THE AGENCY MODEL AND MFN CLAUSES
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Abstract. I provide a general analysis of vertical relations that are intermediated either with wholesale prices or with revenue-sharing contracts. Although revenue-sharing does not eliminate double markups, it nonetheless tends to lower retail prices. Revenue-sharing is extremely attractive to the firm that is able to set the revenue shares, but often makes the other firm worse off. These results hold even when there is imperfect competition at both layers of the supply chain. I also show that retail price-parity restrictions raise industry prices. These results explain why many online retailers have adopted the “agency model” (in which they set revenue shares and suppliers set retail prices) and price-parity clauses. Finally, in an extension that considers private bargaining, I show that unobservable delegations can have equilibrium effects, and identify how pricing discretion can improve a firm’s bargaining position.

It is common for vertical relations to be structured so that one firm sets the “terms of trade” governing exchange with another firm that then sets a retail price to end consumers. These terms of trade stipulate either a per-unit commission or instead a revenue-sharing term. In this article I present a general analysis of such vertical relations. I also investigate the effect of retail price-parity restrictions such as most-favored nation clauses.

My analysis encompasses not only bilateral monopoly but also bilateral imperfect competition (in which there is market power at both stages of supply), and allows for flexibility in which firms in the supply chain set the terms of trade and which set retail prices. Thus, for example, not only am I able to compare how revenue-sharing differs from constant commissions given that the same firms set the terms of trade in either circumstance, but I am also able to assess the effects of a change in which firms set these terms.

This flexibility allows me to evaluate an important shift that is taking place in retailing, especially in online markets. Traditionally, it has been most common for suppliers to set wholesale prices and retailers to set retail prices; this is called the “wholesale model of sales.” With growing prevalence, however, online retailers instead specify a share of revenue that their suppliers will receive, with suppliers then setting the retail price; this is called the “agency model of sales.”

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For example, the online retailer Amazon uses the agency model in operation of its exchange for a vast variety of retail goods called the “Amazon Marketplace,” which accounts for over 40% of the products it sells globally.¹ Amazon’s leading online competitor, eBay, also uses the agency model when merchants sell goods using the fixed-price rather than the auction format (eBay now emphasizes the fixed-price format over the auction format, and it accounts for more than half of all of eBay’s sales). Products as diverse as e-books, applications for mobile devices such as smartphones and tablets, car insurance, and hotel reservations have been sold online using the agency model.

In many of the markets just mentioned, retail most-favored nation contracts (MFNs) have been used. In essence, these terms insist that a supplier cannot set a lower price through one online retailer than through another. The use of MFNs has led to regulatory scrutiny, for example of the Amazon Marketplace, price-comparison websites reselling private motor insurance, the e-book market, and online travel agencies such as Expedia and Booking.com.

Before describing my results, I note that revenue-sharing does not eliminate the double-marginalization problem; double markups remain, but are more subtle. In particular, when the firm setting the revenue share keeps some revenue for itself, the firm setting the retail price bears the entirety of its own costs but receives only a portion of revenue, causing it to act as if its costs are higher than they actually are; this is one markup. The second markup is that which the firm setting retail prices imposes over these perceived marginal costs.

My main results are as follows. First, regardless of the shape of demand, when revenue-sharing is used each firm prefers to be the one that sets revenue shares rather than being the one that sets retail prices. This result is non-obvious because under the commission model, whether it is better to set the terms of trade (given by a wholesale price) or instead the retail price depends on the log-curvature of demand.

Second, despite the fact that double markups do exist under revenue-sharing, the equilibrium retail price tends to be lower. This is true even when there is imperfect competition at both layers of the supply chain. However, in the presence of such competition, the lower prices under revenue-sharing may well lower industry profits.

Even though revenue-sharing may therefore lower industry profits, my third main result is that, given that a firm would set the terms of trade regardless of whether revenue-sharing or instead constant commissions are used, it prefers revenue-sharing under a broad set of circumstances. Indeed, it may even earn higher per-unit margins under revenue-sharing, rather than simply earning higher profit. To be perfectly clear, this profitability result is true even when there is imperfect competition at both layers of the supply chain.

¹See “OFT minded to drop investigation into Amazon pricing policies,” Financial Times, August 29, 2013.
Fourth, the firms that set retail prices are often worse off under revenue-sharing. This is true even if the lower prices associated with revenue-sharing raise industry profits (as is the case under bilateral monopoly). Thus, revenue-sharing can be seen as a weapon that one firm uses to its own advantage but to the detriment of other firms.

When a move to revenue-sharing is accompanied by a switch in which firm sets the retail price, as when there is a move from the wholesale model to the agency model, these results continue to hold (and may be strengthened): a shift from the wholesale model to the agency model tends to make the retailer better off but the supplier worse off.

These results explain why retailers such as Amazon, Apple, and eBay might benefit from competing using the agency model rather than the wholesale model, and why they would wish to set the revenue-sharing terms themselves rather than have suppliers do it.

In essence, the intuition for all of these results is as follows. Suppose that the firm setting retail prices pays the other firm a constant commission per unit sold. When this firm raises the retail price by one, its own per-unit margin therefore increases by one. In contrast, when revenue-sharing is in effect, an increase of one in the retail price is only partially captured by the firm setting that price. That is, its own per-unit margin increases by less than one, because it must share the extra revenue from the price increase with the other firm. This effect diminishes the effectiveness of price increases under revenue-sharing. Thus, compared to the commission-based model of sales, revenue-sharing puts the firm setting retail prices in a difficult position, a fact that the firm setting the revenue-share is able to exploit to its own benefit and the other firm’s harm.

Another main result is that price-parity restrictions in the form of retail MFN contracts tend to raise industry prices. In particular, MFNs kill a retailer’s incentives to compete in the terms of trade that it offers suppliers. The reason is that a retailer who raises the commission it charges (or offers suppliers a lower revenue share) knows that the price set through its store will not increase relative to that at other stores. Indeed, the essence of price parity is that affected suppliers must raise the price that they set through all stores if they are to raise prices at all. This effect encourages retailers to charge higher fees to suppliers, which in equilibrium raises the final retail price and harms consumers.

The prospect that MFNs might lead to higher retail prices—and negate any positive effects from a switch to the agency model—suggests that such contracts ought to be carefully scrutinized. Indeed, concerns have been expressed in multiple antitrust investigations in the US and the EU, as I discuss in more detail later. However, I also discuss potential pro-competitive effects of price-parity clauses.
My final contribution is to consider a private bargaining environment in which the identity of the firms that set retail prices is determined contractually, and in which contracts include fixed fees. This extends the analysis of O’Brien and Shaffer (1992).

In this setting, two main results emerge. First, even though contracting is private, equilibrium prices and hence industry profits depend on which firms have the discretion to set retail prices. This is interesting because it has been argued that the delegation of decision rights can have no effect on equilibrium outcomes when negotiations are private (see Katz (1991)). Thus, I identify a new rationale for why strategic delegation can have equilibrium effects. Second, the equilibrium allocation of pricing discretion affects the distribution of industry profits. In particular, the layer of the supply chain that sets prices receives a higher share of industry profits than the other layer, regardless of the curvature of demand. The reason is that the firm with pricing discretion is able to better respond to out-of-equilibrium contracts, which strengthens its bargaining position. This suggests there may be a tension in the allocation of pricing discretion: an individual firm may prefer such discretion even if it leads to lower industry profits (and even when fixed fees are used).

I now briefly discuss some of the related literature; other contributions are discussed as the opportunity arises. The effect of revenue-sharing alone has been investigated both in the economics and operations research literatures (for example, Mathewson and Winter (1985), Dana and Spier (2001), Cachon and Lariviere (2005), and Krishnan, Kapuscinski, and Butz (2004)). Although very interesting, these articles are quite different in focus and consider neither supplier retail-price setting nor market power at both the manufacturing and retailing stages. There are several related contributions to the economics of platforms. Gans (2012) argues that most-favored nation contracts can resolve a holdup problem. Boik and Corts (2013) argue as I do that MFNs may work to raise final retail prices, but do not consider differences between the agency model and wholesale models or accommodate imperfect competition both at the supplier and retailer levels. Hagiu and Wright (2013) consider the choice between what they call a “marketplace” (the agency model in my terminology) and a “reseller” (the wholesale model in my terminology). They emphasize the importance of non-contractible decisions of suppliers in the determination of optimal supply chain structure.

My paper is organized as follows. Section 1 analyzes bilateral monopoly, which is extended in Section 2 to bilateral imperfect competition. Section 3 assesses price-parity clauses. Section 4 considers the environment with private contracting and general contracts.

Related work on the delegation effects of vertical structure includes, for example, Bonanno and Vickers (1988), Lin (1990), and Rey and Stiglitz (1995). They argue, respectively, that vertical separation, exclusivity provisions, and exclusive territory restrictions may raise retail prices by softening supplier competition. Note that the results that I discussed earlier hold in a model of bilateral monopoly in which such softening effects are necessarily absent.
1. Bilateral Monopoly

In this section I analyze, compare, and contrast the revenue-sharing and constant commissions models of bilateral trade. There are two firms, $D$ and $U$, and end consumers characterized by the strictly decreasing demand curve $Q(p)$, where $p$ is the price of some good. For each unit of the good demanded by consumers, firm $U$ bears a constant marginal cost $c_U > 0$ and firm $D$ bears a constant marginal cost of $c_D > 0$.

A useful measure of the sensitivity of demand is

$$\lambda(p) = -\frac{Q(p)}{Q'(p)} > 0.$$  

If demand is generated by some underlying distribution of consumer valuations, then $\lambda(p)$ is Mills’ ratio or the inverse hazard rate of that distribution.\(^3\) To ensure the quasi-concavity of certain profit functions below, I assume that both $\lambda(p)$ and $\lambda(p) + \lambda(p)(1 - \lambda'(p))$ have slopes strictly less than one.

As is already known, and will be further demonstrated, properties of $\lambda(p)$ are very important in price theory.\(^4\) One such property is the sign of $\lambda'(p)$, which determines the log-curvature of demand (that is, the curvature of log($Q(p)$)): log-concavity, log-convexity, and log-linearity are, respectively, equivalent to $\lambda'(p)$ being (globally) negative, positive, or zero.\(^5\) As shown by Bagnoli and Bergstrom (2005), many common distributions generate demand that is log-concave or log-convex.\(^6\) Canonical illustrations of demand include linear, exponential, and constant-elasticity, which are, respectively, log-concave, log-linear, and log-convex.\(^7\)

In each of the two sales models considered below there is a distinct order of moves, with one firm setting the terms of trade governing exchange between the two firms, and the other firm then setting the retail price. To be as general as possible, I will consider how the order

\(^3\)In particular, suppose consumer valuations $\theta$ are distributed according to $F(\theta)$ with density $f(\theta)$; at price $p$ a consumer $\theta$ buys if $\theta \geq p$, so that $Q(p) \cong 1 - F(p)$ and $Q'(p) \cong -f(p)$, for some common constant of proportionality equal to the size of the market. Thus,

$$\lambda(p) = \frac{1 - F(p)}{f(p)}$$

which is Mills’ ratio of $F$.

\(^4\)For example, whether $\lambda(p)$ is decreasing or instead increasing determines whether the pass-through rate of the change in a monopolist’s marginal cost is less than one or not, and the curvature of $\lambda(p)$ determines whether this pass-through rate is increasing or not.

\(^5\)For example, $Q(p)$ is log-concave if

$$\frac{d^2 \log(Q(p))}{dp^2} \leq 0 \iff \frac{d}{dp} Q'(p) = \frac{1}{Q(p)} \frac{\lambda'(p)}{\lambda(p)^2} \leq 0 \iff \lambda'(p) \leq 0.$$

\(^6\)For example, demand is log-concave when the underlying distribution of consumer valuations is uniform, normal, and certain parameterizations of beta and gamma, just to name a few. Log-convex demand can arise from the Weibull or power distribution, and some parameterizations of the Gamma distribution.

\(^7\)Linear demand $Q(p) = 1 - p$ arises from a uniform distribution of consumer valuations, exponential demand $Q(p) = e^{-\lambda p}, \lambda > 0$, arises from an exponential distribution, and constant-elasticity demand $Q(p) = p^{-1/\sigma}, \sigma \in (0, 1)$, arises from a Pareto distribution.
of moves—and a change to this order—influences market outcomes, and so it is necessary to introduce additional notation. To this end, agree that “firm 1” is the firm which sets the terms of trade, and “firm 2” is the firm that sets the retail price, with marginal costs $c_1$ and $c_2$, respectively. Thus, for example, under the typical wholesale model, firm 1 is $U$ with marginal cost $c_1 = c_U$ and firm 2 is $D$ with marginal cost $c_2 = c_D$. In contrast, under the typical agency model, firm 1 is $D$ with marginal cost $c_1 = c_D$ and firm 2 is $U$ with marginal cost $c_2 = c_U$.

1.1. **Constant commissions.** With constant commissions, one firm sets a wholesale price $w$ and then the other firm sets $p$, taking $w$ as given (and paying $w$ to the first firm for each unit sold). As noted above, I wish to preserve generality and so I do not specify whether it is $U$ or instead $D$ that sets $w$, but instead denote this firm by 1, with marginal cost $c_1$, and the retail price-setting firm by 2, with marginal cost $c_2$.

Taking $w$ as given, 2 chooses $p$ to maximize

$$(p - w - c_2)Q(p).$$

Recalling that $\lambda(p) = -Q(p)/Q'(p)$, the first-order condition can be written as

$$p - w - c_2 = \lambda(p).$$

(1)

Now consider firm 1. Although this firm chooses $w$, a useful alternate view is that it chooses $p$, with the per-unit transfer $w$ equilibrating according to Equation (1); $w = p - c_2 - \lambda(p)$. Thus, a unit increase of $p$ results in an equilibrating increase of $w$ equal to $1 - \lambda'(p)$. (Reciprocally, $(1 - \lambda'(p))^{-1}$ represents the pass-through rate of a monopolist responding to a unit increase in its costs.)

Under this alternate formulation, firm 1’s profit $(w - c_1)Q(p)$ can be written as

$$[p - c_1 - c_2 - \lambda(p)]Q(p),$$

with first-order condition

$$p - c_1 - c_2 = \lambda(p) + \lambda(p)(1 - \lambda'(p)).$$

(2)

If $p^w$ is the equilibrium price then, because $\lambda(p^w)$ is firm 2’s margin, firm 1’s margin is $\lambda(p^w)(1 - \lambda'(p^w))$. Of course, because of the familiar double-marginalization problem, $p^w$ exceeds the price that maximizes joint industry profits.\footnote{Generally, increases in a monopolist’s cost always raise prices. The inequality also $1 - \lambda'(p)$, evaluated at $p^w$, also represents the second-order condition of maximization by firm 2, which I have assumed holds globally.}

\footnote{The price that maximizes joint profits satisfies $p - c_1 - c_2 = \lambda(p)$. The second-order condition of firm 1 is that $\lambda'(p^w) < 1$, and so inspection of Equation 1 reveals that $p^w$ must exceed the joint profit-maximizing price.}
1.2. Revenue sharing. I now consider revenue-sharing contracts. Firm 1 sets the terms of bilateral trade, which are given by a revenue share \( r \in (0, 1) \) that stipulates the share of sales revenue that firm 2 receives; firm 1 keeps the remaining \( 1 - r \) share. Firm 2, taking \( r \) as given, then chooses \( p \).

Consider firm 2. Given that it keeps a share \( r \) of total revenue, it chooses \( p \) to maximize

\[
(rp - c_2)Q(p) = r \left( p - \frac{c_2}{r} \right) Q(p).
\]

Optimization requires that

\[
p - \frac{c_2}{r} = \lambda(p).
\]  

(3)

Thus, firm 2 prices just as it would under constant commissions if it were facing a combined wholesale price and marginal cost equal to \( c_2/r > c_2 \). In other words, because 2 keeps only a share of revenue but bears all of its costs of production, it acts as if its marginal cost is higher than \( c_2 \). I will refer to \( c_2/r \) as the “perceived marginal cost” of firm 2.

Now consider firm 1. Its profits are

\[
[(1 - r)p - c_1]Q(p).
\]

Although firm 1 chooses \( r \), as in the analysis of fixed commissions it is equivalent and convenient to suppose that firm 1 chooses \( p \), with the terms of trade \( r \) then equilibrating according to Equation (3).\(^{10}\) That equation can be rearranged to show that

\[
r = \frac{c_2}{p - \lambda(p)}.
\]  

(4)

Note that lower values of \( r \) are necessarily associated with higher values of \( p \), because lower values of \( r \) increase firm 2’s perceived marginal cost \( c_2/r \).\(^{11}\) Using Equation (4), firm 1’s objective function can be written as

\[
\left[ p - c_1 - c_2 - c_2 \frac{\lambda(p)}{p - \lambda(p)} \right] Q(p).
\]  

(5)

Differentiating and rearranging, the associated optimality condition is

\[
p - c_1 - c_2 = \lambda(p) \left[ 1 + c_2 \frac{p(1 - \lambda'(p))}{(p - \lambda(p))^2} \right].
\]  

(6)

I assume that there is a unique solution \( p^* \) to this first-order condition and that it uniquely maximizes firm 1’s profits. This equilibrium price must exceed that which maximizes joint

\(^{10}\)Note that, because \( r \in (0, 1) \), firm 1 is constrained to pick a price \( p \) exceeding that which 2 would set if firm 2 received all the revenues, that is, higher than a monopolist with marginal cost \( c_2 \) would set. This constraint will not bind, however, because firm 1 earns zero profits if it chooses such a price.

\(^{11}\)Relatedly, one may compute \( dr/dp = -r^2(1 - \lambda'(p))/c_2 < 0 \), where the inequality follows from the second-order condition of firm 2 that \( 1 - \lambda'(p) > 0 \).
profits. To see this, note that the second-order condition of firm 2 is that \( \lambda'(p^w) < 1 \), and so inspection of Equation (6) immediately implies this fact.\(^{12}\)

Therefore, it is not true that revenue-sharing eliminates the double-marginalization problem that exists under constant commissions. Intuitively, the reason is that under revenue-sharing both firms impose markups. Firm 2 does this by setting a markup over its perceived marginal cost \( c_2/r > c_2 \), and firm 1 does this by choosing \( r < 1 \), thereby inflating firm 1’s perceived marginal cost. So long as the equilibrium \( r \) is such that \( c_2/r > c_1 + c_2 \), then the equilibrium price exceeds that which maximizes joint profits.

**Lemma 1.** Under revenue-sharing, the equilibrium retail price strictly exceeds the price that maximizes industry profits. Equivalently, the equilibrium revenue-sharing term \( r \) is such that
\[
\frac{c_2}{r} > c_1 + c_2.
\]

Thus, under either revenue-sharing or constant commissions, there is, in effect, a double-marginalization problem: joint firm profits are not maximized.

### 1.3. Comparing sales models.

Having laid out the basics of the two sales models, here I provide a number of comparative results. I begin by showing that moving first is advantageous under revenue-sharing. That is, being the firm that sets the terms of trade \( r \) is preferable to being the firm that sets the retail price \( p \).

Although it may seem intuitive that it is always better to be the first-mover, in fact this feature is particular to revenue-sharing: with constant commissions, whether it is preferable to move first or instead second depends on the log-curvature of demand. More specifically, under constant commissions a firm prefers to move first if demand is log-concave but prefers to move second if demand is log-convex;\(^{13}\) this observation regarding constant commissions has previously been reported by Weyl and Fabinger (2013) and Adachi and Ebina (2014).\(^{14}\)

To see that moving first is always preferable under revenue-sharing, let \( p^M \) be the price that maximizes joint profits, and \( r^M = c_2/(c_1 + c_2) \) be the corresponding value of \( r \). If firm 1

\(^{12}\)A more conceptual proof of this result is as follows. Suppose that firm 1 were choosing a price strictly less than that which maximizes joint profits. Then a suitable increase in \( p \) would raise joint profits and also lower the share \( r \) of revenue claimed by 2; firm 1’s profit would go up. Similarly, if 1 were choosing \( p \) so as to maximize joint profits, then a small increase in \( p \) would have a negligible first-order effect on profits, but would involve a first-order decrease in \( r \); again, firm 1’s profits would increase.

\(^{13}\)The observation regarding constant commissions follows from the facts that the equilibrium price is invariant to the move order and that the margin of firm 1 is given by \( \lambda(p^w)(1 - \lambda'(p^w)) \) whereas the margin of firm 2 is \( \lambda(p^w) \). Hence, the first-mover earns higher profits when \( \lambda'(p^w) < 0 \) and lower profits when \( \lambda'(p^w) > 0 \). Therefore, by the definition of log-concavity and log-convexity, under constant commissions it is advantageous to move first when demand is log-concave and to move second when demand is log-convex.

\(^{14}\)More precisely, Weyl and Fabinger (2013) provide conditions (pp. 562–563) from which this result can be derived. Adachi and Ebina (2014) provide the condition given above on \( \lambda'(p^w) \) and consider several parametric examples that yield different signs for this value. This argument is also closely related to the observation by Amir, Maret, and Troege (2004) that the pass-through rate of a monopolist is determined by the log-curvature of demand.
chose to implement $p^M$, the industry profit margin would be $p^M - c_1 - c_2 = \lambda(p^M)$, and firm 1’s profits would be

$$[(1 - r^M)p^M - c_1]Q(p^M) = \left[\frac{c_1}{c_1 + c_2}(c_1 + c_2 + \lambda(p^M)) - c_1\right]Q(p^M) = \frac{c_1}{c_1 + c_2}\lambda(p^M)Q(p^M).$$

Thus, firm 1 could ensure itself a fraction $c_1/(c_1 + c_2)$ of maximized industry profits. However, from Lemma 1, it is optimal for firm 1 to choose a strictly higher price (and hence a lower value of $r$), and thereby earn strictly higher profits: in equilibrium, firm 1 earns strictly more than a share $c_1/(c_1 + c_2)$ of maximized industry profits. In turn, this means that for any particular firm $i \in \{D, U\}$, under revenue-sharing $i$ must earn higher profit if it moves first than if it moves second. For example, if $U$ moves first it will claim more than a share $c_U/(c_D + c_U)$ of maximized industry profits, and if instead $D$ moves first then $U$ will secure less than a share $1 - c_D/(c_D + c_U) = c_U/(c_D + c_U)$ of these profits.

**Proposition 1.** Under revenue-sharing, each firm prefers to be the first-mover. In contrast, with constant commissions each firm prefers to be the first-mover if demand is log-concave but prefers to be the second-mover if demand is log-convex.

Proposition 1 confirms that the desire to set the terms of trade is particularly alluring under revenue-sharing, holding regardless of the shape of demand. I note again that the result on move order under constant commissions is already known (see Weyl and Fabinger (2013) and Adachi and Ebina (2014)); my contribution pertains to revenue-sharing.\(^{15}\)

I now assess the effect of revenue-sharing on the retail price. Although Lemma 1 indicates that revenue-sharing does not eliminate the double-marginalization problem, it turns out that under a certain condition on demand this pricing externality is nonetheless mitigated. This condition involves the elasticity of $\lambda(p)$, which is also the elasticity of Mills’ ratio of the underlying distribution of consumer valuations, given by $p\lambda'(p)/\lambda(p).$\(^{16}\)

**Proposition 2.** If the elasticity of Mills ratio is less than one ($p\lambda'(p)/\lambda(p) \leq 1$), then the equilibrium retail price under revenue-sharing is strictly less than that under constant commissions ($p^a < p^w$). This result holds even if the move order differs depending on whether constant commissions or revenue-sharing is used.

As I discuss in more detail below, the condition that $p\lambda'(p)/\lambda(p) \leq 1$ is satisfied so long as demand is not “too convex.” It is satisfied for all log-concave and log-linear demand functions as well as constant-elasticity demand.

\(^{15}\)Note that Proposition 1 does not say that firm 1 earns higher profits than firm 2, but merely that moving first is better than moving second. This is in contrast to the case with constant commissions. The reason for the difference is that, under revenue-sharing, the equilibrium retail price may depend on which firm sets the terms, so that gross profits may depend on the move order.

\(^{16}\)Recall that Mills’ ratio equals $(1 - F(\theta))/f(\theta)$, where $F$ is the underlying distribution of consumer valuations and $f$ is the corresponding density function. The Mills’ ratio is also called the inverse hazard rate.
Proposition 2 is interesting because the goal of firm 1 is not to raise consumer surplus. Indeed, this profit motive implies that firm 1 may face a tradeoff when choosing \( p \) under the two sales models. On the one hand, the optimality condition for firm 2 (Equation (3)) implies that firm 2 receives a margin of only \( r\lambda(p) \) under revenue-sharing rather than \( \lambda(p) \) under constant commissions. Thus, the margin of firm 1 is higher for any given \( p \), encouraging firm 1 to induce a lower price. On the other hand, under revenue-sharing an increase in \( p \) may raise the margin that firm 1 receives by more than it raises the margin that it receives under constant commissions. This force encourages firm 1 to set a higher price.

To see this second effect, suppose that demand has a constant elasticity (so that \( \lambda(p) = \sigma p \) for \( \sigma \in (0, 1) \)) and note that for arbitrary \( p \) firm 1’s margin under constant commissions is \( p - c_1 - c_2 - \lambda(p) = (1 - \sigma)p - c_1 - c_2 \); raising \( p \) raises firm 1’s margin by \( 1 - \sigma < 1 \). In contrast, under revenue-sharing, firm 1’s margin is \( p - c_1 - c_2 - r\lambda(p) \). Using the fact (from Equation (4)) that \( r = c_2/(p - \lambda(p)) = c_2/(1 - \sigma)p \), firm 1’s margin is

\[
p - c_1 - c_2 - c_2 \sigma/(1 - \sigma).
\]

Thus, increasing \( p \) by one raises firm 1’s margin by one—from this perspective, a price increase is more attractive under revenue-sharing.\(^{17}\) Despite this tradeoff, the overall effect of revenue-sharing is to decrease the retail price.\(^{18}\) A related intuition follows Proposition 3.

Note that the sufficient condition \( p\lambda'(p) \leq \lambda(p) \) from Proposition 2 readily admits some log-convex demand functions (most obviously the constant-elasticity one), in addition to log-linear and log-concave ones. Indeed, rewriting the elasticity of Mills’ ratio in terms of the underlying demand function \( Q(p) \), the above condition is equivalent to

\[
\frac{pQ'(p)}{Q(p)} - \frac{pQ''(p)}{Q'(p)} \leq 1.
\]

The left-hand side is the difference between the elasticity of demand and the elasticity of the slope of demand (this second term is also called the adjusted-concavity of demand), and the condition for log-concavity is that this is less than zero. Thus, the condition clearly admits some log-convex demand functions.

I now explore how a move to revenue-sharing influences the profitability of the firms, supposing that the same firm that sets the terms of trade under constant commissions also sets

\(^{17}\)Put instead in terms of traditional pass-through concepts, this says that increasing firm 1’s margin by one leads to an increase in \( p \) of \( 1/(1 - \sigma) > 1 \) under constant commissions, whereas increasing firm 1’s margin by one leads to an increase in \( p \) of only one under revenue-sharing. In this second sense, there is complete pass-through under revenue-sharing, but more-than-complete pass-through with constant commissions.

\(^{18}\)When demand exhibits constant elasticity, both \( p^a \) and \( p^w \) can be explicitly solved for, using the optimality conditions in Equations (2) and (6). This yields

\[
p^a = \frac{(1 - \sigma)c_1 + c_2}{(1 - \sigma)^2} < \frac{c_1 + c_2}{(1 - \sigma)^2} = p^w.
\]
them under revenue-sharing. As I discuss in detail below, the taxation literature already suggests an answer to how firm 1’s profits will change, but does not provide insight into how firm 2’s profits will. Regardless, I significantly generalize these results in the next section where I consider bilateral imperfect competition.

**Proposition 3.** Suppose that the order in which firms D and U move is the same under revenue-sharing as under constant commissions (so that the identities of firm 1 and firm 2 are the same under either sales model). Then:

1. Firm 1 earns higher profits under revenue-sharing than under constant commissions,
2. Firm 2 earns lower profits under revenue-sharing than under constant commissions if demand is either log-linear ($\lambda(p) = \lambda > 0$ for each $p$) or constant-elasticity.

It may seem surprising that firm 2 can be worse off, especially given that industry profits are higher under revenue-sharing. But in another sense, it is not surprising: firm 1 is seeking to maximize its own profits, not those of firm 2.

Note that the log-linear case is a natural one to consider, because with constant commissions firm 1 and firm 2 receive equal profits in equilibrium; neither firm has an advantage over the other. But even with log-concave demand firm 2 can be worse off under revenue-sharing. This is notable because log-concavity implies that firm 2 is at a disadvantage to firm 1 under constant commissions, as shown in Proposition 1, and so one might have thought that firm 2 would tend to prefer revenue-sharing. To see that is not so, suppose that demand is linear, $Q(p) = 1 - p$, and firms have equal costs, $c_1 = c_2 = c$. For the range of costs that lead to positive equilibrium output (that is, $c < 1/2$), the profits of firm 2 under the two sales models are shown in the right-hand panel of Figure 1; the left-hand panel of that figure illustrates the profits of firm 1.\(^{19}\)

An intuition for Proposition 3 emerges from an understanding of a fundamental difference between the two sales models, which is that revenue-sharing reduces the incentives of firm 2 to increase its per-unit margin. To see this, note that under constant commissions firm 2’s margin is $p - w - c_2$ so that if it increases $p$ by one it also raises its margin by one; firm 2 captures the entirety of its increase in the price. In contrast, under revenue-sharing firm 2’s margin is $rp - c_2$ so that an increase in $p$ raises its margin by only $r < 1$; firm 2 captures only a fraction of the increase in $p$, with firm 1 capturing the rest.

The fact that revenue-sharing makes price increases less attractive for firm 2 is exploited by firm 1 in its selection of the revenue-sharing term $r$. Closely related is the fact that at any given $p$ firm 1 must be earning a higher margin than it would under constant commissions.

\(^{19}\)To be precise, this figure illustrates profits for firm 1 and firm 2, under the assumption that the same firm $i \in \{D, U\}$ moves first under both constant commissions and revenue-sharing. Also note that although linear demand is log-concave, log-concave demand by itself does not guarantee that firm 2 is worse off under revenue sharing.
After all, firm 2 would claim a margin of $\lambda(p)$ under constant commissions, but it only receives $r\lambda(p)$ under revenue-sharing. Overall, this benefits firm 1 and consumers, but as indicated firm 2 may be worse off.

![Figure 1. Comparison of the profits of firms 1 and 2 under the two sales models, with linear demand $Q(p) = 1 - p$ and equal costs $c = c_1 = c_2$.](image)

As mentioned briefly above, the first part of Proposition 3 is related to the existing taxation literature. In particular, in a setting where the government sets a tax scheme and then a monopolist chooses a price, it is well-known that an ad-valorem tax can raise at least as much revenue as a per-unit tax (Suits and Musgrave (1953)), and so the first statement in Proposition 3 follows in essence from previous work.\(^{20}\) In the next section, I significantly generalize this result by showing that even with bilateral imperfect competition, in which firms set terms of trade in competition with one another, those firms still prefer revenue-sharing (and indeed may earn higher margins) even though industry profits may be lower.

The fact that firm 2 may earn lower profits under revenue-sharing is very different from the predictions of the taxation literature. In particular, Skeath and Trandel (1994) show that, for a target level of tax revenues, a government can, without harming consumers, find an ad-valorem tax rate that makes the monopolist better off than with a per-unit tax. This is essentially the opposite of what happens in the present context (where the role of monopolist is played by firm 2 and that of the government by firm 1). The difference is driven, of course, by the fact that here the goal of firm 1 is to maximize its own profits.

My final result of this section concerns a change in both move order and the structure of bilateral payments. In particular, I now consider the change from “the wholesale model” to

\(^{20}\)Interpreting firm 1 as the government and setting the target tax revenue equal to firm 1’s equilibrium profit under constant commissions.
“the agency model.” Recall that this corresponds to moving from a situation where firm $U$ sets $w$ and $D$ sets $p$, to one where $D$ sets $r$ and $U$ sets $p$.

**Proposition 4.** If demand is log-concave, log-linear, or constant-elasticity, then a shift from the wholesale model to the agency model raises the profits of $D$ but lowers the profits of $U$.

This proposition, along with its generalization to bilateral imperfect competition in the next section, provides an explanation for why online firms such as Amazon and eBay may be switching to the agency model, and also of course suggests suppliers need not be beneficiaries of this change. Indeed, this result is somewhat stronger than stated, as is readily apparent from inspection of the proof: so long as $p\lambda'(p) \leq \lambda(p)$, $U$ is harmed by the move to the agency model.$^{21}$ Indeed, log-convex demand satisfying this condition emphasizes the strong effects of moving to the agency model: even though $U$ earns lower profits than $D$ does under the wholesale model when demand is log-convex, $U$ nonetheless prefers the wholesale model to the agency model—revenue sharing is too powerful a weapon in the hands of $D$.

2. Bilateral Imperfect Competition

In this section I extend the model and analysis above to a setting in which there is competition at both layers of the supply chain.$^{22}$ Rather than specify any particular model of such competition, I take an approach based on conduct parameters similar to that in the study of pass-through under constant commissions by Weyl and Fabinger (2013).$^{23}$ This allows my results to be stated in terms of properties of the aggregate demand function in the industry, thus providing a close analogue to the study of bilateral monopoly above.

Given the work above, it is easier here to adopt an approach that is slightly more compact. Let $Q(p)$ denote the aggregate demand of consumers, where the interpretation is that each of $M$ substitute products is sold at common price $p$ by each of $N$ substitute retailers. As before, $\lambda(p) = -Q(p)/Q'(p)$. All derivatives in this section measure the effect of an increase in the common price $p$ of the retail prices of all products sold through all stores. Thus, $Q'(p)$ measures the effect on aggregate demand from a small increase in all prices.

My analysis proceeds as follows. First I specify how layer two reacts to arbitrary terms of trade $r$ or $w$, that is, which retail price $p$ is set by the second layer given such terms of trade.

$^{21}$As noted earlier, the condition that $p\lambda'(p)/\lambda(p) \leq 1$ covers log-linear and log-concave demand as well as some log-convex demand functions such as the constant-elasticity one.

$^{22}$Related work that considers market power at multiple stages of production includes Salinger (1988), Reisinger and Schnitzer (2010), and Kourandi and Vettas (2012), consider models that feature both intermediate and final goods markets in their investigations of vertical structure.

$^{23}$In an earlier draft of this paper, I specified a complete micro-foundation for such competition, and showed that moving from the wholesale model to the agency model could lower retail prices, yet also make the downstream firms better off at the expense of upstream firms. A downside of that model was that only very limited forms of demand could tractably be considered. In particular, that model effectively assumed pass-through rates equal to unity.
Second, I specify how the first layer determines these terms of trade, taking as given the response of the second layer.

Suppose that the terms of trade have been set by the first layer. Let \( m_2(p) \) denote the per-unit margin of the second layer, given that all firms of that layer are setting the same price \( p \).

For example, under constant commissions \( m_2(p) = p - w - c_2 \) whereas under revenue-sharing \( m_2(p) = rp - c_2 \). The aggregate profit of the second layer is

\[
m_2(p)Q(p),
\]

with derivative

\[
m_2'(p)Q(p) + m_2(p)Q'(p).
\]

For arbitrary \( r \) or \( w \), I define the value of \( p \) set by layer 2 to be that which implicitly solves

\[
\frac{m_2(p)}{m_2'(p)} = \theta_2 \lambda(p) \implies m_2(p) = \theta_2 m_2'(p) \lambda(p),
\]

where \( \theta_2 \in (0, 1) \) is the conduct parameter of layer 2. Computing the appropriate derivatives \( m_2'(p) \), under constant commissions this says that \( m_2(p) = p - w - c_2 = \theta_2 \lambda(p) \) whereas under revenue-sharing (and using \( m_2'(p) = r \)) this says that \( m_2(p) = rp - c_2 = \theta_2 r \lambda(p) \).

Note that if \( \theta_2 = 1 \) then these equations are precisely what was derived earlier for the optimal response of firm 2 in the case of bilateral monopoly (see Equations (1) and (3)). On the other hand, if \( \theta_2 = 0 \) then these margins are zero. Thus, varying \( \theta_2 \) spans the range of outcomes from perfect competition in layer 2 to perfectly collusive behavior in layer 2.

Now consider the profits of layer 1. As before, it can be supposed that this layer sets \( p \), with the terms of trade with layer 2 then equilibrating. For any final price \( p \), the per-unit margin of layer 1 is denoted by \( m_1 \), and satisfies

\[
m_1(p) = p - c_1 - c_2 - \tilde{m}_2(p),
\]

where \( \tilde{m}_2(p) \) is the margin that layer 2 must be receiving. From the work above, for a given \( p \) the margin of the second layer is \( \tilde{m}_2(p) = \theta_2 \lambda(p) \) under constant commissions. Under revenue-sharing this margin is \( \theta_2 r \lambda(p) \), but using the equilibrium condition given just above for layer 2 (that \( rp - c_2 = \theta_2 r \lambda(p) \)), \( r \) is given by

\[
r = \frac{c_2}{p - \theta_2 \lambda(p)},
\]

a close analogue to Equation (4). Thus eliminating \( r \), under revenue-sharing

\[
\tilde{m}_2(p) = \frac{\theta_2 c_2 \lambda(p)}{p - \theta_2 \lambda(p)}.
\]

For arbitrary \( p \), the overall profitability of the first layer is simply \( m_1(p)Q(p) \), with derivative

\[
m_1'(p)Q(p) + m_1(p)Q'(p).
\]
I define the equilibrium value of \( p \) to be that which satisfies
\[
\frac{m_1(p)}{m'_1(p)} = \theta_1 \lambda(p) \implies m_1(p) = \theta_1 m'_1(p) \lambda(p). \tag{9}
\]
As \( \theta_1 \) ranges from zero to one the resulting price \( p \) ranges from that which gives layer 1 no margin to that which maximizes the profits of layer 1, given how layer 2 will respond.

Noting that \( m'_1(p) = 1 - \tilde{m}'_2(p) \) (using Equation (8)) and computing the value of \( \tilde{m}'_2(p) \) under both constant commissions and revenue-sharing, equations characterizing the industry price are readily derived. As before, let \( p^w \) and \( p^a \) denote the industry prices under the two sales models. The following lemma summarizes the definitions and work above.

**Lemma 2.** Under bilateral imperfect competition with conduct parameters \( \theta_1 \) and \( \theta_2 \), the retail price \( p^w \) under constant commissions satisfies
\[
p^w - c_1 - c_2 = \theta_2 \lambda(p^w) + \theta_1 \lambda(p^w)(1 - \theta_2 \lambda'(p^w)), \tag{10}
\]
and the retail price \( p^a \) under revenue-sharing satisfies
\[
p^a - c_1 - c_2 = \theta_1 \lambda(p^a) + \theta_2 \lambda(p^a) \left[ \frac{p^a (1 - \theta_1 \lambda'(p^a)) + \lambda(p)(\theta_1 - \theta_2)}{(p^a - \theta_2 \lambda(p^a))^2} \right]. \tag{11}
\]
If \( \theta_1 = \theta_2 = 1 \), these equations reduce to those derived for the bilateral monopoly case (Equations (2) and (6)). And, for \( \theta_1 = \theta_2 = 0 \), both of these industry margins equal zero. Finally, note that \( p^w \) is, once again, independent of which firm moves first, as can be seen by expanding the right-hand side of Equation (10).

I will now show that, under broadly similar conditions on the shape of demand, key results derived for bilateral monopoly continue to hold under bilateral competition.

**Proposition 5.** Under bilateral imperfect competition, if the elasticity of Mills ratio is less than one \( (p \lambda'(p)/\lambda(p) \leq 1) \), then the equilibrium retail price under revenue-sharing is strictly less than that under constant commissions \( (p^a < p^w) \). This result holds even if the move order differs depending on whether constant commissions or revenue-sharing is used.

Recall that in the bilateral monopoly case a move to revenue-sharing lowers the retail price, which moves it closer to the price \( p^M \) that maximizes industry profits. In contrast, with bilateral imperfect competition, such a price reduction may lower industry profits. The reason is simply that, depending on the conduct parameters, \( p^w \) may be lower than \( p^M \), so that a further reduction is bad for these profits. This possibility can be seen clearly in Figure 2, which graphs equilibrium prices in comparison to \( p^M \) for a linear-demand specification.

Strikingly, even if industry profits decrease, revenue-sharing nonetheless tends to benefit layer 1 and harm layer 2. Note that because there is competition amongst both layers, the classic result from the taxation literature (Suits and Musgrave (1953)) no longer applies
Figure 2. Comparison of equilibrium prices and $p^M$ (the industry-profit maximizing price), with linear demand $Q(p) = 1 - p$ and equal costs $c_1 = c_2 = 0.1$, and equal conduct parameters $\theta = \theta_1 = \theta_2$.

to the assessment of layer 1’s profits, and so both parts of the following proposition are substantive new results.

**Proposition 6.** Suppose that the order in which layers $D$ and $U$ move is the same under revenue-sharing as under constant commissions. Then aggregate demand $Q(p)$ being log-linear is sufficient for the following results to hold.

1. **Layer one earns higher per-unit margins and higher profits under revenue-sharing,**
2. **Layer two earns lower per-unit margins and lower profits under revenue-sharing.**

As noted earlier, the log-linear case is a natural one to consider because under constant commissions both firms earn the same margin and the same profits. Nonetheless, the same results can hold without log-linear demand. Suppose that demand is linear, a canonical log-concave form. Figure 3 shows, in the left-hand panel, that layer one benefits from revenue-sharing, and shows in the right-hand panel that layer two is worse off. Indeed, the decline in profits for layer 2 can be quite dramatic, in some cases being roughly one-third under revenue-sharing what they are with constant commissions.

As before, these profitability results are strengthened when a shift from the wholesale model to the agency model is considered. To be entirely precise, the following proposition explains what happens to profits, starting from the constant commissions model in which layer $U$ sets $w$ and layer $D$ sets $p$, and moving to the revenue-sharing model in which layer $D$ sets $r$ and layer $U$ sets $p$. 
Proposition 7. Under bilateral imperfect competition, if demand is log-concave or log-linear, then a shift from the wholesale model to the agency model raises the profits of layer D but lowers the profits of layer U.

This proposition again emphasizes the great advantage inherent in setting the terms of trade under revenue-sharing. In fact, this result is somewhat stronger than stated. In particular, layer U is worse off from the move to the agency model whenever \( p\lambda'(p) \leq p \), which is exactly the corresponding condition found for bilateral monopoly (see the discussion following Proposition 4).

3. Most-Favored Nation Contracts

In this section I explore the use of retail price parity clauses such as (retail) most-favored nation (MFN) clauses. MFNs specify that a supplier who is setting retail prices through one retailer cannot charge a lower price through another retailer, if the first has an MFN clause. Such clauses are growing in popularity, especially in online markets, and so understanding their effects is important.\(^{24}\) I show how MFNs can raise retail prices and harm consumers. However, I also discuss potential pro-competitive effects of such clauses. Finally, I describe four examples of online agency markets where MFN clauses have been used.

\(^{24}\)One reason for their popularity is likely that violations of such clauses can be detected because retail prices are typically easily observable, especially relative to more-traditional wholesale MFN contracts that stipulate parity in wholesale prices. See Baker (1996) for a broad discussion of such contracts. Also see McAfee and Schwartz (1994) and DeGraba and Postlewaite (1992) for assessments of the ability of such contracts do limit expropriation resulting from a supplier’s inability to commit in multilateral negotiations.
3.1. **MFN clauses raise industry prices.** To investigate MFNs, I must consider the strategic interactions between retailers in more detail than in Section 2. There are $M$ suppliers who each sell their products through each of $N$ retailers. Throughout, the move order is fixed. First, each retailer simultaneously specifies terms of trade. Second, suppliers set retail prices.

Let $p$ denote the retail price set by suppliers through a given retailer. I assume that this price is a constant markup $\lambda_U > 0$ over their perceived marginal cost of selling through this retailer. Letting $w$ be the commission fee charged by this retailer and $r$ the revenue share offered suppliers by this retailer, this cost is $c_U + w$ under constant commissions and under revenue-sharing it is $c_U/r$. So, the price charged through this retailer is

$$p = w + c_U + \lambda_U$$  \hspace{1cm} (12)

under constant commissions, and

$$p = \frac{c_U}{r} + \lambda_U$$  \hspace{1cm} (13)

under revenue-sharing. In either case, it is as if suppliers set retail prices taking the market share of retailers as given, and also perfectly pass through (perceived marginal) costs.\(^{25}\)

As in earlier analysis, it is convenient to imagine that retailers directly choose the retail prices of goods sold through their stores, with each retailer’s corresponding terms of trade equilibrating according to either Equation (12) or (13), depending on the sales model. Thus, the per-unit margin received by a given retailer setting price $p$ through its store is

$$p - c_D - c_U - \tilde{m}_U(p),$$

where $\tilde{m}_U(p)$ is the per-unit margin that suppliers are receiving.\(^{26}\)

I will consider only symmetric demand and equilibria amongst retailers, and so let $Q(p, \hat{p})$ be the total demand facing any given retailer (across all products) given that $p$ is the price suppliers charge through it and $\hat{p}$ is the common price of goods at all other retailers.

Denoting a given retailer’s own per-unit margin by $m(p) = p - c_D - c_U - \tilde{m}_U(p)$, this retailer chooses $p$ to maximize

$$m(p)Q(p, \hat{p}).$$

I assume that the derivative of this function is decreasing in $p$ when evaluated at $p = \hat{p}$. That is, I assume

$$m'(p)Q(p, p) + m(p)Q_1(p, p)$$

\(^{25}\)Note that, in an earlier version of this paper, I provided complete details of a random-utility specification that generated the above outcome as a process of profit-maximizing behavior by individual retailers. However, here I simply take the above behavior as given.

\(^{26}\)Following earlier work, Equation (12) implies that $\tilde{m}_U = \lambda_U$ under constant commissions, and Equation (13) implies (after also using it to eliminate the revenue share) that $\tilde{m}_U(p) = c_U p/(p - \lambda_U)$. 


is decreasing in \( p \). The price at which it equals zero corresponds to the equilibrium price for the relevant sales model.

Although the goal of this section is to investigate MFN clauses, the model specified here also returns outcomes familiar from earlier analysis. In particular, under the conditions already given, the equilibrium price under revenue-sharing is less than that under constant commissions. If additionally \(-Q(p, p)/Q_1(p, p)\) is decreasing, then retailers earn higher margins and higher profits under revenue-sharing, and suppliers earn lower margins. I demonstrate these claims in the appendix. Thus, the model is quite similar to those analyzed earlier, but allows for precisely defined strategic interactions among competing retailers so that the effect of price-parity restrictions can be rigorously assessed.

Suppose industry-wide MFNs are in effect. This means that the price suppliers set through a given retailer must be the same as the price they set through all other retailers, even if the terms of trade differ across retailers. This influences the incentives retailers face when they set terms of trade. In particular, if a retailer raises the commission \( w \) it charges suppliers or offers suppliers a lower revenue share \( r \), suppliers will respond by raising the retail prices charged through all retailers, not just those charged through this retailer.\(^{27}\)

**Proposition 8.** MFN clauses raise industry prices. This is true whether constant-commissions or instead revenue-sharing is in effect (and MFNs raise the equilibrium wholesale price \( w \) charged to suppliers or lower the share of revenue \( r \) that suppliers receive, respectively).

The intuition is as follows. With MFNs, when a retailer offers suppliers less-attractive terms of trade, it does so with the knowledge that the resulting retail prices will not place it at a relative disadvantage compared to other retailers; there will be retail price parity. This encourages retailers to either raise \( w \) or lower \( r \), which raises the perceived marginal costs of suppliers and in turn raises industry prices.

### 3.2. Possible pro-competitive effects of MFNs.

Although the analysis above indicates how MFNs may raise prices and harm consumers, it is not hard to imagine how MFNs might also be pro-competitive. For example, MFNs might encourage investments.

This force seems particularly relevant for markets where the retail landscape is changing rapidly. In particular, many new online retailers have appeared in recent years, and some of them have chosen to use price-parity restrictions. Some of these retailers have faced substantial costs of developing their online platforms, and it is possible that price-parity restrictions have encouraged their entry and, potentially, benefitted consumers.

Related, price-parity restrictions may also have pro-competitive effects if they encourage efficient investments among retailers and suppliers. For example, suppose that there were

\(^{27}\text{When MFNs are in effect, I suppose suppliers set prices so that their average markup is } \lambda_U.\)
a single vertically integrated supplier and a single independent retailer. If the independent retailer must make investments that benefit the supplier, the incentives for such incentives may be blunted if the supplier undercuts the retailer through its own retail outlet. A price-parity agreement may prevent such behavior and possibly benefit consumers.

3.3. Antitrust cases. In this section I describe four markets in which retail price parity clauses, used in conjunction with the agency model, were investigated by antitrust authorities either in the EU or the US or both. In all four cases, a key concern of authorities was that higher retail prices either could or did result.

The Amazon Marketplace is an online sales platform implemented by Amazon, which covers a wide variety of products and accounts for over 40% of products sold by Amazon. Amazon sets a revenue-sharing contract and suppliers set the retail price that consumers see when they visit Amazon’s website, sharing with Amazon revenue from completed sales. Additionally, Amazon has used price parity clauses.

Both the Bundeskartellamt (Germany’s antitrust agency) and the UK’s Office of Fair Trade (OFT) investigated Amazon’s use of such clauses. The concern was essentially as outlined in my analysis above. Amazon agreed to stop using such clauses in the affected markets.

The OFT also investigated the use of MFN contracts in the private motor insurance market. In that market, private motor insurers (PMIs) set the price of their products sold through price-comparison websites (PCWs) and MFN clauses are used. In its Provisional Findings Report (December 17, 2013), the OFT provides much the same argument that I have made above, in particular that MFNs stifle competition in commissions by PCWs, ultimately leading to higher retail prices. They also present direct evidence that such clauses limit commission competition.

Another investigated market is that for online hotel room booking. Leading online travel sites such as Expedia and Booking.com take a share of revenue from bookings of rooms on their websites, where the prices are set by the owners of the hotel rooms. However, there is some scope for websites to discount offerings, for example by bundling hotel rooms with other travel-related products and offering an advantageous price on the bundle. Both Expedia and Booking.com imposed a combination of MFN clauses and resale price maintenance (RPM) agreements on upstream suppliers. Both the OFT and its French counterpart, The General

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28See “OFT minded to drop investigation into Amazon pricing policies,” Financial Times, August 29, 2013.
29In an August 29, 2013 announcement, the OFT noted the primary concern related to such clauses, saying, “In particular, such policies may raise online platform fees, curtail the entry of potential entrants, and directly affect the prices which sellers set on platforms (including their own websites), resulting in higher prices to consumers.”
30The report reads, “There is direct evidence that wide MFNs harm competition between PCWs. Notably, one PCW has tried to reduce its commission fees with motor insurance providers in exchange for lower premiums. However, motor insurance providers were unable to accept the offer due to the presence of wide MFNs in their contracts with other PCWs. A number of motor insurers also told us they had, as a result of wide MFNs, withdrawn from, or did not consider, such offers from PCWs.”
Directorate for Competition Policy, Consumer Affairs and Fraud Control, have expressed concerns about the contractual provisions used by the booking firms (in the US, a private lawsuit is currently underway). In August, 2013, these firms agreed to stop using such contracts in the UK.

Without question, the most closely watched market was that for e-books. In 2010, Apple entered the e-book market as it introduced its tablet computer, the iPad. Prior to Apple’s entry, Amazon was the only significant player in the market, selling e-books for its dedicated e-book reader, the Kindle. Publishers had been unhappy dealing exclusively with Amazon. One reason is that they thought Amazon’s pricing was contributing to the decline of profits from other formats and channels, such as hardcover and paperback books sold through brick and mortar retailers. Apple convinced publishers to adopt the agency model as a condition of its entry into e-book retailing, and publishers then pressured Amazon to do so as well. Apple also secured an MFN contract for itself prior to entering. Following Apple’s entry, the price of many e-books significantly increased.

In April, 2012, the US Department of Justice accused Apple and five major publishers of conspiring to increase the price of e-books, and a similar investigation began in the EU. Proposition 8 predicts this increase in retail prices. Indeed, the price increase caused by MFNs under the agency model may be sufficient to outweigh the price decrease that is predicted to occur from the shift to the agency model from the wholesale model. That is, it is MFNs not the agency model itself that may have caused the increase in prices.

On the other hand, Amazon and Apple also played important roles in building the e-book market, investing not only in online stores but in hardware devices that encouraged e-book adoption. It is unclear whether Apple would have made these investments without the guarantees provided by MFNs, which possibly would have left Amazon unchecked, and consumers and publishers with fewer options. In other words, it is possible that the entry-inducing effects of MFNs discussed earlier played a role in the e-book market.

4. General Contracts with Bargaining

In this section the identity of the firms that set retail prices is determined contractually, rather than exogenously set. Contracts are also allowed to specify fixed fees in addition to

\[\text{(31)}\] See, for example, “Expedia, Starwood Ask Judge to Dismiss Price-Fixing Suit,” Bloomberg, December 17, 2013.

\[\text{(32)}\] A potential criticism of this interpretation hinges on the question of whether revenue-sharing contracts raise producers’ perceived marginal costs in the e-book market, given that one might feel such marginal costs equal zero. However, suppliers have alternative formats and channels, such as paperback or hardcover books sold through brick and mortar retailers, in which they can sell their content. Such alternatives constitute a positive opportunity cost of selling an e-book, and it is straightforward to show that this opportunity cost plays the same role as a positive marginal cost in the existing analysis.
constant per-unit variable fees. I suppose that terms of trade are determined by private bilateral bargaining, as in O’Brien and Shaffer (1992).

Two main results emerge. First, even when contracting between firms is private, equilibrium prices and hence industry profits depend on which firms have the discretion to set retail prices. This is interesting because it is has been argued that the delegation of decision rights can have no effect on equilibrium outcomes when negotiations are private (see Katz (1991)). Thus, I identify a new rationale for why strategic delegation can have equilibrium effects.

Second, the equilibrium allocation of pricing discretion affects the distribution of industry profits in addition to their level. In particular, the layer of the supply chain that sets prices receives a higher share of industry profits than the other layer, regardless of the curvature of demand and for reasons entirely different from those investigated earlier.

4.1. **Upstream monopoly and downstream competition.** I first consider a monopoly supplier \(U\) whose products are sold by two differentiated retailers (1 and 2) that represent substitute outlets for consumers. Given retail prices \(p_1\) and \(p_2\), retailer \(j\) sells \(Q(p_j, p_{-j})\) units and (symmetrically) \(\bar{Q}(p_{-j}, p_j)\). A retailer’s demand is decreasing in its own price and increasing in its rival’s price, and also more sensitive to its own price than to the price of the other retailer: \(-Q_1(p, p') > Q_2(p, p')\) for all \(p\) and \(p'\). Demand is such that the maximization of industry profits requires symmetric pricing \((p_1 = p_2)\) so that each retailer would sell the same quantity. To solve the model I specify, it will be important that the demand for retailer \(j\) is well-defined if the other retailer is not active, and so denote this demand by \(\bar{Q}(p_j) > Q(p_j, p_{-j})\). Finally, to reduce notation, I assume that all costs are zero.

There are three stages. First, \(U\) contracts separately and simultaneously with retailers 1 and 2. Second, each firm observes (only) the contracts that it has signed (so, \(U\) observes both of its contracts). Third, whichever firm has discretion over \(p_j\) sets \(p_j\), or instead \(p_j\) is set at its contractually specified value, and payoffs are determined.

In detail, a contract between \(U\) and retailer \(j\) specifies the following two things. First, it specifies either that \(U\) or \(j\) has the authority to set \(p_j\) in the third stage, or instead specifies \(p_j\) directly in the contract itself. Second, it specifies terms of trade \((F_j, w_j)\) where \(F_j\) is a fixed transfer from \(U\) to \(j\), and \(w_j\) is a constant non-negative per-unit transfer rate between \(U\) and \(j\). I assume that \(w_j\) is paid from \(j\) to \(U\), unless \(U\) has pricing discretion in which case I assume \(w_j\) is paid from \(U\) to \(j\).

Contracts are determined as follows. \(U\) bargains simultaneously and bilaterally with 1 and 2. When, say, 1 and \(U\) are bargaining, they take as given the contract between \(U\) and 2 and seek to maximize their bilateral surplus. They do this recognizing that the terms of their

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33This says that removing a product from the market raises the other firm’s demand at any given prices.
contract influences not only $p_1$ but also $p_2$ in the event that the contract between $U$ and 2 awards $U$ discretion over $p_2$.

Let $\pi_j$ represent the profits of retailer $j$, excluding the fixed transfer $F_j$, and let $\pi_{Uj}$ represent the profits of $U$ from profits of its good through retailer $j$, excluding $F_j$. Let $\Omega_{-j}$ represent the disagreement point of $U$ when bargaining with $j$, excluding the fixed transfers. Thus, if bargaining breaks down between $U$ and $j$, then $U$ anticipates receiving profits of $\Omega_{-j} + F_{-j}$.

Because the downstream firms only bargain with $U$, their disagreement points are taken to be zero.

I assume Nash bargaining with equal shares, so that the contract between $U$ and $j$ is chosen to maximize

$$[\pi_{U1} + \pi_{U2} + F_1 + F_2 - (\Omega_{-j} + F_{-j})][\pi_j - F_j] = [\pi_{U1} + \pi_{U2} + F_j - \Omega_{-j}][\pi_j - F_j]. \tag{14}$$

In the usual way one can solve for and eliminate $F_j$ from this equation, finding that

$$F_j = \frac{\pi_j - \pi_{U1} - \pi_{U2} + \Omega_{-j}}{2}. \tag{15}$$

Substituting this into Equation (14), it follows that bargaining between $U$ and $j$ entails $w_j$ being chosen to maximize bilateral profits

$$\pi_j + \pi_{U1} + \pi_{U2},$$

taking the contract between $U$ and $i \neq j$ as given. In the event that the contract between $U$ and $i \neq j$ does not give $U$ discretion over $p_i$, then $p_i$ is also taken as given. If $U$ does have discretion over $p_i$, then the contract between $U$ and $j$ is designed assuming that $p_i$ will adjust according the following notion of perfection, of which there are three relevant subcases. First, if the contract with $j$ is such that $U$ has discretion over $p_j$, then $p_i$ is assumed to be what $U$ would optimally set given that it has discretion over both prices taking the per-unit transfer rates as given. Second, if $p_j$ is contractually specified, then $p_i$ is assumed to be what $U$ would optimally set given $p_j$ and also taking the per-unit transfer rates as given. Third, if $j$ has discretion over $p_j$, then $p_i$ is assumed to equal the corresponding Nash equilibrium value that would obtain when $U$ and $j$ simultaneously chose $p_i$ and $p_j$, respectively, given the per-unit transfer rates. I assume that, in any of these cases, there is a unique set of prices that would indeed be realized given any transfer rates.

I now provide several lemmas that characterize the set of equilibria. Agree that an “equilibrium in which discretion over both prices is delegated” is an equilibrium in which neither $p_1$ nor $p_2$ is contractually set.

**Lemma 3.** In any equilibrium in which discretion over both prices is delegated, $U$ has discretion over $p_1$ if and only if $U$ has discretion over $p_2$. 
The above lemma indicates that any equilibrium is symmetric as far as which layer of the supply chain sets prices, assuming that both prices are indeed delegated. It does not rule out the possibility that, say, \( U \) has discretion over \( p_1 \) but \( p_2 \) is contractually set, and indeed such equilibria exist. However, the study of such hybrid equilibria does not provide additional insights and so henceforth I will restrict attention to three types of equilibria: ones in which \( U \) has discretion over both prices, ones in which each retailer has discretion over its price, and ones in which both prices are contractually set.

**Lemma 4.** The following statements are true.

1. There exists an equilibrium in which \( U \) has pricing discretion over both \( p_1 \) and \( p_2 \), and this equilibrium is unique among such equilibria. In it, industry profits are maximized. Also, retailers are compensated entirely with fixed fees.
2. There exist equilibria in which both prices are contractually specified. In one such equilibrium, industry profits are maximized. Also, in this equilibrium, retailers are compensated entirely with fixed fees.
3. There exists an equilibrium in which each retailer \( j \) has pricing discretion over both \( p_j \), and this equilibrium is unique among such equilibria. In it, industry profits are not maximized. Also, \( U \) is compensated entirely with fixed fees.

The above lemma demonstrates that industry profits can be maximized so long as retailers do not have pricing discretion. It is fairly intuitive that such profits can be maximized when \( U \) is awarded pricing discretion because the resulting equilibrium contracts make \( U \) the residual claimant on all profits and so \( U \) selects prices that maximize industry profits.

Making \( U \) the residual claimant on profits is also crucial to maximizing profits when contracts directly specify retail prices. To see why, let \( p_U^* \) be the price that each retailer should set to maximize industry profits, and suppose that the contract between \( U \) and, say, retailer 2 specifies that \( p_2 = p_U^* \) and that \( w_2 = p_U^* \). Thus, \( U \) receives \( p_U^* \) from retailer 2 for each unit this retailer sells, and so \( U \) is the residual claimant on profits generated by this retailer. Now, when \( U \) negotiates with retailer 1 they wish to maximize bilateral profits, which are equal to those generated through retailer 1 plus the profits that \( U \) receives from retailer 2, so that \( U \) and 1 jointly have an incentive to set \( p_1 = p_U^* \), thereby maximizing industry profits.\(^{34}\)

In contrast, allocating pricing discretion to downstream retailers results in a familiar opportunism problem due to the private nature of contracts. In particular, as shown by O’Brien...

\(^{34}\)Note that other prices can be part of an equilibrium in which prices are contractually set. However, I will restrict attention to the one just described in which industry profits are maximized.
and Shaffer (1992), industry-profit maximizing prices cannot be maintained because bilateral profit maximization between $U$ and $j$ results in the expropriation of the other retailer.\(^{35}\)

Overall, it is likely quite intuitive that the equilibrium allocation of pricing discretion to $U$ solves the expropriation problem brought about by private contracting and raises industry profits. Nonetheless, in a sense Lemma 4 is surprising, precisely because contracting is private. In particular, this result says that delegation of a strategic decision (over prices) has equilibrium effects even though the contract itself is unobservable. However, a common critique of the literature on strategic delegation—which argues that delegation over a strategic variable can have equilibrium effects by allowing for precommitment to an otherwise non-equilibrium action—is that parties involved in such delegation have private incentives to “undo” any precommitment so as to respond optimally to the actions of other players. (See Katz (1991) for a more precise discussion, and Bolton and Scharfstein (1990) as well as Katz (1991) for arguments that agency problems between a principal and an agent may allow for effective precommitment even with private contracts.)

In short, private contracting is often taken as a sufficient condition for delegation of actions to have no equilibrium effects. The reason that such delegation can nonetheless have equilibrium effects in the present circumstances is that one of the contracting parties ($U$) is contracting with multiple parties (albeit bilaterally and privately). Moreover, at least with the correct contracts, $U$ has preferences over the profits of both the parties with which it contracts; it is the residual claimant on such profits for some contractual choices.\(^{36}\)

I now show that pricing discretion influences the distribution of industry profits, in addition to possibly affecting the overall level of profits. To this end, consider the profits of $U$, $\pi_{U1} + \pi_{U2} + F_1 + F_2$. The fixed fees are given in Equation (15) and so can be eliminated. As a preliminary step, consider eliminating only a single fixed fee $F_j$, which shows that

$$\pi_{U1} + \pi_{U2} + F_1 + F_2 = (\Omega_{-j} + F_{-j}) + \frac{\pi_{U1} + \pi_{U2} + \pi_j - \Omega_{-j}}{2}.$$  

This says that $U$ receives its disagreement point plus one-half the variable surplus created in its negotiations with $j$.\(^{37}\) Going further and eliminating both fixed fees shows that the

\(^{35}\)The intuition is that $U$ and $j$ neglect any variable profits flowing to $-j$, leading $U$ and $j$ to agree on a contract that leads to an undercutting of the price charged by $-j$. This incentive to so undercut only vanishes when $U$ supplies both retailers with its product at marginal cost (here equal to zero), which means that the competition between retailers is not mitigated by above-cost wholesale prices; industry profits are not maximized.

\(^{36}\)At the risk of saying too much on what may already be quite clear, if $U$ and $2$ believe that the contract $U$ signs with 1 makes $U$ the residual claimant from profits generated by retailer 1, then it becomes optimal for $U$ and 2 to also make $U$ the residual claimant over profits generated by retailer 2. This in turn makes the the contract signed with retailer 1 an optimal response, therefore supporting an equilibrium substantially different from the one in which retailers are the residual claimants.

\(^{37}\)In more detail, note that the term $(\Omega_{-j} + F_{-j})$ gives $U$’s total disagreement point if negotiations with $j$ break down, whereas the other terms equal one-half of the amount by which the total variable profits of $U$ and $j$ exceed the variable portion of $U$’s disagreement point with $j$. 
equilibrium profits of $U$ are
\[ \Omega_1 + \Omega_2 + \frac{\pi_1 + \pi_2 - \Omega_1 - \Omega_2}{2} = \frac{\pi_1 + \pi_2 + \Omega_1 + \Omega_2}{2}. \tag{16} \]

According to the left-hand side of this expression, $U$ receives the sum of (the variable portion of) its disagreement points with the retailers, plus one-half of the variable surplus of retailers that exceeds this sum. (The right-hand side is simply a more compact rewriting.)

The following proposition explains how pricing discretion influences the equilibrium share of profits that $U$ receives.\footnote{This proposition and also Proposition 10 require one further, minor assumption. In particular, if $p_j^*$ is the price that maximizes the total profits generated by retailer $j$ given that $-j$ is charging some price $p_{-j}$, then $p_j^*$ is suboptimal if instead retailer $-j$ is not active in the market. In other words, maximizing the profits generated by a given retailer requires an adjustment of that retailer’s price if the other retailer is not in the market.}

**Proposition 9.** The equilibrium share of industry profits that $U$ receives is highest when it has pricing discretion. Additionally, the equilibrium share of industry profits that $U$ receives when prices are contractually specified (at the levels that maximize industry surplus) is higher than when retailers have pricing discretion.

The intuition for this result leans substantially on the characterization of equilibrium compensation given in Lemma 4. Begin by considering the difference between retailers having price discretion (and hence, by Lemma 4, being residual claimants on all profits) and prices being contractually set (and $U$ being the residual claimant). When retailers have pricing discretion, a breakdown in negotiations between $U$ and $j$ generates “windfall” profits through retailer $-j$ because that firm sells additional units at the equilibrium price by virtue of $j$ not being active in the market. However, $U$ does not receive any of these additional profits because it is compensated only with fixed transfers. In contrast, when prices are contractually set, equilibrium contracts make $U$ the residual claimant, meaning that $U$ receives all of the additional profits generated through $-j$ that result from a breakdown in negotiations with $j$. This means that $U$’s disagreement point is higher in this case and so there is a smaller surplus for retailers to bargain for; $U$ receives a higher share of profits.

The bargaining power of $U$ is enhanced further when it has pricing discretion. The reason is that, in the event of a breakdown in negotiations with retailer $j$, $U$ optimally adjusts the price charged by the other retailer. This further increases the profits that $U$ can collect in such circumstances, and so further reduces what is available for retailers to bargain for.

The following corollary indicates how $U$’s overall profits vary across equilibria.

**Corollary 1.** $U$’s profits are highest when it has pricing discretion, and higher when prices are contractually specified at $p_j^*$ than when retailers have pricing discretion.
That having pricing discretion (or merely being the residual claimant) confers a profitability advantage is in stark contrast to most results from my earlier analysis in Sections 1 (only when demand is log-convex and constant per-unit transfers are in place is setting prices preferred). Moreover, the logic behind this result is completely different. In particular, earlier results focus on how it is often the case that the firm setting the terms of trade (governing variable payments) can use that power to dictate terms that put the firm setting retail prices in a disadvantageous position. Here, the value of pricing discretion (and of being the residual claimant) derives from the position it grants if out-of-equilibrium events occur (namely, breakdowns in negotiations), and also work in essence through fixed transfers.

Considering retailers' profits, it turns out that retailers can be made either better or worse off if they move away from the equilibrium in which they have pricing discretion. On the one hand, such a shift leads to an increase in industry profits, as follows from Lemma 4. On the other hand, the share of industry profits claimed by $U$ increases. For the sake of brevity, I do not walk through all of the computations.\footnote{The profits of retailers can be directly computed along the lines discussed earlier. Doing so for the case in which retailers have pricing discretion and the case in which prices are contractually set allows one to show that the following condition must hold for retailers to be better off when they do not have discretion: 

$$
[p_U Q(p_U^*, p_U^*) - p_D Q(p_D^*, p_D^*)] > p_U [\tilde{Q}(p_U^*) - \tilde{Q}(p_D^*, p_D^*)].
$$

The left-hand side is the increase in the profits generated by a given retailer from an increase in industry prices to $p_U^*$ from $p_D^*$, and the right-hand side is the increase in profits generated by a single retailer charging $p_U^*$ when the other retailer drops out of the market. Given the preceding discussion, it is intuitive that the left-hand side must be larger than the right-hand side for retailers to gain.} After introducing multilateral competition below, I provide a more insightful result regarding the preferences of either retailers or suppliers.

4.2. **Multilateral competition.** I now extend the key results from above into a more general model of multilateral competition. In particular, there are now $M$ differentiated upstream firms and $N$ differentiated downstream firms. Each downstream firm can simultaneously retail each of the products produced by each of the upstream firms. Let $p_{mn}$ be the price of the good produced by supplier $m$ that is sold by retailer $n$ and let $Q_{mn}(p)$ be the demand for that good, where $p$ is the vector of all retail prices. Let $\tilde{Q}_{mn}(p)$ be the corresponding demand in the event that some retailer $k \neq n$ is not carrying $m$’s product.

Demand is symmetric, and I limit attention to symmetric equilibria. To simplify notation, I proceed as follows. For any given upstream firm $m$ and downstream firm $n$, let $\pi_U$ and $\pi_D$ represent the variable profits accruing to $U$ and $D$ from sales of $m$’s good through $n$, where of course these depend on the entire vector of prices and also on the contract that $m$ and $n$ have chosen. Let $\tilde{\pi}_U$ and $\tilde{\pi}_D$ represent the corresponding values that $U$ and $D$ receive from sales involving any other partner, so that for example $\tilde{\pi}_U$ is the variable profits of $U$ from sales of its good through any other retailer $k \neq n$. Likewise, let $F$ represent the fixed transfer
between \( m \) and \( n \) and \( \tilde{F} \) the transfer between \( m \) or \( n \) and any other retailer or supplier, respectively. Finally, let \( \tilde{\Omega}_U \) and \( \tilde{\Omega}_D \) represent the variable profits of the disagreement points of \( m \) and \( n \) in the event that negotiations break down.

To be clear, considering negotiation between upstream firm \( m \) and retailer \( n \), \( m \) is involved with a total of \( N \) retailers and so the total profits of \( m \) are

\[
\pi_U + F + (N - 1)\tilde{\pi}_U + (N - 1)\tilde{F},
\]

whereas \( n \) is involved with a total of \( M \) suppliers and so has total profits of

\[
\pi_D - F + (M - 1)\tilde{\pi}_D - (M - 1)\tilde{F}.
\]

I assume \( m \) and \( n \) negotiate so as to maximize the product of each firms’ total profit minus its disagreement point. Because the disagreement point of \( m \) is \( \tilde{\Omega}_U + (N - 1)\tilde{F} \) and that of \( n \) is \( \tilde{\Omega}_D - (M - 1)\tilde{F} \), the goal of negotiations is to maximize

\[
\left[ \pi_U + F + (N - 1)\tilde{\pi}_U - \tilde{\Omega}_U \right] \left[ \pi_D - F + (M - 1)\tilde{\pi}_D - \tilde{\Omega}_D \right].
\]

Maximizing with respect to \( F \) yields

\[
F = \frac{1}{2} \left[ \pi_D + (M - 1)\tilde{\pi}_D - \pi_U - (N - 1)\tilde{\pi}_U - \tilde{\Omega}_D + \tilde{\Omega}_U \right],
\]

and substitution readily reveals that \( m \) and \( n \) therefore seek to maximize

\[
\pi_D + (M - 1)\tilde{\pi}_D + \pi_U + (N - 1)\tilde{\pi}_U,
\]

where I have excluded terms involving \( \tilde{\Omega}_D \) and \( \tilde{\Omega}_U \) because these are taken as given. Thus, as expected, \( m \) and \( n \) maximize their bilateral surplus.

As a preliminary step in finding the profits of upstream firm \( m \), simply substitute the value of \( F \) derived above into the expression for the total profits of \( m \). Thus,

\[
\pi_U + F + (N - 1)\tilde{\pi}_U + (N - 1)\tilde{F}
\]

\[
= \tilde{\Omega}_U + (N - 1)\tilde{F} + \frac{\pi_D + (M - 1)\tilde{\pi}_D + \pi_U + (N - 1)\tilde{\pi}_U - \tilde{\Omega}_D - \tilde{\Omega}_U}{2}
\]

Again, as one would expect, this says that \( m \) receives its disagreement point plus one-half of the bilateral surplus of \( m \) and \( n \).

Imposing a full symmetric solution, so that \( \tilde{F} = F \), \( \pi_D = \tilde{\pi}_D \), \( \pi_U = \tilde{\pi}_U \), and writing \( \Omega_D \) and \( \Omega_U \) for the equilibrium disagreement values, and then entirely eliminating the fixed fees from the expression for \( m \)’s profits, it follows that the total equilibrium profits of \( m \) are

\[
N\pi_U + NF = N\pi_U \left( 1 - \frac{N}{2} \right) + MN\pi_D + \frac{N}{2}(\Omega_U - \Omega_D).
\]
The following single proposition contains the main results of this section. Note that equilibria in which prices are contractually set do not add much insight beyond what has previously been discussed, and so here I focus on equilibria in which firms have discretion over prices.

**Proposition 10.** The following statements hold.

(1) There exists an equilibrium in which upstream firms have pricing discretion over all products and are residual claimants on all profits, so that retailers are compensated entirely with fixed fees.

(2) There exists an equilibrium in which downstream firms have pricing discretion over all products and are residual claimants on all profits, so that suppliers are compensated entirely with fixed fees.

(3) Upstream firms receive a higher share of industry profits in the equilibrium in which they have pricing discretion than in the equilibrium in which retailers have pricing discretion. (Necessarily, downstream firms receive a higher share of profits in the equilibrium in which they have pricing discretion than in the equilibrium in which suppliers have pricing discretion.)

Proposition 10 generalizes the key results from the analysis of upstream monopoly: (i) the allocation of pricing discretion influences equilibrium prices even when such discretion is unobserved and firms can use fixed fees to transfer surplus, and (ii) having pricing discretion increases the equilibrium share of industry profits that firms capture.

The intuitions are familiar from earlier discussions. For example, the reason that an equilibrium exists in which suppliers have pricing discretion is as follows. When all suppliers have pricing discretion and pay no variable compensation to retailers, then when any supplier $m$ is negotiating with retailer $n$, $\pi_D = 0$ and so from Equation (17) they wish to maximize

$$\pi_D + \pi_U + (N - 1)\pi_U = \sum_{k=1}^{N} p_{mk}q_{mk}(p).$$

Given that all other contracts allocate pricing to upstream firms, all prices are taken as fixed except for those involving $m$’s good. Let $\{p^*_{mk}\}$ maximize the profits above. By making $m$ the residual claimant over the profits of sales of $m$’s good at $n$ (that is, setting $w_{mn} = 0$), and allocating $m$ discretion over $p_{mn}$, it is the case that $\pi_D = 0$ and $\pi_U = p_{mn}q_{mn}(p)$. Thus, in the final stage where prices are actually set, $m$’s variable profits are precisely those that $m$ and $n$ wish to jointly maximize at the contracting stage (given just above). That is, it is possible to implement the prices $\{p^*_{mk}\}$ that maximize $m$ and $n$’s joint surplus. Similar reasoning applies when retailers have pricing discretion.
The intuition for why suppliers receive a higher share of industry profits when they have pricing discretion (and retailers receive a higher share when they instead have pricing discretion) is also the same as in the analysis with a single upstream firm. In particular, consider what happens when suppliers have pricing discretion. In the event of a breakdown between a supplier $m$ and a retailer $n$, extra demand will flow to the good that $m$ sells through other retailers, thereby raising the variable profits that $m$ receives. This means that there is less surplus to be bargained over between $m$ and $n$. Note that this is true even if $m$ does not change the price it charged for its good sold through other retailers. But such pricing discretion allows it to indeed do so, which further increases what $m$ can claim in the event of a breakdown in negotiations with $n$.

Considering Proposition 10, the main difference from the case of upstream monopoly is that industry profits are not maximized here. Indeed, industry profits can readily be higher in the equilibrium where retailers set prices rather than upstream firms, depending on the relative levels of differentiation between suppliers and retailers.

The following simple corollary emphasizes that there is a tension between industry-profit maximization and the desire to seize a large share of such profits. Denote by $\Pi^U$ the industry profits in the equilibrium considered in which upstream firms have pricing discretion, and let $\Pi^D$ denote industry profits in the equilibrium in which retailers have pricing discretion. As just noted, depending on characteristics of demand across products and retailers, it could either be that $\Pi^U \geq \Pi^D$ or instead that $\Pi^D > \Pi^U$. The case of equality ($\Pi^U = \Pi^D$) is interesting because it fixes the level of industry profits across the equilibria considered.

**Corollary 2.** Suppose that $\Pi^D = \Pi^U$. Then upstream firms strictly prefer the equilibrium in which they have pricing discretion, and downstream firms strictly prefer the equilibrium in which they have pricing discretion.

The main implication of this corollary is that a tension exists between layers of the supply chain. For example, even if $\Pi^U < \Pi^D$, then—at least if the difference is not too great—upstream firms prefer the equilibrium in which they have pricing discretion (even though industry profits are lower) because they receive a higher share of industry profits. Thus, even with fairly general contracts (that are set bilaterally), it is not the case that industry participants can necessarily be expected to agree on the allocation of pricing discretion; there may be no Pareto-dominant equilibrium.

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For example, suppose upstream products are essentially identical, but that retailers are well differentiated. Then giving upstream firms pricing discretion leads to intense competition and low profits, whereas giving retailers pricing discretion leads them to internalize intra-product competition within their stores and also to and less-competitive inter-retailer pricing; profits are higher. If instead upstream products were well differentiated but retailers were quite similar, then industry profits would be higher if suppliers set prices.
5. Proofs

Here are proofs of results whose proofs were omitted from the text.

Proof of Proposition 2: I prove the result for fixed move order, but indicate at the end how the proof can be modified if the move order differs when revenue-sharing is used. Suppose for the sake of contradiction that \( p^a \geq p^w \). Under the maintained assumption that \( \lambda(p) + \lambda(p)(1 - \lambda'(p)) \) has a slope less than one, there is a unique solution to Equation (2) and so it must be that

\[
p^a - c_1 - c_2 \geq \lambda(p^a) + \lambda(p^a)(1 - \lambda'(p^a)). \tag{19}
\]

On the other hand, the revenue-sharing optimality condition from Equation (6) indicates that it must be that

\[
p^a - c_1 - c_2 = \lambda(p^a) \left[ 1 + c_2 \frac{p^a(1 - \lambda'(p^a))}{(p^a - \lambda(p^a))^2} \right].
\]

Thus, a contradiction exists if

\[
\lambda(p^a) + \lambda(p^a)(1 - \lambda'(p^a)) > \lambda(p^a) \left[ 1 + c_2 \frac{p^a(1 - \lambda'(p^a))}{(p^a - \lambda(p^a))^2} \right].
\]

This condition reduces to

\[
c_2 p^a < (p^a - \lambda(p^a))^2. \tag{20}
\]

Because \( p^a \geq p^w \) by assumption, and appealing to (a rearrangement of) Equation (19),

\[
c_2 \leq p^a - \lambda(p^a) - \lambda(p^a)(1 - \lambda'(p^a)) - c_1 < p^a - \lambda(p^a) - \lambda(p^a)(1 - \lambda'(p^a)).
\]

Using this inequality, a sufficient condition for Equation (20) to hold is that

\[
p^a[p^a - \lambda(p^a) - \lambda(p^a)(1 - \lambda'(p^a))] \leq (p^a - \lambda(p^a))^2 \iff p^a \lambda'(p^a) \leq \lambda(p^a).
\]

This establishes a contradiction, and so it must be that \( p^a < p^w \). Note, if the move order changes when revenue-sharing is used, the relevant condition analogous to Equation (20) simply involves \( c_1 \) rather than \( c_2 \), but then an upper bound for \( c_1 \) can be found in the same manner as above, leading to the same final condition.

Proof of Proposition 3: The result that firm 1 is better off follows from the following observation. Under revenue-sharing, firm 1 could choose to set \( p = p^w \), and if it did firm 2 would receive a margin of \( r \lambda(p^w) \) (using Equation 3), whereas that firm would receive a margin of \( \lambda(p^w) \) under constant commissions. Thus, firm 1 could assure itself a higher margin and the same level of demand under revenue-sharing than under constant commissions.

The result that firm 2 is worse off under revenue-sharing, when demand is log-linear or constant-elasticity, is actually a corollary of Proposition 4. To see this, note the following
facts. First, because log-linear and constant-elasticity demands are weakly log-convex, under constant commissions this firm does better than if it instead moved first (this follows from Proposition 1 as well as the observation that the retail price and hence total surplus under constant commissions do not depend on the move order). This means that, starting from the position of moving second under revenue-sharing, a shift to constant commissions that also changes the move order is worse than a shift from revenue-sharing to constant commissions that does not change the move order. Hence, if this firm would prefer to move to constant commissions and also have the move order changed, then it necessarily must prefer to move to constant commissions and have the move order fixed (as is the case being considered for the current proposition).

But this is what Proposition 4 shows, for a class of demand functions that include the log-linear and constant-elasticity ones. (Although Proposition 4 is stated in terms of firm 2 under revenue-sharing being $U$, the result doesn’t depend on the labeling of $D$ and $U$.)

**Proof of Proposition 4:** Consider $U$. Under the wholesale model, $U$ moves first and it could (suboptimally) induce $p^a$. Because $D$ would impose a markup of $\lambda(p^a)$ in this case, $U$ would be left with a margin of $p^a - \lambda(p^a) - c_1 - c_2$. Using the optimality condition associated with the agency model, given in Equation (6), and setting $c_2 = c_U$ because this condition is associated with the agency model where $U$ is the second mover, this equals

$$
\frac{c_U p^a \lambda(p^a)(1 - \lambda'(p^a))}{(p^a - \lambda(p^a))^2},
$$

At this price it would sell $Q(p^a)$ units, which is the same sold in equilibrium under revenue-sharing. Under revenue-sharing, $U$, as the second-mover, receives an equilibrium margin of $r\lambda(p^a)$. Using Equation (4) and the fact that $c_2 = c_U$ in this equation because $U$ is the second mover in the agency model, $U$’s equilibrium margin equals

$$
\frac{c_U \lambda(p^a)}{p^a - \lambda(p^a)}.
$$

Algebra reveals that $U$ would earn a higher margin with constant commissions (under the proposed pricing), so long as

$$
\frac{c_2 p^a \lambda(p^a)(1 - \lambda'(p^a))}{(p^a - \lambda(p^a))^2} \geq \frac{c_2 \lambda(p^a)}{p^a - \lambda(p^a)} \iff p^a(1 - \lambda'(p^a)) > p^a - \lambda(p^a) \iff p^a \lambda'(p^a) \leq \lambda(p^a).
$$

From this, it follows that $U$ is worse off under the agency model.

Now consider $D$. If demand is log-concave, the fact that $D$ is better off under the agency model can be shown as follows. Note that the move to the agency model from the wholesale can be decomposed into two steps: first letting $D$ move first rather than second, while
maintaining constant commissions, and next moving to revenue-sharing while fixing the move order. The first step benefits $D$ from Proposition 1, and the second step benefits $D$, also from Proposition 3.

The remaining case is that of $D$’s profits under constant-elasticity, which is log-convex. Suppose under revenue-sharing that $D$ suboptimally chose $r$ to implement the equilibrium price under the wholesale model. This would require setting $r$ such that the perceived marginal cost of firm $U$ is

$$\frac{c_U}{r} = c_D + c_U + \lambda(p^w)(1 - \lambda'(p^w)),$$

where the fact that this would induce $p^w$ follows from inspection of the relevant optimality conditions given in Equations (1) and (3). $U$ would therefore receive a margin of

$$r\lambda(p^w) = \frac{c_U\lambda(p^w)}{c_D + c_U + \lambda(p^w)(1 - \lambda'(p^w))}.$$

I will now show that this margin is less than $U$ would receive under the wholesale model, from which it follows that $D$ must receive a higher margin under the proposed pricing under the agency model. Under the wholesale model, $U$ (as the first-mover) obtains a margin $\lambda(p^w)(1 - \lambda'(p^w))$. Thus the following condition implies that $U$ earns a higher margin under the wholesale model than under the proposed agency pricing.

$$\frac{c_U\lambda(p^w)}{c_D + c_U + \lambda(p^w)(1 - \lambda'(p^w))} < \lambda(p^w)(1 - \lambda'(p^w)) \iff c_U < (1 - \lambda'(p^w))(c_D + c_U + \lambda(p^w)(1 - \lambda'(p^w))).$$

Now, with constant-elasticity demand, $p^w$ can be explicitly solved using Equation (2), giving $p^w = (c_D + c_U)/(\sigma - 1)^2$. Because $\lambda'(p) = \sigma$, the right-hand side of the inequality above reduces to $c_D + c_U$; the condition holds.

**Proof of Proposition 5:** Note that this proof uses different rearrangements of the right-hand side of Equation (10) depending on the circumstance. Now, the proposition is true even if the move order differs depending on which sales model is in effect. The reason is that $p^w$ does not depend on move order. Under either order, the inequality just below arises as a sufficient condition to establish a contradiction to the supposition that $p^a \geq p^w$. So assuming, and using Equations (10) and (11), a contradiction arises if

$$c_2\theta_2\lambda(p^w) \left[p^w(1 - \theta_1\lambda'(p^w)) + \lambda(p^w)(\theta_1 - \theta_2)\right] < \theta_2\lambda(p^w)(1 - \theta_1\lambda'(p^w)).$$

Because $p^a \geq p^w$ by assumption, using (and slightly re-writing) Equation (10) implies that

$$c_2 = p^w - c_1 - \theta_1\lambda(p^w) - \theta_2\lambda(p^w)(1 - \theta_1\lambda'(p^w)) < p^w - \theta_1\lambda(p^w) - \theta_2\lambda(p^w)(1 - \theta_1\lambda'(p^w)).$$
Thusly replacing $c_2$ from the previous inequality, and rearranging, shows that a contradiction arises if

$$\left[(p^w - \theta_2 \lambda(p^w)) - \theta_1 \lambda(p^w)(1 - \theta_2 \lambda'(p^w))\right]\left[(p^w - \theta_1 \lambda(p^w)) + \theta_1 \lambda(p^w) - \theta_2 p^w \lambda'(p^w)\right]$$

$$\leq (1 - \theta_1 \lambda'(p^w))(p^w - \theta_2 \lambda(p^w))^2.$$  

Expanding this and collecting terms containing $p^w - \theta_2 \lambda(p^w)$, this reduces to the condition

$$\theta_1 \lambda(p^w)(1 - \theta_2 \lambda'(p^w))(p^w \lambda'(p^w) - \lambda(p^w)) \leq 0.$$  

Because $\theta_1 \lambda(p^w)(1 - \theta_2 \lambda'(p^w))$ is the margin of layer 1, it is positive, and so this condition reduces to $p^w \lambda'(p^w) - \lambda(p^w) \leq 0$. 

**Proof of Proposition 6:** Because log-linear demand satisfies the assumption of Proposition 5, $p^a < p^w$, and so it follows that a sufficient condition for layer 1 to earn higher profits under revenue-sharing is that its margin under revenue-sharing is weakly higher than under constant commissions.

By definition (Equation 9), layer 1’s equilibrium margin is given by $\theta_1 \lambda m'_1(p)$, where $p = p^a$ or instead $p^w$ as appropriate. From Equation (8), $m'_1(p) = 1 - \tilde{m}'_2(p)$, where the functional form of $\tilde{m}_2(p)$ depends on whether revenue-sharing is in effect (as I further detail below). Thus, if $\tilde{m}'_2(p^a) \leq \tilde{m}'_2(p^w)$ (where these functions are evaluated using the appropriate functional forms depending on the sales model), then layer 1 earns a higher margin under revenue-sharing and the result follows. I now compute these derivatives for the appropriate functional forms.

As described following Equation (8) in the text, under constant commissions $\tilde{m}_2(p) = \theta_2 \lambda$, so that $\tilde{m}'_2(p^w) = 0$ in this case. Under revenue-sharing

$$\tilde{m}_2(p) = \frac{\theta_2 c_2 \lambda}{p - \theta_2 \lambda};$$

so that

$$\tilde{m}'_2(p^a) = -\frac{\theta_2 c_2 \lambda}{(p^a - \theta_2 \lambda)^2} < 0.$$  

This proves that layer 1 earns higher profits under revenue-sharing.

I now show that layer 2 earns lower profits under revenue-sharing. Because $p^a < p^w$, by the assumed quasiconcavity of layer 2’s profits in $p$, an underestimate of layer 2’s profits under constant commissions is $(p^a - c_2 - w)Q(p^a)$, where $w$ is the equilibrium value which is $w = c_1 + \theta_1 \lambda$. To prove the result it is therefore sufficient to compare this unit margin (given by $p^a - c_2 - w = p^a - c_1 - c_2 - \theta_1 \lambda$) to the unit margin under revenue-sharing. That margin is $\theta_2 r \lambda$, and as explained in the text $r = c_2/(p^a - \theta_2 \lambda)$. 


Therefore, a sufficient condition for the result to be true is that 
\[ p^a - c_1 - c_2 - \theta_1 \lambda > \frac{\theta_2 c_2 \lambda}{p^a - \theta_2 \lambda}. \]
Using Equation (11) to rewrite the left-hand side, and then dividing through by \( \theta_2 c_2 \lambda \) and multiplying through by \((p^a - \theta_2 \lambda)^2\), this condition reduces to 
\[ p^a + \lambda (\theta_1 - \theta_2) > p^a - \theta_2 \lambda \iff \lambda \theta_1 > 0, \]
which is true.

**Proof of Proposition 7:** This proof utilizes facts in the text given following Equation (8). Now, with (weakly) log-concave demand, \( p^a < p^w \). Hence, to show that a switch from the wholesale model to the agency model benefits (layer) \( D \), it is enough to show that \( D \)'s per-unit margin is higher under the agency model.

\( D \)'s margin under the wholesale is \( \theta_D \lambda(p^w) \leq \theta_D \lambda(p^a) \), where the inequality follows from the fact that demand is log-concave (so that \( \lambda(p) \) is non-increasing) and \( p^a < p^w \). Under the agency model, from Equation (9) the margin of \( D \) is 
\[ \theta_D \lambda(p^a) m_1'(p^a) = \theta_D \lambda(p^a)[1 - \tilde{m}_2'(p^a)] = \theta_D \lambda(p^a) \left[ 1 - \theta_U c_U \frac{(p^a \lambda'(p^a) - \lambda(p^a))}{(p^a - \theta_U \lambda(p^a))^2} \right]. \]
Because \( p^a \lambda'(p^a) - \lambda(p^a) < 0 \) for log-concave demand, this margin exceeds \( \theta_D \lambda(p^a) \), which is explained above is an upper bound on the margin of \( D \) under the wholesale model. Because \( D \) sells more units under the agency model, the result follows.

I now show that \( U \) is made worse off under the agency model. Because \( p^a < p^w \), an underestimate of layer \( U \)'s profit under the wholesale model is 
\[ m_1(p^a)Q(p^a) = [p^a - c_D - c_U - \tilde{m}_2(p^a)]Q(p^a) = [p^a - c_D - c_U - \theta_D \lambda(p^a)]Q(p^a) \]
\[ = \theta_U c_U \lambda(p^a) \left[ p^a (1 - \theta_D \lambda'(p^a)) + \lambda(p^a)(\theta_D - \theta_U) \right] \frac{1}{(p^a - \theta_U \lambda(p^a))^2} Q(p^a), \]
where the final equality follows from Equation (11), and keeping in mind that that equation is for the price under the agency model in which \( U \) is firm 2. Now consider \( U \)'s profits under the agency model:
\[ r \theta_U \lambda(p^a)Q(p^a) = \frac{\theta_U c_U \lambda(p^a)}{p^a - \theta_U \lambda(p^a)} Q(p^a), \]
so that a sufficient condition for wholesale profits to be higher is that
\[ \frac{p^a (1 - \theta_D \lambda'(p^a)) + \lambda(p^a)(\theta_D - \theta_U)}{(p^a - \theta_U \lambda(p^a))^2} \geq \frac{1}{p^a - \theta_U \lambda(p^a)} \iff \theta_D (p^a \lambda'(p^a) - \lambda(p^a)) \leq 0. \]
Proof of remarks about the MFN model

Here I show that the comments about the behavior of the model in Section 3 are true, namely that $p^a < p^w$, and that retailers earn higher margins and profits under revenue-sharing whereas suppliers earn lower margins. Wherever possible I will suppress retailer-specific subscripts to limit notation.

Under either sales model, a retailer’s profits can be written as $m(p)Q(p, \hat{p})$, where the form of the margin $m(p)$ depends on which sales model is in effect. The first-order condition at an equilibrium (in which $p = \hat{p}$) is

$$m'(p)Q(p, p) + m(p)Q_1(p, p) = 0.$$  

I assume that, under either sales model, this expression is decreasing in $p$. Thus, to show that $p^a < p^w$ it is enough to show that this first-order condition under revenue-sharing is negative when evaluated at $p^w$. This is equivalent to showing that (at $p = p^w$)

$$m'(p) + m(p)\frac{Q_1(p, p)}{Q(p, p)} < 0.$$  

Now, under revenue-sharing, $m(p) = (1 - r)p - c_D$, and using the relation $r = c_U/(p - \lambda_U)$ this gives $m(p) = (p - \lambda_U - c_U)p/(p - \lambda_U) - c_D$. Substituting this into the first-order condition of the retailer, showing that $p^a < p^w$ is equivalent to showing that

$$m'(p) + m(p)\frac{Q_1(p, p)}{Q(p, p)} < 0.$$  

evaluated at $p^w$. Now, under constant commissions, the relevant first-order condition implies that $Q/Q_1 = -(p - \lambda_U - c_U - c_D)$. Also computing $m'(p)$ under revenue-sharing gives

$$m'(p) = \frac{p - \lambda_U)(p - \lambda_U - c_U) + pc_U}{(p - \lambda_U)^2}.$$  

Using this and simplifying, the sought-for condition reduces to

$$-\lambda_U(p - \lambda_U - c_U)(p - \lambda_U - c_U - c_D) - c_D\lambda_U(p - \lambda_U) < 0.$$  

Because this is evaluated at $p^w$, $p - \lambda_U - c_U - c_D > 0$, and so the expression above is negative.

Now to show that retailer margins are higher under revenue-sharing, note that under either mode the first-order condition can be written as

$$m(p) = -m'(p)\frac{Q(p, p)}{Q_1(p, p)},$$  

where the form of $m(p)$ depends on which model is in effect. Because $m'(p) = 1$ under constant commissions, the facts that $p^a < p^w$ and that $-Q(p, p)/Q_1(p, p)$ is non-increasing implies that a sufficient condition for the margin to be higher under revenue-sharing is that $m'(p) > 1$. This derivative is given just above, and can readily be seen to exceed 1. Because
retailer margins are higher and quantity is higher under revenue-sharing, retailers are better off. Likewise, because suppliers only receive a margin \( r \lambda_U < \lambda_U \), their margin is smaller.

**Proof of Proposition 8:** With MFNs, a given retailer’s price \( p \) depends on all the terms of trade, because suppliers are presumed to set prices so as to maintain an average markup of \( \lambda_U \). Let \( w \) and \( r \) be the terms of trade of a given retailer and \( \hat{w} \) and \( \hat{r} \) be the common terms of trade of all other retailers. Thus, under constant commissions

\[
p = c_U + \frac{w + (N - 1)\hat{w}}{N} + \lambda_U, \tag{21}
\]

and under revenue-sharing

\[
p = \frac{Nc_U}{r + (N - 1)\hat{r}} + \lambda_U. \tag{22}
\]

Note that these equations indicate that, at least locally, it can still be taken that the retailer chooses the overall price \( p \) with its terms of trade equilibrating according to the above equations. Let \( M(p) \) denote the per-unit margin of a retailer, so its profits are \( M(p)Q(p,p) \).

Because MFNs are in effect, this retailer effectively chooses the industry price \( p \) and so at the equilibrium values the derivative of its profits with respect to \( p \) is

\[
M'(p)Q(p,p) + M(p)(Q_1(p,p) + Q_2(p,p)) > M'(p)Q(p,p) + m(p)Q_1(p,p),
\]

where the inequality follows from the fact that \( Q_2 > 0 \) and that \( M(p) = m(p) \) (the margin at this price in the absence of MFNs) at an equilibrium (in which case \( w = \hat{w} \) and \( r = \hat{r} \)). If it can be shown that \( M'(p) > m'(p) \), then the result follows given the assumption that the equilibrium without MFNs is defined by the price where \( m(p)Q(p,p) + m'(p)Q_1(p,p) = 0 \) and that this expression is assumed to be decreasing in \( p \).

Thus, consider first the case of constant commissions, where \( m'(p) = 1 \) and \( M(p) = w - c_D \). Using Equation (21), \( M'(p) = dw/dp = N > 1 \), so the result follows in this case.

Now suppose revenue-sharing is in effect. Here \( M(p) = (1 - r)p - c_D \), and so

\[
M'(p) = (1 - r) - \frac{dp}{dr} = (1 - r) + \frac{(r + (N - 1)\hat{r})^2}{Nc_U} = (1 - r) + \frac{Nr^2}{c_U},
\]

using Equation (22) and the fact that \( r = \hat{r} \) in equilibrium. Without MFNs, \( m'(p) = (1 - r) + pr^2/c_U \), and so \( M'(p) > m'(p) \).

**Proof of Lemma 3:** Suppose in equilibrium that \( U \) has discretion over \( p_1 \) with transfer rate \( w_1 \) (and recall this means that \( U \) pays \( w_1 \) to retailer 1 for each unit sold through retailer 1). To prove the lemma it is sufficient to show that it cannot be an equilibrium for \( U \) and 2 to agree to give 2 pricing discretion. To show this, note first that the bilateral surplus of \( U \)
and 2 is

\[(p_1 - w_1)Q(p_1, p_2) + p_2Q(p_2, p_1).\]

Let \(p^*_1\) and \(p^*_2\) be the prices that maximize this surplus, and note that it must be that \(p^*_1 > w_1\) for otherwise \(p_1\) could be increased which would raise profits \(U\)'s profits from sales through retailer 1 and also increase the quantity sold through retailer 2. Also note that \(p^*_2\) satisfies

\[(p_1 - w_1)\frac{\partial Q(p^*_1, p^*_2)}{\partial p_2} + \left[Q(p^*_2, p^*_1) + p^*_2\frac{\partial Q(p^*_2, p^*_1)}{\partial p_2}\right] = 0.\]  

(23)

Now it can be shown that these prices cannot be supported if 2 is given pricing discretion. If 2 is given discretion over \(p_2\) and \(w_2\) is the agreed upon transfer rate (that is paid to \(U\) from 2 for each unit sold by 2), then 2 would in the final stage choose \(p_2\) to maximize \((p_2 - w_2)Q(p_2, p^*_1)\) with first-order condition

\[Q(p_2, p^*_1) + (p_2 - w_2)\frac{\partial Q(p^*_2, p^*_1)}{\partial p_2} = 0.\]

Obviously this is incompatible with Equation (23) for any \(p^*_1 > w_1\) (which must be the case as argued above).

To complete the proof it is now sufficient to show that the prices \(p^*_1\) and \(p^*_2\) can be supported with an alternate contractual choice by \(U\) and 2. Suppose that \(U\) is given discretion over \(p_2\) and that the per-unit fee it pays to 2 is set at zero. Then, in stage three \(U\) chooses \(p_1\) and \(p_2\) to maximize its profits, but these are given by \((p_1 - w_1)Q(p_1, p_2) + p_2Q(p_2, p^*_1)\), which is the bilateral surplus that \(U\) and 2 sought to maximize. Clearly, \(p^*_1\) and \(p^*_2\) are therefore optimal for \(U\) to select. This shows that \(U\) and 2 would do better allocating discretion to \(U\), thereby completing the proof.

\[\blacksquare\]

**Proof of Lemma 4:** Consider part (1) of the result. In the proof of Lemma 3 I showed that if \(U\) has discretion over, say, \(p_1\), then \(U\) and 2 can maximize their bilateral surplus by assigning \(U\) discretion over \(p_2\) and choosing the per-unit transfer rate so that 2 receives no variable compensation. Indeed, amongst contracts that give \(U\) discretion over \(p_2\), that is the only contract that maximizes the bilateral surplus of \(U\) and 2. Thus, there clearly exists an equilibrium in which \(U\) has discretion over both prices, and it is unique among such equilibria. Because \(U\) is the residual claimant on all variable profits at the stage in which it sets prices, clearly it will set prices to maximize industry profits.

Consider part (2). Suppose that \(U\) and, say, retailer 1 have a contract that specifies sets \(p_1\) at the level that would be chosen to maximize industry profits, and which gives \(U\) all variable profits, so that \(p_1 = w_1 = p^*_U\). Then negotiations between \(U\) and 2 seek to maximize bilateral profits which are

\[p^*_UQ(p^*_U, p_2) + p_2Q(p_2, p^*_U).\]
These are simply industry profits given that \( p_1 \) has been chosen optimally, and so setting \( p_2 = p_2^* \) maximizes this and so it is optimal for \( U \) and 2 to indeed agree to a contract that does this. By also setting \( w_2 = p_2^* \), the contract specified between \( U \) and 1 is in turn optimal.

Consider part (3). Suppose that \( U \) and 1 have agreed to give 1 discretion and to set \( w_1 = 0 \) so that \( U \) gets no variable profits from 1. Then in negotiations between \( U \) and 2, bilateral surplus is simply the profits from retailer 2, \( p_2 Q(p_2, p_1) \), where \( p_1 \) is any price set by 1, taken as given by \( U \) and 2. Clearly, one way to maximize these bilateral profits is to give 2 pricing discretion and \( U \) no variable profits, \( w_2 = 0 \). This implies that there is an equilibrium in which both retailers have pricing discretion as described. Because the products are substitutes and retailers obtain inputs from \( U \) at marginal cost, industry profits are not maximized.

All that is left is to show there is no other equilibrium in which both retailers have pricing discretion, that is an equilibrium in which at least one transfer rate \( w_j \) is positive. Consider the negotiation between \( U \) and, say, 2. Their bilateral surplus is \( w_1 Q_2(p_1, p_2) + p_2 Q(p_2, p_1) \). Because \( p_1 \) is set by 1, it is taken as given during negotiations between \( U \) and 2 and thus any desired \( p_2 \) can be achieved via the appropriate selection of \( w_2 \). If bilateral surplus is being maximized, the following first-order condition with respect to \( p_2 \) must hold

\[
w_1 Q_2(p_1, p_2) + [Q(p_2, p_1) + p_2 Q_1(p_2, p_1)] = 0.
\]

However, it is also the case that the actual \( p_2 \) selected by 2 in stage three must maximize 2’s variable profits, which are \((p_2 - w_2) Q(p_2, p_1)\). The first-order condition is

\[
Q(p_2, p_1) + (p_2 - w_2) Q_1(p_2, p_1) = 0.
\]

Simultaneous satisfaction of the two conditions just given requires

\[
w_1 Q_2(p_1, p_2) = -w_2 Q_1(p_2, p_1).
\] (24)

Making a similar argument regarding the contracting process between \( U \) and 1 shows that the following condition must hold

\[
w_2 Q_2(p_2, p_1) = -w_1 Q_1(p_1, p_2).
\] (25)

Note that this condition is the same as Equation (24) above, with indexes on prices and variable fees switched (but with the indexes denoting differentiation unchanged).

Clearly, if either \( w_1 \) or \( w_2 \) is positive, they both are, so for the sake of contradiction suppose both are positive. The following holds:

\[
w_1 Q_2(p_1, p_2) = -w_2 Q_1(p_2, p_1) > w_2 Q_2(p_2, p_1) = -w_1 Q_1(p_1, p_2) \implies Q_2(p_1, p_2) > -Q(p_1, p_2).
\]

where the first equality follows from Equation (24), the inequality follows from the assumption that demand is more sensitive to a retailer’s own price than to that of the other
retailer, and the second equality follows from Equation (25). However, the implication that \( Q_2(p_1, p_2) > -Q(p_1, p_2) \) contradicts the assumption that demand is more sensitive to a retailer’s own price than to that of the other retailer. This contradiction followed from the assumption that either \( w_1 \) or \( w_2 \) was non-zero, and hence all such equilibria (and where retailers have price discretion) can be ruled out.

**Proof of Proposition 9:** First consider the equilibrium where retailers have pricing discretion. From Lemma 4, \( U \) receives no variable compensation and so its profits are derived solely through the fixed fees; \( \pi_{U1} = \pi_{U2} = 0 \), which also means that total industry profits equal the (variable) profits of retailers \( \pi_1 + \pi_2 \). The fact that \( U \) receives only fixed compensation also means that \( \Omega_1 = \Omega_2 = 0 \), because these terms by definition give the disagreement points of \( U \) excluding fixed fees. Hence, according to Equation (16), \( U \)'s total profits equal half of the industry profits.

Now consider the equilibrium in which prices are contractually specified (and equal to those that maximize industry surplus). From Lemma 4, retailers receive no variable compensation (\( \pi_1 = \pi_2 = 0 \)) and so if negotiations between \( U \) and \( j \) break down, \( U \) claims all the variable profits generated through retailer \( -j \). This means that

\[
\Omega_{-j} = p_{U}^* \tilde{Q}(p_{U}^*) > p_{U}^* Q(p_{U}^*, p_{U}^*),
\]

where the inequality follows because by assumption a firm’s demand increases if the other firm’s product is not in the market. Because the right-hand side equals one-half of equilibrium industry profits, then from Equation (16) it must be that \( U \)'s equilibrium profits exceed one-half of industry profits.

Now suppose \( U \) has pricing discretion. In the event of a breakdown with \( j \), \( U \) optimally would adjust the price \( p_{-j} \) to a value other than \( p_{U}^* \). This implies that

\[
\Omega_{-j} \geq p_{U}^* \tilde{Q}(p_{U}^*) > p_{U}^* Q(p_{U}^*, p_{U}^*).
\]

Hence, compared to the case in which prices are contractually set, \( U \) has higher disagreement points and so, once again using Equation (16) and the fact that \( \pi_1 = \pi_2 = 0 \) in either of these two equilibria, \( U \) must be better off when it has pricing discretion.

**Proof of Proposition 10:** The proof of item (1) is in the text, and the proof of item (2) is essentially identical to that but with the roles reversed. Thus, I focus on proving item (3).

Consider the equilibrium payoff of an upstream firm \( m \), given in Equation (18). In the equilibrium in which suppliers set prices, retailers receive no variable profits and so \( \pi_D = \ldots \)
\( \Omega_D = 0 \). Thus, the profits of \( U \) are

\[ N\pi_U + \frac{N}{2}(\Omega_U - N\pi_U). \]

Now, \( \Omega_U \) represents the variable profits of \( m \) if its negotiations with a single downstream firm, say \( n \) break down. Note that, if this occurs, no other firm will change its prices. The reason is that negotiations are private and so only \( m \) and \( n \) will be aware that disagreement occurred, but the downstream firm \( n \) has no price discretion. Let \( \tilde{Q}_{mk}(p), k \neq n, \) denote the demand for \( m \)'s good sold through retailer \( k \), given that retailer \( n \) is not carrying \( m \)'s good. Because different retailers carrying the same good are substitutes, \( \tilde{Q}_{mk}(p) > Q_{mk}(p) \).

Thus, although it would be suboptimal, \( m \) could also choose not to adjust the prices of its goods sold through retailers \( k \neq n \). Thus, taking \( p \) to be the vector of equilibrium prices, and keeping in mind that upstream firms claim all variable profits in this equilibrium,

\[ \Omega_U > \sum_{k \neq n} p_{mk} \tilde{Q}_{mk}^n(p) \geq \sum_{k \neq n} p_{mk} Q_{mk}(p) = (N - 1)\pi_U. \]

Using this inequality in the expression for \( m \)'s equilibrium profits, it follows that \( m \)'s equilibrium profits exceed

\[ N\pi_U - \frac{N}{2} \pi_U = \frac{N}{2} \pi_U. \]

Again using the fact that suppliers capture all variable profits, \( \pi_U \) represents the entire value of profits from the sale of good \( m \) through any single retailer \( n \). Of course, there are a total of \( N \) retailers, and so this says that \( m \) captures more than one-half of the entire surplus generated by sales of its good across all retailers. As this is true for all suppliers, suppliers as a whole capture more than one-half of the industry surplus.

Now I will show that suppliers capture less than one-half of the profits generated by sales of their goods, in the equilibrium in which retailers have pricing discretion. This, combined with what was just shown, will prove the desired result.

In the equilibrium in which retailers have pricing discretion they receive all variable profits and so \( \pi_U = \Omega_U = 0 \). Thus, once again using Equation(18), the equilibrium profits of an upstream firm \( m \) are

\[ \frac{N}{2}(M\pi_D - \Omega_D). \]

But, similar to the logic given above, in the case of a breakdown between \( m \) and some retailer \( n \), no prices except for those of goods at \( n \) change, because only \( m \) and \( n \) are aware of the disagreement but \( m \) has no pricing discretion. Although it would be optimal for \( n \) to vary its prices in this event, it could choose not to. Thus,

\[ \Omega_D \geq \sum_{k \neq m} p_{kn} \tilde{Q}_{kn}^n(p) > \sum_{k \neq m} p_{kn} Q_{kn}(p) = (M - 1)\pi_D. \]
Using this bound on $\Omega_D$ in the expression for the supplier’s profit given above, it follows that each supplier earns equilibrium profits strictly lower than $N\pi_D/2$. Given that suppliers receive zero variable profits, $\pi_D$ is the profit from sales of $m$ through any given retailer. Because each supplier sells through $N$ retailers, this says that this supplier earns less than one-half of the overall profits generated by sales of its good, and hence the same is true of all suppliers together. This proves the result.

\section*{References}


