[ \frac{M}{2a_i} - \frac{M}{2} \left( \frac{1-a_i}{1-s_i a_i} \right) \right] - d(s_i - 1 + z)^2
\]

\[
\pi(s, a_{i-1}, 1) = V + \left[ \frac{M}{2(1-a_{i-1})} \right] - \frac{M}{2} \left( \frac{a_{i-1}}{s_i (1-a_{i-1})} \right) - d(s_i - z)^2
\]

We first see if \( a_i = 1 - a_{i-1} \), then \( \pi(s, 0, a_i) \) and \( \pi(s, a_{i-1}, 1) \) are the same function by treating \( 1-s_i \) as a variable. Hence the optimal choice defined by (2) satisfies \( s_i = 1 - s_i \). And it is easy to see that when \( a_i = 1 - a_{i-1} \) and \( s_i = 1 - s_{i-1} \), \( \pi(s, 0, a_i) = \pi(s, a_{i-1}, 1) \) The structure of the proof for \( a_i = 1 - a_{i-1} \), \( s_i = 1 - s_{i-1} \) and so on is similar. Q.E.D.

**Proof of Lemma 2**

Based on Proposition 1, when \( x = 1 \) the set of dividing points are \( a_0 = 0 \), \( a_1 = 1 \) and \( s_1 = \frac{1}{2} \). It is obvious (3) and (4) of Proposition 1 are satisfied. We want to check whether \( s_1 = \frac{1}{2} \) is optimal for the medium under consumers’ expectations. Since a consumer’s expected utility is

\[
E[u_b(N, s_i, p \mid s_i)] = V + \frac{M}{2} - d(s_i - b)^2 - p,
\]
the highest price the media can charge is

\[
p = V + \frac{M}{2} - d(s_i - 1 + z)^2.
\]
Since all consumers purchase at this price, this is also represents the
media’s profit. We can directly see that \( s_i = \frac{1}{2} \) maximizes profit and hence this set of dividing points and the media stance satisfies (1) and (2) of Proposition 1. What’s left is to show that consumers’ decision rule \( \Psi_r(.) = 1 \) is optimal for all consumers when reporting rules and beliefs are specified as (1) to (4) of Proposition 1. We can see not buying can lead to lower payoff for consumer since \( E[u_h(N, s_i, p | s_i)] \geq 0 \) for all consumers. The profit follows immediately from (1) in Proposition 1.

We now show there exists a cutoff point \( M_1 \) such that when \( 0 < M < M_1 \), it is impossible to have a partially informative equilibrium. Let \( (a_i)_{i \in \cdot} \) denote a set of diving points, with \( x \geq 2 \) as characterized in Proposition 1. Suppose \( t \in [0, \frac{1}{2}] \) and consider any interval \([a_{i-1}, a_i] \subset [0, \frac{1}{2}]\). For \( t \) in this interval, the optimal stance \( s \in [a_{i-1}, a_i] \) must solve the first-order condition for the maximization in (2) of Proposition 1:

\[
\frac{\partial \pi(s, a_{i-1}, a_i)}{\partial s} = \frac{M}{a_i - a_{i-1}} \left[ \frac{a_{i-1}^2}{2s^2} - \frac{(1-a_i)^2}{2(1-s)^2} \right] - 2d(s-1+z) \leq 0.
\]

This has a solution when \( M \leq 4d(1-z-a_i)a_i^2/(a_{i-1} + a_i) \), a condition which must hold for all intervals \([a_{i-1}, a_i] \subset [0, \frac{1}{2}]\). Define \( M_1 = \min_{a_i \in \cdot} [4d(1-z-a_i)a_i^2/(a_{i-1} + a_i)] \),

It is immediate that \( M_1 \geq 0 \). In fact \( M_1 \) is positive except possibly for \( a_i = 0 \) or \( a_i = 1-z \). But Lemma 1 established that there does not exist a fully informative equilibrium, which implies that that \( a_i \neq 0 \). And, \( a_i \leq \frac{1}{2} \) and \( z < \frac{1}{2} \) imply that \( a_i < 1-z \). Hence, \( M_1 > 0 \) for \( t \in [0, \frac{1}{2}] \).

We now show that when \( 0 < M < M_1 \), there is no partially informative equilibrium. Based on the first order condition above, for all intervals \([a_{i-1}, a_i] \) lying to left of \( \frac{1}{2} \), we have \( s_i = a_i \). If \( x = 2 \) then \( s_1 = s_2 = \frac{1}{2} \), which implies that consumers cannot update about their beliefs about which interval contains the state. Hence, there does not exist a partially informative equilibrium with \( x = 2 \). When \( x = 3 \), \( s_i = a_i \) when \( t \in [a_0, a_i] \) means that the number of reportable facts is \( N_1 = \frac{1}{2a_i} [1-(\frac{1-a_i}{1-s_i})^2] = \frac{1}{2} \). For the interval \([a_i, a_2] \) since \( a_i = 1-a_2 \) because of symmetry, the optimal stance \( s_2 = \frac{1}{2} \), which means \( N_2 > \frac{1}{2} \). Therefore, \( \pi(s_2 = \frac{1}{2}, a_1, a_2) > \pi(s_1 = a_i, a_0, a_1) \).

Finally, when \( x > 3 \), \( N_1 = \frac{1}{2} \), which is less than any other \( N_i = \frac{1}{2} + \frac{a_{i-1}}{2a_i} \) when \( a_i \leq \frac{1}{2} \). Therefore
\[ \pi(s_i = a_i, a_{-i}, a_i) > \pi(s_i = a_i, a_0, a_i), \] which violates the requirement of equal profitability across intervals (condition (1) of Proposition 1).

**Proof of Proposition 2**

We first show when \( M > M_2 \), there exists a partially informative equilibrium with \( x = 2 \). Define \( M_2 = 2d(\frac{1}{2} - z) \). If there are two intervals in equilibrium, then the first order condition of the maximization in (2) of Proposition 1 is

\[ \frac{\partial \pi(s,0,a_i)}{\partial s} = -\frac{M(1-a_i)^2}{2a_i(1-s)^2} - 2d(s-1+z) = 0. \]

When \( M > M_2 \), the optimal \( s_i < \frac{1}{2} \) for \( t \in [0, \frac{1}{2}] \), and \( s_2 > \frac{1}{2} \) for \( t \in [\frac{1}{2}, 1] \). It is easy to verify this specified media reporting rules satisfies the requirement of PBE in Proposition 1. Hence we showed that when \( M > M_2 \), there exists a partially informative equilibrium with \( x = 2 \). Also given the definition of \( M_1 \) in the proof of Lemma 2, we know \( M_1 \leq M_2 \).

Next we show when \( M > M_2 \), the media is strictly more profitable in the partially informative equilibrium than in the uninformative equilibrium. When the monopoly media reports uninformatively, the expected profit is \( \Pi^U = V + \frac{M}{2} - d(\frac{1}{2} - z)^2 \). However, in the partially informative equilibrium with \( x = 2 \), \( \Pi^I(s_1,0,\frac{1}{2}) = V + M - \frac{M}{d(1-s_1)} - d(1-z-s_1)^2 \) with \( s_1 \in [0, \frac{1}{2}] \).

Notice that if \( s_i \) is an interior solution to the maximization condition of part (2) of Proposition 1, then it must satisfy \( s_2 = \arg\max \Pi^I(s_1,0,\frac{1}{2}) \). Hence, \( \Pi^I(s_1,0,\frac{1}{2}) > \Pi^I(\frac{1}{2},0,\frac{1}{2}) = \Pi^U \). Q.E.D.

**Proof of Proposition 3**

First we can see if both media report truthfully, they will engage into Bertrand competition and obtain zero profit. But if one media deviates, consumers believe both are uninformative and the position differentiation permits a markup for the media. So the deviation is profitable. Therefore a truthful revealing equilibrium is never optimal for both media. To show there doesn’t exist a fully informative equilibrium in duopoly, we followed Lemma 1 in Battaglini (2002) such that no truthful informative equilibrium implies no fully informative equilibrium exists. See Battaglini (2000, 2002) for further details. Q.E.D.

**Proof of Proposition 4**
We analytically show when \( z < \frac{5}{12} \), there does not exist any partially informative equilibrium with two media. When \( \frac{5}{12} \leq z < \frac{1}{2} \), it is impossible to analytically prove the non-existence of partially informative reporting equilibrium with two media. We therefore establish this case numerically. By contradiction, assume there exists a partially informative equilibrium with two media with a set of dividing points \((a_i)_{i=0,\ldots,x}\) in \([0,1]\), and a set of media stance choice \((s^A_i, s^B_i)_{i=1,\ldots,x} \in [a_{i-1}, a_i]\), where \(s^A_i \neq s^B_i\). We know given the choice of media stances, consumers first formalize belief about the number of facts \(N^A_i\) and \(N^B_i\), then make purchase decision based on the price \(p^A\) and \(p^B\). The demand for medium \(j\) and \(k\) is given by

\[
D^A_j = \frac{[s^A_i + s^B_i + \frac{M(N^A_i - N^B_i) + p^B - p^A}{d(s^B_i - s^A_i)}] - 2z}{2(1-2z)} \quad \text{and} \\
D^B_j = 1 - \frac{[s^A_i + s^B_i + \frac{M(N^A_i - N^B_i) + p^B - p^A}{d(s^B_i - s^A_i)}] - 2z}{2(1-2z)}.
\]

The profit of each medium is \(\pi^A_i = D^A_i p^A_i\) and \(\pi^B_i = D^B_i p^B_i\). The respective first order conditions on prices imply:

\[
\bar{p}^A = \frac{1}{3}[d(s^B_i - s^A_i)(s^A_i + s^B_i + 2 - 6z) + \Delta M]
\]

\[
\bar{p}^B = \frac{1}{3}[d(s^B_i - s^A_i)(4 - 6z - s^A_i - s^B_i) - \Delta M]
\]

and with corresponding profits

\[
\pi^A_i = \frac{[d(s^B_i - s^A_i)(s^A_i + s^B_i + 2 - 6z) + \Delta M]^2}{18d(s^B_i - s^A_i)(1-2z)}
\]

\[
\pi^B_i = \frac{[d(s^B_i - s^A_i)(4 - 6z - s^A_i - s^B_i) - \Delta M]^2}{18d(s^B_i - s^A_i)(1-2z)}
\]

where \(\Delta \equiv (N^A_i - N^B_i)\). Now consider the first interval \([0, a_i]\), in this interval the expected number of facts

\[
N^A_i = \int_0^{a_i} \frac{t}{s^A_i} f(t) \, dt + \int_{a_i}^{s^A_i} \frac{(1-t)}{(1-s^A_i)} f(t) \, dt = \frac{1}{2a_i} [1 - (1-a_i)^2/(1-s^A_i)],
\]

\[
N^B_i = \frac{1}{2a_i} [1 - (1-a_i)^2/(1-s^B_i)], \quad \text{and}
\]
\[ \Delta = \frac{(1 - a_i)^2}{2a_i^2} \left[ \frac{1}{(1 - s_i^a)} - \frac{1}{(1 - s_i^b)} \right]. \]

Therefore \( \Delta > 0 \) if \( s_i^A < s_i^B \). We can see medium \( B \) is at a competitive disadvantage. Therefore we want to check whether medium \( B \) has incentive to deviate. Consider the deviation in which medium \( B \) “jams” \( A \)’s stance by choosing a stance outside the interval \([0, a_i]\). In the assumed equilibrium, we know \( B \) earns

\[ \pi_i^B(s_i^A, s_i^B, \Delta > 0) = \frac{[d(s_i^b - s_i^A)(4 - 6\Delta - s_i^A - s_i^b) - \Delta M]^2}{18d(s_i^b - s_i^A)(1 - 2\Delta)}, \]

which is obviously larger when \( \Delta = 0 \). Hence if there exist such a \( \tilde{s}^B > a_i \) such that

\[ \pi_i^B(s_i^A, \tilde{s}^B, \Delta = 0) \geq \pi_i^A(s_i^A, s_i^B, \Delta = 0), \]

then medium \( B \) can profitably deviate by jamming \( A \). We know when \( \Delta = 0 \), the optimal stance choice of medium \( B \) is given by the best response function

\[ s_i^A - 3\tilde{s}^B + 4 - 6\Delta = 0, \]

or \( \tilde{s}^B = \frac{1}{3}(s_i^A + 4 - 6\Delta) \). If \( z < \frac{\Delta - a_i}{6} \), then \( \tilde{s}^B > a_i \) for any \( s_i^A \in [0, a_i] \).

Under this condition medium \( B \) has incentive to jam \( A \). In order to determine when this condition is met, we consider the case of \( t \in [a_{x-1}, 1] \) with \( s_x^A < s_x^B \). In this case

\[ N_x^A = \frac{1}{2(1 - a_{x-1})} \left[ 1 - \frac{a_x^2}{s_x^A} \right] \quad \text{and} \quad N_x^B = \frac{1}{2(1 - a_{x-1})} \left[ 1 - \frac{a_x^2}{s_x^B} \right]. \]

We can see \( N_x^A < N_x^B \) so that \( \Delta < 0 \) and \( \pi_x^A(s_x^A, s_x^B, \Delta < 0) < \pi_x^A(s_x^A, s_x^B, \Delta = 0) \). In a manner similar to finding \( \tilde{s}^B \) above, we find a \( \tilde{s}^A < a_{x-1} \) such that \( \pi_x^A(s_x^A, s_x^B, \Delta = 0) \leq \pi_x^A(s_x^A, \tilde{s}^A, \Delta = 0) \). Maximizing \( \pi_x^A(s_x^A, s_x^B, \Delta = 0) \) over \( s_x^A \) implies \( \tilde{s}^A = \frac{1}{7}(s_x^B + 6\Delta - 2) \). And under the condition that

\[ z < \frac{1}{6}(3a_{x-1} + 1), \]

\( \tilde{s}^A < a_{x-1} \) for any \( s_x^B \in [a_{x-1}, 1] \) and jamming is a profitable deviation for \( A \).

Together we know that if \( z < \max\left\{ \frac{1 - a_i}{6}, \frac{3a_{x-1} + 1}{6} \right\} \) some media will find jamming a profitable deviation. Since \( a_i \), the unconditional minimum upper bound for \( z \) is when \( a_i = a_{x-1} = \frac{1}{2} \), so that

\[ \min_{0 \leq a_i, 0 \leq a_{x-1} \leq 1} \left\{ \max\left( \frac{1 - a_i}{6}, \frac{3a_{x-1} + 1}{6} \right) \right\} = \frac{5}{12}. \]

Therefore we have proved that \( z < \frac{5}{12} \) is sufficient for the non-existence of a partially informative equilibrium.
When \( \frac{z}{12} \leq z < \frac{1}{2} \), we are unable to use the argument above to rule out this equilibrium. So we conduct a grid search to determine whether there exists a medium which can benefit from reporting from another interval. We first numerically solve each medium’s profit if they stay in the same interval, then compute the maximum profit each medium can obtain if it deviates to any other interval. Numerical results show that, as long as \( M > 0 \), there always exist one medium that can benefit from deviating to other intervals and jam the other, which establishes our previous result. Q.E.D.

Proof of Proposition 5

We show by contradiction that the partially informative equilibrium with one media cannot exist for \( M > M_3 = 4d(4 - 6z) \). Without loss of generality, we assume medium A is partially informative with two intervals (the proof of more than two intervals is similar therefore is omitted) while medium B is uninformative. In equilibrium \( N^A > \frac{1}{2} \) and \( N^B < \frac{1}{4} \). Hence \( \Delta > \frac{1}{4} \). Similar to Proposition 4, the optimal profit equations are given by:

\[
\pi^A = \frac{[d(s^b - s^A)(s^A + s^b + 2z) + \Delta M]^2}{18d(s^b - s^A)(1 - 2z)},
\]

and

\[
\pi^B = \frac{[d(s^b - s^A)(4 - 6z - s^A - s^b) - \Delta M]^2}{18d(s^b - s^A)(1 - 2z)}.
\]

We know for media B to earn positive profit, it must be the case that

\[ p^b = \frac{1}{2}[d(s^b - s^A)(4 - 6z - s^A - s^b) - \Delta M] > 0 \]

we need \( M < \frac{d(s^b - s^A)(4 - 6z - s^A - s^b)}{\Delta} \) in order to guarantee the positive profit of media B. Then, if \( M > M_3 \), then \( p^b \) is negative regardless other variables. This essentially means that the disadvantage of remaining uninformative is so big that media B cannot obtain positive profit when consumers believe another media is partially informative. Under this case, media B will always deviate from \( \hat{s}^B \) to use signal jamming strategy and the partially informative with one media doesn’t exist either.

Now consumers expect neither medium to provide an informative stance about the state of the world. Then the only possibility is both media are uninformative so that consumers have no updates on the expected number of facts. Under this case, \( N^A = N^B = \frac{1}{4} \), therefore \( \Delta = 0 \), with the mutually profit maximizing stances \( \tilde{s}^A = \max\{\frac{1}{2} - \frac{3}{4}(1 - 2z), 0\} \) and

\( \tilde{s}^B = \min\{\frac{1}{2} + \frac{3}{4}(1 - 2z), 1\} \). Since consumers believe neither medium is informative in any deviation,
no firm can improve consumers’ expectation about the number of facts in its report by choosing a different stance. Hence, there is no profitable deviation for either firm. Q.E.D.

**Proof of Proposition 6**

We start with first part of the proposition. Assume there exists an equilibrium in which only one media is partially informative. It suffices to show that the uninformative medium always has incentive to deviate by jamming the rival through symmetrically imitating the rival’s stance. Note first that if this equilibrium satisfies the favorable criterion, then there exists at least one favorable out-of-equilibrium belief that reduces the media’s payoff if a medium deviates. From now on we restrict our attention to equilibria such that both media stances are optimal given these beliefs. We first analytically show when \( M \to 0 \) the uninformative medium has incentive to jam the other. For the range of \( M \) that we can’t analytically compare, numerical analysis has been conducted to confirm the result.

Without loss of generality, we assume media \( A \) is partially informative while media \( B \) is uninformative. In equilibrium \( N^A > \frac{1}{2} \) and \( N^B < \frac{1}{2} \). Hence \( \Delta > 0 \). Similar to Proposition 4, the optimal profit equations are given by:

\[
\pi^A = \frac{(s^A - s^B)(s^A + s^B + 2 - 6z) + \Delta M}{18d(s^A - s^B)(1 - 2z)},
\]

\[
\pi^B = \frac{(s^A - s^B)(4 - 6z - s^A - s^B) - \Delta M}{18d(s^A - s^B)(1 - 2z)}.
\]

First, we can see that, in equilibrium we need \( s^A_e \in [a_{i-1}, a_i] \) and \( s^B_e \in [0, 1] \) satisfies that

\[
s^A_e = \text{argmax}_{s^A} \pi^A(s^A, a_{i-1}, a_i) \quad \text{and} \quad s^B_e = \text{argmax}_{s^B} \pi^B(s).
\]

We start with the case when \( x = 2 \). Although it is impossible to find closed form solutions of media stances for partially informative equilibrium, we can still show analytically how the media optimally response when \( M \to 0 \). Later we discuss the case for larger \( M \). When \( M \) is zero, consumers don’t value facts at all. Solve the profit maximum for both media we have

\[
\tilde{s}^A = \max\{\frac{1}{2} - \frac{3}{4}(1 - 2z), 0\} \quad \text{and} \quad \tilde{s}^B = \min\{\frac{1}{2} + \frac{3}{4}(1 - 2z), 1\}.
\]

When \( M > 0 \) and medium \( A \) is partially informative, we know \( N^A > \frac{1}{2} \), and \( N^B < \frac{1}{2} \) so that \( \Delta > 0 \). Because there is no closed form solution for the optimal stances, we cannot directly compare the profit of medium \( B \) under jamming or non-jamming. Instead we focus the relative profit change for \( B \) when \( M \) changes from 0 to \( dM > 0 \) in both cases. We start from the non-
jamming case when $s^A < s^B$, which occurs if $t \in [0, \frac{1}{3}]$. We denote the optimal stance choices when $M > 0$ and $B$ stays uninformative are $\hat{s}^A_1$ and $\hat{s}^B_1$. The analysis when $t \in [\frac{1}{2}, 1]$ is similar therefore is omitted.

We calculate medium B’s profit changes in two cases: the profit change when $B$ jams $A$ by choosing $s^B = 1 - \hat{s}^A_1$ so that consumers don’t know which one is partially informative; and the profit change when $B$ stays uninformative and chooses the optimal media stance $s^B = \hat{s}^B_1$.

We first know $\frac{d\hat{s}^A_1}{dM}|_{M \to 0} > 0$, and $\frac{d\hat{s}^B_1}{dM}|_{M \to 0} > 0$. Hence, when $M$ increases from 0 to $dM > 0$, if $B$ stays uninformative, then medium $A$’s stance increases from $\tilde{s}^A$ to $\hat{s}^A_1$, and $\tilde{s}^B \rightarrow \hat{s}^B_1$.

If medium $B$ jams $A$ with $\tilde{s}^B = 1 - \hat{s}^A_1$ so that the two media stances are symmetric and consumers don’t know which medium is informative. Its profit is: $\pi_{jam}^B = \frac{d^2(1 - 2\hat{s}^A_1)(3 - 6\tilde{s})}{18d(1 - 2\tilde{s})}$.

Let’s denote $ds^A = \hat{s}^A_1 - \tilde{s}^A$. Then the profit change of $B$ from $\tilde{s}^B$ to $\tilde{s}^B = 1 - \hat{s}^A_1$ by jamming the signal is $d\pi_{jam}^B = \pi_{jam}^B(\hat{s}^A_1, 1 - \hat{s}^A_1) - \pi^B(\tilde{s}^A, \tilde{s}^B) = \frac{2d^2(3 - 6\tilde{s})^2(ds^A)}{18d(1 - 2\tilde{s})}$.

If medium $B$ doesn’t jam $A$, it’s optimal choice is $s^B = \hat{s}^B_1$. To calculate the profit change due to the change of $M$, we take the total differentiation of $M$ on $\pi^B$:

$$\frac{d\pi^B}{dM} = \frac{\partial \pi^B}{\partial M} + \frac{\partial \pi^B}{\partial s^B} \frac{ds^B}{dM} + \frac{\partial \pi^B}{\partial \tilde{s}^A} \frac{d\tilde{s}^A}{dM};$$

When $M \rightarrow 0$, $\frac{\partial \pi^B}{\partial M}|_{M \to 0} = \frac{1}{18d(1 - 2\tilde{s})}[-2d\Delta(3 - 6\tilde{s})]$; $\frac{\partial \pi_{jam}^B}{\partial \tilde{s}^A}|_{M \to 0} = \frac{1}{18d(1 - 2\tilde{s})}[-2d^2(3 - 6\tilde{s})]$.

Therefore, $\frac{d\pi^B}{dM} = -\frac{1}{18d(1 - 2\tilde{s})}[2d\Delta(3 - 6\tilde{s})] + \frac{d\pi_{jam}^B}{dM};$

Or: $d\pi_{nojam}^B = -\frac{1}{18d(1 - 2\tilde{s})}[2d\Delta(3 - 6\tilde{s})]dM + d\pi_{jam}^B$;

Hence $d\pi_{nojam}^B < d\pi_{jam}^B < 0$, which means when $M$ changes from 0 to $dM$, $\pi_{jam}^B(\hat{s}^A_1, 1 - \hat{s}^A_1) > \pi^B(\hat{s}^A_1, \hat{s}^B_1)$.

So medium $B$ has strict incentive to use media stance $\tilde{s}^B = 1 - \hat{s}^A_1$ to jam the report instead of

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25 Detailed analyses are omitted for brevity and available upon request.
using $\tilde{s}^B$, which proves that the equilibrium of partially informative with one medium doesn’t exist with $x=2$ when $M$ is small.

This result shows once $M > 0$, medium $B$ suffers by remaining uninformative because $N^A > N^B$. In fact, the reduction of $B$’s profit is so big that medium $B$ chooses to jam $A$, to balance the expected number of facts between them. Now if there exists a partially informative equilibrium with $x > 2$, we can see there must exist an interval $[a_{i-1}, a_i]$ such that $\tilde{s}^A \in [a_{i-1}, a_i]$. It is easy to verify that the net difference of number of facts between media $A$ and $B$ is becoming larger in this case. The loss of medium $B$ by staying as the only uninformative media is even larger. So $B$ will have stronger incentive to jam $A$ when $t \in [a_{i-1}, a_i]$. This shows that when $M$ is close to zero, it is impossible to find an equilibrium with only one partially informative medium.

Last, for the range of $M \in (0, M_3]$, profit comparisons between jamming and no-jamming can’t be signed analytically. We conducted a grid search to determine whether the uninformative medium has incentive to jam. We numerically solve the optimal $\tilde{s}^A_1$ and $\tilde{s}^B_1$ for given $M$, then compare the jamming and no-jamming profit of medium $B$. The numerical analysis confirms the previous result. We showed, as $M$ increases, the uninformative medium’s profit is lower by remaining uninformative. Therefore, jamming is a profitable deviation when $M \in (0, M_3]$.

Proposition 4 and the result above assure us that, consumers do not expect an equilibrium in which one or both media provide informative stances. To establish the last part of the proposition’s claim, therefore, we need only show that the uninformative equilibrium characterized in Proposition 5 survives the favorable criterion. If a medium deviates from this equilibrium, by choosing a different stance, when consumers believe this medium is still uninformative, then its profits strictly decreases since the original equilibrium stance choices are mutual maximum. Hence, we show there is a favorable belief for which any deviation strictly lowers profit. Hence this equilibrium satisfies the favorable criterion. Q.E.D.

Proof of Proposition 7

When $0 < M < M_1$, the monopoly & duopoly media generate uninformative reports. Hence competition doesn’t increase the informativeness of media reporting when $M$ is small. However, when $M > M_2$, the monopoly’s report is partially informative with at least two intervals, while
duopoly media generate uninformative reports. Therefore competition decreases the informativeness when $M$ is large. 

**Proof of Proposition 8**

When $0 < M < M_1$, the expected number of facts provide by the monopoly is $\frac{1}{2}$. While $M > M_2$, the expected number of facts by the monopoly is $\frac{1}{2} < n < 1$. Under competition, both media are uninformative so $N^A = N^B = \frac{1}{2}$. Hence each medium provides fewer facts when $M > M_2$. Q.E.D.

**Proof of Proposition 9**

Under duopoly if $z \to \frac{1}{2}$, then $s^A \to \frac{1}{2}$ and $s^B \to \frac{1}{2}$ and $MP_{Duo} \to 0$. The monopoly’s report is influenced by $M$. When $M > M_3$, there exists partially informative with at least two intervals. Also notice as $M$ increases, the stance by monopoly moves further away from $\frac{1}{2}$ and $MP_{Mon} > 0$. Therefore when $z$ and $M$ are large enough the monopoly’s stance is more polarized than in duopoly. But as $z \to 0$, $s^A \to 0$ and $s^B \to 1$ in duopoly so that $MP_{Duo} \to 1/2$. While under monopoly, when $M < M_1$, the stance is $\frac{1}{2}$ so that $MP_{Mon} = 0$. Therefore, when $z$ and $M$ is small, monopoly is less polarized than duopoly. Q.E.D.

**Proof of Proposition 10**

Under uninformative reporting $s = \frac{1}{2}$, so

$$MB_{Mon}^I = \int_0^1 (t - \frac{1}{2})^2 dt = \frac{1}{12}.$$ 

Under partially informative with 2 intervals, $a_i = \frac{1}{2}$ and

$$MB_{Mon}^I = \int_0^1 (t - s_i)^2 2dt + \int_1^2 (t - s_2)^2 2dt.$$ 

It is readily seen that when $M \geq 8d(1 - z)$, $s_1 = 0$ and $s_2 = 1$, therefore $MB_{Mon}^I = \frac{1}{6} > MB_{Mon}^I$. Q.E.D.

**Proof of Proposition 11**

When $M > 8d(1 - z)$, from Prop 10 we know the media bias of the monopolist is

$$MB_{Mon} = \int_0^1 (t - 0)^2 2dt + \int_{\frac{1}{2}}^1 (t - 1)^2 2dt = \frac{1}{6}.$$ 

The duopolists, however, remain uninformative with stance $s^A = \max\{\frac{1}{2} - \frac{1}{4}(1 - 2z), 0\}$ and $s^B = \min\{\frac{1}{2} + \frac{3}{4}(1 - 2z), 1\}$. We can see when $z \to \frac{1}{2}$, then $s^A \to \frac{1}{2}$ and $s^B \to \frac{1}{2}$, hence the average media bias $MB_{Duo} = \int_0^1 (t - \frac{1}{2})^2 dt \to \frac{1}{12}$. Therefore when $z$
and $M$ are large enough, at least the partially informative equilibria under monopoly with $x = 2$ have higher media bias than duopoly.

When $z$ decreases, media bias in duopoly increases. For example, when $z \leq \frac{x}{2}$, then $\xi^1 = 0$ and $\xi^2 = 1$. Hence $MB_{Duo} = \int_0^1 (t - 0)^2 \, dt = \frac{1}{4}$ on average. When $M < M_1$, the monopoly provides uninformative report regardless of $z$. $MB_{Mon} = \int_0^1 (t - \frac{x}{2})^2 \, dt = \frac{1}{12}$. Therefore we showed when $z$ and $M$ is small enough, monopoly has lower media bias than duopoly. Q.E.D.