Cooperative Search Advertising

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Abstract

Retailing is among the top spenders in search advertising, and channel coordination in search advertising is becoming an important managerial decision for both manufacturers and retailers. Manufacturers and retailers cooperate in search ad spendings, while at the same time, compete in search ad auctions. Cooperative search advertising is new to study, because all channel members’ advertisements compete with each other in position auctions, in contrast to the usual complementary effect in traditional cooperative advertising. We build a game-theoretic model to answer the following research questions: Why would a manufacturer sponsor several retailers to bid for his product at the same time? Isn’t that indeed competing with himself and burning his own money? Given higher margin from direct sales than via retailers, why sometimes manufacturers do not advertise by themselves? What is a manufacturer’s optimal cooperative search advertising strategy?

We focus on a manufacturer and multiple retailers’ intra-brand competition and coordination in search advertising, but also count for the inter-brand competition with advertisers of other brands. We consider various model setups where the manufacturer can sponsor one or multiple retailers to bid higher by providing positive participation rates, and may also bid by himself. We find inefficiency of the participation rate mechanism in the sense that the manufacturer under-supports the retailer since he does not takes the retailer’s profit margin into account when making the decision. We also find that given two retailers, the manufacturer will always help the retailer with higher total channel profit per click get a higher position in equilibrium. With two symmetric retailers, the manufacturer may optimally choose to cooperate with only one of them. When the manufacturer can bid directly, he may still sponsor the retailer to get a higher position than himself as long as the total channel profit per click is higher at the retailer’s website.
1. Introduction

Search advertising is growing quickly and has become a major advertising channel. Compared to a 6.0% growth of the entire advertising industry, search advertising is growing fast at 16.2% in 2015, and the worldwide expenditure on search advertising has reached $80 billion dollars. Channel coordination is an important managerial problem in search advertising, as most top spending industries on search advertising involve channel relationship between manufacturers and retailers. The No. 1 spender on Google Adwords is Amazon, and retailing is among the top spending industries in search advertising. Retailing and general merchandise, as well as several top spenders like home and garden, computer and consumer electronics, and vehicles, together contribute more than one quarter of Google’s revenue. In all these industries, channel coordination on search advertising, or the so-called cooperative search advertising is necessary.

Cooperative search advertising is also very complicated. On one hand, both manufacturer and its retailers compete directly with each other for a better position on the search engine platforms. On the other hand, it is common for manufacturers to subsidize retailers’ spending in search advertising. As a result of this conflicting vertical relationship, the competitive landscape of the market looks complex. For example, in a search query of “laptop” on Google (Figure 1), we see that some manufacturers advertise products via their own e-commerce websites (Microsoft Surface, Samsung and Google Chromebook), some advertise products via retailers (Asus, Apple, Lenovo, Dell, and Toshiba), and some, such as HP, do both. Further explorations reveal more heterogeneities in the market structure. In the laptop example above, both manufacturers and retailers participate in advertising; while at other times, only retailers advertise, as shown in the example of washers in Figure 2. It appears that sometimes several retailers can post search ads for the same brand. For example, in Figure 2, both AJ Madison and Home Depot advertise Samsung washers.

In this paper, we build a game-theoretic model to study how manufacturers and retailers coordinate in search advertising, and answer the following research questions. Why would a manufacturer sponsor several retailers to bid for his product, while the manufacturer himself may also bid at the same time? Isn’t that indeed competing with himself and burning his own money? When should a manufacturer get a better position for search ad himself, and when should he help retailers get a better position?

Two features of search advertising make the problem of channel coordination distinct from that on traditional advertising. First, while for traditional advertising, it is reasonable to model de-

\footnotesize
\begin{itemize}
\item[2] This is according to a breakdown of Google’s 2011 revenue provided by WordStream.com.
\end{itemize}
Figure 1: An example of Google ad on “laptop”. Google provide two kinds of search ads: AdWords ads in texts and product listing ads in pictures.

Figure 2: An example of Google ad on “washer”.
mand as a smooth concave function of advertising expenditure (e.g., Kim and Staelin 1999), the expenditure on search advertising affects demand in a unique and more complex way, determined through position auction mechanism. Second, in search advertising, a manufacturer and his retailers compete directly in bidding for a better position. In contrast, for traditional advertising, a manufacturer’s ad and his retailers’ ads usually complement with each other. We take these features into considerations when building our model.

Following the literature of channel coordination in traditional advertising (e.g. Bergen and John 1997) and the practice in industry, we consider the coordination mechanism that a manufacturer will cover a fixed percentage (called participation rate) of a retailer’s spending on search advertising. A manufacturer and his retailers play a Stackelberg game: the manufacturer decides the participation rate for each retailer first, and then retailers (and possibly the manufacturer) decide their bids. We focus on the intra-brand competition and coordination in search advertising among a focal manufacturer and his retailers, but also count for the inter-brand competition with advertisers of other brands. We analyze the channel coordination strategy in equilibrium.

We find that in general, there exists inefficiency in the participation rate mechanism as the retailer’s position in equilibrium is lower than that of an integrated channel. The intuition is that the manufacturer does not take into account the retailer’s profit margin when choosing the participation rate. Despite of the channel inefficiency, we find that when there are multiple retailers, their relative positions are determined by the channel profit per click at each retailer, rather than the manufacturer’s or retailer’s own profit per click. With two symmetric retailers, a manufacturer may optimally cooperate with only one of them. When the manufacturer can submit bids directly, he may still sponsor a retailer to get a higher position than himself as long as the total channel profit per click is higher through the retailer’s website, even if the manufacturer can get a higher profit margin via direct sales on his own website. We also consider an extension in which wholesale prices and participation rates are determined at the same stage. In that case, adjusting wholesale prices rather than increasing participating rate is in general more profitable for the manufacturer. In equilibrium the manufacturer will never sponsor both retailers at the same time.

Despite the growing literature in search advertising, the problem of cooperative search advertising has not been addressed yet so far as we know. For analytical modeling of search advertising, most extant works from economics and information systems focus on auction mechanism design and equilibrium properties (e.g., Edelman et al. 2007, Varian 2007, Feng et al. 2007, Feng 2008, Chen and He 2011, Athey and Ellison 2011). In marketing literature, Wilbur and Zhu (2009) consider the impact of click fraud on advertiser’s bidding strategy and search engine’s revenue. Zhu and Wilbur (2011), and Dellarocas (2012) compare the equilibrium properties of pay-per-click, pay-
per-impression, and hybrid advertising auctions. Xu et al. (2011) studies the interaction of firms’ advertising auction and price competition. Katona and Sarvary (2010) study the interplay between organic and sponsored links, and Berman and Katona (2013) further investigate the impact of search engine optimization on the competition between advertisers for organic and sponsored search results. Jerath et al. (2011) study the bidding strategies of vertically differentiated firms for sponsored links. Sayedi et al. (2014) and Desai et al. (2014) consider when a company should buy a competitor’s keyword and whether the search engine should allow such practice. Lu et al. (2015) investigates how advertisers’ budget constraints affect their profits and publisher’s revenue. Among the empirical literatures in search advertising, many of them focus on the relationship between positions and important behavior measures such as click-through rates, sales, etc. (e.g., Ghose and Yang 2009, Yang and Ghose 2010, Agarwal et al. 2011, Rutz and Trusov 2011, Abhishek et al. 2015, Narayanan and Kalyanam 2015).

This paper is also closely related to the literature of cooperative advertising in traditional media. The early work by Berger (1972) proposed cooperative advertising plan as a solution to the “double marginalization” problem. Desai (1992) proposes a co-op advertising plan in which franchisees pool their advertising budget together and let the franchisor make the advertising decision so as to overcome the free-riding problems among franchisees. Bergen and John (1997) examines co-op plans in a conventional channel setting, where manufacturers choose a participation rate in retailers’ advertising spending, and retailers decide the amount of local advertising. They show that co-op plans can combat the under-advertising problem of retailers caused by intrabrand competition and spillovers in local advertising. Kim and Staelin (1999) consider a setting in which manufacturers give retailers side payments for local advertising, but retailers can decide how much of side payments are actually used in local brand promotion. They show that interbrand competition within a retailer induces manufacturers to do that even without getting higher profits. Xie and Wei (2009) examines a similar problem setting as Bergen and John (1997), but takes both national advertising level chosen by manufacturer and local advertising level chosen by retailers into account.

2. Model

Consider a channel with one manufacturer and its $n$ retailers ($n \geq 1$). The manufacturer sells one product, either via retailers or directly to consumers. The manufacturer first signs a wholesale contract with each retailer. The wholesale contract between the manufacturer and each retailer $i$ essentially determines the manufacturer’s and retailer $i$’s unit profit margins for each product sold by the retailer, which are denoted as $m_i$ and $r_i$ respectively. When the manufacturer sells the product directly to consumers, his unit profit margin is denoted as $m_0$. Since in most channel
relationship the pricing decision is made before the decision on search advertising, the value of $m_i$, $r_i$, and $m_0$ can be regarded as exogenously given when the manufacturer and retailers make their search advertising decisions. Later on we will consider the case that pricing and search advertising decisions are made simultaneously, so that pricing decisions become endogenous.

A search ad appears on the result page of a specific keyword as a sponsored link to the webpage of the advertiser, and the search advertising platform determines the rank of search ads for each keyword through auctions. The rank is determined by generalized second-price auctions, which is a commonly used mechanism by search engine platforms like Google (Varian 2007, Edelman et al. 2007). We consider the case that the number of bidders are greater than the number of slots. Under a generalized second-price (GSP) position auction with $S$ slots, the bidder with the $i$-th highest bid wins the $i$-th position, and pays the price that is equal to the $(i + 1)$-th highest bid, for $1 \leq i \leq S$. Consistent with most search engine platforms’ practice, we consider a pay-per-click mechanism. That is, an advertiser pays for each click at the sponsored link. The click through rate (CTR) of the $i$-th position is denoted as $d_i$, and it is assumed that $d_i > d_{i+1}$ for $1 \leq i \leq S - 1$. Following the literature of position auction (Varian 2007, Edelman et al. 2007), the bidding is a complete-information simultaneous game. Since the bidding takes place a very high frequency, after many rounds of bidding the bidders can get to know the value of each other, so it is reasonable to assume they have complete information. Although the bidding takes place repeatedly, stable bids must be best response to each other, so we focus on a one-shot simultaneous game.

We consider both intra-brand and inter-brand competitions in the auction, where the bidders consist of the retailers (and possibly the manufacturer) of the focal brand, as well as the outside advertisers representing other brands. If a retailer carries multiple brands, we assume that her bidding decisions across brands are independent.

Participation rate is a common form of contract for cooperative advertising, where the manufacturer will reimburse a pre-determined percentage of its retailer’s ad spending. Following the tradition of literature in channel coordination, we consider a Stackelberg game with the manufacturer as the leader and retailers as followers. First, the manufacturer decides the participation rate $\alpha_i \in [0, 1)$ for each retailer $i$, which means that the manufacturer will contribute $\alpha_i$ percentage of the retailer $i$’s spending in search advertising, and retailer $i$ only needs to pay the rest $1 - \alpha_i$.

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3 In reality, search engines uses modified GSP mechanism, which adjusts the ranking according to a quality score which summarizes an advertiser’s match with the keyword, attractiveness of its website, etc. We do not consider this complexity here to make the focus clear.

4 As shown below, this mechanism is not efficient in the sense that it does not maximize channel profit. The rationale we provide to justify this mechanism despite of its inefficiency is similar to the one proposed to justify linear pricing despite of double marginalization (e.g., Villas-Boas 1998)—it is prevalent in the industry practice, and it is simple and robust enough to accommodate against changing environment or incomplete contract.
portion. Second, each retailer $i$ decides her bid $b_i$, and the manufacturer may also bid $b_0$ for his own e-commerce site if he has one. Lastly, given everyone’s bid, the auction outcome realizes, the demand realizes, and the profits realize.

The conversion rate at retailer $i$’s website is denoted as $\theta_i$, which means that out of all clicks on the retailer’s website, $\theta_i$ of them will lead to purchases eventually. The conversion rate at the manufacturer’s website is denoted as $\theta_0$. We assume that the conversion rate is independent of the position and hence the bid of the sponsored link. This assumption is consistent with some recent empirical findings (e.g., Narayanan and Kalyanam (2015)).

For tractability of the model, we consider a position auction with two positions ($S = 2$) and three bidders. We set up the model progressively. First we suppose the manufacturer cannot submit bids by himself. We start with the simplest case that the manufacturer has only one retailer and there are two outside advertisers (denoted as 1R). This helps us understand when and to what extent the manufacturer should sponsor a retailer in search advertising. Then we consider the case that the manufacturer has two retailers and there is one outside advertiser (denoted as 2R), which allows us to investigate when the manufacturer should sponsor multiple retailers. Lastly, we consider the case that the manufacturer can submit bids by himself. He has one retailer and there is one outside advertiser (denoted as 1M1R). This case allows us to investigate when the manufacturer should get a higher position himself and when the manufacturer should help the retailer get a higher position.

Before digging into cooperative search advertising strategy, we first recap Varian (2007) and Edelman et al. (2007)’s analysis of equilibrium in position auctions. Lemma 1 gives the equilibrium when three independent bidders compete for two positions.

**Lemma 1:** Consider three independent bidders with payoff per click $v_1 \geq v_2 \geq v_3$ competing for two positions. In equilibrium, bidder 1 will get the first position, bidder 2 will get the second position, and bidder 3 will not get a position (or equivalent, the third position with zero CTR). The equilibrium bids by bidder 2 and 3 are,

$$b_2^* = \frac{d_1 - d_2}{d_1} v_2 + \frac{d_2}{d_1} v_3,$$

$$b_3^* = v_3. \quad (1)$$

To understand $b_2^*$, let us consider bidder 2’s one potential deviation—by bidding higher, he may be able to take the first position. To guard against this deviation, we must have $d_2(v_2 - b_3) \geq d_1(v_2 - b_1)$. Varian (2007) proposed the concept of symmetric Nash equilibrium by requiring that $d_2(v_2 - b_3) \geq d_1(v_2 - b_2)$, which is a stronger condition because $b_1 \geq b_2$. Therefore, symmetric Nash equilibrium (SNE) is a subset of Nash equilibrium (NE). In general, it turns out this condition may
not be binding, and after counting for all other possible deviations, there are still infinite Nash equilibria. Varian (2007) further proposed the equilibrium selection criterion LB (short for lower bound, because it gives the lower bound of bids that satisfy SNE): if it happens that bidder 1 bid so low that bidder 2 slightly exceeded the bidder 1’s bid and he moved up to the first position, bidder 2 would earn at least as much profit as how much he makes now at the second position. LB rule implies that \( d_1(v_2 - b_2) \geq d_2(v_2 - b_3) \). Together, we have \( d_2(v_2 - b_3^*) = d_1(v_2 - b_2^*) \), by which we can get the expression of \( b_2^* \) in equation (1). One can verify can \( b_3^* = v_3 \) is also an SNE and satisfies LB equilibrium selection rule. Therefore, under Varian (2007)’s SNE and LB equilibrium selection criteria, the first bidder that does not win a position will bid truthfully, just like what happens in a second-price auction.\(^5\)

In order to simplify notations, we use position 1, 2, 3 to denote the first, second, and third position respectively. The third position will not get displayed thus get zero CTR.

2.1. Model 1R

In this subsection, we consider the case that the manufacturer has only one retailer (\( n = 1 \)) and there are two outside advertisers. The manufacturer cannot bid by himself. We start with the benchmark case where the channel is integrated, so that the manufacturer and the retailer will bid like a single agent with the payoff per click as \( \theta_1(m_1 + r_1) \). The two outside advertisers A and B’s profit per click are denoted as \( v_A, v_B \) (we assume \( v_A > v_B \)). We consider three cases below.

In the first case, the integrated channel will get position 1 in equilibrium. This happens when \( \theta_1(m_1 + r_1) \geq v_A > v_B \). As given by equation (1), advertiser A’s bid in equilibrium is \( (d_1 - d_2)/d_1 \cdot v_A + d_2/d_1 \cdot v_B \), and it is what the integrated channel needs to pay for each click, so the integrated channel profit will be

\[
\Pi_C = d_1 \left[ \theta_1(m_1 + r_1) - \left( \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_B \right) \right] = d_1 [\theta_1(m_1 + r_1) - v_A] + d_2(v_A - v_B). \tag{3}
\]

In the second case, the integrated channel will get position 2 in equilibrium. This happens when \( v_A > \theta_1(m_1 + r_1) \geq v_B \). The integrated channel pay \( v_B \) for each click, so the integrated channel profit will be

\[
\Pi_C = d_2 \left[ \theta_1(m_1 + r_1) - v_B \right]. \tag{4}
\]

\(^5\)Edelman et al. (2007) propose the concept of “locally envy-free equilibria” which requires that each bidder cannot improve her payoff by exchanging bids with the bidder ranked one position above her, and it yields the same same result as the LB of SNE in Varian (2007).
In the third case, the integrated channel will get position 3 in equilibrium. This happens when \( v_A > v_B > \theta_1(m_1 + r_1) \). The integrated channel will not get any click, so the channel profit from search advertising will be zero.

To summarize, we have the integrated channel profit as the following,

\[
\Pi_C = \begin{cases} 
  d_1 \left[ \theta_1(m_1 + r_1) - v_A \right] + d_2(v_A - v_B), & \theta_1(m_1 + r_1) \geq v_A > v_B, \text{ position 1} \\
  d_2 \left[ \theta_1(m_1 + r_1) - v_B \right], & v_A > \theta_1(m_1 + r_1) \geq v_B, \text{ position 2} \\
  0, & v_A > v_B > \theta_1(m_1 + r_1), \text{ position 3}. 
\end{cases}
\]  

(5)

Now, suppose the channel is decentralized. Given the manufacturer’s participation rate \( \alpha_1 \), it is easy to show that the retailer’s equivalent payoff per click in the position auction will be \( \theta_1 r_1 / (1 - \alpha_1) \), in the sense that her bidding strategy will be the same with the case as if her payoff per click was \( \theta_1 r_1 / (1 - \alpha) \) and she got not support from the manufacturer.\(^6\) Below, we focus on the case that \( v_A > v_B \geq \theta_1 r_1 \), i.e., without support from the manufacturer, the retailer cannot win a position by herself. This is the most interesting case to study because this is the case when different levels of the participation rates can move the retailer to positions with different CTRs. In any real search ad auctions, the number of bidders are usually large, but our model captures the essence that the manufacturer decides how much to help push up the retailer by choosing participation rates. Consistent with the benchmark case, we consider three cases below.

In the first case, the retailer gets position 1. In order to help the retailer get position 1, the manufacturer needs to choose \( \alpha_1 \) such that \( \frac{\theta_1 r_1}{1 - \alpha_1} \geq v_A > v_B \). The manufacturer’s profit will be

\[
\pi_M(\alpha_1) = d_1 \left[ \theta_1 m_1 - \alpha_1 \left( \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_B \right) \right],
\]

which decreases with \( \alpha_1 \), so the manufacturer will choose the smallest \( \alpha_1 \) that satisfies \( \frac{\theta_1 r_1}{1 - \alpha_1} \geq v_A \). His optimal choice will then be,

\[
\alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A}.
\]

(7)

Correspondingly, the manufacturer’s profit will be,

\[
\pi_M(\alpha_1^*) = d_1 \left[ \theta_1 m_1 - \alpha_1^* \left( \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_B \right) \right] = d_1 \left[ \theta_1(m_1 + r_1) - v_A \right] + d_2 \frac{v_A - \theta_1 r_1}{v_A} (v_A - v_B),
\]

(8)

\(^6\)To show the equivalence, consider this retailer will get position \( i \) and pay \( p_i \) in the equilibrium. Then for any \( j \neq i \), we should have \( d_i[\theta_1 r_1 - (1 - \alpha_1)p_i] \geq d_j[\theta_1 r_1 - (1 - \alpha_1)p_j] \), which is equivalent to \( d_i[\theta_1 r_1/(1 - \alpha_1) - p_i] \geq d_j[\theta_1 r_1/(1 - \alpha_1) - p_j] \).
Table 1: Participation Rate, Manufacturer Profit and Channel Profit in 1R (Decentralized)

<table>
<thead>
<tr>
<th>Position</th>
<th>$\alpha_1^*$</th>
<th>$\pi_M(\alpha_1^*)$</th>
<th>$\pi_C(\alpha_1^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1 - \frac{\theta_1 r_1}{v_A}$</td>
<td>$d_1 \left[ \theta_1 (m_1 + r_1) - v_A \right] + d_2 \frac{v_A - v_B}{v_A} (v_A - v_B)$</td>
<td>$d_1 \left[ \theta_1 (m_1 + r_1) - v_A \right] + d_2 (v_A - v_B)$</td>
</tr>
<tr>
<td>2</td>
<td>$1 - \frac{\theta_1 r_1}{v_B}$</td>
<td>$d_2 \left[ \theta_1 (m_1 + r_1) - v_B \right]$</td>
<td>$d_2 \left[ \theta_1 (m_1 + r_1) - v_B \right]$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

and the retailer’s profit will be

$$\pi_R(\alpha_1^*) = d_1 \left[ \theta_1 r_1 - (1 - \alpha_1^*) \left( \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_B \right) \right] = d_2 \frac{\theta_1 r_1}{v_A} (v_A - v_B).$$  \hspace{1cm} (9)

The total channel profit will be,

$$\pi_C(\alpha_1^*) = \pi_M(\alpha_1^*) + \pi_R(\alpha_1^*) = d_1 \left[ \theta_1 (m_1 + r_1) - v_A \right] + d_2 (v_A - v_B).$$ \hspace{1cm} (10)

Similarly, we can work out the case that the retailer gets position 2 and position 3. Table 1 summarizes the optimal participation rate $\alpha_1^*$ as well as the corresponding manufacturer and channel profit in each case.

Comparing the manufacturer’s profit in the three cases, we know that in equilibrium the manufacturer will sponsor the retailer to get position 1 when $\theta_1 (m_1 + r_1) \geq v_A + \frac{d_2}{d_1 - d_2} \frac{v_A - v_B}{v_A} \theta_1 r_1$, and get position 2 when $v_A + \frac{d_2}{d_1 - d_2} \frac{v_A - v_B}{v_A} \theta_1 r_1 > \theta_1 (m_1 + r_1) \geq v_B$. When $v_B > \theta_1 (m_1 + r_1)$, the manufacturer will not sponsor the retailer and let her stay at position 3.

Comparing total channel profit under the integrated case and that under the decentralized case as well as the condition for the retailer to get the first and second position, we get the following result regarding to channel coordination inefficiency.

**Theorem 1:** Consider a position auction participated by two outside advertisers and one retailer who is supported by a manufacturer with optimally chosen participation rate. Given the retailer’s position, the total channel profit is the same as that of an integrated channel. However, compared to the integrated channel, the manufacturer is less likely to push the non-integrated retailer from position 2 to position 1. In other words, the retailer’s equilibrium position and the total channel profit can be lower than those of the integrated channel.

The intuition behind this inefficiency is that in the decentralized case, the manufacturer does not take the retailer’s profit into account when choosing the participation rate. Specifically, when deciding whether to help the retailer move from position 3 to position 2, the manufacturer acts as if the channel is integrated, because he chooses the participation rate such that the retailer’s
equivalent payoff per click equals what she pays for each click, and thus the manufacturer collects all the channel profit at position 2. However, when deciding whether to help the retailer move from position 2 to position 1, the condition for the decentralized channel is stricter than that for the integrated channel. In other words, the channel coordination can be inefficient. This is because the manufacturer needs to choose the participation rate such that the retailer’s equivalent payoff per click equals $v_A$, but the price per click is less than $v_A$; hence the retailer gets positive profit at position 1. As a result, the manufacturer’s incentive to push the retailer from the second to position 1 is weaker compared to that of integrated channel.

2.2. Model 2R

In this subsection, we consider the case that the manufacturer has two retailers ($n = 2$) and there is one outside advertiser. The manufacturer still cannot bids by himself. The outside advertiser A’s payoff per click is denoted as $v_A$. Without loss of generality, we assume $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$, i.e., the channel profit per click at retailer 1 is higher than at retailer 2. Similarly as in the 1R model, we still focus on the case that $v_A \geq \theta_i r_i$ ($i = 1, 2$) below. Under this condition, the two retailers are not able to win position 1 without the manufacturer’s support, and different levels of participation rates can potentially move each to different positions. This allows us to investigate whether it is optimal for the manufacturer to sponsor one or both retailers, and how much support he should provide to each retailer. There are three possible cases in equilibrium.

In the first case, the outside advertiser gets position 1. Let us first assume that the retailer 1 gets position 2 and retailer 2 gets position 3, and later we will show that this is the unique possible outcome when the two retailer take the second and third positions. This equilibrium outcome requires $v_A \geq \theta_1 r_1/(1 - \alpha_1) \geq \theta_2 r_2/(1 - \alpha_2)$.

According to Lemma 1, retailer 2’s bid at position 3 will be her equivalent payoff per click, $\theta_2 r_2/(1 - \alpha_2)$, and this is the price for each click at retailer 1. The manufacturer’s profit will be,

$$\pi_M(\alpha_1, \alpha_2) = d_2 \left( \theta_1 m_1 - \alpha_1 \frac{\theta_2 r_2}{1 - \alpha_2} \right),$$

which decreases with both $\alpha_1$ and $\alpha_2$. Therefore, the manufacturer will choose the smallest $\alpha_1$ and $\alpha_2$ that satisfy $v_A \geq \theta_1 r_1/(1 - \alpha_1) \geq \theta_2 r_2/(1 - \alpha_2)$. The optimal choice will be,

$$\alpha_1^* = \max \left\{ 1 - \frac{\theta_1 r_1}{\theta_2 r_2}, 0 \right\},$$

$$\alpha_2^* = 0.$$
Correspondingly, the manufacturer’s profit under the optimal participation rates will be,

\[
\pi_M(\alpha^*_1, \alpha^*_2) = d_2 \left[ \theta_1(m_1 + r_1) - \max\{\theta_1 r_1, \theta_2 r_2\} \right].
\]  

(14)

According to the equation above and by symmetry, we can see that given \(\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)\), the manufacturer cannot gain higher profit by exchanging the positions of retailer 1 and retailer 2. Therefore, given that the two retailers take the second and third positions, it must be that retailer 1 gets position 2, and retailer 2 gets position 3. Accordingly, profits for the two retailers and the channel under \(\alpha^*_1\) and \(\alpha^*_2\) are,

\[
\pi_{R_1}(\alpha^*_1, \alpha^*_2) = d_2 \left[ \theta_1 r_1 - (1 - \alpha^*_1) \frac{\theta_2 r_2}{1 - \alpha^*_2} \right] = \max\{\theta_1 r_1 - \theta_2 r_2, 0\},
\]  

(15)

\[
\pi_{R_2}(\alpha^*_1, \alpha^*_2) = 0,
\]  

(16)

\[
\pi_C(\alpha^*_1, \alpha^*_2) = \pi_M(\alpha^*_1, \alpha^*_2) + \pi_{R_1}(\alpha^*_1, \alpha^*_2) + \pi_{R_2}(\alpha^*_1, \alpha^*_2) = d_2 \left[ \theta_1(m_1 + r_1) - \theta_2 r_2 \right].
\]  

(17)

To summarize, we have analyzed the case when the outside advertiser gets position 1. We find that the retailer with relatively higher total channel profit per click will get position 2. The manufacturer needs to provide positive participation rate to support this retailer to get position 2 only when this retailer’s own profit per click is lower than that of the other retailer; otherwise, the manufacturer does not need to provide support for any retailer.

Now we turn to the second case where the outside advertiser takes position 2. Similarly we presume that retailer 1 gets position 1, and retailer 2 gets position 3, and we will verify this is the unique possible equilibrium outcome in the second case. This equilibrium outcome requires \(\theta_1 r_1/(1 - \alpha_1) \geq v_A \geq \theta_2 r_2/(1 - \alpha_2)\). According to Lemma 1, retailer 2 will bid her equivalent value \(\theta_2 r_2/(1 - \alpha_2)\), and the outside competitor will bid \((d_1 - d_2)/d_1 \cdot v_A + d_2/d_1 \cdot \theta_2 r_2/(1 - \alpha_2)\). The manufacturer’s profit will be,

\[
\pi_M(\alpha_1, \alpha_2) = d_1 \left[ \theta_1 m_1 - \alpha_1 \left( \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \frac{\theta_2 r_2}{1 - \alpha_2} \right) \right],
\]  

(18)

which decreases with both \(\alpha_1\) and \(\alpha_2\). Therefore, the manufacturer will choose the smallest participation rates that ensure \(\theta_1 r_1/(1 - \alpha_1) \geq v_A \geq \theta_2 r_2/(1 - \alpha_2)\). The optimal choice will be,

\[
\alpha^*_1 = 1 - \frac{\theta_1 r_1}{v_A},
\]  

(19)

\[
\alpha^*_2 = 0.
\]  

(20)
Correspondingly, the manufacturer’s profit under the optimal participation rates will be,

\[ \pi_M(\alpha_1^*, \alpha_2^*) = d_1 \left[ \theta_1(m_1 + r_1) - v_A \right] + d_2 \frac{(v_A - \theta_1 r_1)(v_A - \theta_2 r_2)}{v_A}. \]  

(21)

According to the equation above and by symmetry, similarly, we can see that given \( \theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2) \), the manufacturer indeed gains higher profit when retailer 1 instead of retailer 2 gets position 1. Accordingly, profits for the two retailers and the channel under \( \alpha_1^* \) and \( \alpha_2^* \) are,

\[ \pi_{R_1}(\alpha_1^*, \alpha_2^*) = d_1 \left[ \theta_1 r_1 - (1 - \alpha_1^*) \left( \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \theta_2 r_2 \right) \right] = d_2 \theta_1 r_1 \left( 1 - \frac{\theta_2 r_2}{v_A} \right), \]  

(22)

\[ \pi_{R_2}(\alpha_1^*, \alpha_2^*) = 0, \]  

(23)

\[ \pi_C(\alpha_1^*, \alpha_2^*) = d_1 \left[ \theta_1(m_1 + r_1) - v_A \right] + d_2 \left[ v_A - \theta_2 r_2 \right]. \]  

(24)

To summarize, we have analyzed the case when the outside advertiser gets position 2. We find that the retailer with higher total channel profit per click will get position 1. The manufacturer will provide positive participation rate to this retailer in order to help her get this position. The manufacturer will not provide support for the other retailer.

Lastly, we study the third case when the outside advertiser gets position 3. Similarly, we presume that retailer 1 gets position 1, and retailer 2 gets position 2. We will verify this is the only possible equilibrium outcome in the third case. This outcome requires \( \theta_1 r_1/(1 - \alpha_1) \geq \theta_2 r_2/(1 - \alpha_2) \geq v_A \). According to Lemma 1, the outside advertiser will bid \( v_A \), and retailer 2 will bid \( (d_1 - d_2)/d_1 \cdot \theta_2 r_2/(1 - \alpha_2) + d_2/d_1 \cdot v_A \). The manufacturer’s profit will be,

\[ \pi_M(\alpha_1, \alpha_2) = d_1 \left[ \theta_1 m_1 - \alpha_1 \left( \frac{d_1 - d_2}{d_1} \frac{\theta_2 r_2}{1 - \alpha_2} + \frac{d_2}{d_1} v_A \right) \right] + d_2 \left( \theta_2 m_2 - \alpha_2 v_A \right), \]  

(25)

which decreases with both \( \alpha_1 \) and \( \alpha_2 \). Therefore, the manufacturer will choose the smallest participation rates that ensure \( \theta_1 r_1/(1 - \alpha_1) \geq \theta_2 r_2/(1 - \alpha_2) \geq v_A \). The optimal choice will be,

\[ \alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A}, \]  

(26)

\[ \alpha_2^* = 1 - \frac{\theta_2 r_2}{v_A}. \]  

(27)

Correspondingly, the manufacturer’s profit under the optimal participation rates will be,

\[ \pi_M(\alpha_1^*, \alpha_2^*) = d_1 \left[ \theta_1(m_1 + r_1) - v_A \right] + d_2 \left[ \theta_2(m_2 + r_2) - v_A \right]. \]  

(28)

Similarly, we can see that given \( \theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2) \), the manufacturer indeed gains higher
profit when retailer 1 instead of retailer 2 gets position 1. Therefore, we have again verified that in equilibrium, retailer 1 will get a higher position than retailer 2. Accordingly, profits for the two retailers and the channel under $\alpha_1^*$ and $\alpha_2^*$ are,

\[
\begin{align*}
\pi_{R_1}(\alpha_1^*, \alpha_2^*)& = 0, \quad (29) \\
\pi_{R_2}(\alpha_1^*, \alpha_2^*)& = 0, \quad (30) \\
\pi_C(\alpha_1^*, \alpha_2^*) & = d_1 \left[ \theta_1(m_1 + r_1) - v_A \right] + d_2 \left[ \theta_2(m_2 + r_2) - v_A \right]. \quad (31)
\end{align*}
\]

To summarize, we have analyzed the case when the outside advertiser gets position 3. We find that the retailer with higher channel profit per click will get position 1, and the other retailer will get position 2. The manufacturer will provide positive participation rates to both retailers. Specifically, he can carefully choose the participation rates such that the equivalent values of both retailers are infinitely close to $v_A$, which also equals the price for each click at both positions. As a result, both retailers earn zero profit, and the manufacturer collects the entire channel profit.

Comparing the third case with the second case, we find that although supporting both retailers seems like having the two retailers competing with each other and burning the manufacturer’s own money, the manufacturer does not need to provide a higher participation rate to retailer 1 when moving retailer 2 from position 3 to position 2 because he keeps the value at position 2 equal to $v_A$. The price for each click at position 1 increases from $d_1 - d_2 v_A + d_2 \theta_2 r_2$ to $v_A$, but the manufacturer also benefits from an increase in demand from retailer 2. This is the tradeoff faced by the manufacturer when deciding whether to sponsor both retailers or retailer 1 only, and it is optimal for the manufacturer to sponsor both retailers as long as the increase in payment for retailer 1 is less salient than the increase in demand from retailer 2.

So far, we have identified the equilibrium given all possible position configurations of retailers. The following theorem summarizes a main finding.

**Theorem 2:** Consider a position auction with one outside advertiser and two retailers supported by the focal manufacturer with optimally chosen participation rates. In equilibrium, the retailer with higher total channel profit per click will always take a higher position than the other retailer. Supporting both retailers can be an equilibrium outcome. When it is optimal for the manufacturer to support only one retailer, he will choose to support the retailer with higher total channel profit.

The theorem above shows that the manufacturer makes the decision of which retailer to support based on total channel profit per click rather than his own profit per click, and the relative position of the two retailers in equilibrium is determined by total channel profit per click instead of retailers’ own profit per click. Therefore, when deciding the two retailers’ relative positions, the manufacturer
acts as if the channel is integrated, although the channel is not fully coordinated under the participation rate mechanism. (We have shown the inefficiency in model 1R, and we will show similar inefficiency results in model 2R below.) The intuition is that, in cooperative search advertising, the manufacturer needs to consider not only his own profit per click, but also how much he needs to pay for the retailer to get a good position. When the retailer’s own profit per click is relatively high, she already has relatively high willingness-to-pay for the position, and thus the manufacturer can help her get the good position with relatively low cost.

In order to know which position configuration will be the chosen by the manufacturer in equilibrium, we only need to compare the manufacturer’s profit among the three cases. Intuitively, when sponsoring retailer 1 to move from position 2 to the first, the manufacturer gets higher demand, but at the same time the price for each click gets higher and he bears higher participation rate. Similarly, by supporting retailer 2 to move from position 3 to the second, the manufacturer gets more demand from retailer 2, but at the same time he bears not only higher price per click and higher participation rate for retailer 2 but also higher price per click for retailer 1. In general, the manufacturer faces a tradeoff between larger demand versus higher bidding cost and higher participation rate when choosing different position configurations. Given equations (14), (21), and (28), it is straightforward to write down the condition for each position configuration to be chosen by the manufacturer, but the conditions are not very intuitive to understand. Therefore we will show a special case that leads to more intuitive result—the case when the two retailers are symmetric, i.e., $\theta_1 = \theta_2 = \theta$, $m_1 = m_2 = m$, and $r_1 = r_2 = r$.

Figure 3 characterizes a manufacturer’s optimal cooperative search advertising strategy for two symmetric retailers. Roughly speaking, the manufacturer provides positive participation rates to both retailers when the total channel profit per click is relatively high; he provides no support to both retailers if both his and each retailer’s profit per click are relatively low. More interestingly, we find that the manufacturer will provide positive participation rate to only one retailer when his profit per click is relatively high but the retailers’ profit per click is relatively low. In this case, retailers need high participation rates to move up to a higher position, but it is too expensive for the manufacturer to support both retailers. In traditional cooperative advertising, advertisements from different retailers usually have positive spillover effects (Bergen and John 1997) and thus are complementary to each other. However, in cooperative search advertising, advertisements from different retailers have no spillover effects, and retailers directly compete with each other in bidding. As a result, in the traditional cooperative advertising, the manufacturer usually set up a co-op program and make it open to all his retailers; while in cooperative search advertising, a manufacturer may optimally offer the co-op program only to a subset of retailers, even when the retailers are homogeneous. When retailers are heterogeneous, we know that the manufacturer should offer the co-op program
to the retailer with higher total channel profit per click according to Theorem 2.

Figure 3: Manufacturer’s optimal cooperative search advertising strategy, given two symmetric retailers.

**Integrated Channel**

So far, we have analyzed the 2R model when the manufacturer and two retailers are decentralized decision makers. It is interesting to study the search advertising strategy of an integrated channel and compare the results. When the channel is integrated, the two retailers will collude in the position auction, so the equilibrium result given by Lemma 1 based on the assumption of independent bidders will fail. We conduct the equilibrium analysis for position auctions with colluding bidders by inheriting Varian (2007)’s concept of SNE and LB equilibrium selection criterion. We first notice that given $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$, retailer 1 will always take a higher position than retailer 2, because otherwise the channel could gain higher profit by exchanging the two retailers’ bids. We analyze three cases below, and identify the equilibrium condition for each case. The outside advertiser A’s bid is denoted as $b_A$.

In the first case, the outside advertiser takes position 1, and the channel takes positions (2,3). The channel’s possible deviations are positions (1,3) and positions (1,2); the outside advertiser’s possible deviations are position 2 and 3. To guard against all these possible deviations, we have the
following condition for a Nash equilibrium (NE),

\[ d_2 [\theta_1(m_1 + r_1) - b_2] \geq d_1 [\theta_1(m_1 + r_1) - b_A], \tag{32} \]

\[ d_2 [\theta_1(m_1 + r_1) - b_2] \geq d_1 [\theta_1(m_1 + r_1) - b_1] + d_2 [\theta_2(m_2 + r_2) - b_A], \tag{33} \]

\[ d_1 (v_A - b_1) \geq d_2 (v_A - b_2), \tag{34} \]

\[ d_1 (v_A - b_1) \geq 0. \tag{35} \]

To guarantee the Nash equilibrium is a symmetric Nash equilibrium (SNE), we impose the following stronger conditions,

\[ d_2 [\theta_1(m_1 + r_1) - b_2] \geq d_1 [\theta_1(m_1 + r_1) - b_1], \tag{36} \]

\[ d_2 [\theta_1(m_1 + r_1) - b_2] \geq d_1 [\theta_1(m_1 + r_1) - b_1] + d_2 [\theta_2(m_2 + r_2) - b_2], \tag{37} \]

\[ d_1 (v_A - b_1) \geq d_2 (v_A - b_2), \tag{38} \]

\[ d_1 (v_A - b_1) \geq 0. \tag{39} \]

Moreover, the LB equilibrium selection rule implies that,

\[ d_2 [\theta_1(m_1 + r_1) - b_2] \leq d_1 [\theta_1(m_1 + r_1) - b_1]. \tag{40} \]

Combining the SNE and LB conditions together, we have,

\[ b_1 = \frac{d_1 - d_2}{d_1} \theta_1(m_1 + r_1) + \frac{d_2}{d_1} b_2, \tag{41} \]

\[ b_2 \leq \theta_2(m_2 + v_2), \tag{42} \]

\[ v_A \geq \frac{d_1 b_1 - d_2 b_2}{d_1 - d_2}. \tag{43} \]

The channel’s problem is to maximize profit \( d_2 [\theta_1(m_1 + r_1) - b_2] \) subject to the above three conditions, so we have the equilibrium as,

\[ b_1^* = \frac{d_1 - d_2}{d_1} \theta_1(m_1 + r_1), \tag{44} \]

\[ b_2^* = 0. \tag{45} \]

This equilibrium exists if and only if \( v_A \geq \theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2) \). We notice that the bidder with a higher profit per click still takes a higher position even though two of the bidders are colluding.
Correspondingly, the channel’s profit in the equilibrium is,

\[ \Pi_C = d_2 \theta_1 (m_1 + r_1). \]  \hfill (46)

Similarly, we can work out the other two cases where the outsider advertiser takes position 2 and position 3.

In the second case, the outside advertiser takes position 2, and the channel takes position (1,3). We have the equilibrium bids as,

\[ b^*_A = \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_2, \] \hfill (47)
\[ b^*_2 = v_2. \] \hfill (48)

This equilibrium exists if and only if \( \theta_1 (m_1 + r_1) \geq v_A \geq \theta_2 (m_2 + r_2) \). Correspondingly, the channel’s profit in the equilibrium is,

\[ \Pi_C = d_1 [\theta_1 (m_1 + r_1) - v_A] + d_2 [v_A - \theta_2 (m_2 + r_2)]. \] \hfill (49)

In the third case, the outside advertiser takes position 3, and the channel takes position (1,2). We have the equilibrium bids as,

\[ b^*_2 = v_A, \] \hfill (50)
\[ b^*_A = v_A. \] \hfill (51)

This equilibrium exists if and only if \( \theta_1 (m_1 + r_1) \geq \theta_2 (m_2 + r_2) \geq v_A \). Correspondingly, the channel’s profit in the equilibrium is,

\[ \Pi_C = d_1 [\theta_1 (m_1 + r_1) - v_A] + d_2 [\theta_2 (m_2 + r_2) - v_A]. \] \hfill (52)

To summarize, when the channel is integrated, the retailer with a higher channel profit per click still gets a higher position than the other retailer, which is the same as the decentralized channel. However, when the channel is integrated, the rank of the three bidders in equilibrium is exactly consistent with the rank of their payoffs per click. This originates from the SNE and LB equilibrium conditions, and is different from the decentralized case.

Given each position configuration, we compare the profit of the decentralized and the integrated channel. When the two retailers take positions (1,2), both retailers earn zero profits in the decen-
tralized case, and the channel profits are the same for the decentralized and integrated cases. When the two retailers take positions (2,3), retailer 1 earns positive profit in the decentralized case, and we find that the channel profit in the decentralized case is strictly less than that in the integrated case. When the two retailers take positions (1,3), retailer 1 earns positive profit in the decentralized case, and we get a striking result that the channel profit in the decentralized case is strictly greater than that in the integrated case. This comes from the requirement of SNE, which has the lowest bidder always bid her true value. In the decentralized case, retailer 2 will bid $\theta_2 r_2$, while in the integrated case, retailer 2’s true value becomes $\theta_2(m_2 + r_2)$. Higher bid from retailer 2 ends up with higher price per click for retailer 1, and thus lowers channel profit. In fact, we have this similar counter-intuitive finding when the position configuration is not given. Figure 4 shows the comparison of the channel profits with two symmetric retailers in the decentralized case and the integrated case. When the two retailers are symmetric, $\theta_1(m_1 + r_1) = \theta_2(m_2 + r_2)$, which is either greater or smaller than $v_A$, so the two retailers will never take position 1 and 3 in the integrated case. Still, we find that the decentralized channel profit can be strictly greater than the integrated channel profit.

![Figure 4: Comparison of channel profit in the decentralized case $\Pi^D_C$ and that in the integrated case $\Pi^I_C$, given two symmetric retailers.](image-url)
Before concluding the 2R model, we analyze a benevolent manufacturer who maximizes channel profit instead of his own profit by providing cooperative advertising plans to two retailers. In the previous studies of channel coordination of wholesale prices (e.g., Telser 1960), and cooperative advertising (e.g., Bergen and John 1997), the manufacturer is able to achieve integrated channel profit by carefully choosing wholesale price or participation rate respectively. This implies that by combining a lump-sum payment with a linear pricing scheme, a two-part tariff will be able to fully coordinate channel. We find that this is not the case in our model of cooperative search advertising.

We first argue that, given the positions of the two retailers, the participation rates that maximize manufacturer’s own profit also maximize total channel profit. In fact, as shown by our analysis of the 2R model above, given a position configuration, the optimal participation rate for each retailer is the lowest one that ensures this retailer’s profit per click is no less than that of the bidder just below her. Now consider a manufacturer who maximizes channel profit. Suppose retailer 1 takes a higher position than retailer 2. Given retailer 1’s position, the participation rate to retailer 1 only affects her bid but not her demand nor her price per click, and retailer 1’s bid will not affect retailer 2 who takes a lower position. Therefore, given retailer 1’s position, any participation rate to her renders the same channel profit. Similarly, the participation rate to retailer 2 only affects her bid but not her demand nor her price per click. Meanwhile we notice that, higher participation rate to retailer 2 will lead to higher bid by retailer 2, which in turn will lead to a higher price per click for retailer 1, who takes a higher position. Therefore, to maximize channel profit, the manufacturer will also choose the lowest participation rate which is exactly the same with his choice when he is maximizing his own profit. Given the above observation, we must have the same equilibrium bids and hence the same channel profits no matter whether the manufacturer is maximizing his own profit or channel profit. This also implies that when the manufacturer maximizes channel profit, the retailer with higher channel profit per click will still get a higher position.

However, when the position configuration is not given, the manufacturer’s choice of participation rates will be different. For a manufacturer who maximizes his own profit, he chooses to let retailers take position 2 and 3 if and only if \( \pi_M(\alpha_1^*, \alpha_2^*) \) in equation (14) is greater than that in equations (21) and (28); in contrast, For a manufacturer who maximizes total channel profit, he chooses to let retailers take position 2 and 3 if and only if \( \pi_C(\alpha_1^*, \alpha_2^*) \) in equation (17) is greater than that in equations (24) and (31). Obviously, \( \pi_C(\alpha_1^*, \alpha_2^*) \) in equations (17), (24) and (31) are different with the channel profits in the case of integrated channel in equations (46), (49) and (52), therefore by choosing participation rates, the manufacturer will never be able to fully coordinate channel.

Figure 5 shows a benevolent manufacturer’s cooperative search advertising strategy for two
symmetric retailers. The gray lines denote the manufacturer’s optimal strategy when he maximizes his own profit (inherited from Figure 3), while the black lines denote the manufacturer’s optimal strategy when he maximizes channel profit. We find that, compared with the regions partitioned by the gray lines, some part of both “Support Both” and “Support None” now become “Support One” when the strategy space is partitioned by black lines. This implies that when a manufacturer maximizes his own profit rather than maximizing channel profit, he is less likely to support one retailer, but more likely to support two retailers. The reason is that, when the manufacturer only supports one retailer, the retailer will earn a positive profit; when he supports both retailers, both earn zero profit. When moving retailer 1 and 2 from positions (1,3) to (1,2), for the channel the benefit is gaining more demand from retailer 2, and the cost is that retailer 1 needs to pay a higher price for each click. For the manufacturer, the benefit from retailer 2 is the same as that of the channel, but the cost from retailer 1 is less since the manufacturer squeezes retailer 1’s profit to zero and retailer 1 takes more of the cost. Therefore, the manufacturer is more willing to support both retailers when maximizing her own profit.

![Figure 5: A benevolent manufacturer’s optimal cooperative search advertising strategy, given two symmetric retailers.](image)

2.3. Model 1M1R

In this subsection, we study the case that the manufacturer has one retailer and there is one outside advertiser, and the manufacturer submits bid by himself.
Details TBD. We are able to prove the following theorem.

**Theorem 3:** Consider the position auction with one manufacturer, one retailer, and one outside advertiser. In equilibrium, the channel member with higher total channel profit per click will get a higher position.

Even if the manufacturer may earn a higher profit from selling through his own website than supplying the retailer, the manufacturer may still sponsor the retailer and let the retailer get a position above himself. The reason is that, when bidding for his own website, the manufacturer needs to bear all the payment, and when supplying the retailer, the manufacturer only needs to pay a proportion. If the retailer already has a relatively high value from each click, the manufacturer only needs to undertake a small proportion of the retailer’s payment. Considering both the profit from each click and the cost for each click, it is optimal for the manufacturer to let the retailer get a higher position if selling through the retailer’s website leads to a higher profit for the channel. In other words, when deciding the position ranking within the channel, the manufacturer act as if the channel is integrated.

### 3. Endogenous Prices

In this section, we consider the cooperative advertising problem under the 2R model but with endogenous prices. In previous sections, both wholesale prices and retail prices have been determined before the manufacturer and his retailers make decisions on search advertising, which applies for industries in which retail prices are relatively stable. For industries in which retailers can change prices frequently, since a manufacturer’s coop programs requires heavy administrative support and thus cannot be adjusted frequently, it is more reasonable to consider a game that the manufacturer decides wholesale prices and participation rates first, and then the retailer determines retail prices and bidding strategy. As a result, prices cannot be regarded as exogenously given when we consider the search advertising strategy. Figure 6 illustrates the timeline of events. Since retailers can adjust their bids continuously with automatic support system, it is reasonable to regard retail prices as given when retailers are deciding bids. We implicitly assume that all players’ actions made in last period become the common knowledge in this period.

We consider a linear wholesale price contract. The manufacturer chooses unit wholesale prices $w_1, w_2$ and participation rates $\alpha_1, \alpha_2$ for the two retailers. Given the manufacturer’s choice, the two retailers decide retail prices $p_1, p_2$ and bids $b_1, b_2$ for search advertising. We focus our analysis on text ads as shown in Figure 1. For text ads, the price is usually not displayed in the ads, so it
should not influence the click through rate directly given a bidder’s position. However, the price will affect how many consumers will finally purchase the product after they click the sponsored link. That is, the price will influence the conversion rate. We assume that out of all consumers that click retailer $i$’s ads, $\bar{\theta}_i$ of them will seriously consider purchasing the product if the price is low enough. Consumers’ valuation of the product, $v$, is assumed to be uniformly distributed in $[0, 1]$. Therefore, the conversion rate of retailer $i$, $\theta_i = \bar{\theta}_i(1-p_i)$. Retailer $i$’s profit margin is $r_i = p_i - w_i$. We consider ex ante symmetric retailers with $\bar{\theta}_1 = \bar{\theta}_2 = \bar{\theta}$.

Manufacturer chooses wholesale prices $w_1, w_2$, and participation rates $\alpha_1, \alpha_2$. Retailers choose bids $b_1$ and $b_2$ respectively. Retailers choose prices $p_1$ and $p_2$ respectively. Positions in the auction realize; consumer clicks and purchases realize.

Figure 6: Timeline of events.

We solve the equilibrium by backward induction. In fact, in the last stage when the wholesale prices, participation rates, and retail prices have been determined, retailers face the exactly same problem as 2R model above, and we have solved for the retailers’ optimal bids given their payoff per click. Now, let us consider retailers’ decisions of retail prices. We first notice that $p_i$ enter into retailer $i$’s profit function only via $\theta_i r_i$. Let us define retailer $i$’s equivalent profit per click as,

$$v_i \equiv \frac{\theta_i r_i}{1-\alpha_i} = \frac{\bar{\theta}(1-p_i)(p_i-w_i)}{1-\alpha_i}, \quad (53)$$

which takes the maximum value $v_i^* = \frac{\bar{\theta}(1-w_i)^2}{4(1-\alpha_i)}$ when $p_i = (1+w_i)/2$, and takes the minimum value as zero when $p_i$ is equal to $w_i$ or 1. Therefore, when choosing the retail price $p_i \in [w_i, 1]$, retailer $i$ is essentially choosing $v_i \in [0, v_i^*]$. Given the two retailers’ choice of retail prices, their positions will be determined by the order of $v_1$, $v_2$, and $v_A$. We show that the positions in equilibrium are completely determined by $v_1^*$, $v_2^*$, and $v_A$.

Lemma 2: In equilibrium, retail prices are set at $p_i^* = (1+w_i)/2$, and the positions of retailer 1, retailer 2 and the outside advertiser are given by the rank of $v_1^*$, $v_2^*$, and $v_A$.

Proof. Suppose $v_1^* > v_2^*$, we first prove that retailer 1 will have a higher position than retailer 2 in equilibrium. We prove by contradiction. Suppose in equilibrium retailer 1 has a lower position than retailer 2, then we must have $v_1 < v_2$. If retailer 1 is in position 3, she can deviate by setting
$v_1$ equal to $v_1^*$, and earn positive profit at position 2 or 1. Thus, it must be that retailer 1 takes position 2, and retailer 2 takes position 1. In that case, retailer 1’s profit $\pi_{R_1} = (1-\alpha_1)d_2(v_1 - v_A)$, which increases with $v_1$, so retailer 1 will choose the largest possible $v_1$ given her position. We have $v_1 = v_2^*$, and $\pi_{R_1} = (1-\alpha_1)d_2(v_2^* - v_A)$. On the other hand, if retailer 1 deviates by setting $v_1 = v_1^*$, she will take the first position, and retailer 2 will take the second position. Given her position, retailer 2 will set $v_2 = v_2^*$ to maximize her profit. As a result, retailer 1 will earn,

$$\pi'_{R_1} = (1-\alpha_1)d_1[v_1^* - ((d_1 - d_2)/d_1 \cdot v_2^* + d_2/d_1 \cdot v_A)] > (1-\alpha_1)d_1[v_2^* - ((d_1 - d_2)/d_1 \cdot v_2^* + d_2/d_1 \cdot v_A)] = \pi_{R_1}.$$ 

This implies that it is not an equilibrium for retailer 1 to take the second position while retailer 2 is taking the first position. To summarize, we have proved that in equilibrium, when $v_1^* > v_2^*$, retailer 1 has a higher position than retailer 2.

Next, suppose $v_1^* > v_2^*$, and we need to prove that (i) when $v_A > v_1^*$, the outside advertiser will take the first position; (ii) when $v_1^* > v_A > v_2^*$, the outside advertiser will take the second position; and (iii) when $v_2^* > v_A$, the outside advertiser will take the third position.

(i) is obvious: given $v_A > v_1^* \geq v_1$, the outsider advertiser must take a higher position than retailer 1.

(ii): When $v_1^* > v_A > v_2^*$, we have $v_A > v_2^* \geq v_2$, so the outside advertiser will always take a higher position than retailer 2. Suppose she takes position 1, then we must have $v_A > v_1$. In this case, retailer 1’s profit is $\pi_{R_1} = (1-\alpha_1)d_2(v_1 - v_2)$. Similarly as above, we can show that retailer 1 can earn a higher profit by setting $v_1$ as $v_1^*$ and take the first position instead. Then (ii) is proved.

(iii) is straightforward to prove by contradiction. Suppose $v_2^* > v_A$ and the outside advertiser takes the second position. In this case, retailer 2 will take position 3 and earn zero profit, and she can deviate by setting $v_2$ at $v_2^*$ and taking the second position instead, which will give her positive profit.

Given that the positions are determined by the rank of $v_1^*, v_2^*$, and $v_A$, we know that each retailer $i$’s profit function increases with $v_i$, so in equilibrium retailer $i$ will set retail price $p_i^* = (1+w_i)/2$, under which $v_i$ takes the maximum value $v_i^*$.

Now we consider the manufacturer’s problem. The manufacturer’s margin for retailer $i$ is $m_i = w_i - c$, where $c$ is the marginal production cost. Similar with 2R model, we consider three cases.
In the first case, the outside advertiser takes the first position. The manufacturer’s problem is,

\[
\begin{align*}
\max_{\alpha_i, w_i} & \ d_2 \left[ \frac{1}{2} \theta (1 - w_1) (w_1 - c) - \alpha_1 \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)} \right] \\
\text{s.t.} & \quad v_A \geq \frac{\bar{\theta}(1 - w_1)^2}{4(1 - \alpha_1)} \geq \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)}, \\
& \quad 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2.
\end{align*}
\]  

The optimal solution is

\[
\begin{align*}
\begin{cases}
\begin{aligned}
w_1^* &= \frac{c+1}{2}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1) \quad &\text{when } v_A \geq \frac{\bar{\theta}(1-c)^2}{16}, \\
w_1^* &= 1 - \sqrt{\frac{4v_A}{\bar{\theta}^2}}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1) \quad &\text{when } \frac{\bar{\theta}(1-c)^2}{16} \geq v_A.
\end{aligned}
\end{cases}
\end{align*}
\]  

Correspondingly, the manufacturer’s profit is

\[
\pi^*_M = \begin{cases}
d_2 \frac{\bar{\theta}(1-c)^2}{8} & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2}{16}, \\
d_2 \left[ (1 - c) \sqrt{v_A \theta} - 2v_A \right] & \text{when } \frac{\bar{\theta}(1-c)^2}{16} \geq v_A.
\end{cases}
\]

To summarize, in this case, the manufacturer essentially only sells to retailer 1, and he provides zero participation rate for retailer 1. Retailer 2 is not competitive at all, so the manufacturer can set the wholesale price for retailer 1 at the manufacturer-profit-maximizing level. When the outside advertiser is too weak, the manufacturer needs to further increase the wholesale price in order to keep retailer 1 at position 2.

In the second case, the outside advertiser takes the second position. The manufacturer’s problem is,

\[
\begin{align*}
\max_{\alpha_i, w_i} & \ d_1 \left[ \frac{1}{2} \theta (1 - w_1) (w_1 - c) - \alpha_1 \left( \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \bar{\theta}(1 - w_2)^2 \right) \right] \\
\text{s.t.} & \quad \frac{\bar{\theta}(1 - w_1)^2}{4(1 - \alpha_1)} \geq v_A \geq \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)}, \\
& \quad 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2.
\end{align*}
\]

The optimal solution is

\[
\begin{align*}
\begin{cases}
\begin{aligned}
w_1^* &= \frac{d_1+c}{d_1+d_2}, w_2^* = 1, \alpha_1^* = 1 - \frac{\bar{\theta}(1-c)^2d_2^2}{4(d_1+d_2)^2v_A}, \alpha_2^* \in [0, 1), \quad &\text{when } v_A \geq \frac{\bar{\theta}(1-c)^2d_2^2}{4(d_1+d_2)^2}, \\
w_1^* &= 1 - \sqrt{\frac{4v_A}{\theta d_2^2}}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), \quad &\text{when } \frac{\bar{\theta}(1-c)^2d_2^2}{4(d_1+d_2)^2} > v_A \geq \frac{\bar{\theta}(1-c)^2}{16} \quad (58) \\
w_1^* &= \frac{1+c}{2}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), \quad &\text{when } \frac{\bar{\theta}(1-c)^2}{16} > v_A.
\end{aligned}
\end{cases}
\end{align*}
\]
Correspondingly, the manufacturer’s profit is

\[
\pi^*_M = \begin{cases} 
\frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1 + d_2)} - (d_1 - d_2)v_A & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1 + d_2)} \\
\frac{\bar{\theta}(1-c)^2}{8} & \text{when } \frac{\bar{\theta}(1-c)^2}{4(d_1 + d_2)^2} > v_A \geq \frac{\bar{\theta}(1-c)^2}{16} \\
\frac{1}{2} (1 - c) \sqrt{v_A \bar{\theta} - 2v_A} & \text{when } \bar{\theta}(1-c)^2 > v_A
\end{cases}
\]

(59)

To summarize, in this case, the manufacturer essentially only sells to retailer 1. When the outside advertiser is relatively weak \((v_A \text{ small})\), the manufacturer can charge retailer 1 manufacturer-profit-maximizing wholesale price and provide her zero participation rate at the same time. When the outside advertiser is relatively strong, he lowers the wholesale price but still provides zero participation rate. Lowering the wholesale price not only increases the retailer’s profit margin and thus can help the retailer beat the outside advertiser, but also increases the demand as the retailing price goes down, while increasing participation rate only has the first effect. Therefore it is more profitable for the manufacturer to lower the wholesale price rather than to raise participation rate in order to help the retailer. When the outside advertiser is even stronger, further lowering the wholesale price while keeping participation rate at zero is no more profitable, so the manufacturer will provide positive participation rate to retailer 1 and slightly increase the wholesale price.

In the third case, the outside advertiser takes the third position. The manufacturer’s problem is

\[
\max_{\alpha_i, w_i} d_1 \left[ \frac{\theta - w_1}{2}(w_1 - c) - \alpha_1 \left( \frac{d_1 - d_2 \bar{\theta}(1-w_2)^2}{d_1} + \frac{d_2}{d_1}v_A \right) \right] + d_2 \left[ \frac{\theta - w_2}{2}(w_2 - c) - \alpha_2 v_A \right] \\
\text{s.t. } \frac{\bar{\theta}(1-w_1)^2}{4(1 - \alpha_1)} \geq \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)} \geq v_A, \\
0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2.
\]

(60)

The optimal solution is

\[
\begin{cases} 
\alpha_1^* = \alpha_2^* = 1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\
\alpha_1^* = \alpha_2^* = 0, & \text{when } \frac{\bar{\theta}(1-c)^2}{3} > v_A \geq \frac{\bar{\theta}(1-c)^2}{16} \\
\alpha_1^* = \alpha_2^* = \frac{1 + c}{2}, & \text{when } \frac{\bar{\theta}(1-c)^2}{16} > v_A
\end{cases}
\]

(61)
Correspondingly, the manufacturer’s profit is

\[
\pi_M^* = \begin{cases} 
(d_1 + d_2) \left[ \frac{\theta(1-c)^2}{4} - v_A \right] & \text{when } v_A \geq \frac{\theta(1-c)^2}{4} \\
(d_1 + d_2) \left[ (1-c)\sqrt{v_A \theta} - 2v_A \right] & \text{when } \frac{\theta(1-c)^2}{4} > v_A \geq \frac{\theta(1-c)^2}{16} \\
(d_1 + d_2)^{\frac{\theta(1-c)^2}{8}} & \text{when } \frac{\theta(1-c)^2}{16} > v_A
\end{cases}
\] (62)

To summarize, in this case, the manufacturer sells to both retailers. The wholesale prices and participation rates for the two retailers are “symmetric”, i.e., the manufacturer will only provide a marginally lower wholesale price or a marginally higher participation rate to retailer 1 to let her beat retailer 2. When the outside advertiser is weak, the manufacturer charges manufacturer-profit-maximizing wholesale price and provides zero participation rate to both retailers. When the outside advertiser is relatively strong, the manufacturer lowers wholesale prices but still provides zero participation rates. When the outside advertiser is even stronger, the manufacturer only charges marginal production cost as wholesale prices, and provides positive participation rates to both retailers.

We compare the manufacturer’s profit in the three cases, and get the manufacturer’s strategy in equilibrium as follows: taking positions (1,2) when \( v_A \leq \frac{d_1^2}{4(d_1+d_2)^2} \theta(1-c)^2 \), taking positions (1,3) when \( \frac{d_1^2}{4(d_1+d_2)^2} \theta(1-c)^2 < v_A \leq \frac{d_1^2+d_2^2}{8(d_1+d_2)} \theta(1-c)^2 \), and taking positions (2,3) when \( v_A > \frac{d_1^2+d_2^2}{8(d_1+d_2)} \theta(1-c)^2 \). The detailed strategy is summarized below (For conciseness, we omit \( \alpha_i^* \) if it equals zero or \( w_i^* = 1 \), and we denote \( s = \theta(1-c)^2, d = d_1 + d_2 \)).

\[
\begin{align*}
(1,2), \quad w_1^* &= w_2^* = \frac{1+c}{2}, \quad \pi_M^* = \frac{ds}{8} \quad \text{when } v_A \leq \frac{s}{16} \\
(1,2), \quad w_1^* &= w_2^* = 1 - \sqrt{\frac{4v_A}{\theta}}, \quad \pi_M^* = d \left[ (1-c)\sqrt{v_A \theta} - 2v_A \right] \quad \text{when } \frac{s}{16} < v_A \leq \frac{s}{2} \\
(1,3), \quad w_1^* &= \frac{(1-c)d_1+d_2}{d}, \quad w_2^* = 1, \quad \alpha_1^* = 1 - \left( \frac{d_2}{d} \right)^2 \frac{s}{4v_A}, \quad \pi_M^* = \frac{sd_2^2}{4d} - (d_1 - d_2)v_A \quad \text{when } \frac{s}{2} < v_A \leq \frac{s}{4} \\
(2,3), \quad w_1^* &= \frac{c+1}{2}, \quad w_2^* = 1, \quad \pi_M^* = \frac{ds}{8} \quad \text{when } v_A > \frac{s}{4}
\end{align*}
\] (63)

We see that when the outside advertiser is relatively weak, the manufacturer sells to both retailers and sets the wholesale price such that both retailers can get a position without his support; when the outside advertiser is strong, the manufacturer only sells to retailer 1 with the wholesale price at the manufacturer-profit-maximizing level, and retailer 1 can get position 2 without help from the manufacturer; when the outside advertiser’s value per click is in the middle range, the manufacturer only sells to retailer 1 and provides positive participation rate to her such that she can get position 1. Therefore, when the wholesale price and participation rates are determined simultaneously, the
manufacturer actually should never sponsor both retailers at the same time. He will only sell to and sponsor one retailer when the competitor is not too strong nor too weak.
References


