Add-on Policies under Vertical Differentiation:
Why Do Luxury Hotels Charge for Internet
Whereas Economy Hotels Do Not?

Song Lin
Hong Kong University of Science and Technology
June 2015

Abstract

This paper examines firms’ product policies when they sell an add-on (Internet service) to a base good (hotel rooms) under vertical differentiation (four- vs. three-star hotels). Theoretical analysis uncovers the differential roles of add-ons for vertically differentiated firms. A firm with higher base quality always sells an add-on as optional so that higher-taste consumers self-select to buy it. This incentive to price discriminate applies to a lower-quality firm who, however, has to lower the add-on price to lure consumers considering the higher-quality base good without the add-on. If providing the add-on is not costly, the lower-quality firm may sell it as standard. This equilibrium potentially explains the puzzle that lower-end hotels are more likely than higher-end ones to offer free Internet service. Empirical examination of a sample of hotels that are likely to be in a monopoly or vertical duopoly market provides suggestive evidence for this prediction.

Surprisingly, an optional add-on can intensify competition, in contrast to standard conclusions in the literature. If both firms sell an optional add-on, they will price aggressively to compete for consumers who trade off the higher-quality base versus the lower-quality base good plus the add-on. Although selling the add-on as optional is unilaterally optimal, equilibrium profits may reduce - a Prisoner’s Dilemma outcome.
1 Introduction

Although Internet service is a necessity and is available in most hotels, only 54% of luxury hotels provide it for free. In contrast, the percentage grows to 72%, 81%, 93%, and 91% for upscale, mid-priced, economy, and budget hotels, respectively.\footnote{The phenomenon also applies to other hotel amenities such as breakfast and local phone calls. The data source is 2012 Lodging Survey by American Hotel & Lodging Association. In the empirical section I provide details of the survey data.} This phenomenon appears to be counter-intuitive, attracting considerable public attention and media coverage.\footnote{To name a few: “The price of staying connected” (New York Times, 5/6/2009), “Hotel guests crave free Wi-Fi” (Los Angeles Times, 9/6/2010), “Luxury hotels free up Wi-Fi” (Wall Street Journal, 5/5/2011), “Wi-Fi in hotels: the most unkindest charge of all” (The Economist, 5/16/2011), “Some hotels with free Wi-Fi consider charging for it” (USA Today, 6/22/2012), “Wi-Fi fees drag hotel satisfaction down” (CNN, 7/25/2012).} Why does the phenomenon persist? The answer to the question may have important managerial implications. In many industries, it is common for firms to sell a base good or service as their primary business, and then sell a complementary item or upgrade (hereafter, “add-on”).\footnote{Some add-ons are better described as surcharges, which are mandatory or necessary after the purchase of a base good. Examples include taxes for most services and goods, fuel surcharges for airlines, concession recovery fees for car rental at airport locations, etc. A recent stream of behavioral research on partitioned pricing explores how consumers react under this situation (e.g., Morwitz et al. [1998], Cheema [2008]). This paper restricts attention to add-ons that are not mandatory or not necessary on the purchase of a base good.} Examples include airlines selling drinks and snacks on a flight, car manufacturers selling upgrades such as GPS and leather seats on top of a base model, and mobile applications offering in-app purchases or premium upgrades. Solving the puzzle elucidates what product policies firms should adopt: Should they sell an add-on separately from the base good as optional, sell it as standard (i.e., free), or not sell it at all?

Existing pricing theories do not offer adequate explanations to this puzzle. Conventional wisdom from monopoly pricing suggests that selling an add-on as optional allows a firm with market power to price discriminate to enhance profit. This argument, however, contradicts the practice of lower-end hotels. A simple explanation would be that consumers staying at higher-end hotels are less price-sensitive.\footnote{There are several related explanations along the same line. For example, one may argue that consumers} However, this argument would suggest that
the higher-end hotels charge a higher total price rather than separate Internet charges from room rates. Shugan and Kumar (2014) compare the hotel industry to the airline industry and argue that it is optimal for a monopolist with a product line of base services to bundle add-ons with its lower-end base to decrease the base differentiation when the differentiation is large (hotel industry), and to unbundle add-ons with its lower-end base when the base differentiation is small (airline industry). However, this theory does not explain why the phenomenon persists even when higher-end and lower-end hotels are not owned by the same company, and why policies are different for different types of add-ons within an industry (e.g., mini-bar, laundry, or airport shuttle services in the hotel industry). I propose a different but complementary explanation that vertical differentiation between competing firms plays a role. I develop a competitive theory of add-on pricing to examine how vertical differentiation can lead to a divergence in product policy. The theory leads to three interesting insights.

First, the theory identifies the differential roles of add-ons for vertically differentiated firms. A firm with higher base quality behaves more or less like a monopolist. Selling an add-on as optional at a high price serves as a screening or segmentation device so consumers with higher tastes for quality self-select to buy the expensive add-on. This incentive to screen consumers also applies to a firm with lower base quality. However, the lower-quality firm is incentivized to lower the add-on price to lure those consumers, who may consider buying the higher-quality base without paying for the add-on, to switch to the lower-quality base with the add-on. This vertical differentiation role of the add-on is absent from extant literature on add-on pricing, which focuses on unobserved add-on prices with horizontal or no differentiation (Lal and Matutes 1994, Verboven 1999, Ellison 2005, Gabaix and Laibson 2006). Due to the trade-off between screening and differentiation, the lower-quality firm’s at higher-end hotels have corporate accounts covering their expenses whereas consumers at lower-end hotels travel with their own accounts. Another related argument is that higher-end hotels customers are typically business travelers whereas lower-end hotels have more leisure travelers. 

The term “add-on pricing” has been used to refer to a specific situation with unobserved add-on prices (Ellison 2005). In this paper the term refers to broader problems that involve pricing of a base good and an
policy is sensitive to the efficiency of supplying the add-on. The firm will not sell the add-on if it is too costly to provide the add-on to improve quality because it has to charge a high add-on price that discourages consumers from buying (e.g., mini-bar). In equilibrium, the lower-quality firm sell the add-on only if it is not too costly. If the cost becomes sufficiently small (e.g., Internet service), the add-on price will be so low that all consumers who buy the base from the lower-quality firm will also pay for the add-on. Consequently, the higher-quality firm will sell the add-on as optional, whereas the lower-quality firm will sell it as standard. This equilibrium outcome provides one possible explanation of the stylized fact.

Examining a sample of monopoly and duopoly markets with vertical differentiation in the American hotel industry, I find suggestive evidence for the theoretical predictions when the cost of an add-on is very small (i.e., Internet service). On the one hand, a hotel at the higher end of a vertical duopoly market is as likely as a monopoly hotel to charge for Internet service, consistent with the theory that higher-quality firms focus on screening consumers, behaving like a monopolist. On the other hand, a hotel is more likely to offer free Internet service if it is at the lower end of a vertical duopoly market. This finding supports the theory that vertical differentiation introduces a trade-off for lower-quality firms, which find it optimal to sell an add-on as standard rather than sell it as optional. Conclusions are robust even after controlling for a number of potentially confounding factors such as hotel segment, location, size, age, operation, etc.

Second, the theory leads to a surprising prediction that selling an add-on as optional intensifies competition. Since a higher-quality firm sells an add-on to its higher-taste consumers, leaving some lower-taste consumers who do not buy the add-on, it creates an opportunity for a lower-quality firm to lower its add-on price to induce switching. The firms then price aggressively to compete for these marginal consumers who trade off the higher-quality base good versus the lower-quality base good including the add-on. Although the optional-add-on, regardless of the observability of the add-on price.
add-on policy is unilaterally optimal, a Prisoner’s Dilemma emerges in which both firms lose profits in equilibrium. The result is striking. Extant literature predicts that selling an add-on as optional either has no impact on firm profits under competition (Lal and Matutes 1994), or softens price competition (Ellison 2005). The profit-irrelevant result is based on the argument that any profit earned from selling a high-priced add-on is competed away on the base price. The competition-softening result hinges on the idea that with the add-on prices unobserved naturally, firms create an adverse selection problem that makes price-cutting unappealing, thereby raising equilibrium profits. In contrast, I identify a mechanism by which selling an add-on hurts firm profits. The mechanism does not rely on the unobservability of add-on prices. It is the interaction between the screening effect and the differentiation effect that reverses standard conclusions on the profitability of selling an add-on. Due to the intensified competition, the higher-quality firm may want to commit to a standard-add-on policy, and the lower-quality firm to commit to a no-add-on policy, if such commitments are possible. Luxury cars, for example, are more likely than economy cars to offer some advanced features such as standard leather seats, GPS navigation, side airbags, etc.

Third, when consumers do not observe add-on prices, hold-up problems arise naturally. However, unlike other settings in the literature, vertical differentiation moderates the effects of hold-up on firm profits in this context. The higher-quality firm’s policy is unaffected by the unobservability of the add-on price, because its consumers already expect the add-on to be expensive due to the firm’s screening incentive. The hold-up effect coincides with the screening effect for the higher-quality firm. Anticipating being held up by the lower-quality firm, some consumers refrain from buying from it and switch to the higher-quality base good without paying for the add-on. Consequently, the higher-quality firm demands a higher base price, but the lower-quality firm is forced to lower its base price while keeping the add-on price high. In equilibrium the higher-quality firm is better off, whereas the lower-quality firm is worse off, suggesting that the higher-quality firm has no incentive to advertise the add-on.
price, whereas the lower-quality firm has a strict incentive to advertise. This prediction contrasts sharply with extant results that a hold-up problem has no impact on firm profits under competition because profits earned by holding up consumers \textit{ex post} are competed away by lowering base prices (i.e., “loss leader”; Lal and Matutes 1994).

\textbf{Related Literature}

This study relates to broader literature on price discrimination and multi-product pricing. Not surprisingly, add-on pricing can be a form of \textit{second-degree} price discrimination. A base good including an add-on versus the base good alone can be viewed as two quality levels. If firms sell an add-on as standard, they essentially sell the same quality to all consumers with no price discrimination. If firms instead adopt an optional-add-on policy, they sell the bundle (i.e., the higher-quality level) to consumers with higher tastes for quality while selling only the base (i.e., the lower-quality level) to lower-taste consumers. In \textit{monopoly} markets, it is generally optimal for firms to price discriminate. The problem is much harder to analyze under imperfect competition. Extant studies of \textit{competitive second-degree} price discrimination (Stole 1995, Armstrong and Vickers 2001, Rochet and Stole 2002, Ellison 2005, Schmidt-Mohr and Villas-Boas 2008) examine settings in which firms are symmetric, finding that firms engage in price discrimination if sufficient consumer heterogeneity exists. They do not consider the possibility of vertical pressure competing firms face, a scenario that arises often in the real world. One exception is Champsaur and Rochet (1989), who study differentiated firms in a duopoly market competing by offering ranges of quality to heterogeneous consumers. After firms’ investments in quality ranges, they determine the optimal nonlinear pricing policies. The price-setting game, given differential quality ranges, is similar in spirit to the pricing game I study here. However, unlike those authors, I study the context of add-on pricing in which quality is discrete, and derive implications of competitive price discrimination on profitability.
The result that selling an add-on intensifies competition also relates to several findings in the literature. Although it is generally true that monopolists find it optimal to price discriminate among consumers, this conclusion is no longer robust under competitive settings. Price discrimination can intensify competition, thereby hurting profits. One mechanism by which this result arises is competitive third-degree price discrimination (Thisse and Vives 1988, Shaffer and Zhang 1995, Corts 1998). Corts (1998) points out that when firms have a divergent view on the ranking of consumer segments (i.e., a strong market for one firm is weak for the other), it is possible that price discrimination leads to lower prices in all market segments, reducing profits. Another mechanism giving rise to this result is competitive mixed bundling in the two-stop shopping framework (Matutes and Regibeau 1992, Anderson and Leruth 1993, Armstrong and Vickers 2010). Practicing mixed bundling might trigger fierce price competition, lowering prices for component products. Consequently, profits reduce. This paper identifies an alternative mechanism that differs substantially in terms of modeling framework and business context. The add-on pricing problem studied here differs from third-degree price discrimination because firms do not know consumers’ preferences, and thus rely on incentive compatibilities to practice price discrimination. The problem is also different from multi-product bundling because an add-on is only available and valuable conditional on the purchase of a base good. Nevertheless, the recurring theme underlying the competition-intensifying result appears to be that competing firms price aggressively to acquire consumers who trade off between buying a higher-quality level from one firm versus buying a lower-quality level from another.

Finally, this paper relates to growing academic and industrial interest in drip pricing. Many cases exist in which this condition does not hold, and thus competing firms are better off in equilibrium (e.g., Chen et al. 2001, Shaffer and Zhang 2002). When competing firms offer multiple products, they can either sell the component products alone (i.e., component pricing), or sell the products in a bundle (i.e., pure bundling), or both (i.e., mixed bundling). Consumers cannot buy the add-on without buying the base good, but they can buy the base good alone without buying the add-on. For example, a consumer cannot access Internet service in a hotel if she does not stay at the hotel, but she can stay in the hotel room without using the Internet service. The Federal Trade Commission defines drip pricing as “a pricing technique in which firms advertise only...”

---

6 Many cases exist in which this condition does not hold, and thus competing firms are better off in equilibrium (e.g., Chen et al. 2001, Shaffer and Zhang 2002).

7 When competing firms offer multiple products, they can either sell the component products alone (i.e., component pricing), or sell the products in a bundle (i.e., pure bundling), or both (i.e., mixed bundling).

8 Consumers cannot buy the add-on without buying the base good, but they can buy the base good alone without buying the add-on. For example, a consumer cannot access Internet service in a hotel if she does not stay at the hotel, but she can stay in the hotel room without using the Internet service.

9 The Federal Trade Commission defines drip pricing as “a pricing technique in which firms advertise only...”
The leading theory that explains why firms adopt drip pricing is that it is profitable to exploit myopic consumers who do not anticipate the hidden cost. Gabaix and Laibson (2006) show that firms may choose not to advertise add-on prices (i.e., shrouding) even under perfect competition. Shulman and Geng (2012) extend the mechanism to the situation in which firms are ex ante different in both horizontal and vertical dimensions. Dahremöller (2013) introduces a commitment decision of shrouding or unshrouding, which can alter underlying incentives to unshroud. Muir et al. (2014) provide empirical evidence of drip pricing that driving schools take advantage of the fact that many students are not aware of extra driving school fees when they take a base course of driving instruction. Unlike these authors, I examine long-run market outcomes in which consumers know or correctly anticipate add-on prices in equilibrium. This assumption is not unreasonable, given that consumers might learn about prices through repeat purchases and/or word-of-mouth and in many cases, firms advertise add-on policies because they care about reputation or regulations require disclosure. It is unlikely that the motivating stylized fact is driven by consumer myopia, given that repeat purchases are common in the hotel industry and Internet service is a highly expected and frequently used feature. Results from this study suggest that vertical differentiation not only has profound impacts on add-on policies even under complete price information, but also interacts with firms’ incentives to advertise add-on prices when they are unobservable.

---

7

---

10 For example, having stayed at Marriott and learned that Internet service costs $13, a consumer may keep this in mind the next time she books a room at the same hotel or at another location of the same chain. 11 If consumers are boundedly rational, then Internet fees should be shrouded and higher than marginal costs at both the luxury and economy hotels.
2 A Theory of Add-on Policy under Vertical Differentiation

In this section, I present a theory that explains why and when a divergence in product policy arises as an equilibrium outcome. I first write down the simplest model that illustrates the key underlying mechanism, and then relax some of the simplifying assumptions to demonstrate that they do not alter the main message of the mechanism.

2.1 Model Setup

There are two firms $j \in \{l, h\}$, that differ in terms of the quality of the base good $V$ such that $V_h > V_l > 0$. The quality difference, or quality premium, is $\Delta V$. The marginal cost of the base good is normalized to zero for both firms. In an extension discussed later, I allow the marginal cost to be different for the two firms. In addition to the base good, an add-on technology is available. The add-on has value $w$ and costs $c$, the same for both firms. Again, the symmetric assumption is made only to simplify analysis. Qualitative conclusions remain largely unaffected in an extension with asymmetric add-on considered later. The efficiency of supplying the add-on is measured by the cost-to-value ratio, $\alpha = c/w$, which plays an important role in the equilibrium analysis. It is assumed that the quality premium is greater than the value of the add-on, $\Delta V > w$, allowing interesting equilibria to arise. Each firm can set base price $P_j$ and add-on price $p_j$.

A continuum of consumers differ in their marginal valuation, or taste, for quality. The taste parameter, $\theta$, is distributed uniformly with $\theta \in [\underline{\theta}, \bar{\theta}]$ and $\underline{\theta} > 0$.

Two assumptions are made throughout the analysis. First, $\bar{\theta} > 2\underline{\theta}$, so there is a sufficient amount of consumer heterogeneity in the market. Second, $\bar{\theta} > \alpha$, so there are positive sales of the add-on in

\[ \text{[12]} \text{The assumption that the lower bound is positive, combined with a sufficiently large base quality } V_j, \text{ ensures that the market is fully covered. It simplifies analysis by focusing on the interaction between the two firms, assuming away the outside option of not buying from any firm.} \]
equilibrium. The utility of buying from firm $j$ for type-$\theta$ consumer is

$$U_{\theta j} = \begin{cases} 
\theta V_j - P_j & \text{if only the base good is purchased;} \\
\theta(V_j + w) - P_j - p_j & \text{if both the base good and the add-on are purchased.}
\end{cases}$$

(1)

The base and add-on values $V$ and $w$ are both common knowledge to all parties. This is not an unreasonable assumption for industries such as the hotel industry in which consumers possess sufficient knowledge or information due to say repeat purchases.\(^{13}\) However, consumers know their own tastes $\theta$, but the firms do not. The firms only know the distribution of tastes, and hence rely on incentive compatibilities to screen consumers or price discriminate.

It is worth noting that the assumption that the unobserved consumer preferences are summarized entirely in one dimension, $\theta$, may appear strong, but it keeps the model tractable. An alternative interpretation is that tastes are the inverse of price sensitivities.\(^{14}\) The implicit restriction behind this setup is that willingness-to-pay for the base good and for the add-on ($\theta V_j$ and $\theta w$) correlate perfectly. Nevertheless, the mechanism does not rely on this assumption, as shown in an extension with imperfectly correlated tastes in the Appendix.

What is necessary is that there is unobserved heterogeneity in both the base good and add-on, enabling consumers to trade off between the add-on and the base quality.\(^{15}\) For price discrimination to arise, the single-crossing property is necessary such that higher tastes for the value of an add-on and for a base quality imply higher willingness-to-pay for the add-on

\(^{13}\) There are, of course, situations in which consumers experience uncertainty about $V$ and/or $w$. If firms have superior knowledge on these values, then the add-on can signal the base quality. For example, Bertini et al. (2009) show that add-on features can influence consumers’ evaluations of a base good about which they are uncertain. How firms design product policies under this situation is an interesting direction to explore, but it is beyond the scope of this paper. If firms are also uncertain about these values, insights from this simpler specification of consumer utility may apply.

\(^{14}\) One can specify an equivalent preference model such that the utility of buying both a base and an add-on is given by $U_{\theta j} = V_j + w - (P_j + p_j)/\theta$.

\(^{15}\) This feature distinguishes the paper from Dogan et al. (2010). In their model of competitive second-degree price discrimination with vertical differentiation in the context of rebates, there is no unobserved heterogeneity in tastes for the base quality.
and for the higher-quality base.

The strategic interaction is modeled as a full-information simultaneous-move game. Both firms announce prices simultaneously, \((P_h, p_h)\) and \((P_l, p_l)\). Consumers observe all prices and decide which firm to visit and whether to pay for an add-on from the chosen firm. The formulation naturally builds on two stylized models. If there is no add-on, the model reduces to a duopoly model of vertical differentiation \([\text{Shaked and Sutton 1983}]\). If there is no competition, the model reduces to a monopoly model of nonlinear pricing \([\text{Mussa and Rosen 1978}]\) with continuous-type consumers and discrete qualities. Each reduced model is straightforward to solve. However, combining the two features complicates equilibrium analysis dramatically due to the many possibilities of market outcome. Specifically, each firm can price its base and add-on to implement any of the following three outcomes, taking into account consumers’ incentive compatible decisions:

**No Add-on:** No consumer buys the add-on. All consumers buy just the base good.

**Standard Add-on:** All consumers buy the add-on. It is essentially bundled with the base good because only the total or bundle price matters.

**Optional Add-on:** Some but not all consumers buy the add-on. The higher-type consumers buy it, whereas the lower-type consumers buy only the base.

There are nine possible market configurations, depending on the implementations by both firms. Each configuration constitutes a possible equilibrium profile. To prove the existence of an equilibrium for each profile, one has to examine, for each firm, all non-local deviations that lead to any form of the remaining eight possible market configurations. However, the solution to the game can be simplified by observing that the higher-quality firm always serves the highest-type consumers and thus may have a strong incentive to sell the add-on as optional in equilibrium. The next sub-section formalizes this intuition.
2.2 The Higher-quality Firm Focuses on Screening

If the higher-quality firm implements the optional-add-on policy, it essentially divides its pool of consumers into two segments. The higher-type consumers, \( \theta \in [\hat{\theta}_h, \bar{\theta}] \), buy both the base and the add-on. Type \( \hat{\theta}_h \) is the intra-marginal consumer who is indifferent between buying the bundle and buying only the base, and it is equal to \( p_h/w \). The remaining lower-type consumers, \( \theta \in [\hat{\theta}_{hl}, \hat{\theta}_h] \), buy the base only. This segmentation is a result of the single-crossing property that higher-type consumers self-select to buy the add-on. Type \( \hat{\theta}_{hl} \) is the marginal consumer who is indifferent between buying from the higher-quality or lower-quality firm. This marginal consumer is only affected by base price \( P_h \), and the add-on price is irrelevant.

The firm’s profit decomposes into two additive profit components

\[
\Pi_h = (\bar{\theta} - \hat{\theta}_{hl})P_h + (\bar{\theta} - \hat{\theta}_h)(p_h - c) \, \pi_h(p_h). \tag{2}
\]

The first is the profit from selling the base good, independent from the add-on price since it is irrelevant to the marginal consumer \( \hat{\theta}_{hl} \). The second is the additional profit from selling the add-on, independent from the base price. Without considering incentive constraint \( \hat{\theta}_h \geq \hat{\theta}_{hl} \), to maximize profit, the higher-quality firm simply chooses the appropriate base and add-on prices to maximize the two components separately. If, however, the incentive constraint is binding, all consumers will buy the bundle. In this case, the firm is selling the add-on as standard and a single bundle price \( P_h^+ \) suffices. Since the firm serves consumers with the highest tastes in the market, it relaxes the pressure for a binding constraint. Just as in a monopoly market, the higher-quality firm can easily find the optional-add-on policy strictly better than the standard-add-on policy. The following lemma formalizes this observation.

(All proofs are provided in the Online Appendix.)

**Lemma 1.** For the higher-quality firm, selling the add-on as standard is strictly dominated
by selling the add-on as optional, if:

1. \( P_1 > -\alpha \Delta V \) when the lower-quality firm does not sell the add-on, or

2. \( P_{1}^{+} > -\alpha (\Delta V - w) \) when the lower-quality firm sells the add-on.

The intuition behind this lemma is simple. When the higher-quality firm sells the add-on as standard, the add-on price is set sufficiently low so that all of its consumers buy it. Consider a local deviation whereby the firm increases the add-on price by a small amount \( \epsilon \) and decreases the base price by the same amount. Then some consumers refrain from paying for the add-on (i.e., the firm implements the optional-add-on policy). On the one hand, increasing the add-on price does not lose many consumers who have originally bought the add-on, but it generates additional revenue from those higher-type consumers who continue to pay for it. On the other hand, lowering the base price expands the market. Acquired consumers are of the lower types so the loss due to the lower base price is quite limited. The total profit is then increased. Note that the argument requires that there is sufficient number of different types (more than two) so that a local deviation by separating the prices is profitable.\(^{16}\) This intuition is the similar to that in a monopoly market. Despite facing competitive pressures from the lower-quality firm, the higher-quality firm behaves like a monopolist. The first main proposition follows.

**Proposition 1.** In any equilibrium, if it exists, the higher-quality firm sells the add-on as optional.

The mechanism for the higher-quality firm accords with what many managers have in mind. One reason luxury hotels charge for Internet service is “because they can”. Selling optional Internet service is so lucrative that these hotels do not want to give up this source of revenue. One driving force is the firm’s incentive to screen consumers, an effective way to

\(^{16}\) With just two types of consumers, it can easily lead to the conclusion that selling an add-on as standard is optimal, as in Ellison (2005) and Shugan and Kumar (2014).
boost short-term profits. This incentive should also apply to the lower-quality firm, given that it also serves a pool of consumers with heterogeneous tastes. Why does the lower-quality firm behave differently? The next sub-section provides the answer.

2.3 The Lower-quality Firm Trades off Screening and Differentiation

Having shown that the only possible implementation of the higher-quality firm is the optional-add-on policy, there are only three possible equilibrium profiles to be considered, depending on the lower-quality firm’s implementation. In what follows I develop the equilibrium in which the lower-quality firm sells the add-on as optional. The other two possible equilibria become straightforward given this development.

The lower-quality firm now targets consumers of lower types, \( \theta \in [\hat{\theta}_l, \hat{\theta}_{hl}] \). Similar to the higher-quality firm, it segments consumers into two groups. The lower-type consumers, \( \theta \in [\hat{\theta}_l, \hat{\theta}_l] \), buy only the base good. The intra-marginal consumer, who is indifferent between buying the bundle and buying only the base, is given as \( \hat{\theta}_l = p_l/w \). This consumer does not react to the base price of the higher-quality firm. Consumers of higher types, \( \theta \in [\hat{\theta}_{hl}, \hat{\theta}_{hl}] \), buy both the base and add-on. The marginal consumer, who is indifferent between the two firms, \( \hat{\theta}_{hl} \), depends on the lower-quality firm’s total price of the base and add-on, \( P_l^+ = P_l + p_l \). The profit is

\[
\Pi_l = (\hat{\theta}_{hl} - \theta)P_l + (\hat{\theta}_{hl} - \hat{\theta}_l)(p_l - c).
\]

Like its rival, the lower-quality firm is incentivized to set a high add-on price to allow the consumers self-select. Those who have a higher taste for quality consider the high-priced add-on, and the lower-type consumers consider only the base. Unlike its rival, however, the lower-quality firm also uses the add-on to attract consumers who consider only the base from the competitor. The firm attempts to keep its add-on price reasonably low to attract these potential switchers. To see the optimal pricing strategy that resolves this trade-off, it
is instructive to rewrite the firm’s profit as

\[ \Pi_l = (\hat{\theta}_{hl} - \theta)(P_l^+ - c) - (\hat{\theta}_l - \theta)(p_l - c). \]  

(3)

In this decomposition, the first component depends only on bundle price \( P_l^+ \), and the second depends only on add-on price \( p_l \). The unconstrained problem is solved by maximizing each component separately. The intuition can be understood as follows. The lower-quality firm advertises an attractive bundle package (e.g., a 3-star hotel offers “Internet included” or “stay-connected” packages) to its potential consumers, \( \theta \in [\hat{\theta}_l, \hat{\theta}_{hl}] \), and convinces all to visit. Once consumers accept the offer, the firm excludes the lowest-type consumers, \( \theta \in [\hat{\theta}_l, \hat{\theta}_{hl}] \), from consuming the add-on by subsidizing them to not consume it. In this way, the firm attracts its most valuable consumers, \( \theta \in [\hat{\theta}_l, \hat{\theta}_{hl}] \), while avoiding unnecessary costs of supplying the add-on to the lowest-type consumers, who do not value it much.

The strategic interaction between the two firms reduces to the competition between the lower-quality firm’s bundle and the higher-quality firm’s base good. They compete for the marginal consumer who is indifferent between the two, given by \( \hat{\theta}_{hl} = (P_h - P_l^+)/\Delta V - w \). The competition is the same as a duopoly model of vertical differentiation\(^{17}\) except that the quality premium is \( \Delta V - w \) instead of \( \Delta V \), and that the lower-quality firm’s price is \( P_l^+ \) instead of \( P_l \). In equilibrium, the prices are

\[ P_h^* = \frac{1}{3}(\Delta V - w)(2\hat{\theta} - \theta) + \frac{1}{3}c, \quad \text{and} \quad P_l^{**} = \frac{1}{3}(\Delta V - w)(\bar{\theta} - 2\hat{\theta}) + \frac{2}{3}c, \]  

(4)

and the marginal consumer is

\[ \hat{\theta}_{hl}^* = \frac{1}{3}(\bar{\theta} + \theta) - \frac{c}{3(\Delta V - w)}. \]  

(5)

\(^{17}\) See Tirole (1988) for a stylized model.
which determines the equilibrium market share of each firm. Note that equilibrium marginal consumer, $\hat{\theta}_{hl}^*$, decreases with add-on value $w$. As the value grows, two opposite effects occur. On the one hand, there is a direct effect of increasing demand for the lower-quality firm, and decreasing demand for the higher-quality firm, because consumers obtain higher utility from the lower-quality firm due to the added value of the add-on. For fixed prices, the marginal consumer moves upward as $w$ increases. On the other hand, there is an indirect effect stemming from strategic price responses to changes in product quality. The higher-quality firm lowers its base price as value $w$ increases. Contrarily, the lower-quality firm raises its bundle price to exploit acquired consumers who have higher tastes. The price gap is reduced, moving the marginal consumer downward. In equilibrium, which effect dominates depends on the cost of the add-on relative to the cost of the quality premium. In the current setup, the strategic effect dominates given that the cost of the quality premium is small (i.e., assumed to be zero), and hence the marginal consumer becomes lower as $w$ increases.

Independent of strategic interactions that determine equilibrium market shares, the firms set an optimal add-on price. Add-on prices $p_h$ and $p_l$ are chosen to maximize $\pi_h(p_h)$ and $\pi_l(p_l)$ in Equations (2) and (3) respectively, which lead to

$$p_h^* = \frac{1}{2}(\bar{\theta} + \alpha)w,$$

and

$$p_l^* = \frac{1}{2}(\theta + \alpha)w,$$

with resulting intra-marginal consumers: $\hat{\theta}_h^* = \frac{1}{2}(\bar{\theta} + \alpha)$ and $\hat{\theta}_l^* = \frac{1}{2}(\theta + \alpha)$. The optimal add-on prices reflect underlying consumer tastes. Although the add-on is the same for both firms, the price is higher at the higher-quality firm. This accords with the casual observation that Internet fees at higher-end hotels are higher than those at lower-end hotels (if they charge for it). Furthermore, the add-on price at the higher-quality firm is higher than the marginal cost, suggesting that it is a profitable business. However, the price is considerably

---

18 In the extension in which the marginal cost of the base good is asymmetric, this conclusion holds as long as the cost of the add-on is greater than the cost of the quality premium.
lower at the lower-quality firm, even lower than the marginal cost. In fact, the lower-quality firm prices the add-on significantly lower than what it would have charged if there were no competition. To see that, imagine that the lower-quality firm is the monopolist for a market of consumers with types \( \theta \in [\underline{\theta}, \bar{\theta}] \). The maximization problem would lead to an add-on price of \( \tilde{p}^*_l = \frac{1}{2}(\bar{\theta}_l + \alpha)w \), which is greater than \( p^*_l \) in the equilibrium under vertical differentiation because \( \bar{\theta}_l > \bar{\theta} \). This is true even though \( \bar{\theta}_l \) can be much smaller than \( \bar{\theta} \). The fact that the lower-quality firm serves the consumers with lower tastes is not the main driver of the low add-on price; vertical differentiation forces the firm to price it considerably low, even below the marginal cost.

2.4 Incentive Compatibility and Equilibrium Outcomes

The preceding analysis uncovers the equilibrium pricing under the scenario in which both firms sell the add-on as optional. This equilibrium exists as long as the following incentive constraints hold:

\[
\begin{align*}
(1) & \quad \hat{\theta}^*_h > \hat{\theta}^*_hl, \\
(2) & \quad \hat{\theta}^*_hl > \hat{\theta}^*_l, \\
(3) & \quad \hat{\theta}^*_l > \underline{\theta}.
\end{align*}
\]

The first incentive constraint holds as long as \( \Delta V > w \). Intuitively, this means that as long as the quality premium is greater than the add-on value the lower-quality firm provides, some consumers prefer the higher-quality base good to the lower-quality bundle.

The last two constraints depend crucially on the cost of supplying the add-on relative to its value, and determine whether the lower-quality firm sells the add-on, or sell it as standard or as optional. On the one hand, the marginal consumer who is indifferent between the two firms, \( \hat{\theta}^*_hl \), decreases in marginal cost of add-on \( c \) for a fixed value of the add-on, according to Equation (5). This implies that \( \hat{\theta}^*_hl \) decreases as \( \alpha \) increases. Intuitively, as the cost of the add-on increases, so does the bundle price of the lower-quality firm. Some consumers would rather buy only the base from the higher-quality firm. On the other hand, the intra-marginal consumer for the lower-quality firm, \( \hat{\theta}^*_l \), increases with \( \alpha \). As the add-on becomes
more costly to provide, it is optimal for the lower-quality firm to exclude more consumers who do not value it much. Consequently, the segment of consumers who buy the bundle from the lower-quality firm shrinks. As cost-to-value ratio $\alpha$ becomes sufficiently large, the firm excludes all consumers from buying the add-on, thereby not selling it.\footnote{More generally, not selling the add-on is strictly dominated by selling the add-on provided that $\alpha$ is sufficiently small. This result is summarized in a lemma, analogous to Lemma 1, in the proof of the next proposition in the Appendix.} Therefore, incentive constraint $\hat{\theta}_{hl}^* > \hat{\theta}_l^*$ ensures that the firm sells the add-on in equilibrium, leading to

$$\alpha < \frac{1}{3}(2\bar{\theta} - \theta) \quad \text{and} \quad \Delta V > \frac{2\bar{\theta} - \theta - \alpha}{2\bar{\theta} - \theta - 3\alpha} \cdot w \equiv \Delta_1. \quad (6)$$

If however cost-to-value ratio $\alpha$ becomes smaller, $\hat{\theta}_{hl}^*$ increases whereas $\hat{\theta}_l^*$ decreases. The segment of consumers who buy the bundle from the lower-quality firm expands, deriving from two sources. One source of switchers comes from those who originally considered only the base from the higher-quality firm, and are now drawn to the lower-quality firm due to its lower bundle price. The other switchers originally considered only the base from the lower-quality firm, and are now drawn to the add-on because it becomes affordable. As $\alpha$ becomes sufficiently small, all consumers who decided to buy from the lower-quality firm are willing to buy the add-on. The add-on is then essentially standard, or free, because all consumers pay just the bundle price. In this case, $\hat{\theta}_l^* \leq \theta$, which is equivalent to $\alpha \leq \theta$. Constraints (2) and (3) now reduce to $\hat{\theta}_{hl}^* > \theta$, leading to

$$\Delta V > \frac{\bar{\theta} - 2\theta + \alpha}{\bar{\theta} - 2\theta} \cdot w \equiv \Delta_2. \quad (7)$$

The following proposition summarizes pure-strategy Nash equilibrium of the game.

**Proposition 2.** 1. If $\alpha \geq \frac{1}{3}(2\bar{\theta} - \theta)$, there exists an equilibrium in which the higher-quality firm sells the add-on as optional whereas the lower-quality firm does not sell it;
2. If $\theta < \alpha < \frac{1}{3}(2\theta - \theta)$, there exists an equilibrium when $\Delta V > \Delta_1$, in which both firms sells the add-on as optional;

3. If $\alpha \leq \theta$, there exists an equilibrium when $\Delta V > \Delta_2$, in which the higher-quality firm sells the add-on as optional whereas the lower-quality firm sells it as standard;

4. If $\alpha < \frac{1}{3}(2\theta - \theta)$ and $\Delta V \leq \max\{\Delta_1, \Delta_2\}$, there is no pure-strategy equilibrium.

2.5 Remarks

A few remarks are in order. The theory explains the relative difference in add-on policies between higher-quality and lower-quality firms. While a higher-quality firm concentrates on using an add-on to screen consumers, a lower-quality firm’s policy is much more sensitive to the cost-to-value ratio of providing the add-on because of its trade-off between screening and differentiation. Internet service has arguably a very low marginal cost and/or a large value, making cost-to-value ratio $\alpha$ very small. Consumers who visit lower-end hotels always pay for Internet service. In this sense, Internet service is essentially bundled with room rates, and thus hotels are selling it as standard or for free, and quoting only the total price. For an add-on with a larger cost-to-value ratio, the add-on price has to be higher to recover the cost, discouraging consumers from buying. It is then optimal for a lower-quality firm to offer it as optional. This case can explain why lower-end hotels are equally likely as higher-end hotels to offer add-ons such as laundry or airport shuttle services as optional. It can also explain why lower-end airlines or cruise lines are equally likely as their higher-end competitors to charge for Internet service, because the costs of providing Internet access remain large based on today’s technology\footnote{Unlike hotels, airlines use ground stations or satellites to provide Internet access on a flight and most cruise lines use satellite technology to provide Internet access. Currently in-flight Internet fees range from $10 to $20 per hour, and Internet charges on a cruise ship can be as high as 90 cents per minute.} For an add-on with a very large cost-to-value ratio, the add-on price may be so high that very few consumers choose to pay for it from the lower-quality firm. This case can explain why lower-end hotels are less likely to offer amenities such as
mini-bar or room services, compared to higher-end hotels which often sell them as optional at high prices.

Clearly the competition between higher-quality and lower-quality firms for marginal consumers who trade off a higher-quality base and a lower-quality bundle is driving the result. This competition is the reason why a higher-quality firm is able to concentrate on price discrimination whereas its lower-quality rival has to think more in terms of using an add-on to differentiate. If both firms are owned by the same parent company, then it is optimal not to allow the two brands to price aggressively for the marginal consumers. One can show that in a monopoly model with a product line, under the same set of assumptions, selling the add-on for the lower-quality brand will cannibalize profit of the higher-quality brand.\textsuperscript{21} Therefore, absent other forces, the monopolist will sell the add-on as optional for the higher-quality brand but does not sell it for the lower-quality brand.

The theory assumes no horizontal differentiation to focus on the mechanism of vertical differentiation. Real-world competitions are likely to involve both aspects. If two firms are differentiated horizontally, they will both have an incentive to lower add-on prices. Without asymmetry generated by quality difference, however, the competing firms will tend to adopt the same policy. Therefore, horizontal differentiation alone is unlikely to explain why two competing firms diverge in their add-on policies. An alternative explanation is that horizontal differentiations both at a higher-end market and at a lower-end market can lead to different market outcomes driven by different characteristics of the two markets. Applying theories of competitive second-degree price discrimination with horizontal differentiation, under the assumption that consumers’ brand preferences are independent of their preferences for quality, may predict that an add-on is sold at the marginal cost (Verboven 1999). However, marginal costs for Internet service are arguably small and similar at both the higher-end

\textsuperscript{21} Due to page limit, the proofs of this model and all remaining extensions considered in this section are not included in the appendix but are available upon request.
and lower-end markets, suggesting that hotels at both ends will offer Internet service as standard, contradicting the stylized fact. If the independence assumption is violated, as Ellison (2005) shows, firms will bundle an add-on when consumers are less heterogeneous and unbundle when they are more heterogeneous. The theory can then explain the stylized fact, if consumers are more heterogeneous at the higher-end markets than at the low-end markets. However, this argument relies on the modeling assumption that only two types of consumers exist (in terms of their tastes for a higher quality), which leads to the result that firms prefer to sell an add-on to both types. The analysis of the higher-quality firm in Section 2.2 implies that the incentive to unbundle is generally strong provided a sufficient number of types exists.

I focus on the simplest setting in which there is only one add-on with one quality level. In reality, firms can supply multiple add-ons (e.g., breakfast, local calls, or airport shuttle), or various qualities of an add-on (e.g., high-speed Internet access). These applications share the common feature that consumers who value a higher quality more self-select to buy more add-ons or the higher-quality level of the add-on. The primary intuition of the mechanism applies. For example, “all-inclusive” hotels are typically not the most luxurious; less-than-luxurious hotels are more likely to offer all-inclusive services. It is increasingly common that higher-end hotels use tiered pricing to charge for Internet service. They offer complimentary Internet access for basic use such as emailing but charge for higher-speed Internet or heavy use such as video conferencing and streaming movies. This practice is rarely adopted by lower-end hotels.22

An add-on may evolve due to, for example, technology improvement or changing consumer preferences. This may reduce the cost of supplying the add-on and/or enhance the value of the add-on. For example, the cost of Internet service has decreased, and the value

22 See, for example, a recent industry report “Hotel chains play WiFi follow the leader” (http://www.hotelnewsnow.com/Article/15186/Hotel-chains-play-Wi-Fi-follow-the-leader).
has increased over time. Based on Proposition 2, it is straightforward to predict the dynamics of add-on policies:

**Corollary 1.** *As the cost of an add-on decreases, or its value increases, over time, the lower-quality firm’s policy changes from no add-on, to optional add-on, and eventually to standard add-on.*

Recall the three simplifying assumptions. First, the add-on is homogeneous even though the firms are differentiated vertically with respect to the base good. This may be a reasonable assumption for some applications like Internet service. More realistically, the add-on may be asymmetric across firms in terms of cost and/or value. For example, breakfast may be of higher quality but it costs more at a 5-star hotel than at a 4-star hotel. Second, the marginal cost of the base good is assumed equal for both firms even though the base quality differs. This assumption may be reasonable if the quality premium originates from the fixed costs of investing in the product design of the base good. A 5-star hotel, for example, can invest in better locations and views, swimming pools, fitness centers, the costs of which may be substantially higher than those at a 4-star hotel, but less so for the marginal costs. Nevertheless, it is more realistic to allow asymmetric marginal costs. Third, consumers have the same marginal utility or taste for the base good and for the add-on, making the preferences for the two perfectly correlated. It is more realistic to assume that consumers have separate tastes, one for the base good and the other for the add-on. However, it is not unreasonable to allow for some degree of positive correlation between the two tastes, perhaps through price sensitivity. For example, a less price-sensitive consumer may be willing to pay more for a more comfortable room and for Internet service. Each of these convenient assumptions can be relaxed without fundamentally altering the main conclusions.
3 Profitability of Add-on Policies

The focus of analysis thus far has been on why and when a divergence of product policy arises as an equilibrium outcome. The analysis has uncovered the profound effect of vertical differentiation between competing firms. A higher-quality firm behaves like a monopolist, who finds it optimal to sell an add-on as optional to screen consumers. A lower-quality firm faces a trade-off between screening and differentiation. It sells an add-on as optional in equilibrium only when it is not too costly to supply. That selling an optional add-on is unilaterally optimal to both firms does not necessarily imply that equilibrium profits improve over situations in which neither firm sells it as optional. This is because strategic interaction may render the optional-add-on policy unprofitable.

To investigate the firms’ ex ante incentives to implement add-on policies, I introduce, before firms set price levels, a commitment stage in which they can simultaneously choose whether to commit to a no-add-on policy or standard-add-on policy. By committing to a no-add-on policy, firms do not introduce an add-on, and charge only for the base good. By committing to a standard-add-on policy, firms always bundle the base and add-on, and charge only for the bundle. Once they commit to either of two policies, they are unable to price the add-on and the base separately in the second stage of price setting. By not committing to these policies, however, firms retain the flexibility of selling an add-on as optional to screen the consumers. The timing of the game is such that both firms first simultaneously decide their commitment choices during the first stage, and then compete in prices during the second stage, given their chosen policies. This formulation allows us to compare equilibrium profits when both firms sell an add-on as optional to those when neither firm does.

By introducing the commitment stage, the model no longer fits the business environment of the hotel industry which motivates the story. However, other industries may well be
consistent with the modeling assumptions if selling an add-on involves a large fixed cost of investment. The automobile industry seems to be a good example. Many advanced features such as side airbags, GPS navigation, and leather seats require large investments in production. It may be harder for manufacturers to sell these features as options once investments have been made.

Nine commitment outcomes are possible in the first stage, and each can lead to a pricing equilibrium in the second-stage pricing game. To allow equilibrium in which both firms sell the add-on as optional, I focus on the case with moderate add-on cost: $\theta < \alpha < \frac{1}{3}(2\overline{\theta} - \theta)$. The equilibrium of the full game is found by first solving the second-stage pricing equilibrium for each of the 9 commitment outcomes, and then finding the equilibrium commitment choices taking into account the second-stage subgame outcomes. The following proposition summarizes the equilibrium of the full game.

**Proposition 3.** Suppose $\theta < \alpha < \frac{1}{3}(2\overline{\theta} - \theta)$. There exists threshold $\Delta^*$ such that when $\Delta V > \Delta^*$, the higher-quality firm commits to the standard-add-on policy, whereas the lower-quality firm commits to the no-add-on policy in equilibrium. Equilibrium prices are

$$P_h^{**} = \frac{1}{3}(\Delta V + w)(2\overline{\theta} - \theta) + \frac{2}{3}c,$$

and

$$P_l^{**} = \frac{1}{3}(\Delta V + w)(\overline{\theta} - 2\overline{\theta}) + \frac{1}{3}c.$$

The result that in equilibrium neither firm has an incentive to sell an optional add-on is striking. This contrasts sharply with the insight from monopoly settings in which the optional-add-on policy is always profit enhancing (weakly). As shown in Section 2, fixing its rival’s pricing, each firm finds it optimal to sell an add-on as optional. However, taking into account rivals’ reactions, firms find themselves trapped in a Prisoner’s Dilemma; they can both be better off to commit not to sell an add-on as optional.

To understand why this result arises, it is helpful to examine what might have happened if, under the equilibrium of Proposition 3, firms fail to commit. When commitment fails,
Lemma 1 implies that it is optimal for the higher-quality firm to separate the base and add-on prices, with the best-response base price given by $P_h^{(1)} = (P_l^* + \Delta V \bar{\theta})/2$. In response, the lower-quality firm sells the add-on to those who buy only the base from its rival, inducing them to switch and get the extra benefit of the add-on. The firm chooses optimal bundle price $P_l^{+ (1)}$ that responds to the higher-quality firm’s base price $P_h^{(1)}$, and optimal add-on price $p_l^{(1)}$ that minimizes the cost of over selling the add-on:

$$P_l^{+ (1)} = \frac{1}{2}(P_h^{(1)} + c - (\Delta V - w)\bar{\theta}), \quad \text{and} \quad p_l^{(1)} = \frac{1}{2}(w\bar{\theta} + c).$$

Further, in response to its rival’s bundle price, the higher-quality firm sets optimal base price $P_h^{(2)} = (P_l^{+ (1)} + (\Delta V - w)\bar{\theta})/2$, which is lower than its previous price $P_h^{(1)}$ given that $\alpha < \frac{1}{3}(2\bar{\theta} - \theta)$:

$$P_h^{(2)} - P_h^{(1)} = -\frac{w}{8}(5\bar{\theta} - 4\theta - \alpha) < 0.$$ 

The lower base price further motivates the lower-quality firm to lower its bundle price. This dynamic iterates and converges to an equilibrium in which both firms end up being worse off, even though they both sell the add-on as optional eventually.

The fact that the two firms end up being maximally differentiated under the two-stage game may bear some similarity to the same result that arises from a stylized duopoly model of vertical differentiation with ex ante product-design decisions (i.e., firms can choose to invest in $V$). However, the results of this analysis suggest that even with the presence of potential benefits from screening consumers, the maximal-differentiation principle remains dominating. In fact, what the screening does is the opposite of what the firms expect; it opens the opportunity for fiercer price competition, thereby hurting profits. The negative result of the optional-add-on policy on profitability presents a challenge for firms selling an add-on. In the short run, firms may be better off selling an add-on as optional. In the long run, however, profits may be damaged if firms are vertically differentiated. Hence, it
is valuable for firms to have commitment powers. For many add-ons in the hotel industry (Internet, phone calls, breakfast, etc.), firms lack such commitment powers because it is quite flexible for them to add or remove these items. For many features in the automobile industry (side airbags, GPS navigation, leather seats, etc.), however, firms cannot flexibly add these features once a base model is built. These features tend to be standard in luxury cars but not in economy cars, a phenomenon consistent with Proposition 3.

4 Unobserved Add-on Prices

Thus far, I have assumed add-on prices are observable by consumers. This is not unreasonable given that consumers may learn about prices through repeat purchases or word-of-mouth, and that in many cases, firms advertise add-on policies because either they care about reputation or regulations require disclosure. Much of the focus in the literature, however, has been on situations in which add-on prices are unobserved by consumers. For example, many consumers do not know about ATM and minimum balance fees when they open bank accounts (Cruickshank 2000). Prices of mini-bar items are often unknown to consumers before they book a hotel. In this section, I explore how the assumption of unobserved add-on prices influences equilibrium outcomes and firm profits. To that end, I model the game as follows:

- at $t = 0$, both firms set prices for the base and add-on;
- at $t = 1$, consumers observe only base prices $P_h$ and $P_l$, and decide which firm to buy from, and pay the base price;
- at $t = 2$, consumers visit the firms they have chosen; the add-on price is revealed and they decide whether to pay for the add-on.

Consumers have rational expectations about add-on prices $p^e_h$ and $p^e_l$. In this setup, consumers cannot learn about add-on prices by searching. Though somewhat strict, the as-
sumption is sufficient to illustrate the problem. In an alternative specification, consumers can incur positive search costs to discover add-on prices. According to the standard argument by [Diamond (1971)](Diamond1971), even though search costs may be very small, in equilibrium, firms still enjoy monopoly power *ex post* after consumers patronize, and thus the add-on is charged at a monopoly price. The equilibrium outcome is the same as the one presented here. I examine each firm’s problem in turn and derive the *sequential equilibrium* in which both firms sell an add-on.

4.1 The Higher-quality Firm

The higher-quality firm’s problem is analyzed backwards. Suppose a fraction of consumers $[\hat{\theta}_{hl}, \theta]$ decide to visit the higher-quality firm at time $t = 1$. The marginal consumer $\hat{\theta}_{hl}$ is a function of base price $P_h$ and the total price of buying from the lower-quality firm. Among these consumers, the higher types, $\theta \in [\hat{\theta}_{hl}, \theta]$, buy the add-on if add-on price $p^e_h$ is not too high. The intra-marginal consumer is given by $\hat{\theta}_{h} = p^e_h / w$. The firm charges optimal price $p^e_h$ that maximizes add-on profit, $\pi_h = (\theta - \hat{\theta}_{hl}) (p^e_h - c)$. This leads to the monopoly price, $p^e_h = \frac{1}{2}(\theta w + c)$, and the equilibrium intra-marginal consumer becomes $\hat{\theta}^*_h = \frac{1}{2}(\theta + \alpha)$. Note that this price is independent of the lowest-type consumer, $\hat{\theta}_{hl}$, as long as the solution is interior. Therefore, at the beginning period $t = 0$, maximizing total profit is equivalent to maximizing base profit, $(\theta - \hat{\theta}_{hl})P_h$. The key observation is that the higher-quality firm’s problem is essentially the same as the one with observable add-on price. The add-on price is chosen to maximize *ex-post* profit given the firm’s local monopoly power. Consumers expect that even though the price is unobservable.

4.2 The Lower-quality Firm

At time $t = 1$, remaining consumers $\theta \in [\theta, \hat{\theta}_{hl}]$ decide to pay the base price to visit the lower-quality firm. At time $t = 2$, the consumers observe add-on price $p_l$ and are faced with the decision of whether to buy it. Given these consumers, the firm maximizes *ex-post* profit,
\[ \pi_l = (\hat{\theta}_l - \hat{\theta}_i)(p_l^e - c) \] with \( \hat{\theta}_l = p_l^e / w \). It is useful to change the control variable from add-on price \( p_l^e \) to intra-marginal consumer \( \hat{\theta}_l \) who is indifferent regarding buying the add-on. The optimal intra-marginal consumer is chosen as a function of the marginal consumer such that \( \hat{\theta}_l^* (\hat{\theta}_h) = (\hat{\theta}_h + \alpha) / 2 \). Taking into account the second-period problem, the firm’s problem at time \( t = 0 \) is to find the optimal marginal consumer \( \hat{\theta}_h \) that maximizes its total profit

\[
\max_{\hat{\theta}_h} (\hat{\theta}_h - \hat{\theta}_i)(P_h - \hat{\theta}_h(\Delta V - w) - c) - (\hat{\theta}_l^*(\hat{\theta}_h) - \theta)(w \cdot \hat{\theta}_l^*(\hat{\theta}_h) - c).
\]

Given the higher-quality firm’s base price \( P_h \), the best-response for the lower-quality firm is to choose a marginal consumer of

\[
\hat{\theta}_{BR}^{hl} = \frac{2P_h - 2c + (2\Delta V - w)\theta}{4\Delta V - 3w}.
\]

This leads to a total price of \( P_l + p_l^e = P_h - \hat{\theta}_{BR}^{hl}(\Delta V - w) \). Combined with the best response of the higher-quality firm, the resulting equilibrium profile is

\[
\hat{\theta}_{hl}^* = \frac{(\bar{\theta} + \theta)\Delta V - (2\bar{\theta} + \theta + 2\alpha)w}{6\Delta V - 5w}; \quad P_h^* = \frac{2(2\bar{\theta} - \theta)\Delta V - (3\bar{\theta} - \theta - 2\alpha)w}{6\Delta V - 5w} \cdot (\Delta V - w).
\]

Equilibrium outcomes are summarized in the following proposition.

**Proposition 4.** There exist equilibria in which the higher-quality firm always sells the add-on as optional, whereas the lower-quality firm’s policy depends on cost-to-value ratio \( \alpha \) such that

1. if \( \alpha > \frac{1}{3}(2\bar{\theta} - \theta) \), then the firm does not sell the add-on;
2. if \( \alpha \in (\theta, \frac{1}{3}(\bar{\theta} + \theta)) \), then the firm sells the add-on as optional, when \( \Delta V > \Delta_1^{(u)} \);
3. if \( \alpha \leq \theta \), then the firm sells the add-on as standard when \( \Delta V \in [\Delta_0^{(u)}, \Delta_2^{(u)}] \), and sells it as optional when \( \Delta V > \Delta_2^{(u)} \).

27
This proposition reveals that the interaction between screening and vertical differentiation remains critical even when add-on prices are unobserved. The trade-off between the two forces at the lower-quality firm renders its add-on policy sensitive to the cost of providing the add-on. The unobserved add-on price at the lower-quality firm leads to the hold-up problem that keeps the add-on price high. As a result, the lowest types refrain from buying the expensive add-on, and hence the firm is less likely to sell it as standard even when it is not costly to provide. The next sub-section further investigates the impacts of this hold-up effect on equilibrium outcomes and profits.

4.3 Unobserved-price Equilibrium versus Observed-price Equilibrium

How does the equilibrium under unobservable prices differ from the one under observable prices? To facilitate the comparison, I restrict analysis to an add-on with moderate cost so both firms sell it as optional under either scenario.

**Proposition 5.** Consider a moderately costly add-on such that $\alpha \in (\frac{1}{3}(\Theta + \bar{\Theta}), \Theta)$ and a sufficiently large quality premium such that $\Delta V > \Delta V_{u1}^{(u)}$. Compared to the observed-price equilibrium, under the unobserved-price equilibrium,

1. the lower-quality firm has a smaller market share and less sales of the add-on;
2. the higher-quality firm charges a higher base price but the same add-on price, whereas the lower-quality firm charges a lower base price but a higher add-on price; the total prices for both firms are higher;
3. the higher-quality firm’s profit improves, whereas the lower-quality firm’s profit reduces.

The main reason for the difference in equilibrium outcomes and profits is the different effects of the hold-up problem on the vertical differentiated firms. For the higher-quality firm, the hold-up problem has no effect because consumers anticipate an *ex-post* high add-on
price charged to exploit the highest-type consumers. The hold-up problem at the lower-quality firm is more subtle. The higher-type consumers at the lower-quality firm anticipate being held up at a high price, and hence will consider switching to the higher-quality firm and buy the base only, without paying for the add-on. Consequently, the market share of the lower-quality firm falls. This further relaxes the higher-quality firm’s competitive pressure, thereby increasing its base price. In response, the lower-quality firm also increases its total price. Clearly, the higher-quality firm benefits from the add-on price being unobserved at the lower-quality firm. Although the lower-quality firm’s total price increases, its segment of consumers who pay for the expensive add-on shrinks. Eventually the firm’s profit drops.

This result suggests that vertically differentiated firms experience disparate incentives regarding whether to advertise add-on prices. The higher-quality firm has no incentive to do so because its higher-type consumers expect the add-on to be expensive because of their higher willingness-to-pay. Its lower-type consumers are uninterested in the add-on, so whether the price is advertised is irrelevant to them. Contrarily, the lower-quality firm has an incentive to advertise to its rival’s consumers that it has a better deal by lowering the bundle price. Despite the fact that the total price drops, the gains from acquiring new consumers who originally bought only the higher-quality base good and from persuading more lower-type consumers to buy the add-on, make advertising profitable. Given the incentive to advertise, the equilibrium will converge to the observed-price equilibrium.

5 Empirical Evidence

The theory is constructed to explain the motivating stylized fact that higher-end hotels are more likely to charge for Internet service, but its implications go beyond the hotel industry. There are of course other plausible explanations for the stylized fact. It is beyond the scope of the paper to test each of them. In what follows I provide some suggestive evidence that is consistent with the theoretical predictions.
Prediction. Comparing a monopoly market to a duopoly market with vertical differentiation when the cost-to-value ratio of the add-on is small, the theory suggests: (a) there is no difference between the monopolist and the higher-quality firm in the vertical duopoly; (b) the lower-quality firm in the duopoly is more likely than the others to sell the add-on as standard.

5.1 Data

The primary data source came from a lodging survey study conducted by the American Hotel and Lodging Association (AH&LA) every two years. The surveys present respondents (hotel managers) with a list of amenities, and ask whether their properties provide them. The list is comprehensive, ranging from in-room and bathroom amenities such as high-definition TV, coffee makers, and Internet services, to general services such as swimming pools, airport shuttles, and guest parking. For a few amenities (e.g., Internet service, breakfast, local calls), respondents report whether they are provided free. The surveys also obtain relevant hotel information such as open dates, and numbers of guest rooms and floors. The survey population includes all properties in the United States with 15 or more rooms (more than 52,000).

I obtained the individual-level data for the most recent four surveys (2006 to 2012) from the research company, Smith Travel Research (STR), which implemented the study on behalf of AH&LA. The total number of respondents for the four surveys was 25,179. The company groups hotels into five price segments of approximately equal size based on actual or estimated average room rates: luxury, upscale, mid-priced, economy, and budget. In addition, the company provides numbers of census properties and rooms within each zip code. There are 163 general markets and 11,154 zip codes in the data.

To isolate the hotels most likely to be monopolists or vertically differentiated duopolists, I focused on small markets with one or two hotels within a zip code. I obtained a dataset from the company that contained the number of hotels for each price segment within each

\[\text{The overall response rate is 23% with more than 12,000 participants, believed representative of the U.S. lodging industry.}\]

\[\text{Among them, 65% completed one survey, 24% two, 9% three, and 2% all four.}\]
zip code. This allowed me to identify monopoly and duopoly markets, and infer the vertical relationship within a duopoly market. There were 4,229 observations in these markets, 1,441 of which had no information concerning in-room Internet service. According to the company, not all properties completed all questions for a variety of reasons. One major reason could be that the surveys were long, fatiguing respondents. There is no strong evidence that would suggest the pattern of missing information correlates with the hotels’ incentives to offer free Internet service.\footnote{However, it is worth noting that lower-end hotels were less likely than higher-end hotels to answer questions on the surveys.}

Analysis was performed on the remaining 2,269 observations identified to provide Internet service, either free or otherwise.\footnote{I collected additional public data from a major online travel agency, Expedia, which had more complete information, and found very similar results. There were 93% of monopoly markets offering free Internet service, not statistically different from the higher-end hotels in duopoly markets (91%), but statistically lower than the lower-end hotels in duopoly markets (98%). Details of the data and analysis are not included in the appendix but are available upon request.}

There were 1,285 hotels in monopoly markets, 705 hotels at the higher end of vertical duopoly markets, and 279 hotels at the lower end of vertical duopoly markets. The 519 hotels in horizontal duopoly markets in which both hotels fall into the same price segment were excluded from analysis.\footnote{The relatively high ratio of vertical duopoly to horizontal duopoly is also an evidence that hotels are often vertically differentiated.}

5.2 Preliminary Results

Figure 1 visualizes the likelihood of offering free Internet service under various market conditions. 82% of the hotels in the monopoly markets provided free Internet service. Higher-end hotels in the duopoly markets were equally likely (81%) to offer Internet free. In contrast, the likelihood was much higher for lower-end hotels in the duopoly markets: 87% provided free Internet service. These descriptive statistics are consistent with the theoretical predictions. Notice that the absolute percentages are quite high across all three market conditions. The primary reason is that these markets (i.e., zip codes) with one or two hotels are most likely in suburban or small towns rather than in big cities. These hotels are less likely to be luxury or upscale, and hence are more likely to face upward competition from other markets.
Furthermore, it is likely that many higher-end hotels offer basic Internet access for free but charge for heavy uses. These hotels might report free Internet policy even though they were practicing price discrimination. Therefore, the observed difference in Internet policy between lower-end and higher-end hotels might be smaller than the actual difference, suggesting that the analysis is conservative.

A number of factors might have influenced Internet policies. For example, many hotels have VIP or club floors that target consumers with higher willingness-to-pay. The VIP floors typically charge customers for higher room rates, but provide additional benefits that likely include Internet access. These hotels are likely not to offer free Internet service to all consumers. Another example is that hotels vary in terms of number of rooms. A larger size likely increases setup, maintenance, or labor costs of supplying Internet service, or the cost of implementing price discrimination. Other potential confounding factors include the age, location (i.e., airport, interstate, resort, small town, suburban, or urban), and type of operation (i.e., chain operated, franchise, or independent) of a hotel.\(^{28}\)

I performed regression analysis and control for these possible confounds. The dependent variable was a binary indicator of whether a hotel offered free Internet service. Independent variables were dummies for three market conditions: monopoly, high-end in a vertical duopoly, and low-end in a vertical duopoly. Monopoly markets were treated as a benchmark group. The first column of Table 1 suggests that (a) the likelihood of offering free Internet service at a high-end hotel in a duopoly market was not significantly different from that at a monopoly hotel, and (b) the likelihood was significantly higher at a lower-end hotel in a duopoly market in comparison to a monopoly hotel. The second column of the table reports regression results after controlling for potential confounding factors. Conclusions remained robust even when these confounds were controlled. Effects of the confounds varied. As ex-

\(^{28}\) There were some hotels with missing information on VIP floor or age. These hotels were flagged in the regressions.
pected, hotels with a VIP floor were less likely to offer free Internet service. Larger hotels were also less likely to offer free Internet service. However, no trend was apparent over the eight years.

5.3 Higher- versus Lower-end Hotels within a Duopoly

One limitation of the preceding analysis is that market-specific factors are not controlled. This leads to a noisy comparison among hotels (e.g., a three-star hotel in Boston is compared to a four-star hotel in New York). It would be useful to examine whether the two hotels in a vertical duopoly behave differently, a more direct test of the stylized fact. To make within-market comparison, I restricted attention to duopoly markets in which both hotels reported their Internet policies. There were 86 such duopoly markets. As shown in Figure 3, 74% of the relatively higher-end hotels provided free Internet service, whereas 88% of the lower-end competing hotels provided it for free. A paired t-test suggests that this difference was statistically significant ($p = 0.022, t = 2.325$). This result suggests that in a market with vertical differentiation, the lower-quality firm is significantly more likely than its higher-quality competitor to sell an add-on as standard when it has a very small unit cost.

5.4 Restricting Analysis to Upscale Hotels

Another limitation of the preceding analysis is that the patterns found might be attributed to the differences in price segments. Next I focused on the segment of upscale hotels. A hotel in this restricted sample could be in the higher-end condition if there is a mid-priced or below hotel in the same zip code, or in the lower-end condition if there is a luxury hotel nearby. A hotel could also be the monopolist in a zip code. There were 646 observations, of which 355 were monopolists, 250 higher-end, and 41 lower-end.

Figure 2 summarizes the descriptive statistics. The third and fourth columns of Table 1 report the coefficients of the logistic regressions. The results are qualitatively similar to

29 In the preceding analysis, the objective was to compare duopoly markets to monopoly markets. It did not require both hotels in the same market reported their Internet policies.
the preceding analysis. Notice that the higher-end hotels appeared to be more likely to offer free Internet service than monopoly hotels did. This is most likely because hotels in the duopoly condition tend to be in urban or suburban areas, which potentially have more luxury hotels forcing them to free Internet service. The significance of the difference, however, was weakened when control variables such as location types were included.

Restricting the analysis to the same segment of upscale hotels allows us to rule out several alternative explanations. One is that higher-end hotels face more heterogeneous consumers than lower-end hotels do. Presumably the upscale hotels have very similar levels of consumer heterogeneity across different market conditions. The variation in heterogeneity is unlikely to explain the patterns observed here. It is also plausible that higher-end hotels tend to attract consumers who do not anticipate Internet charges, and thus charge a high Internet price to exploit myopic consumers. Again, the restriction to upscale hotels implies that the patterns are not attributed to the difference in consumer biases. Another alternative argument would be that implementing price discrimination costs more at lower-end markets than at higher-end markets due to efficiency of management, so lower-end hotels tend to offer free Internet service (no price discrimination). However, the costs presumably do not differ substantially across the upscale hotels in the sample, and hence the cost explanation does not seem to support the observed patterns.

6 Concluding Remarks

Motivated by the seemingly counter-intuitive phenomenon that higher-end hotels are more likely than lower-end ones to charge for Internet service, I examine the role of vertical differentiation in add-on policies, an element that has been left unexplored in the literature. The theory uncovers the differential roles of add-ons for vertically differentiated firms. Because the firms primarily competing for marginal consumers who trade off a higher-quality base

\[30\] The cost explanation is, to some extent, controlled by the variable hotel size (number of rooms in a hotel) in the regressions.
good alone versus a lower-quality base good including an add-on, the firm with the higher base quality focuses on screening consumers, whereas the lower-quality firm has to trade off screening and differentiation. Surprisingly, selling an add-on as optional can intensify competition, because the firms price aggressively to attract these marginal consumers.

These insights are particularly relevant to product policy decisions that managers face in many industries. When a firm sells an add-on in addition to a primary base product, managers must decide whether they should sell the add-on separately from the base as optional, or sell it as standard (i.e., free), or not sell it at all. Firms often face competition in the real-world, both horizontal and vertical. Ignoring any of the two aspects can lead to misleading policy recommendations. This paper sheds some light on what product policies a firm should adopt in the presence of vertical differentiation. Managers should evaluate their add-on policies based on the positioning of their base good. They should consider whether the base good is differentiated vertically from competitors before designing their add-on product policy. They should be cautious regarding whether an optional-add-on policy hurts profit under competition in the long run.

A number of questions remain unaddressed, and may be worth future investigation. First, the comparative statics implied by Proposition 2 with respect to the cost-to-value ratio of an add-on provide a direction for future empirical work. Detailed data with exogenous changes in the cost or value of an add-on are needed for such a study. Second, the proposition that selling an add-on as optional can hurt profits of vertically differentiated firms is an interesting hypothesis to test. Unfortunately, the data used in this paper were unsuitable to test it. Future empirical research into this topic is worth exploring. Third, the theory focuses on pure vertical differentiation, assuming away horizontal differentiation. In reality, firms are often differentiated both vertically and horizontally. Further research allowing for both aspects is interesting.
References


Figure 1: Vertical Duopoly vs. Monopoly in Small Markets

Figure 2: Vertical Duopoly vs. Monopoly in Small Markets (Upscale Hotels)

Figure 3: Higher- vs. Lower-end Within a Vertical Duopoly
### Table 1: Regression Results

<table>
<thead>
<tr>
<th></th>
<th>All Hotels</th>
<th>Upcale Hotels Only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Controls</td>
<td>With Controls</td>
</tr>
<tr>
<td>High-end in Duopoly</td>
<td>-0.020</td>
<td>0.101</td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
<td>(0.149)</td>
</tr>
<tr>
<td>Low-end in Duopoly</td>
<td>0.381**</td>
<td>0.416*</td>
</tr>
<tr>
<td></td>
<td>(0.191)</td>
<td>(0.226)</td>
</tr>
<tr>
<td>VIP Floor</td>
<td>-1.486***</td>
<td>-1.668***</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.320)</td>
</tr>
<tr>
<td>No Info on VIP Floor</td>
<td>-1.813***</td>
<td>-2.315***</td>
</tr>
<tr>
<td></td>
<td>(0.206)</td>
<td>(0.428)</td>
</tr>
<tr>
<td>Year 2008</td>
<td>-0.023</td>
<td>-0.388</td>
</tr>
<tr>
<td></td>
<td>(0.214)</td>
<td>(0.413)</td>
</tr>
<tr>
<td>Year 2010</td>
<td>0.271</td>
<td>0.202</td>
</tr>
<tr>
<td></td>
<td>(0.224)</td>
<td>(0.434)</td>
</tr>
<tr>
<td>Year 2012</td>
<td>-0.120</td>
<td>-0.306</td>
</tr>
<tr>
<td></td>
<td>(0.233)</td>
<td>(0.460)</td>
</tr>
<tr>
<td>Location - Interstate</td>
<td>-0.042</td>
<td>1.256</td>
</tr>
<tr>
<td></td>
<td>(0.422)</td>
<td>(0.781)</td>
</tr>
<tr>
<td>Location - Resort</td>
<td>-0.076</td>
<td>1.673*</td>
</tr>
<tr>
<td></td>
<td>(0.456)</td>
<td>(0.980)</td>
</tr>
<tr>
<td>Location - Small Town</td>
<td>-0.116</td>
<td>1.198*</td>
</tr>
<tr>
<td></td>
<td>(0.364)</td>
<td>(0.706)</td>
</tr>
<tr>
<td>Location - Suburban</td>
<td>-0.414</td>
<td>0.944</td>
</tr>
<tr>
<td></td>
<td>(0.321)</td>
<td>(0.675)</td>
</tr>
<tr>
<td>Location - Urban</td>
<td>-0.180</td>
<td>1.813**</td>
</tr>
<tr>
<td></td>
<td>(0.355)</td>
<td>(0.755)</td>
</tr>
<tr>
<td>Operation - Franchised</td>
<td>0.948***</td>
<td>0.229</td>
</tr>
<tr>
<td></td>
<td>(0.205)</td>
<td>(0.499)</td>
</tr>
<tr>
<td>Operation - Independent</td>
<td>0.676***</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td>(0.258)</td>
<td>(0.559)</td>
</tr>
<tr>
<td>Size - 75 to 149 rooms</td>
<td>-0.309</td>
<td>0.660*</td>
</tr>
<tr>
<td></td>
<td>(0.210)</td>
<td>(0.392)</td>
</tr>
<tr>
<td>Size - 150 to 299 rooms</td>
<td>-1.777***</td>
<td>-1.556***</td>
</tr>
<tr>
<td></td>
<td>(0.228)</td>
<td>(0.448)</td>
</tr>
<tr>
<td>Size - 300 to 500 rooms</td>
<td>-2.726***</td>
<td>-2.744***</td>
</tr>
<tr>
<td></td>
<td>(0.297)</td>
<td>(0.622)</td>
</tr>
<tr>
<td>Size - &gt;500 rooms</td>
<td>-3.447***</td>
<td>-3.753**</td>
</tr>
<tr>
<td></td>
<td>(0.603)</td>
<td>(1.605)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.003</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>No Info on Age</td>
<td>0.482</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.280)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** This table reports the coefficients of the logit model. The dependent variable is whether a hotel offers free Internet service (1 - yes, 0 - no). The benchmark group is the monopoly markets. Robust standard errors are reported. Significantly different from zero: * for p < .1, ** for p < .05, and *** for p < .01.
Online Appendix

to
Add-on Policies under Vertical Differentiation

List of Contents

A. Proof of Lemma

B. Proof of Proposition

C. Proof of Proposition

D. Proof of Proposition

E. Proof of Proposition

F. Proof of Proposition
A Proof of Lemma 1

When the Lower-quality Firm Does not Sell the Add-on

Suppose the higher-quality firm sells standard add-on at price $P_h^+$. Then the marginal consumer indifferent between the two firms is: $\hat{\theta}_{hl} = (P_h^+ - P_l)/(\Delta V + w)$. The higher-quality firm’s profit is $\Pi_h = (\bar{\theta} - \hat{\theta}_{hl})(P_h^+ - c)$. This strategy is exactly equivalent to separating the bundle price $P_h^+$ into a base price $P_h$ and an add-on price $p_h$ such that $P_h^+ = P_h + p_h$, and that

$$\frac{p_h}{w} = \frac{P_h^+ - P_l}{\Delta V + w} = \frac{P_h - P_l}{\Delta V}. \quad (A-1)$$

Consider a small increase of the add-on price to $p_h' = p_h + \epsilon$ with $\epsilon > 0$, and a small decrease of the base price to $P_h' = P_h - \epsilon$ by the same amount. The bundle price $P_h^+$ is then unchanged. The profit function now becomes

$$\Pi_h(P_h', p_h') = (\bar{\theta} - \frac{P_h' - P_l}{\Delta V})P_h' + (\bar{\theta} - \frac{p_h'}{w})(p_h' - c).$$

Evaluating this function around the original prices $(P_h, p_h)$ by Taylor series yields:

$$\Pi_h(P_h', p_h') = \Pi_h(P_h, p_h) - \epsilon \cdot \Pi_{P_h}(P_h, p_h) + \epsilon \cdot \Pi_{p_h}(P_h, p_h) + \frac{1}{2} \left[ \epsilon^2 \cdot \Pi_{P_hP_h}(P_h, p_h) + 2\epsilon \cdot \Pi_{P_hp_h}(P_h, p_h) + \epsilon^2 \cdot \Pi_{p_hp_h}(P_h, p_h) \right] + R_2$$

$$= \Pi_h(P_h, p_h) + \frac{\epsilon}{\Delta V} (P_h + \alpha \Delta V) - \frac{\epsilon^2}{w \Delta V} (\Delta V + w),$$

where the second equality follows from the fact that the remainder term $R_2$ is zero because the higher order derivatives are all zero, and Equation (A-1). Therefore, $M > 0$ for small $\epsilon$ when $P_l + \alpha \Delta V > 0$.

When the Lower-quality Firm Sells the Add-on

Suppose the higher-quality firm sells standard add-on at price $P_h^+$. Then the marginal consumer indifferent between the two firms is $\hat{\theta}_{hl} = (P_h^+ - P_l)/\Delta V$. The higher-quality firm’s profit becomes $\Pi_h = (\bar{\theta} - \hat{\theta}_{hl})(P_h^+ - c)$. This strategy is exactly equivalent to separating the bundle price $P_h^+$ into a base price $P_h$ and an add-on price $p_h$ such that $P_h^+ = P_h + p_h$, and that

$$\frac{p_h}{w} = \frac{P_h^+ - P_l^+}{\Delta V} = \frac{P_h - P_l^+}{\Delta V - w}. \quad (A-2)$$
Consider a small increase of the add-on price to \( p'_h = p_h + \epsilon \), and a small decrease of the base price to \( P'_h = P_h - \epsilon \) by the same amount. The bundle price \( P^+_h \) is then unchanged.

The profit function now becomes

\[
\Pi_h(P'_h, p'_h) = (\theta - \frac{P'_h - P^+_l}{\Delta V - w})P'_h + (\theta - \frac{p'_h}{w})(p'_h - c).
\]

Evaluating this function around the original prices \((P_h, p_h)\) by Taylor series yields:

\[
\Pi_h(P'_h, p'_h) = \Pi_h(P_h, p_h) - \epsilon [\theta - \frac{P_h - P^+_l}{\Delta V - w}] + \epsilon [\theta - \frac{p_h}{w}] + \frac{\epsilon^2}{2} \left( -\frac{2}{\Delta V - w} - \frac{2}{w} \right) w
\]

where the last equality follows from Equation (A-2). Therefore, \( M > 0 \) for small positive \( \epsilon \) when \( L^+_l + \alpha(\Delta V - w) > 0 \).

B Proof of Proposition 1

Suppose the statement is not true, then in equilibrium the higher-quality firm either (1) sells standard add-on, or (2) does not sell the add-on at all. In each case there are two possible equilibria, depending on whether the low quality firm sells the add-on. Next I show that in each case the higher-quality firm finds it profitable to deviate to selling optional add-on.

When the Higher-quality Firm Sells Standard Add-on

Case (a): the lower-quality firm sells the add-on. The marginal consumer indifferent between the two firms is \( \hat{\theta}^{hl} = \frac{(P^+_h - P^+_l)}{\Delta V} \). Equilibrium prices are

\[
P^{**}_h = \frac{1}{3}\Delta V(2\bar{\theta} - \theta) + c, \quad p^{**}_h < w\hat{\theta}^{hl}; \quad P^{**}_l = \frac{1}{3}\Delta V(\bar{\theta} - 2\bar{\theta}) + c, \quad p^{**}_l < w\hat{\theta}^{hl},
\]

with threshold \( \hat{\theta}^{*}_{hl} = (\bar{\theta} + \theta)/3 > \bar{\theta} \). Notice that whether or not the lower-quality firm sells only the base or the bundle to its lowest-type consumers (i.e., whether \( p^{*}_l > \bar{\theta} \)) does not affect the argument below. Because \( P^{**}_l > 0 \), by Lemma 1 the higher-quality firm can profitably deviate to selling optional add-on by increasing the add-on price while lowering the base price.

Case (b): the lower-quality firm does not sell the add-on. The marginal consumer indif-
ferent between the two firms is \( \hat{\theta}_{hl} = (P_h^+ - P_l)/(\Delta V + w) \). Equilibrium prices are

\[
P_{h}^{**} = \frac{1}{3}(\Delta V + w)(2\bar{\theta} - \theta) + \frac{2}{3}c, \quad p_h^* < w\hat{\theta}_{hl}; \quad P_l^* = \frac{1}{3}(\Delta V + w)(\bar{\theta} - 2\bar{\theta}) + \frac{1}{3}c, \quad p_l^* > w\hat{\theta}_{hl},
\]

with threshold \( \hat{\theta}^* = (\bar{\theta} + \theta)/3 + c/(3\Delta V + 3w) > \theta \). Because \( P_l^* > 0 \), by Lemma 1 the higher-quality firm can profitably deviate to selling optional add-on by increasing the add-on price while lowering the base price.

**When the Higher-quality Firm Does not Sell the Add-on**

The higher-quality firm’s profit becomes \( \Pi_h = (\bar{\theta} - \hat{\theta}_{hl})P_h \). The threshold \( \hat{\theta}_{hl} \) is either \((P_h - P_l)/(\Delta V - w)\) or \((P_h - P_l)/\Delta V\), depending on whether the lower-quality firm sells the add-on to its highest-type consumers. The best-response prices lead to a threshold \( \hat{\theta}_{hl}^* < \bar{\theta} \). The higher-quality firm would need to set a high add-on price \( p_h^* > \bar{\theta}w \) so that no one buys the add-on. However, it is always profitable for the higher-quality firm to lower its add-on price \( p_h < \bar{\theta}w \) without affecting the profit from selling the base as long as \( p_h > w\hat{\theta}_{hl}^* \). Therefore, no equilibrium exists where the higher-quality firm sells only the base.

**C Proof of Proposition 2**

I first establish a lemma for the lower-quality firm and then proceed to prove the two cases of the proposition.

**C.1 A Lemma for the Lower-quality Firm**

**Lemma 2.** For the lower-quality firm, not selling the add-on while leaving some consumers to buy from the higher-quality firm is strictly dominated by selling the add-on, if:

1. \( P_h > \alpha \Delta V \) when the higher-quality firm sells optional add-on, or
2. \( P_h^+ > \alpha(\Delta V + w) \) when the higher-quality firm sells standard add-on.

The proof of the lemma entails a similar argument as in the proof of Lemma 1: (1) assume the low-quality firm sells only the base, (2) find the equivalent bundle price and add-on price, (3) consider a small deviation by lowering the bundle price and add-on price, (4) show that the deviation is profitable. The detail of the proof is available upon request.

**C.2 Equilibrium When \( \alpha \geq \frac{1}{3}(2\bar{\theta} - \theta) \)**

In this case the equilibrium profile consists of the following pricing strategies:

\[
P_h^* = \frac{1}{3}(2\bar{\theta} - \theta)\Delta V, \quad p_h^* = w\hat{\theta}_h^*; \quad P_l^* = \frac{1}{3}(\bar{\theta} - 2\bar{\theta})\Delta V, \quad p_l^* > w\hat{\theta}_{hl}^*.
\]
where the thresholds are given by \( \hat{\theta}_{hl}^* = \frac{1}{3}(\bar{\theta} + \bar{\theta}) \) and \( \hat{\theta}_h^* = \frac{1}{2}(\bar{\theta} + \alpha) \). Next I establish that there is no profitable deviation for either firm.

**No Profitable Deviation for the Lower-quality Firm**

This is established by considering three possible non-local deviations.

*Case (a): the lower-quality firm does not sell the add-on and leaves no consumers buy \( H \).* The marginal consumer indifferent between the two firms is \( \hat{\theta}_{hl}^* = \frac{P^*_h - P^*_l}{\Delta V + w} \). For this situation to arise, we need that \( \hat{\theta}_{hl}^* \geq \frac{p^*_h}{w} \) so that no consumer buys \( H \). By contrast the marginal consumer in the equilibrium profile satisfies:

\[
\hat{\theta}_{hl}^* < \hat{\theta}_h^* \iff \frac{P^*_h - P^*_l}{\Delta V + w} < \frac{p^*_h}{w},
\]

(C-1)

The lower-quality firm’s profit is given by \( \Pi_l = (\hat{\theta}_{hl}^* - \hat{\theta}_l^*)P_l \). It turns out that the optimal profit under this case is achieved by the corner solution \( \hat{\theta}_{hl}^* = \frac{p^*_h}{w} \). To see this, note that the first order condition of the Lagrangian is given by \( P_l = \frac{P^*_h - P^*_l}{\Delta V + w} - \frac{p^*_h}{w} \). The constraint becomes:

\[
\hat{\theta}_{hl} - \frac{p^*_h}{w} = \frac{P^*_h - P^*_l}{\Delta V + w} - \frac{p^*_h}{w} < \frac{P^*_l - P^*_l}{\Delta V + w} = \frac{1}{2(\Delta V + w)}(-p^*_h + w\bar{\theta} + \lambda),
\]

where the inequality follows from Equation (C-1) and the last equality is obtained by substituting the best responses of the lower-quality firm. Because \( p^*_h > w\bar{\theta} \), to guarantee the constraint is nonnegative, we need \( \lambda > 0 \). This implies that the constraint has to be binding by complementary slackness. This corner solution coincides with the corner solution for the problem under equilibrium because:

\[
\frac{P^*_h - P^*_l}{\Delta V + w} = \frac{p^*_h}{w} \implies \frac{P^*_l}{\Delta V} = \frac{p^*_h}{w}.
\]

Therefore the deviation does not improve the profit.

*Case (b): the lower-quality firm sells the add-on and leaves some consumers buy \( H \).* The market is divided into four consumer segments: the consumers \( \theta \in [\hat{\theta}_h, \bar{\theta}] \) buy \( H^+ \), the consumers \( \theta \in [\hat{\theta}_{hl}, \hat{\theta}_h] \) buy \( H \), the consumers with \( \theta \in [\hat{\theta}_l, \hat{\theta}_{hl}] \) buy \( L^+ \), and the consumers with \( \theta \in [\bar{\theta}, \hat{\theta}_l] \) buy \( L \). The thresholds are given by \( \hat{\theta}_h = \frac{p^*_h}{w}, \hat{\theta}_{hl} = \frac{(P^*_h - P^*_l - p_l)}{(\Delta V - w)}, \) and \( \hat{\theta}_l = \frac{p_l}{w} \). It will later become evident that when the lower-quality firm sells the add-on to all of its consumers (i.e., \( \hat{\theta}_l < \bar{\theta} \)) the profit is not improved. Given the higher-quality firm’s equilibrium pricing the lower-quality firm maximizes profit \( \Pi_l = (\hat{\theta}_{hl} - \theta)(P^+_l - c) - \).
\((\hat{\theta}_l - \bar{\theta})(p_l - c)\) subject to the constraints: (1) \(\hat{\theta}_hl \geq \hat{\theta}_l\), (2) \(\hat{\theta}_l \geq \bar{\theta}\), and (3) \(\hat{\theta}_h \geq \hat{\theta}_hl\).

The objective function is concave and thus the necessary and sufficient condition for the optimization program is Karush-Kuhn-Tucker conditions. Let \(\lambda_1\), \(\lambda_2\), and \(\lambda_3\) be the Lagrangian multipliers. The first-order conditions lead to \(P^+_l = (P^*_h - (\Delta V - w)\bar{\theta} + c - \lambda_1 + \lambda_3)/2\) and \(p_l = (w\bar{\theta} + c - \lambda_1 + \lambda_2)/2\). Therefore, Constraint (1) is given by

\[
\hat{\theta}_hl - \hat{\theta}_l = \frac{1}{2w(\Delta V - w)} \left[ w\Delta V \left( \frac{1}{3}(2\bar{\theta} - \theta) - \alpha \right) - \lambda_2(\Delta V - w) - \lambda_3 w + \lambda_1 \Delta V \right].
\]

Because \(\alpha > (2\bar{\theta} - \theta)/3\) and both \(\lambda_2\) and \(\lambda_3\) are nonnegative, the term \(M\) in the bracket is strictly negative \((M < 0)\). To guarantee the constraint is nonnegative, we need \(\lambda_1 > 0\). This implies that Constraint (1) has to be binding. Therefore the profit is no greater than that obtained from not selling the add-on (i.e., the equilibrium strategy).

It remains to verify that the lower-quality firm would not deviate to selling the add-on to all of its consumers. In this case its profit is \(\Pi_l = (\hat{\theta}_hl - \bar{\theta})(P^+_l - c)\). This profit can be strictly improved if the firm sets the add-on price \(p_l\) to be anywhere between \([\bar{\theta}w, c]\) so that some consumers decide not to buy the add-on. The incremental benefit is equal to \(- (\hat{\theta}_l - \bar{\theta})(p_l - c)\) which is strictly positive.

\textit{Case (c): the lower-quality firm sells the add-on but leaves no consumers buy} \(H\). The marginal consumer indifferent between the two firm is \(\hat{\theta}_hl = (P^*_{hl} - P^+_l)/\Delta V\). To ensure that no consumers buy \(H\), it is necessary that \(\hat{\theta}_hl \geq p^*_l/w\). Consider that the lower-quality firm maximizes profit \(\Pi_l = (\hat{\theta}_hl - \bar{\theta})(P^+_l - c) - (\hat{\theta}_l - \bar{\theta})(p_l - c)\). The profit differs from Case (b) only in terms of the profit of selling the bundle, \((\hat{\theta}_hl - \bar{\theta})(P^+_l - c)\). Maximizing this bundling profit under the constraint yields \(P^+_l = (P^*_{hl} - \Delta V\bar{\theta} + c - \lambda)/2\). Substituting this best response back to the constraint gives

\[
\hat{\theta}_hl - \frac{p^*_l}{w} = \frac{P^*_{hl} - P^+_l}{\Delta V} - \frac{p^*_l}{w} = \frac{1}{2\Delta V} \left[ -\frac{1}{3}(\bar{\theta} - 2\theta + 3\alpha)\Delta V + \frac{1}{2}(\bar{\theta} - \alpha)w + \lambda \right].
\]

Because \(M < 0\) when \(\alpha > (2\bar{\theta} - \theta)/3\), for the constraint to be nonnegative, it is necessary that \(\lambda > 0\). This implies that the constraint has to be binding by complementary slackness. Therefore the profit is no greater than that obtained from Case (b), which is not a profitable deviation.
No Profitable Deviation for the Higher-quality Firm

Case (a): the higher-quality firm sells optional add-on and leaves some consumers buy $L^+$. The marginal consumer indifferent between the two firm becomes $\hat{\theta}_{hl} = (P_h - P_l^* - p_l^*) / (\Delta V - w)$. Consider the higher-quality firm’s maximization problem $\Pi_h = (\bar{\theta} - \hat{\theta}_{h1})P_h + (\bar{\theta} - \hat{\theta}_h)(p_h - c)$ subject to the constraint $\hat{\theta}_{hl} \geq p_l^*/w$. Notice that the profit function is the same as the equilibrium one except the threshold $\hat{\theta}_{hl}$. It then suffices to compare the profit from selling the base only. The first-order condition of the constrained problem is given by $P_h = (P_l^* + p_l^* + (\Delta V - w)\bar{\theta} + \lambda) / 2$. Therefore,

$$\hat{\theta}_{hl} - \frac{p_l^*}{w} = \frac{1}{2(\Delta V - w)} \left[ \frac{2}{3} (\bar{\theta} + \hat{\theta}) \Delta V - \bar{\theta}w - \frac{p_l^*}{w} (2\Delta V - w) + \lambda \right]$$

$$\leq \frac{1}{2(\Delta V - w)} \left[ \frac{2}{3} (\bar{\theta} + \hat{\theta}) \Delta V - \bar{\theta}w - \frac{1}{3} (\bar{\theta} + \hat{\theta})(2\Delta V - w) + \lambda \right]$$

$$= \frac{1}{2(\Delta V - w)} (-\frac{1}{3} (2\bar{\theta} - \hat{\theta})w + \lambda),$$

where the inequality holds when $p_l^* > (\bar{\theta} + \hat{\theta})w/3$. Because $(2\bar{\theta} - \hat{\theta})$ is positive, to ensure the constraint is nonnegative it is necessary that $\lambda > 0$. By complementary slackness the constraint must be binding, implying that the higher-quality firm leaves no demand for $L^+$. Therefore, the optimal profit obtained in this case is no greater than the equilibrium profit.

Case (b): the higher-quality firm sells standard add-on and leaves some consumers buy $L^+$. Because $P_l^+ > 0$, by Lemma $[1]$ this strategy is strictly dominated by a local deviation whereby the higher-quality firm sells optional add-on by increasing the add-on price while lowering the base price. This deviation is exactly in the form of Case (a), which yields no greater profit than the equilibrium one.

Case (c): the higher-quality firm sells standard add-on but leaves no consumers buy $L^+$. Because $P_l^* > 0$, by Lemma $[1]$ this strategy is strictly dominated by a local deviation whereby the higher-quality firm sells optional add-on by increasing the add-on price while lowering the base price. This local deviation is exactly in the form of the maximization problem in the equilibrium. Therefore, there is no profitable deviation.

Case (d): the higher-quality firm does not sell the add-on at all. This deviation can take two forms. First, the higher-quality firm can leave no consumer buy the add-on from the lower-quality firm. The optimal base price is then the same as the equilibrium strategy $P_h^*$. However, since the deviation forgoes the profit from selling the add-on, the deviation profit is smaller. Second, the higher-quality firm can leave some consumers buy the add-on from the
lower-quality firm. The profit is smaller than that in case (a) because a small positive sales of the add-on can improve profit without affecting profit from the base good. Therefore, in either case, the deviation is not profitable.

C.3 Equilibrium When \( \alpha < \frac{1}{3}(2\bar{\theta} - \theta) \)

Recall that the equilibrium profile consists of the following pricing strategies:

\[
P_h^* = \frac{1}{3}(2\bar{\theta} - \theta)(\Delta V - w) + \frac{1}{3}c, \quad p_h^* = w\hat{\theta}_h^*, \quad P_l^{++*} = \frac{1}{3}(\bar{\theta} - 2\theta)(\Delta V - w) + \frac{2}{3}c,
\]

and \( p_i^* = w\hat{\theta}_i^* \) if \( \alpha > \bar{\theta} \) but \( p_i^* \leq w\hat{\theta}_i^* \) if \( \alpha \leq \bar{\theta} \). The thresholds are given by \( \hat{\theta}_hl = (\bar{\theta} + \alpha)/2, \hat{\theta}_h^* = (\bar{\theta} + \alpha)/2, \) and \( \hat{\theta}_l = (\bar{\theta} + \alpha)/2 \). The next two subsections establish the second and third statements of the proposition by showing that neither firm will deviate when \( \Delta V > \max\{\Delta_1, \Delta_2\} \). The last subsection shows that no pure strategy equilibrium exists when \( \Delta V \leq \max\{\Delta_1, \Delta_2\} \).

No Profitable Deviation for the Lower-quality Firm

Case (a): the lower-quality firm does not sell the add-on while leaving some consumers to buy \( H \). By Lemma 2 this is not a profitable deviation as long as \( P_h^* > \alpha \Delta V \). Notice that

\[
P_h^* - \alpha \Delta V = \frac{1}{3}[(2\bar{\theta} - \theta - 3\alpha)\Delta V - (2\bar{\theta} - \theta - \alpha)w],
\]

which is positive exactly when \( \alpha < \frac{1}{3}(2\bar{\theta} - \theta) \) and \( \Delta V > \Delta_1 \). The deviation is not profitable.

Case (b): the lower-quality firm does not sell the add-on and leaves no consumers to buy \( H \). By Lemma 2 this is not a profitable deviation as long as \( P_h^{++*} > \alpha(\Delta V + w) \). Notice that

\[
P_h^{++*} - \alpha(\Delta V + w) = \frac{1}{3}[(2\bar{\theta} - \theta - 3\alpha)\Delta V - (1/2\bar{\theta} - \theta + 1/2 \alpha)w] > \frac{1}{3}[(2\bar{\theta} - \theta - 3\alpha)\Delta V - (2\bar{\theta} - \theta - \alpha)w],
\]

where the inequality follows from \( \alpha < \bar{\theta} \). The last equation is again positive when \( \alpha < \frac{1}{3}(2\bar{\theta} - \theta) \) and \( \Delta V > \Delta_1 \).

Case (c): the lower-quality firm sells the add-on but leaves no consumers to buy \( H \). The marginal consumer indifferent between the two firms now becomes \( \hat{\theta}_hl = (P_h^{++*} - P_i^+)/\Delta V \). To ensure that no one buys \( H \), the lower-quality firm has to set the bundle price \( P_i^* \) low enough so that \( \hat{\theta}_hl \geq p_h^*/w \). By contrast the marginal consumer in the equilibrium profile satisfies:

\[
\hat{\theta}_hl < \hat{\theta}_h^* \iff \frac{P_h^* - P_i^*}{\Delta V - w} < \frac{p_h^*}{w} \Rightarrow \frac{P_h^{++*} - P_i^{++*}}{\Delta V} < \frac{p_h^*}{w}.
\]

(C-2)
The lower-quality firm’s profit is given by \( \Pi_l = (\hat{\theta}_hl - \theta)(P_l^+ - c) - (\hat{\theta}_l - \theta)(p_l - c) \). This profit is different from the equilibrium one only in terms of the bundling component \((\hat{\theta}_hl - \theta)(P_l^+ - c)\).

The first order condition of the Lagrangian for the constrained maximization of the bundling profit is given by \( P_l^+ = (P_h^* - \Delta V \bar{\theta} + c - \lambda)/2 \). The constraint becomes

\[
\hat{\theta}_hl - \frac{p_h^*}{w} = \frac{P_h^* - P_l^+}{\Delta V} - \frac{p_h^*}{w} < \frac{P_h^* - P_l^+}{\Delta V} = \frac{1}{2\Delta V}(-p_h^* + w\bar{\theta} + \lambda),
\]

where the inequality follows from Equation (C-2) and the last equality is obtained by substituting the best responses of the lower-quality firm. Because \( p_h^* > w\bar{\theta} \), to guarantee the constraint is nonnegative, we need \( \lambda > 0 \). This implies that the constraint has to be binding. This corner solution coincides with the one for the problem under the equilibrium because

\[
\frac{P_h^* - P_l^+}{\Delta V} = \frac{p_h^*}{w} \Rightarrow \frac{P_h^* - P_l^+}{\Delta V} = \frac{p_h^*}{w}.
\]

Hence, the deviation profit is no greater than that obtained from the equilibrium strategy.

**No Profitable Deviation for the Higher-quality Firm**

*Case (a): the higher-quality firm sells optional add-on but leaves no demand for \( L^+ \).* The marginal consumer indifferent between the two firms is \( \hat{\theta}_hl = (P_h - P_l^*)/\Delta V \). The high-quality firm maximizes profit \( \Pi_h = (\bar{\theta} - \hat{\theta}_hl)P_h + (\bar{\theta} - \hat{\theta}_h)(p_h - c) \) subject to the constraint, \( \hat{\theta}_hl \leq \frac{p_l^*}{w} \), so that there is no demand for consumers who buy \( L^+ \). Note that the profit is also separable into a component for the base good and a component for the add-on price. It suffices to examine the profit component of the base good which is different from the equilibrium one. The first-order condition for the constrained optimization problem of the base profit is \( P_h = (P_l^* + \Delta V \bar{\theta} - \lambda)/2 \). There are two cases to consider. First, if \( \alpha > \bar{\theta} \), then the equilibrium add-on price is \( p_l^* = (\bar{\theta}w + c)/2 \) and satisfies \( \hat{\theta}_hl > \hat{\theta}_l^* \). The constraint becomes

\[
\frac{p_l^*}{w} - \hat{\theta}_hl = \frac{p_l^*}{w} - \frac{P_h - P_l^*}{\Delta V} < \frac{P_h^* - P_h}{\Delta V} = \frac{1}{2\Delta V}(p_l^* - \bar{\theta}w + \lambda),
\]

where the inequality follows from \( \hat{\theta}_hl > \hat{\theta}_l^* \), and the last equality is obtained by substituting the best responses of the higher-quality firm. Because \( p_l^* < \bar{\theta}w \), for the constraint to be nonnegative we need \( \lambda > 0 \), implying that the constraint must be binding. The corner solution is the same as the one under the equilibrium problem because:

\[
\frac{P_h - P_l^*}{\Delta V} = \frac{p_l}{w} \Rightarrow \frac{P_h - P_l^+}{\Delta V} = \frac{p_l}{w}.
\]
Therefore the deviation profit is no greater than the equilibrium profit.

Second, if \( \alpha \leq \theta \), then \( p_1^* \leq \theta w \). There is no demand for the lower-quality firm at all. The equilibrium strategy is optimal.

**Case (b): the higher-quality firm sells standard add-on and leaves some demand for \( L^+ \).** Because \( P_l^{**} > 0 \), by Lemma 1 this strategy is strictly dominated by selling optional add-on by increasing the add-on price and lowering the base price. This is exactly the equilibrium strategy. Therefore, deviation in this case is not profitable.

**Case (c): the higher-quality firm sells standard add-on to and leaves no demand for \( L^+ \).** Because \( P_l^* > 0 \), by Lemma 1 this strategy is strictly dominated by selling optional add-on by increasing the add-on price and lowering the base price. This is exactly the strategy in Case (a), the profit of which is no greater than the equilibrium profit.

**Case (d): the higher-quality firm does not sell the add-on at all.** This deviation can take two forms. First, the higher-quality firm can leave no one buy the add-on from the lower-quality firm. The profit is smaller than that in case (a) because a small positive sales of the add-on can improve profit without affecting profit from the base good. Second, the higher-quality firm can leave some consumers buy the add-on from the lower-quality firm. The optimal base price is then the same as the equilibrium strategy \( P_h^* \). However, since the deviation forgoes the profit from selling the add-on, the deviation profit is smaller than the equilibrium profit. Therefore, in either case, the deviation is not profitable.

**No Pure Strategy Equilibrium Exists When \( \Delta V \leq \max\{\Delta_1, \Delta_2\} \)**

Because in equilibrium the higher-quality firm will always sell the add-on as optional by Proposition 1, there are two possible equilibrium outcomes depending on the lower-quality firm’s implementation: (a) the lower-quality firm sells the add-on, and (b) the firm does not sell the add-on at all. Neither case is sustainable when \( \Delta V \leq \max\{\Delta_1, \Delta_2\} \).

**Case (a): the lower-quality firm sells the add-on.** The equilibrium profiles are the same as those in the second and the third statements of the proposition. When \( \alpha > \bar{\theta} \), both firms sell optional add-on. The fraction of consumers who buy the bundle from the lower-quality firm is given by

\[
\hat{\theta}_l - \hat{\theta}_l^* = \frac{1}{6(\Delta V - w)} \left[ (2\bar{\theta} - \theta - 3\alpha)\Delta V - (2\bar{\theta} - \theta - \alpha)w \right],
\]

which is positive only if \( \Delta V > \Delta_1 \).\(^{31}\)

When \( \alpha \leq \bar{\theta} \), the lower-quality firm will sell the add-on as standard, and the fraction of \(^{31}\)Note that \( \Delta_1 > \Delta_2 \) when \( \alpha > \bar{\theta} \), and \( \Delta_2 > \Delta_1 \) when \( \alpha < \bar{\theta} \).
consumers who buy the bundle from the lower-quality firm becomes

\[ \hat{\theta}_{hl}^* - \bar{\theta} = \frac{1}{3(\Delta V - w)} \left[ (\bar{\theta} - 2\bar{\theta}) \Delta V - (\bar{\theta} - 2\bar{\theta} + \alpha)w \right]. \]

which is positive only if \( \Delta V > \Delta_2 \).

Case (b): the lower-quality firm does not sell the add-on at all. In this case, the equilibrium profile is the same as that in the first statement of the proposition. The higher-quality firm’s base price is then \( P_h^* = (2\bar{\theta} - \bar{\theta}) \Delta V/3 \). Since \( \alpha < (2\bar{\theta} - \bar{\theta})/3 \), we have \( P_h^* > \alpha \Delta V \). Therefore, by Lemma 2 the lower-quality firm can profitably deviate by selling the add-on.

D Proof of Proposition 3

D.1 Second-stage Pricing Game

First observe that the higher-quality firm will never commit to no-add-on policy. By committing to not selling the add-on, the higher-quality firm always uses the base good to compete with the lower-quality firm in the second stage. Deviating to not making the commitment allows it to sell the add-on to a fraction of consumers who have the highest taste for quality. This does not affect the profit from selling the base good but can gain some additional benefit from selling the add-on. Profit is then strictly improved. Therefore, committing to no-add-on policy is always strictly dominated by not committing.

Second, when the higher-quality firm does not commit to standard add-on, it retains the flexibility of selling the add-on as optional. The equilibrium outcomes are the same as those summarized in Proposition 2. The difference is that here the lower-quality firm can not deviate to selling optional add-on in the second-stage pricing game once it has made the commitment to either standard-add-on or no-add-on policies. The equilibrium outcome depends on each of the three commitment choices of the lower-quality firm.

When the lower-quality firm commits to no-add-on policy, the equilibrium pricing in the second stage is the same as that in the first case of Proposition 2. The resulting profits are given by \( \Pi_h^{(opt,\text{opt})} \) and \( \Pi_l^{(opt,\text{opt})} \).

When the lower-quality firm commits to standard-add-on policy, the equilibrium pricing is the same as that in the third case of Proposition 2. The resulting profits are given by \( \Pi_h^{(opt,\text{std})} \) and \( \Pi_l^{(opt,\text{std})} \).

When the lower-quality firm does not commit, the equilibrium pricing is the same as that in the second case of Proposition 2. The resulting profits are given by \( \Pi_h^{(opt,\text{no})} \) and \( \Pi_l^{(opt,\text{no})} \).

Third, when the higher-quality firm commits to standard-add-on policy, it commits to a
uniform price $P^*_h$ that bundles the base good and the add-on. In the second-stage pricing game, the firm cannot sell the add-on separately to screen consumers. The equilibrium outcome depends on each of the three commitment choices of the lower-quality firm.

When the lower-quality firm commits to no-add-on policy, the equilibrium pricing in the second stage now becomes

$$P^*_h = \frac{1}{3}(2\bar{\theta} - \theta)(\Delta V + w) + \frac{2}{3}c, \quad P^*_l = \frac{1}{3}(\bar{\theta} - 2\theta)(\Delta V + w) + \frac{1}{3}c,$$

and the marginal consumer is given by $\hat{\theta}^*_{hl} = (\bar{\theta} + \theta)/3 + c/(3\Delta V + 3w)$. The incentive constraints $\theta > \hat{\theta}^*_hl > \theta$ hold given that $\theta > 2\theta$. The resulting profits are given by $\Pi^{(std,no)}_h$ and $\Pi^{(std,no)}_l$.

When the lower-quality firm commits to standard-add-on policy, the second-stage pricing becomes

$$P^*_h = \frac{1}{3}(2\bar{\theta} - \theta)\Delta V + c, \quad P^*_l = \frac{1}{3}(\theta - 2\bar{\theta})\Delta V + c,$$

and the marginal consumer is given by $\hat{\theta}^*_{hl} = (\bar{\theta} + \theta)/3$. The incentive constraints $\bar{\theta} > \hat{\theta}^*_hl > \bar{\theta}$ hold given that $\bar{\theta} > 2\theta$. The resulting profits are given by $\Pi^{(std,std)}_h$ and $\Pi^{(std,std)}_l$.

When the lower-quality firm does not commit, the second-stage pricing becomes

$$P^*_h = \frac{1}{3}(2\bar{\theta} - \theta)\Delta V + c, \quad P^*_l = \frac{1}{3}(\bar{\theta} - 2\theta)\Delta V + c, \quad p^*_l = w\hat{\theta}^*_l,$$

where the marginal consumers are given by $\hat{\theta}^*_hl = (\bar{\theta} + \theta)/3$ and $\hat{\theta}^*_l = (\theta + \alpha)/2$. The incentive constraints $\bar{\theta} > \hat{\theta}^*_hl > \hat{\theta}^*_l > \theta$ hold given that $\bar{\theta} > 2\theta$ and $\bar{\theta} < \alpha < (2\bar{\theta} - \theta)/3$. The resulting profits are given by $\Pi^{(std, opt)}_h$ and $\Pi^{(std, opt)}_l$.

D.2 First-stage Commitment Choices

The Higher-quality Firm Commits to Standard Add-on

First, when the lower-quality firm commits to not selling the add-on, the higher-quality firm is better off to commit to standard add-on because the improvement in profit is positive

$$\Pi^{(std,no)}_h - \Pi^{(opt,no)}_h = \frac{w}{9} \left[ (2\bar{\theta} - \theta)^2 - 2\alpha(2\bar{\theta} - \theta) - \frac{9}{4}(\bar{\theta} - \alpha)^2 + \frac{\alpha^2w}{\Delta V + w} \right] > 0.$$

Second, when the lower-quality firm sells the add-on, the higher-quality firm’s profit is unaffected by whether its rival commits to standard add-on or not. This is because in either
case the higher-quality firm is competing with its rival’s bundle good. The firm, however, benefits from committing to standard add-on as long as the quality premium is large enough. The improvement in profit becomes

$$\Pi_h^{(\text{std,opt})} - \Pi_h^{(\text{opt,opt})} = \frac{w}{9} \left[ \frac{(2\bar{\theta} - \bar{\theta})^2}{\Delta V - w} - \frac{2\theta(2\bar{\theta} - \bar{\theta})}{\Delta V - w} - \frac{9}{4}(\theta - \bar{\theta})^2 - \frac{\alpha^2w}{\Delta V - w} \right].$$

(D-1)

which is increasing in the quality premium \(\Delta V\) and is positive if \(\Delta V = (1 + \alpha^2/X)w \equiv \Delta_3\).

**The Lower-quality Firm Commits to No Add-on**

Given that the higher-quality firm commits to standard add-on, it remains to consider the three possibilities depending on the lower-quality firm’s commitment choices. First, note that committing to standard add-on is dominated by not committing, \(\Pi_l^{(\text{std, opt})} > \Pi_l^{(\text{std, std})}\). This is because the lower-quality firm is selling the add-on to its higher-type consumers anyway, the commitment choice does not matter for the competition with its rival. This is very different from the higher-quality firm’s problem.

Second, selling optional add-on is dominated by committing to not selling it. To see that, the benefit from commitment is given by

$$\Pi_l^{(\text{std, no})} - \Pi_l^{(\text{std, opt})} = \frac{w}{9} \left[ \frac{(\bar{\theta} - 2\bar{\theta})^2}{\Delta V - w} - \frac{2\theta(\bar{\theta} - 2\bar{\theta})}{\Delta V - w} - \frac{9}{4}(\theta - \bar{\theta})^2 + \frac{\alpha^2w}{\Delta V - w} \right].$$

(D-2)

which is also increasing in the quality premium \(\Delta V\). It can be shown that \(Y > 0\) when \(\theta < \alpha < \frac{1}{3}(2\bar{\theta} - \bar{\theta})\). Therefore, \(\Pi_l^{(\text{std, no})} > \Pi_l^{(\text{std, opt})}\).

Therefore, there exists an equilibrium of the full game such that the higher-quality firm commits to standard add-on, while the lower-quality firm commits to no add-on, provided the quality premium is large enough such that \(\Delta V > \max\{\Delta_1, \Delta_3\}\).

**E Proof of Proposition 4**

**E.1 When \(\alpha < (\bar{\theta} + \theta)/3\)**

For the equilibrium in which the low-quality firm sells the add-on as *optional* to exist, the following incentive constraints have to be satisfied: (a) \(\hat{\theta}_{hl}^* > \hat{\theta}_{hl}^*\); (b) \(\hat{\theta}_{lh}^* > \theta \Rightarrow \hat{\theta}_{hl}^* > 2\bar{\theta} - \alpha\).

For the equilibrium in which the low-quality firm sells the add-on as *standard* to exist, it requires that (c) \(\hat{\theta}_{hl}^* > \underline{\theta}; (d) \hat{\theta}_{lh}^* < \underline{\theta}\). Note that (a) and (b) imply (c). Define the following
quantities:
\[ \Delta_0^{(u)} = \frac{\theta - 2\theta + \alpha}{\theta - 2\theta} \cdot w, \quad \Delta_1^{(u)} = \frac{2\theta + \theta - 3\alpha}{2(\theta + \theta - 3\alpha)} \cdot w, \quad \text{and} \quad \Delta_2^{(u)} = \frac{2\theta - 9\theta + 7\alpha}{2(\theta - 5\theta + 3\alpha)} \cdot w. \]

Constraint (a) is equivalent to \( \hat{\theta}_h^* > \alpha \) since \( \hat{\theta}_l^* = \frac{1}{2}(\hat{\theta}_h^* + \alpha) \). Note that \( \hat{\theta}_h^* > \alpha \) is positive as long as \( \Delta V > \Delta_1^{(u)} \). Constraint (b) is equivalent to \( \hat{\theta}_l^* > 2\theta - \alpha \) since \( \hat{\theta}_l^* = \frac{1}{2}(\hat{\theta}_h^* + \alpha) \). Note that \( \hat{\theta}_h^* - (2\theta - \alpha) \) is positive as long as \( \Delta V > \Delta_2^{(u)} \). Note further that \( \Delta_1^{(u)} > \Delta_2^{(u)} \) is equivalent to \( 2(\alpha - \theta)(\theta - 2\theta + 6\alpha) > 0 \), which holds if and only if \( \alpha > \theta \). Therefore, if the lower-quality firm sells optional add-on in equilibrium, then either one of the following must hold: (1) \( \alpha > \theta \) and \( \Delta V > \Delta_1^{(u)} \), or (2) \( \alpha < \theta \) and \( \Delta V > \Delta_2^{(u)} \).

Finally, if the lower-quality firm sells standard add-on in equilibrium, Constraint (c) gives
\[ \hat{\theta}_h^{(u)} - \theta = \frac{(\theta - 2\theta)\Delta V - (\theta - 2\theta + \alpha)w}{6\Delta V - 5w} > 0 \iff \Delta V > \Delta_0^{(u)}. \]

To satisfy Constraint (d), it suffices that \( \Delta V < \Delta_2^{(u)} \). Note that \( \Delta_2^{(u)} > \Delta_0^{(u)} \) is equivalent to \( (\theta - \alpha)(\theta - 2\theta + 6\alpha) > 0 \), which always holds when \( \alpha < \theta \). It remains to verify that there is no profitable deviation from either firm.

**No Profitable Deviation for the Lower-quality Firm**

The only non-local deviation is that the lower-quality firm does not leave demand for \( H \). Fixing the higher-quality firm’s equilibrium prices, \( (P_h^*, p_h^*) \), and given the lower-quality firm’s total price, \( P_l^+ \), the consumers with \( \theta \in [\theta, \hat{\theta}_h] \) choose to pay the base price to visit the lower-quality firm at time \( t = 1 \). The marginal consumer is given by \( \hat{\theta}_h = (P_h^* - P_l^+)/V \). At time \( t = 2 \), these consumers observe the ad-on price \( p_l \) and decide whether to buy it or not. The firm maximizes its \( \text{ex post} \) profit, \( \pi_l = (\hat{\theta}_h - \hat{\theta}_l)(p_l - c) \) with \( \hat{\theta}_l = p_l^*/w \). Again, the optimal strategy by the firm is to set \( \hat{\theta}_l \) as a function of the marginal consumer \( \hat{\theta}_h \): \( \hat{\theta}_l(\hat{\theta}_h) = (\hat{\theta}_h + \alpha)/2 \).

The firm’s problem at time \( t = 0 \) is then finding the optimal \( \hat{\theta}_h \) that maximizes the total profit taking into account the second-period problem
\[
\max_{\hat{\theta}_h} (\hat{\theta}_h - \theta)(P_h^* - \hat{\theta}_h)\Delta V - c) - (\hat{\theta}_l(\hat{\theta}_h) - \theta)(w \cdot \hat{\theta}_l(\hat{\theta}_h) - c).
\]
The (unconstrained) optimal choice of the marginal consumer is
\[ \hat{\theta}_{hl}^d = \frac{2P_{hl}^{++} - 2c + (2\Delta V + w)\hat{\theta}}{4\Delta V + w}. \]

This deviation requires that no consumer would buy only the base good from the higher-quality firm, that is, \( \hat{\theta}_{hl}^d \geq \hat{\theta}_{hl}^* \). However, notice that
\[ \hat{\theta}_{hl}^d - \hat{\theta}_{hl}^* = \frac{-4(\theta - 2\hat{\theta} + 3\alpha)\Delta V^2 - (\theta - 2\theta - 5\alpha)w\Delta V + \frac{7}{2}(\theta - 2\theta + \alpha)w^2}{(4\Delta V + w)(6\Delta V - 5w)}, \]

which is negative because \( \Delta V > w \). Therefore, the optimal solution is binding such that \( \hat{\theta}_{hl} = \hat{\theta}_{hl}^* \). Hence, the lower-quality firm has no incentive to deviate.

**No Profitable Deviation for the Higher-quality Firm**

The only non-local deviation that could be profitable is that the higher-quality firm implements the standard add-on policy. Since it is always optimal for the higher-quality firm to set the add-on price, \( p_{hl}^* = (\theta w + c)/2 \), that maximizes ex post profit, the deviation is achieved by increasing the base price so that no consumer will buy the base good. In the first case, the firm deviates by charging the total price \( P_{hl}^+ = P_h + p_{hl}^* \) at \( t = 0 \) while leaving some demand for \( L^+ \). The optimal deviation is \( P_{hl}^{+d} = \frac{1}{2}(P_{hl}^{++} + \Delta V\theta + c) \). The profit from this deviation, \( \Pi_{hl}^d \), is no greater than the equilibrium profit because
\[ \Pi_{hl}^* - \Pi_{hl}^d = \frac{1}{4(\Delta V - w)w\Delta V} \left[ w[P_{hl}^{++} + \theta(\Delta V - w)] - (\Delta V - w)(\theta w - c) \right]^2. \]

is non-negative. In the second case, the firm charges \( P_{hl}^+ \) such that there is no demand for \( L^+ \). The optimal deviation is \( P_{hl}^{+d} = (P_{hl}^* + \Delta V(\theta w + c))/2 \). The profit from this deviation, \( \Pi_{hl}^d \), is no greater than the deviation profit in the first case, because
\[ \Pi_{hl}^d - \Pi_{hl}^* = \frac{1}{4\Delta V} \left[ P_{hl}^{++} + \theta(\Delta V - c) \right]^2 - \frac{1}{4(\Delta V + w)} \left[ P_{hl}^* + \theta(\Delta V + w) - c \right]^2 \]
\[ = \frac{B^2(\Delta V + w) + 2AB(\Delta V + w) + A^2w}{4(\Delta V + w)\Delta V} \]

is non-negative given that \( A + B > 0 \) and \( B < 0 \). Therefore, there is no profitable deviation for the higher-quality firm.
E.2 When $\alpha \geq (\bar{\theta} + \theta)/3$

In this case, the equilibrium prices are exactly the same as case (1) in Proposition 2 (see subsection C.2 in the Appendix), except that the add-on price at the higher-quality firm is unobserved but correctly expected by rational consumers. Therefore, as long as $\alpha \geq (2\bar{\theta} - \theta)/3$, there is no profitable deviation from either firm. However, when $\alpha \in ((\bar{\theta} + \theta)/3, (2\bar{\theta} - \theta)/3)$, the lower-quality firm can profitably deviate by selling the add-on to some consumers.

F Proof of Proposition 5

Let superscripts “o” and “u” denote the observed-price and the unobserved-price cases.

F.1 Market Shares

The difference in the marginal consumers indifferent between the two firms is

$$\hat{\theta}^{(u)}_{hl} - \hat{\theta}^{(o)}_{hl} = -\frac{[(\bar{\theta} - 2\theta)\Delta V - (\bar{\theta} - 2\theta + \alpha)w]w}{3(\Delta - w)(6\Delta V - 5w)} < 0.$$  

The inequality follows from $\Delta V > \Delta^{(u)}_0$. The difference in the marginal consumers indifference between buying the add-on and not buying is

$$\hat{\theta}^{(u)}_{il} - \hat{\theta}^{(o)}_{il} = \frac{1}{2}(\hat{\theta}^{(u)}_{hl} - \theta) = \frac{[\bar{\theta} - 2\theta)\Delta V - (\bar{\theta} - 2\theta + \alpha)w}{6\Delta V - 5w} > 0.$$  

The inequality holds if and only if $\Delta V > \Delta^{(u)}_0$. Therefore the lower-quality firm is losing consumers who buy the add-on.

F.2 Prices

The difference in the base prices of the higher-quality firm is given by:

$$P^{(u)}_h - P^{(o)}_h = \frac{w(\Delta V - w)(\bar{\theta} - 2\theta + 6\alpha)}{3(6\Delta V - 5w)} > 0.$$  

Therefore the higher-quality firm charges higher base price when the add-on prices are not observed. The difference in the add-on prices of the lower-quality firm is given by:

$$p^{(u)}_l - p^{(o)}_l = \frac{1}{2}(\hat{\theta}^{(u)}_{hl} - \theta)w = \frac{[(\bar{\theta} - 2\theta)\Delta V - (\bar{\theta} - 2\theta + \alpha)w]w}{6\Delta V - 5w} > 0,$$  

55
where the inequality follows from $\Delta V > \Delta_{0}^{(u)}$. Therefore the lower-quality firm charges a higher add-on price under the unobserved-price equilibrium.

Note that under either unobserved-price or observed-price equilibrium, the indifference condition and the higher-quality firm’s best response satisfy:

$$\begin{cases} P_{l}^{+*} = P_{h}^{*} - \hat{\theta}_{hl}^{*}(\Delta V - w) \\ P_{h}^{*} = \frac{1}{2}(P_{l}^{+*} + \bar{\theta}(\Delta V - w)) \end{cases} \Rightarrow P_{l}^{+*} = (\Delta V - w)(\bar{\theta} - 2\hat{\theta}_{hl}^{*}). \quad (F-1)$$

With some algebras, the difference in the total prices is positive: $P_{l}^{+*(u)} - P_{l}^{+*(o)} = 2(\Delta V - w)(\hat{\theta}_{hl}^{*(o)} - \hat{\theta}_{hl}^{*(u)}) > 0$. The difference in the base price is given by

$$P_{l}^{*(u)} - P_{l}^{*(o)} = 2(\Delta V - w)(\hat{\theta}_{hl}^{*(o)} - \hat{\theta}_{hl}^{*(u)}) - \frac{1}{2}(\hat{\theta}_{hl}^{*(u)} - \bar{\theta})w = -\frac{w[(\bar{\theta} - 2\hat{\theta})\Delta V - (\bar{\theta} - 2\bar{\theta} + \alpha)w]}{3(6\Delta V - 5w)},$$

which is negative given that $\Delta V > \Delta_{0}^{(u)}$.

**F.3 Profits**

The higher-quality firm’s profit is clearly increased because it sells the base to more consumers at a higher price while keeping the add-on profit unchanged. For the lower-quality firm, the profit difference is given by

$$\Delta \Pi_{l} = (\hat{\theta}_{hl}^{*(u)} - \bar{\theta})(P_{l}^{+*(u)} - c) - (\hat{\theta}_{hl}^{*(u)} - \bar{\theta})(P_{l}^{*(u)} - c) - (\hat{\theta}_{hl}^{*(o)} - \bar{\theta})(P_{l}^{+*(o)} - c) + (\hat{\theta}_{hl}^{*(o)} - \bar{\theta})(P_{l}^{*(o)} - c).$$

Let

$$B = \frac{[(\bar{\theta} - 2\hat{\theta})\Delta V - (\bar{\theta} - 2\bar{\theta} + \alpha)w]}{3(\Delta - w)(6\Delta V - 5w)} w.$$ 

Then from the above results imply $\hat{\theta}_{hl}^{*(u)} = \hat{\theta}_{hl}^{*(o)} - B$, $\hat{\theta}_{l}^{*(u)} = \hat{\theta}_{l}^{*(o)} + 3B(\Delta - w)/w$, $P_{l}^{+*(u)} = P_{l}^{+*(o)} + 2(\Delta - w)B$, and $p_{l}^{*(u)} = p_{l}^{*(o)} + 3(\Delta - w)B$. Substituting these quantities into the profit difference yields $\Delta \Pi_{l} = -(3\Delta V - 2w)(\Delta V - w)B^{2}/w < 0$, where the equality follows from $3\Delta V > 2w$. 

"