A firm’s cost of its products is increasingly transparent to consumers as third parties publish such information. Cost transparency stymies the ability of low-cost firms to intertemporally price discriminate because consumers delay purchases as they are more certain of future price drops as the low-cost firm cannot pool with the high cost firm. Using a dynamic model with forward-looking consumer choice and firm pricing, the paper assesses how cost transparency impacts the firm’s pricing strategy and the resultant impact on consumer surplus and firm profits. Cost transparency narrows the profit differential between the low-cost and high-cost firms. We show that, interestingly, cost transparency can hurt both consumers and low-cost firms ex post. However, in expectation, cost transparency leads to a win-win for both the firm and consumers, which implies that cost transparency can foster a firm’s new product innovation investment. Finally, we consider future price drops arising from potential future entry and find that the profit can be independent of the firm’s cost for a range of cost values.

Key words: signaling, dynamic pricing, strategic consumer, cost information transparency, pricing strategy, consumer surplus, market structure
1. Introduction

A firm’s cost of its products is increasingly transparent to consumers due to the rise of many intermediaries focused on the provision of such information. For example, consumers can estimate the car dealer’s cost of a vehicle by referring to the factory invoice price disclosed by truecar.com. Many other websites also provide cookbook guides or online calculators to help consumers gauge the dealer’s true cost of getting a car from the manufacturer. Consumers can also find cost-breakdown analysis reports for many electronic devices. For example, electronic360.com provides a detailed cost-breakdown analysis of Jawbone Mini Jambox, a Bluetooth speaker, suggesting that a Mini Jambox, sold at $179.99, costs Jawbone only $24.32 to produce.¹ By contrast, some products have narrower gaps between their costs and their prices. For instance, a report from electronic360.com reveals that the total material cost of Xiaomi Millet TV 2 is $613.35, while its price is only $650. This paper investigates how an increase in cost transparency impacts consumer purchase timing, the firm’s pricing trajectory and the resultant impact on consumer surplus and firm profits.

We begin with building intuition for why cost transparency will impact consumer purchase timing and the firm’s pricing trajectory to motivate our model and analysis. Suppose that a consumer has a valuation of \( v \) for a product, which is priced at \( p \). If the consumer is myopic and does not consider future prices, she will purchase it if and only if \( v \geq p \), regardless of the cost of the product. In this case, her knowledge of the firm’s cost will not affect her purchase decision. However, if consumers are strategic and take the firm’s future pricing into consideration, the purchase decision will change. After a firm has sold products to consumers with relatively high

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valuation, it has an incentive to cut its price to target the low-valuation consumers. This phenomenon, termed as *intertemporal price discrimination*, has been well documented in the extant literature (e.g., Bridges et al., 1995; Coase, 1972; Conlisk et al., 1984; Narasimhan, 1989; Stokey, 1979, 1981). As Stokey (1979) pointed out,

“Many new products, like the successive generations of pocket calculators, are very expensive when they first appear on the market; then over a period of time the price declines. Similarly, new books often appear first in hardcover and later as less expensive paperbacks...The falling price...is appeared to be, at least in part, for the purpose of exploiting differences in consumers’ reservation price.”

When consumers are strategic, they will anticipate that the firm will drop the price in the future and may choose to postpone purchase if they are patient and believe that the firm will reduce its price significantly in the future. Their belief about how much the firm’s price will fall in the future is affected by the firm’s cost. If consumers know or believe that the firm has a high cost and a fairly low profit margin, they will expect any dramatic price drop in the future to be unlikely and thus will be more likely to purchase the product in the current period. By contrast, if they know or believe that the firm has low cost and a relatively high profit margin, they may expect the price to drop very significantly in the future and choose to postpone their purchase. For instance, as a technology blog noted, “While prices for sub-50-inch HDTVs are flat, prices for displays larger than 60 inches are already heading down.” It then explained, “TV-makers are looking to stimulate sales at the high-end [HDTVs], where profit margins are the greatest, while keeping prices steady for sub-50-inch models that already have razor-thin margins. On the low-end of the market, the
margins are too small and pricing will stay flat.” On another online tech-forum, one user suggested that consumers should buy new computers right away rather than wait because “… with hardware profit margins already pretty low, I wouldn’t expect massive reductions [of computer prices].” Such strategic buying behavior by the consumer can also help to explain the experimental findings in Mohan et al. (2014) that cost transparency by a firm can increase a consumer’s interest in buying the firm’s product as long as the firm’s profit margin is not too high.

Without cost-information transparency, i.e., when consumers do not know the firm’s cost, consumers’ belief about the future price may depend on the firm’s current price. Intuitively, a high-cost firm tends to charge a high price. When a consumer sees a high price, she may infer that the firm has a high cost and expect that the future price will also be relatively high. In this case, she is more likely to make an immediate purchase, which is beneficial to the firm. As a result, a low-cost firm may have an incentive to also charge a high initial price to make consumers infer that the cost is high so they will be more willing to buy the product right away. However, cost-information transparency reveals the firm’s cost to consumers so firms will have limited ability to manipulate consumers’ expectation of future prices.

This paper analyses a two-period game-theoretic model to study how cost-information transparency affects the firm’s optimal pricing strategy and profit. Without cost-information transparency, the firm’s cost is its private information and consumers may infer the firm’s cost from its current price, which affects their expectation of the future price and hence their current purchase decisions. In this case, the low-cost firm has an incentive to mimic the high-cost firm by charging the same price as the high-cost firm does in the first period, and the high-cost firm has an

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2 http://www.pcworld.com/article/2020284/cracking-the-gadget-pricing-code-buy-now-or-wait.html
3 http://www.tomshardware.com/forum/305519-28-should-expect-computer-prices-drop-released
incentive to increase its first-period price to avoid the low-cost firm’s mimicry. With cost-information transparency, the firm’s cost is common knowledge and consumers’ belief about the future price drop will be based on the firm’s true cost (rather than the inferred cost) and its current profit margin. In this case, since the low-cost firm can no longer exploit mimicking the high-cost firm, the low-cost firm will be worse off and the high-cost firm will be better off.

To introduce a new product to the market, firms often need to incur significant fixed R&D costs before the successfully developed product can be put into production. So, when a firm is in the early stage of developing a new product, it typically will not know its exact production cost. Its product-introduction or market-entry decision will be based on the expected profit. So, the effect of cost transparency on the firm’s product-introduction or market-entry decisions depends on how cost transparency affects the firm’s expected profit. If the firm’s cost turns to be high, cost transparency will increase the firm’s profit, and if its cost turns to be low, cost transparent will reduce the firm’s profit. Our analysis reveals that cost transparency will increase the firm’s expected profit. This implies that cost-information transparency will make the firm more likely to invest (the fixed R&D cost) to develop new products or enter new markets. In other words, cost transparency tends to foster product innovation or market entry. Our analysis also shows that cost transparency reduces the difference in profit between a low-cost firm and a high-cost firm. This suggests that with cost transparency, a high-cost firm may have lower incentives to make cost-reduction investment.

Moreover, policy makers may want to know whether cost transparency benefits or harms consumers. Conventional wisdom suggests that consumers will benefit from more knowledge about the firm. In our setting, consumers will benefit from cost transparency when the firm has a low cost, since such knowledge will result in the firm charging lower prices. However, if the firm
has a high cost, cost transparency may reduce the consumer’s surplus since cost transparency makes a low-cost firm’s pooling impossible hence the high-cost firm will charge higher prices. Our analysis shows that cost transparency will overall increase the expected consumer surplus. Therefore, cost transparency is ex ante a win-win situation for both the firm and consumers.

Intertemporal price discrimination may not be the only reason for a firm to reduce its price over time. Future entrants can intensify competition in a market and drive the future price down. The strength of future competition depends on the level of barrier-to-entry of the market. In markets with high entry barriers, the future competition may be weak so intertemporal price discrimination might be the main driving factor of the price decrease over time. By contrast, markets with low entry barriers will have intense future competition, which may also drive prices down over time. It is important to check whether our findings are robust to different reasons of price decreases when the level of market entry barrier changes.

To test the robustness of our results, we compare the results of two models representing two limiting cases of high barrier-to-entry market and low barrier-to-entry market. In the first model, we assume that the firm is a monopoly in both periods, which is the limiting case where the barrier-to-entry is prohibitively high. In the second model, we assume that the firm is a monopoly in the first period but the market becomes perfectly competitive in the second period, which is the limiting case where the barrier-to-entry is negligibly low so that fast-follower companies can soon introduce competing products to the monopolist new product. We find that most of our results are qualitatively the same in both cases, suggesting the robustness of our results.

In addition, we find that if the firm faces competition in the second period and if its cost is known to consumers, its profit may be independent of its cost. This result is surprising because in general one would expect that a higher cost will negatively impact on the firm’s profit. However,
when consumers are strategic and anticipate the future price change, a higher cost also positively affects the firm’s profit because it makes consumers more willing to purchase the product right away. Our analysis shows that these two effects can offset each other so the firm’s profit may be independent to its marginal cost when the marginal cost is low enough. This result suggests that when consumers know the firm’s cost and the market’s entry barrier is low, the negative effect of higher marginal cost on the firm’s profit can be largely mitigated if consumers can anticipate that the price will drop less dramatically in the future.

Our paper is organized as follows: Section 2 reviews the related literature. Section 3 sets up the model. Section 4 investigates the effects of cost-information transparency in a market with high barrier-to-entry, where the firm remains a monopolist. Section 5 analyzes a market with low barrier-to-entry, where the firm is a monopolist in the first period but faces perfect competition in the second period. Section 6 concludes the paper.

2. Literature Review

Our research contributes to intertemporal price discrimination and dynamic pricing literature. In a seminal paper, Coase (1972) proposes that if a monopolist selling durable products is able to decrease its price on an infinitely frequent basis or on an infinite time horizon, consumers will not accept any price higher than the product’s marginal cost and the firm earns zero profit as a result. However, it has been shown that when the firm can pre-commit its future price or it has increasing cost with respect to time, it can charge prices higher than its marginal cost (Kahn, 1986; Stokey, 1981). In our paper, the dynamic equilibrium prices can be higher than the firm’s marginal cost when the time horizon is finite. Several other papers also focus on firm’s optimal intertemporal price discrimination strategies under different settings (Aviv and Pazgal, 2008; Besanko and Winston, 1990; Conlisk et al., 1984; Levin et al., 2008; Narasimhan, 1989; Stokey, 1979).
Our research complements the aforementioned literature on intertemporal price discrimination in that we study cost uncertainty and information asymmetry, which most of the above researches have neglected. Only a few studies investigate the impact of uncertainty and information asymmetry on the firm’s intertemporal price discrimination decisions. For example, Png (1991) shows that when the firm has demand uncertainty, high-valuation consumers may want to gamble on a price cut in the following period. To mitigate this problem, the firm can offer most-favored-customer protection which guarantees customers who buy early that they will benefit from subsequent reduction in prices. To the best of our knowledge, our paper is the first paper that incorporates the firm’s cost as its private information into the analysis of intertemporal price discrimination.

This paper also contributes to signaling literature in marketing (e.g. Desai and Srinivasan 1995; Jiang et al. 2011; Moorthy and Srinivasan 1995; Simester 1995; Soberman 2003). Several papers have studied how firms should signal their cost information to consumers through the price or the quality of their products. Simester (1995) considers duopolistic firms whose marginal costs are their private information. Due to limitation of advertising space, the firms cannot advertise all their product prices. The firm with low marginal cost may advertise low prices for some products to signal that prices of unadvertised products are also low, since costs of different products are positively correlated. Balachander and Srinivasan (1998) consider a firm selling a durable good whose cost decreases due to learning-by-doing over time. The more products a firm sells in the introductory period, the lower its unit cost in the post-introductory period. Firms are differentiated in their efficiencies of learning-by-doing, which consumers do not know a priori. A high-experience firm tends to sell more products in the first period by charging a lower introductory price to reduce its cost rapidly in the post-introductory period, which lowering consumers’
expectation of the future price. The low-experience firm can strategically reduce its first-period demand by charging a high introductory price to signal its low learning efficiency and that the future price will not drop dramatically. Shin (2005) examines how a store’s “vague advertisements,” which commits only the minimum price of a product, may construct a credible price image if consumers do not know the store’s selling cost. If a store has to incur a high selling cost for every customer visit, it will be dissuaded from advertising low minimum prices but charging a higher price, since doing so will attract too many customers who are less likely to make a purchase. He shows that “vague advertisements” can be credible if there is a substantial difference in retailers’ costs or when the selling cost is high. Guo and Jiang (2015) consider the case in which a consumer feels unfair when the firm’s profit margin is too high relative to her surplus. Hence, a high-cost firm may have an incentive to use the price and product quality to signal that it has high cost. They find that product quality may be non-monotone with respect to the degree of consumers’ inequity aversion. They also find that as the expected cost-efficiency of the market decreases, both product quality and social welfare may increase rather than decrease. Our paper proposes another reason for firms to signal their high cost. We argue that when the firm’s cost is not known to consumers, the high-cost firm has an incentive to signal its high cost to convince consumers that its future price will also likely be relatively high, thus making consumers more willing to purchase the product right away.

This paper also contributes to cost information disclosure literature. This literature is typically focused on whether a firm share its private cost information with competitors or within a supply chain. For example, Fried (1984) shows that in a duopoly setting where both firms sell homogeneous products, each firm will commit to share its cost information to the other firm because such commitment can help coordinate both firms’ production quantity, while Gal-Or
(1986) shows that firms selling differentiated products in a Bertrand market may prefer not to disclose their cost information to their opponents. Shapiro (1986) focuses on firms’ information exchanging behavior when firms can decide whether to commit to disclose cost information prior to knowing their own cost in an oligopolistic market. He finds that firms always want to commit to share cost information and social welfare increases, because cost sharing can increase the market share of lower cost firms. However, consumer surplus is diminished. Yao et al. (2008) considers a supply chain with one supplier and two value-adding retailers, and each retailer possesses private information of its own cost structure. They find that when retailers have low efficiency, they will reveal their cost information to the supplier so the supplier will set a low wholesale price in response. Our paper augments this stream of literature by examining the effects of cost-information transparency to consumers.

Lastly, this paper is related to consumer search literature (e.g., Athey and Ellison, 2011; Brynjolfsson and Smith, 2000; Chade and Smith, 2006; Moorthy et al., 1997; Stahl, 1989; Weitzman, 1979). For example, Stahl (1989) studies an oligopolistic market with homogeneous products, where consumers search for firms’ prices sequentially. He finds that the equilibrium prices in the market are distributed from the firms’ marginal cost (normalized to 0) to the monopoly price. In such search settings, consumers generally make decisions on whether to stop searching and buy the product now, or to conduct more search for prices or information on other product attributes. Similarly, in our setting, consumers make decisions on whether to buy the product now or to wait for future price drops. However, our model is fundamentally different from the traditional search models (e.g., Stahl 1989), where firms have made pricing or other product attribute decisions, but consumers do not costless observe the firms’ product attributes and can incur a search cost to learn a firm’s product attributes (e.g., its current price, quality and features).
before making a purchase decision. By contrast, in our paper, at any given time, consumers perfectly know these attributes when deciding whether to purchase the product immediately. Note that in our model consumers cannot directly observe the firm’s future price, since the firm will dynamically decide its future price in the later period. If consumers have perfect knowledge about the firm’s cost (as in the case of cost transparency), consumers can rationally anticipate the firm’s exact optimal future price when they decide to buy now or later. However, if consumers do not observe the firm’s cost, they will make the current-period purchase decision based on the expected future price, rationally inferred from the firm’s current price and consumers’ belief about the firm’s cost. Thus, in contrast with the search literature, our model entails a signaling problem in a dynamic pricing setting.

3. Model

We consider a monopoly firm selling a durable product in two periods \((t = 1, 2)\). Consumers will purchase at most one unit of the product, either in the first period or the second period. Consumers are uncertain about the firm’s marginal cost but know its probability distribution. For simplicity, we assume that the firm’s marginal cost is either high or low—it is high (i.e., \(c_H\)) with probability \(\alpha\), and low (i.e., \(c_L\)) with probability \(1 - \alpha\), where \(c_H > c_L\). That is, the firm can be of two cost types \(i \in \{H, L\}\). We normalize \(c_L = 0\) and denote \(c_H = c\). So, \(c\) represents the cost difference between the two types of firms. We assume that the firm’s cost is constant across the two periods (i.e., there is no learning-by-doing); please see Section 6 for discussion regarding the possibility of cost decreases over time.

Let \(p_{i1}\) and \(p_{i2}\) denote the \(i\) -type firm’s first-period price and second-period price, respectively. If consumers purchase the product in the first period, they can get a total usage value
of \( \nu \) from the product. We assume that \( \nu \sim \text{uniform}(0,1) \) in the consumer population. Note that for many innovative new products, consumers tend to derive some extra satisfaction from being innovators or early adopters. For example, the consumer’s satisfaction from using this year’s latest model of a smartphone or fashion products tends to be larger than her satisfaction from using the same new product a year later. To capture this in our model, we assume that if consumers buy the product in the second period, besides the time discount of their future utility, they will derive only a fraction \( \mu \in (0,1) \) of the total usage value \( \nu \), because they no longer enjoy the “newness” of the product or the satisfaction of being early adopters.\(^4\) That is, the consumer obtains a net present utility of \( u_1 = \nu - p_{t,1} \) if buying the product in the first period; she gets a net present utility of \( u_2 = \delta(\mu \nu - p_{t,2}) \) if buying the product in the second period, where \( \delta \) is the discount factor. Our analysis will focus on the “newness” factor \( \mu \) rather than the time-discounting of the firm or consumers, so we will assume both the firm and the consumers have the same discount factor \( \delta = 1 \). If the consumer does not purchase the product in either period, she gets zero utility (from the outside option). We normalize the total population of consumers to 1.

The sequence of the game is as follows: In the first period, the firm sets \( p_{t,1} \), and then given \( p_{t,1} \), consumers decide whether to purchase the product in the first period based on their expectation about the firm’s second-period price (i.e., \( \bar{p}_{t,2} \)). In the second period, the firm chooses \( p_{t,2} \) and consumers who have not purchased the product can decide whether to purchase it in the second period.

4. **Analysis**

\(^4\) In this respect, our model is more relevant for hedonic products (or product with a significant hedonic attribute) rather than for pure utilitarian products.
4.1. Firm’s Cost is not Transparent to Consumers

When consumers do not know the firm’s cost, a low-cost firm may have an incentive to mimic a high-cost firm’s pricing strategy in the first period to induce the consumers to make immediate purchases rather than wait. If the firm has a high cost, it would want consumers to know its high cost so that consumers will be less likely to postpone their purchase. To credibly convince consumers that it has a high cost, the firm needs to sufficiently raise its first-period price such that a low-cost firm would not want to mimic. Note that a low-cost firm will not want to choose a very high price even if consumers believe that the firm has a high cost. The opportunity cost of a high price is higher for a low-cost firm than for a high-cost firm, because for a given price, the low-cost firm’s profit margin is higher and hence it loses more profit when losing a sale (due to the high price) than a high-cost firm. So, intuitively if the high-cost firm’s opportunity cost from the high price is not too high, it may find it profitable to choose a high first-period price, \( p_{1,H} \) to separate; otherwise it will pool with the low-cost firm.

Since a firm prefers that consumers believe it has a high cost, one might wonder whether the firm may want to intentionally raise its cost. Our analysis reveals that the low-cost firm’s equilibrium profit is always higher than that of the high-cost firm, and that even if the firm’s cost is directly observable to consumers, it will not be able to improve its profit by raising its actual cost. Thus, although a low-cost firm may want consumers to believe that it has a high cost, it will not want to increase its actual cost.

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5 The high-cost firm may want to directly reveal its cost to consumers. However, the firm’s own claim of high costs may lack credibility and be considered cheap talk. Also, in practice, credibly disclosing cost information may be costly for the firm or can sometimes violate financial regulations for the publicly traded companies. In contrast, many third-party infomediaries, such as 360electronics.com, can examine or take apart the new products to provide reliable cost information.
We derive the Perfect Bayesian Equilibria (PBE) of the game. Two types of PBE are possible in our setting: In separating equilibria, different types of firms charge different $p_1$ so consumers can correctly infer the firm’s type after observing $p_1$. In pooling equilibria, the firm will choose the same $p_1$ regardless of its cost, so after observing $p_1$ consumers are still not able to pin down the firm’s cost. There are infinitely many possible PBE outcomes based on different off-equilibrium beliefs of consumers. We use the Lexicographical Maximum Sequential Equilibrium (LMSE) concept, proposed by Mailath et al. (1993), to refine the multiple equilibria. LMSE and the related Undefeated Equilibrium concepts have been widely used in marketing and management research (e.g., Gill and Sgroi 2012; Jiang et al. 2014; Miklós-Thal and Zhang 2013; Nan and Wen, 2014; Taylor, 1999). In our setting, the LMSE selects among all PBE the outcome that is most profitable for the type of firm that wants its type revealed (i.e., the high-cost firm), and subsequently, if there are still multiple outcomes, then among those selects the outcome that gives the low-type firm the highest profit.\(^6\)

We derive the LMSE of our game by following these three steps: First, we determine the PBE that results in the highest profit for the high-cost firm among all separating equilibria, $\sigma^*_\text{sep}$. Second, we find the PBE giving the highest profit to the high-cost firm among all pooling equilibria, $\sigma^*_\text{pool}$. Finally we compare $\sigma^*_\text{sep}$ and $\sigma^*_\text{pool}$ to determine the LMSE outcome. Throughout this section, we assume $c < \frac{(2-\mu)^2}{4-3\mu} \mu$ such that in equilibrium a high-cost firm will serve some consumers in the second period. Otherwise, the model will de facto reduce to a one-period game and the results are not interesting.

\(^6\) We adopt LMSE as the equilibrium refinement criterion instead of other criteria mainly because, in our model, other equilibrium refinement criteria, such as the Intuitive Criterion, still leave a continuum of PBE and do not identify a unique equilibrium outcome. By contrast, all PBE surviving LMSE correspond to the same equilibrium outcome.
The unique equilibrium outcome is summarized below. When the prior probability of the firm having a high cost is below than a threshold, i.e., $\alpha < \alpha^*$, the equilibrium is separating.

Under the separating equilibrium, if the firm has a high cost, its first-period and second period prices are $p_{H,sep,1}^* = \frac{(2-\mu)^2 + 2c(1-\mu) + (2-\mu)\sqrt{c^2 + 4c(1-\mu)}}{2(4-3\mu)}$ and $p_{H,sep,2}^* = \frac{c^2(4-5\mu) + (2-\mu)^2 + 2c(8+\mu(11\mu-20+\sqrt{c^2 + 4c(1-\mu)})}}{4(2-\mu)(4-3\mu)}$, and its total profit is $\pi_{H,sep}^* = \frac{(2-\mu)^2 + c(2-\mu + 2\alpha(1-\mu))}{2(4-3\mu)}$.

If the firm has a low cost, its first-period and second-period prices are $p_{L,sep,1}^* = \frac{(2-\mu)^2}{2(4-3\mu)}$ and $p_{L,sep,2}^* = \frac{\mu(2-\mu)}{2(4-3\mu)}$, and its total profit is $\pi_{L,sep}^* = \frac{(2-\mu)^2 + 2\mu^2 + 2\alpha(1-\mu)}{4(4-3\mu)}$.

When $\alpha > \alpha^*$, the equilibrium is pooling: both types of firms charge $p_{pool,1}^* = \frac{(2-\mu)^2 + c(2\alpha(1-\mu))}{2(4-3\mu)}$ in the first period. If the firm has a high cost, its profit is $\pi_{H,pool}^* = \frac{\mu(2-\mu)^2 + c^2(4-2(2+2\alpha-a^2))\mu - 2c\mu(6-5\mu-2\alpha(1-\mu))}{4\mu(4-3\mu)}$; if it has a low cost, its profit is $\pi_{L,pool}^* = \frac{(2-\mu)^2 + 4(1-\mu)\alpha(1-\alpha^2)c^2}{4(4-3\mu)}$.

**Lemma 1.** Suppose that consumers do not know the firm’s cost. A unique LMSE outcome exists; it is separating if $\alpha < \alpha^*$, and pooling if $\alpha \geq \alpha^*$, where $\alpha^* =$

\[
\sqrt{\frac{4 - \frac{8c}{1-\mu} - 2\left(\frac{c}{1-\mu}\right)^2 + \frac{4c}{1-\mu}\sqrt{\frac{c}{1-\mu}\left(2 + \frac{c}{1-\mu}\right)} - 2\left(1 - \frac{c}{1-\mu}\right)}{2c\frac{2c}{1-\mu}}}
\]
Figure 1 depicts the equilibrium outcome in different parameter regions in terms of $\alpha$ and $\frac{c}{1-\mu}$. Lemma 1 shows that the equilibrium is separating if and only if $\alpha < \alpha^*$. This result is intuitive.

When $\alpha$ is low enough, if the high-cost firm pools with the low-cost firm, many consumers may postpone their purchases because the firm will very likely have a low cost and will significantly reduce its price in the second period. In other words, when $\alpha$ is very low, the high-cost firm will find the opportunity cost for pooling very high, so it will prefer incurring the signaling cost (of raising its first-period price) to separate itself from the low-cost firm.

Lemma 1 can be equivalently expressed as: The equilibrium is separating if and only if $c > c^*$. Recall that when the firm has a low cost, its marginal cost is normalized to zero. So, the high-cost firm’s marginal cost ($c$) represents the difference between the two types of firms’ marginal costs. Hence, Lemma 1 suggests that when the difference in costs between the two types of firms is high, the high-cost firm will prefer separating itself from the low-cost firm.

\[ c^* \text{ is implicitly determined by } \frac{2c\alpha}{1-\mu} = \sqrt{4 - \frac{8c}{1-\mu} - 2 \left( \frac{c}{1-\mu} \right)^2 + \frac{4c}{1-\mu} \sqrt{\frac{c}{1-\mu} \left( \frac{2 + \frac{c}{1-\mu}}{1-\mu} \right)} - 2 \left( 1 - \frac{c}{1-\mu} \right) \]
Next we consider how the firm’s profit is affected by $c$.

**Corollary 1.** The high-cost firm’s profit $\pi_H^*$ decreases with $c$. The low-cost firm’s profit $\pi_L^*$ increases with $c$ when $c \leq \frac{2(1-\mu)\alpha}{1-\alpha^2}$, decreases with $c$ when $\frac{2(1-\mu)\alpha}{1-\alpha^2} < c \leq c^*$, and remains unchanged when $c > c^*$.

Corollary 1 shows that the high-cost firm’s profit strictly decreases with its marginal cost ($c$), which is intuitive. In contrast, the low-cost firm has zero marginal cost but its profit changes non-monotonically with $c$. When $c \leq c^*$, the equilibrium is pooling. Under this condition, a higher $c$ has two opposite effects on the low-cost firm’s profit. First, a higher $c$ tends to increase the consumers’ expectation of the second-period price hence consumers will be more likely to purchase the product in the first period. This effect increases the low-cost firm’s profit. Second, when $c$ increases, the high-cost firm tends to increase its first-period price, which the low-cost firm will mimic under pooling. This higher pooling price will increase the low-cost firm’s profit margin but reduce its first-period demand. Note that when the pooling price is already high, the low-cost firm’s profit margin is very high, hence the increase in the pooling price (due to an increase in $c$) will have a smaller marginal effect on increasing the firm’s profit margin than on reducing the firm’s demand. As a result, when $c \leq \frac{2(1-\mu)\alpha}{1-\alpha^2}$, the first effect dominates so the low-cost firm’s profit increases with $c$. By contrast, when $\frac{2(1-\mu)\alpha}{1-\alpha^2} < c \leq c^*$, the low-cost firm’s profit decreases with $c$. When $c > c^*$, the equilibrium is separating so the low-cost firm’s profit does not depend on the high-cost firm’s marginal cost $c$.

**4.2. Firm’s Cost is Transparent to Consumers (Perfect-information Case)**
With cost-information transparency, the firm’s cost is common knowledge and the consumers’ belief about future prices will be based on the firm’s true cost. We use “∼” over a variable to indicate this perfect-information setting.

**Lemma 2.** *When the firm’s cost is common knowledge, its first-period and second-period (perfect-information) prices are* \( \bar{p}_{1}^{*} = \frac{(2-\mu)^2}{2(4-3\mu)} + \frac{c_i}{2} \) and \( \bar{p}_{2}^{*} = \frac{\mu(2-\mu)}{2(4-3\mu)} + \frac{c_i}{2} \), respectively, and its total profit is \( \bar{\pi}_i^{*} = \frac{(1-\mu)^2}{4-3\mu} + \frac{(\mu-c_i)^2}{4\mu} \), where \( i \in \{H, L\} \).

We can determine the effect of cost-information transparency by comparing the market outcomes with versus without cost transparency.

**Proposition 1.** *(1)* Cost-information transparency will lower the high-cost firm’s first-period and second-period prices when \( \alpha < \alpha^* \), and will increase its first-period and second-period prices when \( \alpha \geq \alpha^* \); cost-information transparency will not affect the low-cost firm’s prices when \( \alpha < \alpha^* \) and will lower its first-period and second-period prices when \( \alpha \geq \alpha^* \). *(2)* Cost transparency will increase the high-cost firm’s total profit and (weakly) reduce the low-cost firm’s total profit.

Proposition 1 examines how cost transparency affects the firm’s pricing strategies and its total profit. When the firm’s cost is not known to consumers, if \( \alpha < \alpha^* \), the equilibrium is separating. The high-cost firm has to set a first-period price higher than its perfect-information price to reduce its demand enough to make mimicking unprofitable for a low-cost firm. Note that under the separating equilibrium, the high-cost firm’s second-period price is also higher than its second-period price under perfect information (i.e., under cost transparency). This is because in the separating equilibrium the second-period customer base (those consumers who have not purchased in the first period) has higher valuations than the second-period customer base in the case with
cost transparency. When $\alpha \geq \alpha^*$, the equilibrium is pooling. Because consumers cannot tell the firm’s cost, their expectation of the second-period price is lower than the price that the high-cost firm will actually charge in the second period. Therefore, consumers are less willing to purchase the product in the first period, so the equilibrium prices are lower than the high-cost firm’s prices in the perfect-information case. With cost transparency, the high-cost firm can charge its perfect-information price, making higher profits than when its cost is not known to consumers.

By contrast, cost transparency can make the low-cost firm worse off because its low cost is known to consumers. When $\alpha < \alpha^*$ and consumers do not know the firm’s cost, the equilibrium is separating, where consumers can correctly infer the firm’s cost. In this case, the low-cost firm will charge its perfect-information price. Therefore, cost transparency will not affect its profit when $\alpha < \alpha^*$. When $\alpha \geq \alpha^*$, the equilibrium is pooling and consumers cannot infer the firm’s cost from $p_1$. Note that in this case a low-cost firm can always deviate by charging its perfect-information price and earn the same profit as it does in the perfect-information case, so its profit in the pooling equilibrium must be higher than its profit under cost-information transparency. Hence, cost transparency will reduce the low-cost firm’s profit. Table 1 exhibits an example of the equilibrium outcome (the firm’s optimal prices, its demand in each period and its total profit) using $c = 0.2$ and $\mu = 0.75$. 
Table 1 Numerical Example of Equilibrium Outcomes \((c = 0.2, \mu = 0.75)\)

<table>
<thead>
<tr>
<th></th>
<th>Firm's Cost</th>
<th>Cost Transparency</th>
<th>(p_1^*)</th>
<th>(p_2^*)</th>
<th>Demand</th>
<th>(\pi^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(t = 1)</td>
<td>(t = 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Separating</strong></td>
<td>High</td>
<td>w/o</td>
<td>0.6499</td>
<td>0.4299</td>
<td>0.1202</td>
<td>0.3066</td>
</tr>
<tr>
<td></td>
<td></td>
<td>w/</td>
<td>0.5464</td>
<td>0.3678</td>
<td>0.2857</td>
<td>0.2238</td>
</tr>
<tr>
<td>((\alpha = 0.35))</td>
<td>Low</td>
<td>w/o</td>
<td>0.4464</td>
<td>0.2678</td>
<td>0.2857</td>
<td>0.3571</td>
</tr>
<tr>
<td></td>
<td></td>
<td>w/</td>
<td>0.4464</td>
<td>0.2678</td>
<td>0.2857</td>
<td>0.3571</td>
</tr>
<tr>
<td><strong>Pooling</strong></td>
<td>High</td>
<td>w/o</td>
<td>0.5364</td>
<td>0.3828</td>
<td>0.2457</td>
<td>0.2438</td>
</tr>
<tr>
<td></td>
<td></td>
<td>w/</td>
<td>0.5464</td>
<td>0.3678</td>
<td>0.2857</td>
<td>0.2238</td>
</tr>
<tr>
<td>((\alpha = 0.65))</td>
<td>Low</td>
<td>w/o</td>
<td>0.5364</td>
<td>0.2828</td>
<td>0.2457</td>
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<td></td>
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<td>0.4464</td>
<td>0.2678</td>
<td>0.2857</td>
<td>0.3571</td>
</tr>
</tbody>
</table>

Proposition 2 summarizes how cost transparency affects the consumer surplus.

**PROPOSITION 2.** When \(\alpha < \alpha^*\), cost-information transparency will increase the consumer surplus if the firm has a high cost, and it will not affect the consumer surplus if the firm has a low cost. When \(\alpha \geq \alpha^*\), cost-information transparency may lower the consumer surplus if the firm has a high cost and increase the consumer surplus if the firm has a low cost.

Suppose that the firm has a high cost. When consumers do not know the firm’s cost, if \(\alpha < \alpha^*\), \(\sigma^*\) is separating. As discussed earlier, the firm’s price will be higher than its perfect-information price. Hence, cost-information transparency lowers the high-cost firm’s price and increases the consumer surplus. When \(\alpha \geq \alpha^*\), \(\sigma^*\) is pooling and the equilibrium price is lower than the high-cost firm’s perfect-information price. Hence, when consumers know the firm’s cost, it does not
need to lower its price anymore, so consumer surplus may be diminished. Similarly, in the low-cost firm case, consumer surplus increases when $\alpha < \alpha^*$ and remains unchanged when $\alpha \geq \alpha^*$.

So far we have examined the effect of cost transparency on the firm’s profit and consumer surplus given the firm’s cost. If the firm’s cost is high, cost transparency will benefit the firm but may either benefit or harm consumers. By contrast, if the firm’s cost is low, cost transparency makes the firm worse off but consumers better off. In practice, when a firm is in the early stage of new product development, it may not know its exact production cost in the future (for example, the future input material or labor costs may be uncertain when the firm is making its product-development or market-entry investment decisions). What is the effect of cost-information transparency on the firm’s product-development or market-entry decisions? If the firm knows that third-party infomediaries will reveal its costs to consumers in the future, will the firm be more or less likely to invest to introduce the new product to the market? For such decisions, the firm needs to compare its ex ante—before learning its production cost—expected profit with the fixed cost needed for product development or market entry. Policy makers may also be interested in the overall effect of information transparency on the consumer’s welfare (i.e., the expected consumer surplus).

**Proposition 3.** Cost-information transparency will lower the expected price for both the first period (i.e., $\alpha p_{H,1} + (1-\alpha)p_{L,1}$) and the second period (i.e., $\alpha p_{H,2} + (1-\alpha)p_{L,2}$). However, it will increase both the firm’s expected profit (i.e., $\alpha \pi_H + (1-\alpha)\pi_L$) and the expected consumer surplus.

Proposition 3 examines how cost transparency affects the expected price, the expected profit and consumer surplus in the market. We find that cost transparency will reduce the expected prices in both periods. One might intuit that the lower expected price will lead to a lower expected profit
for the firm. However, our analysis reveals that both the firm and consumers will actually be better off in expectation. In the separating parameter region, without cost transparency, if the firm has a high cost, it will raise its price to signal its cost, but with cost transparency the firm will choose a lower price (its perfect-information price). Note that in the separating parameter region (when $\alpha < \alpha^*$ as shown in Figure 1), if the firm has a low cost, it will charge its perfect-information price regardless of there is cost transparency. Thus, overall cost transparency will lower the expected price in the separating parameter region. In contrast, in the pooling parameter region (when $\alpha \geq \alpha^*$), cost transparency will lower the expected price mainly because with cost transparency the low-cost firm will significantly lower its price from the high pooling price to its low perfect-information price. Hence, cost transparency will lower the firm’s prices in expectation, resulting in a higher expected consumer surplus.

Cost transparency will increase the firm’s ex ante expected profit for the following reason. In the separating parameter region (when $\alpha < \alpha^*$), if the firm has a low cost, cost transparency will have no effect on its profit, but if the firm has a high cost, cost transparency will allow the firm to make a higher profit (i.e., its perfect-information profit) since the firm no longer needs to distort its price to separate from the low-cost firm. Thus, overall, in the separating parameter region, cost transparency will give the firm a higher expected profit. Similarly, in the pooling parameter region ($\alpha \geq \alpha^*$), cost transparency will also increase the firm’s ex ante expected profit, mainly because the firm’s benefit from having a high cost revealed dominates its profit loss from having a low cost revealed.

Proposition 3 implies that cost transparency increases the likelihood that the firm will invest the required fixed cost to introduce a new product. In other words, cost transparency tends to foster product innovation or market entry. By contrast, cost transparency will tend to lower the firm’s
incentives in making cost-reduction investments (e.g., process innovation). This is because cost transparency will increase a high-cost firm’s profit and reduce a low-cost firm’s profit, leading to a smaller profit difference between a high-cost firm and a low-cost firm.

5. **Extension: Market with Low Barriers to Entry**

In Section 4, we implicitly assumed the existence of high entry barriers in the market such that the firm is a monopoly in both periods and so intertemporal price discrimination is the reason for its price decrease over time. In reality, competition may be another reason for a firm to reduce its price over time. Future entrants can intensify competition in the market and drive down the future price. The strength of future competition depends on the level of barrier-to-entry in the market. Some markets have high barrier-to-entry and firms can maintain their monopoly power for a long period of time because they are protected by patents, government policies, geographical barriers, etc. Other markets may have relatively low barrier-to-entry and a monopolist’s market power will diminish significantly by subsequent market entrants. In markets with high entry barriers, future competition may be weak, so intertemporal price discrimination may be the main driving force of price reductions over time. By contrast, markets with low entry barriers will have intense future competition, driving prices down over time. We check whether our qualitative findings in Section 4 continue to hold when the consumers’ expectation of lower future prices is a result of future competition (as in a market with a low entry barrier). It is also of practical relevance for managers to know how the effects of cost-information transparency may vary with the level of competition or entry barriers in the market. How should they adjust their prices differently based on the competition characteristics? How are their profits affected?

This section studies a limiting case where the entry barrier is very low so that the market becomes perfectly competitive in the second period. This contrasts our assumption in Section 4
that the firm remains a monopoly in the second period, which represents the other limit where the entry barrier is prohibitively high. In reality, entry barriers or the level of competition in the market may lie somewhere between these two cases; analyzing these two limits will help us better gauge the boundary of our results.

Let us assume that in the first period the firm is a monopoly but in the second period new entrants will enter the market producing the product with the same marginal cost \( c_i \). Therefore, the equilibrium price drops to \( c_i \) in the second period. Other aspects of the model are the same as those in the core model. We will focus on the interesting case of \( c \leq \frac{\mu}{2} \) such that even if the firm has a high cost, it still in equilibrium serves some consumers in the second period. Lemma 3 presents the equilibrium results when consumers do not know the firm’s cost.

**Lemma 3.** Suppose that the firm’s cost is not known to consumers. A unique LMSE outcome exists: it is separating if \( \alpha < \alpha^*_{\text{comp}} \), and pooling if \( \alpha \geq \alpha^*_{\text{comp}} \), where \( \alpha^*_{\text{comp}} = 1 - \frac{1 - \left( \frac{c}{\sqrt{1-\mu(1-2\mu)}} - \frac{c}{1-\mu} \right)^2}{\frac{c}{1-\mu}} \).

When the prior probability of the firm having a high cost is below a threshold (i.e., \( \alpha < \alpha^*_{\text{comp}} \)), the equilibrium is separating. Under the separating equilibrium if the firm’s cost is high, its optimal first-period price will be \( p^*_{H,\text{sep},1} = \frac{1-\mu+c+\sqrt{c(c+2(1-\mu))}}{2} \) with a corresponding total profit of \( \pi^*_{H,\text{sep}} = \frac{1-\mu-2c(c+1-\mu-\sqrt{c(c+2(1-\mu))})}{4(1-\mu)} \); if the firm’s cost is low, its optimal first-period price is \( p^*_{L,\text{sep},1} = \frac{1-\mu}{2} \) with a corresponding total profit of \( \pi^*_{L,\text{sep}} = \frac{1-\mu}{4} \). When \( \alpha > \alpha^*_{\text{comp}} \), the equilibrium

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8 The subscript “comp” stands for “competitive”. 
is pooling, and both types of firms will charge \( p_{pool,1}^* = \frac{(1+\alpha)c+1-\mu}{2} \) in the first period. If the firm has a high cost, its total profit will be \( \pi_{H, pool}^* = \frac{(1-\mu-(1-\alpha)c)^2}{4(1-\mu)} \); if the firm has a low cost, its total profit will be \( \pi_{L, pool}^* = \frac{(ac+1-\mu)^2-c^2}{4(1-\mu)} \). Figure 2 illustrates the equilibrium outcome in different parameter regions in terms of \( \alpha \) and \( \frac{c}{1-\mu} \). One can see that the boundary separating the pooling and separating parameter regions is qualitatively the same as that in Figure 1.

**Figure 2 Equilibrium Outcome – Market with Low Entry Barriers**

![Equilibrium Outcome](image)

Corollary 2 shows that when the market has low barriers-to-entry, the high-cost firm’s profit decreases with its marginal cost \( (c) \), but interestingly, the high-cost firm’s marginal cost \( (c) \) has a non-monotonic effect on the low-cost firm’s profit. This is qualitatively similar to Corollary 1.
COROLLARY 2. The high-cost firm’s total profit $\pi_H^*$ decreases with $c$. The low-cost firm’s total profit $\pi_L^*$ increases with $c$ when $c \leq \frac{(1-\mu)\alpha}{1-\alpha^2}$, decreases with $c$ when $\frac{(1-\mu)\alpha}{1-\alpha^2} < c \leq c_{\text{comp}}^*$, and is independent of $c$ when $c > c_{\text{comp}}^*$.

LEMMA 4. With cost transparency, the firm’s optimal first-period price is $\bar{p}_{l,1}^* = c_i + \frac{1-\mu}{2}$, and its profit is $\bar{\pi}_i^* = \frac{1-\mu}{4}$, where $i \in \{H, L\}$.

Lemma 4 depicts the equilibrium outcome with cost-information transparency. We highlight one interesting new finding from Lemma 4 in the following proposition.

PROPOSITION 4. With cost transparency and low entry barriers in the market, the firm’s equilibrium profit is independent of its marginal cost $c_i$ for $c_i < \frac{\mu}{2}$.

One might expect that a firm’s profit should strictly decrease with its marginal cost. However, Proposition 4 shows that, counterintuitively, the firm’s profit may remain unchanged with its marginal cost as long as that cost is not too high. This is because a higher marginal cost has two opposing effects on the firm. On the one hand, it tends to lower the firm’s profit margin. On the other hand, it also raises the consumers’ expected second-period price ($\bar{p}_2$), which tends to increase the likelihood that consumers will buy the product in the first period. Note that when consumers know the firm’s cost, $\bar{p}_2$ will be equal to the firm’s marginal cost $c_i$ (since the market is perfectly competitive in the second period). Also one can show that the consumer’s first-period purchase decision will remain unchanged if both the first-period price $p_1$ and the expected second-period price $\bar{p}_2$ increase by the same amount (not too large). So, when $c_i$ increases, the firm can raise its

\[ c_{\text{comp}}^* \text{ is implicitly determined by } \frac{\left( c_{\text{comp}}^* \left( \frac{c_{\text{comp}}^*}{1-\mu} + 2 \right) - \frac{c_{\text{comp}}^*}{1-\mu} \right)^2}{\sqrt{1 \left( 1 - \frac{c_{\text{comp}}^*}{1-\mu} \left( \frac{c_{\text{comp}}^*}{1-\mu} + 2 \right) - \frac{c_{\text{comp}}^*}{1-\mu} \right)^2}} = (1 - \alpha) \frac{c_{\text{comp}}^*}{1-\mu} \]
first-period price by the same amount, making the firm’s profit margin and its market coverage stay the same in the first period. Since the firm earns zero profit in the second period, the firm’s total profit is just its first-period profit, which will not be affected by the change of marginal cost. In other words, when the market has very low entry barriers, the two opposing effects of a cost increase can cancel out when there is cost transparency in the market, so the firm’s total profit will be unchanged.

Proposition 4 shows the invariance of the firm’s profit with respect to its marginal cost under cost-information transparency, assuming perfect competition in the future. In practice, the level of competition in the market is often not perfect, so these two effects of a cost change may not perfectly cancel out, and the firm’s profit can decrease with its marginal cost. We should interpret Proposition 4 as that under cost transparency the negative effect of an increase in a firm’s marginal cost may be significantly mitigated because it can reduce the consumer’s strategic delay of product purchase.

Proposition 4 is shown based on two assumptions. First, the competition is perfect in the second period. Second, the consumers’ valuation $v$ is uniformly distributed on $[0,1]$. One might wonder whether Proposition 4 will still hold when these two assumptions are relaxed. We have checked the robustness of Proposition 4. First, instead of perfect competition in the second period, we assume that each entrant needs to incur some fixed cost $F$ to enter the market. In the second period, the incumbent firm and all new entrants will engage in Cournot competition, and the equilibrium number of entrants in the second period is such that each entrant’s gross profit in the second period just covers its market-entry cost $F$. Our original assumption that the market is perfectly competitive is a special case of $F = 0$. The second robustness check we conduct is to use a general
cumulative distribution function, $\mathcal{F}(\nu)$, for $\nu$, rather than restricting $\nu \sim \text{uniform}(0,1)$. In the Appendix we show that Proposition 4 still holds under both of these alternative models.

Our analysis shows that all other results (Propositions 1, 2 and 3) stay qualitatively the same in the current case of very low barrier-to-entry. That is, the qualitative effects of cost transparency on the firm’s profit and consumer surplus are robust to different levels of entry barriers in the market.

We also compare the results in Section 4 (high entry barriers) and Section 5 (low entry barriers) to determine how the level of entry barriers will influence the magnitude of the effects of cost transparency. In both the high entry barrier and the low entry barrier cases, we calculate the percentage increase in the firm’s expected profit due to cost transparency (i.e., $\frac{\alpha^H + (1-\alpha)L}{\alpha^H + (1-\alpha)L} - 1$), which is shown in Figure 3.

**Figure 3 Percentage Increase in Expected Profit due to Cost Transparency**

![Figure 3 Percentage Increase in Expected Profit due to Cost Transparency](image)

We have two observations. First, regardless of the level of entry barriers, the impact of cost-information transparency on the firm’s profit is the strongest when $\alpha$ is in the middle range.
Intuitively, when $\alpha$ is very high (low), the firm’s cost uncertainty is low since consumers are almost sure that the firm has a high cost (low cost) even without cost transparency. Thus, the impact of cost transparency will be relatively small.

Second, the increase in the firm’s expected profit due to cost-information transparency is higher when the barrier-to-entry is low than when it is high, i.e., the effect of cost transparency is stronger when the market has a low entry barrier. This is because with a very high entry barrier, the firm is a monopolist in both periods. When consumers do not know the firm’s cost, although a high-cost firm will not be able to earn its perfect-information profit in the first period, in the second period it can still recover some profit by strategically lowering its second-period price. Similarly, if the firm has a low cost, it can also still exploit the unserved consumers in the second period. By contrast, with a very low entry barrier, competition in the second period will drive the market price down to the marginal cost, so a high-cost firm suffers more and a low-cost firm benefits less from the cost-information asymmetry. In other words, cost-information transparency will lead to a higher increase in the firm’s expected profit when the barrier-to-entry is low than when it is high.

We also examine the percentage change in the firm’s optimal price by focusing on the case where $\alpha < \min\{\alpha^*, \alpha_{comp}^*\}$, such that the equilibrium is separating in both the low entry barrier and the high entry barrier cases. We find that cost-information transparency will reduce the high-cost firm’s optimal price by a higher percentage when the market’s barrier-to-entry is low than when it is high. This is because when the market has a low entry barrier, all of the firm’s profit comes from the first period. Thus, convincing consumers to purchase in the first period is crucially important, giving the low-cost firm a stronger incentive to mimic the high-cost firm when the firm’s cost is not known to consumers. As a result, without cost transparency, when the market has a low entry barrier, the high-cost firm has to further distort its price upwards from its perfect-
information price to separate. So, put differently, the price-decreasing effect of cost transparency on the high-cost firm is higher when the barrier-to-entry is low than when it is high. That is, in a market with a lower entry barrier (i.e., with more intense future competition), a high-cost firm should reduce its price more (in percentage) in the presence of cost transparency.

6. Conclusion

This paper examines how cost-information transparency affects the firm’s pricing strategy, profitability and consumers’ surplus. We show that when the prior probability that the firm has a high cost ($\alpha$) is low, cost transparency increases the high-cost firm’s profit as well as consumer surplus. The low-cost firm’s profit is unchanged. When $\alpha$ is high, although ex post the firm or consumers may be worse off, cost transparency ex ante makes both parties better off in expectation. This result suggests that in expectation cost-information transparency can foster firms’ investment in new product innovation and market entry and improve consumers’ well-being. We also find that in markets with low entry barriers, cost transparency may significantly mitigate the negative effect of an increase in a firm’s marginal cost. We check the robustness of our result by comparing the results of our analysis when the barrier-to-entry is high with when it is low, finding that most of our results are qualitatively the same in both cases. Moreover, our analysis shows that cost transparency increases the firm’s profit more when the entry barrier is low.

This paper provides several empirically testable results. First, we predict that as the firm’s production-cost information becomes more transparent to consumers, the average price charged by the firm tends to decrease. Secondly, the firm’s expected profit tends to increase due to cost-information transparency. Moreover, in markets with lower barrier-to-entry, the average profit increase of the firm and the average price decrease due to cost transparency are likely to be higher. One can check the validity of our model by testing these predictions with appropriate data.
In reality, production cost of a product tends to decrease over time due to technology advancement or learning-by-doing. In this case, consumers are more willing to postpone purchasing because the cost reduction will drive the price down. However, if the high-cost firm’s future cost is still higher than the low-cost firm’s future cost, the high-cost firm still has an incentive to reveal its high-cost and our main intuition will still hold.

Our analysis can be extended in the following ways: First, throughout the paper we have assumed that all consumers are strategic and anticipating future price change when they make purchase decisions. It will be interesting to investigate what happens if some of the consumers are myopic and do not anticipate future price changes. Previous research shows that the existence of myopic consumers increases the firm’s profit when there is no information asymmetry (Aviv and Pazgal, 2008). Future research can focus on whether this result will still hold when information asymmetry exists and consumers do not know the firm’s cost.

Second, in this paper we analyze a two-period model. We do not investigate a multi-period model or a continuous-time model. Previous research shows that as the number of period increases, firms have stronger and stronger incentive to acquire a good reputation, for example, “being tough”, “being benevolent” (Kreps and Wilson, 1982). If their arguments can be applied to our research, a firm tends to have a stronger incentive to be considered as having high costs. Hence, the low-cost firm has stronger incentive to mimic the high-cost firm and the high-cost firm needs to raise its price more to prevent being mimicked. As a consequence, we conjecture that if the number of periods increases, the high-cost firm will benefit more from cost transparency but the low-cost firm is worse off to a larger extent. However, analyzing firms’ signaling behavior in more-than-two-period or continuous-time model is technically demanding and is worth its own study, which we leave for future research.
Appendix

PROOF OF LEMMA 1 AND 2: See Technical Appendix.

PROOF OF PROPOSITION 1. (1) We compare the firm’s optimal price and profit when its cost is not known to consumers with that when the cost is known.

\[
\bar{p}_{H,1} - p^*_{H,sep,1} = \frac{(2-\mu)^2}{2(4-3\mu)} + \frac{c}{2} - \frac{(2-\mu)^2 + 2c(1-\mu) + (2-\mu)\sqrt{c(2+4(1-\mu))}}{2(4-3\mu)} = \frac{(2-\mu)(c - \sqrt{c(2+4(1-\mu))})}{2(4-3\mu)} < 0.
\]

\[
\bar{p}_{H,2} - p^*_{H,sep,2} = \frac{\mu}{2-\mu} (\bar{p}_{H,1} - p^*_{H,sep,1}) < 0.
\]

\[
\bar{p}_{H,1} - p^*_{pool,1} = \frac{(2-\mu)^2}{2(4-3\mu)} + \frac{c}{2} - \frac{(2-\mu)^2 + c(2-\mu + 2\alpha(1-\mu))}{2(4-3\mu)} = \frac{2c(1-\mu)(1-\alpha)}{2(4-3\mu)} > 0.
\]

\[
\bar{p}_{H,2} - p^*_{pool,2} = \frac{\mu}{2-\mu} (\bar{p}_{H,1} - p^*_{pool,1}) > 0.
\]

\[
\bar{p}_{L,1} = p^*_{L,sep,1}, \bar{p}_{L,2} = p^*_{L,sep,2}.
\]

\[
\bar{p}_{L,1} - p^*_{pool,1} = \frac{(2-\mu)^2}{2(4-3\mu)} - \frac{(2-\mu)^2 + c(2-\mu + 2\alpha(1-\mu))}{2(4-3\mu)} = -\frac{c(2-\mu + 2\alpha(1-\mu))}{2(4-3\mu)} < 0.
\]

\[
\bar{p}_{L,2} - p^*_{pool,2} = \frac{\mu}{2-\mu} (\bar{p}_{L,1} - p^*_{pool,1}) > 0.
\]

\[
\bar{\pi}_H - \pi^*_{H,sep} = c \left( \frac{c + 2(1-\mu)}{2(4-3\mu)} \right) = \frac{(1-\mu)^2 c}{1-\mu} \left( \frac{c}{1-\mu} + 2 \frac{c}{\sqrt{1-\mu}} \right) > 0.
\]

\[
\bar{\pi}_H - \pi^*_{H,pool} = \frac{(1-\alpha)c(4-4\mu - (1-\alpha)c)}{4(4-3\mu)}. \text{ Because } c < \frac{(1-\mu)^2}{(2-\mu)(4-3\mu)}, \bar{\pi}_H - \pi^*_{H,pool} > 0.
\]

\[
\bar{\pi}_L = \pi^*_{L,sep}.
\]

\[
\bar{\pi}_L - \pi^*_{L,pool} < 0 \text{ because } \bar{\pi}_L = \pi^*_{L,pool,dev} \leq \pi^*_{L,pool}.
\]
PROOF OF PROPOSITION 3.

Cost transparency strictly increases the firm’s expected first-period and second-period prices:

\[
(\alpha \tilde{p}_{H,1}^* + (1 - \alpha) \tilde{p}_{L,1}^*) - (\alpha p_{H,sep,1}^* + (1 - \alpha) p_{L,sep,1}^*) = \alpha (\tilde{p}_{H,1}^* - p_{H,sep,1}^*) + \\
(1 - \alpha)(\tilde{p}_{L,1}^* - p_{L,sep,1}^*) < 0.
\]

\[
(\alpha \tilde{p}_{H,1}^* + (1 - \alpha) \tilde{p}_{L,1}^*) - (\alpha p_{pool,1}^* + (1 - \alpha) p_{pool,1}^*) = \alpha (\tilde{p}_{H,1}^* - p_{pool,1}^*) + \\
(1 - \alpha)(\tilde{p}_{L,1}^* - p_{pool,1}^*) = \frac{2c (1-\mu)(1-\alpha)}{2(4-3\mu)} - (1 - \alpha) \frac{c(2-\mu+2\alpha(1-\mu))}{2(4-3\mu)} = \frac{2-\mu}{2(4-3\mu)} < 0. \quad \blacksquare
\]

Cost transparency strictly increases the firm’s expected profit:

\[
(\alpha \tilde{\pi}_H^* + (1 - \alpha) \tilde{\pi}_L^*) - (\alpha \pi_{H,sep}^* + (1 - \alpha) \pi_{L,sep}^*) = \alpha (\tilde{\pi}_H^* - \pi_{H,sep}^*) + (1 - \alpha)(\tilde{\pi}_L^* - \pi_{L,sep}^*) > 0.
\]

\[
(\alpha \tilde{\pi}_H^* + (1 - \alpha) \tilde{\pi}_L^*) - (\alpha \pi_{pool}^* + (1 - \alpha) \pi_{pool}^*) = \alpha (\tilde{\pi}_H^* - \pi_{pool}^*) + \\
(1 - \alpha)(\tilde{\pi}_L^* - \pi_{pool}^*) = \frac{c(a-c(4\mu-1-\alpha)c)}{4(4-3\mu)} + (1 - \alpha) \frac{c(a-c(1-\alpha)4\mu))}{4(4-3\mu)} = \frac{(1-\alpha)^2c^2}{4(4-3\mu)} > 0.
\]

Cost transparency strictly increases the expected consumer surplus:

We define \( CS_{i,t} \) as consumer surplus in period \( t \) if the firm’s cost is \( c_i \), and \( CS_i = CS_{i,1} + CS_{i,2} \) is the total consumer surplus. \( \overline{CS}_i = \overline{CS}_{i,1} + \overline{CS}_{i,2} = \int_{v_{i,H}}^1 (s - \tilde{p}_{i,1}^*) ds + \int_{v_{i,L}}^1 (\mu s - \tilde{p}_{i,2}^*) ds \), \( CS_{i,sep} = CS_{i,sep,1} + CS_{i,sep,2} = \int_{v_{i,sep,1}}^1 (s - p_{i,sep,1}^*) ds + \int_{v_{i,sep,2}}^1 (\mu s - p_{i,sep,2}^*) ds \) and \( CS_{i,pool} = CS_{i,pool,1} + CS_{i,pool,2} = \int_{v_{i,pool,1}}^1 (s - p_{i,pool,1}^*) ds + \int_{v_{i,pool,2}}^1 (\mu s - p_{i,pool,2}^*) ds \), where \( i \in \{H, L\} \). The expected consumer surplus is \( E[\overline{CS}] = \alpha \cdot \overline{CS}_H^* + (1 - \alpha)\overline{CS}_L^* \) when the cost is
known to consumers, is $E[CS_{sep}] = \alpha \cdot CS_{H,sep}^* + (1 - \alpha)CS_{L,sep}^*$ when the cost is not known to consumers and $\sigma^*$ is separating, and is $E[CS_{pool}] = \alpha \cdot CS_{H,pool}^* + (1 - \alpha)CS_{L,pool}^*$ when the cost is not known to consumers and $\sigma^*$ is pooling.

First, we will show that when $\sigma^*$ is separating, cost transparency strictly increases consumer surplus. $\tilde{CS}_H^* > CS_{H,sep}^*$ since $\tilde{p}_{H,1}^* < p_{H,sep,1}^*, \tilde{p}_{H,2}^* < p_{H,sep,2}^*, \tilde{v}_{H,1}^* < v_{H,sep,1}^*$ and $\tilde{v}_{H,2}^* < \tilde{v}_{H,sep,2}^*$. Intuitively, when consumers know the firm’s cost, consumer surplus is higher because prices in both periods are lower and the amount of first-period consumers and the amount of total consumers both increase. Moreover $\tilde{CS}_L^* = CS_{L,sep}^*$ since $\tilde{p}_{L,1}^* = p_{L,sep,1}^*, \tilde{p}_{L,2}^* = p_{L,sep,2}^*$. Therefore, $E[CS] = E[CS_{sep}] > 0$.

Next we claim that when $\sigma^*$ is pooling, cost transparency strictly increases consumer surplus. Let $\hat{p}_1 = \alpha \hat{p}_{H,1}^* + (1 - \alpha)\hat{p}_{L,1}^*$. One can see that $\hat{p}_1 < p_1$ from section (2) of the proof of proposition 1. Now suppose both the high-cost firm and the low-cost firm have to charge a price $\hat{p}_1$ in the first period. Given this first period price $\hat{p}_1$, type-i firm charges $\hat{p}_{2,i}^*$ to maximize its second period profit $\tilde{\pi}_{2,i}$. Since both types of firms charge the same first period price, at the end of the first period consumers still cannot tell the firm’s cost type. One can show that $\hat{p}_{2,H}^* = \frac{\mu(2\hat{p}_1 - ac)}{2(2 - \mu)} + \frac{c}{2}$ and $\hat{p}_{2,L}^* = \frac{\mu(2\hat{p}_1 - ac)}{2(2 - \mu)}$. The marginal consumer in the first period is $\hat{v}_1 = \frac{2\hat{p}_1 - ac}{2 - \mu}$, the marginal consumer in the second period if the firm has high cost is $\hat{v}_{2,H} = \frac{2\hat{p}_1 - ac}{2(2 - \mu)} + \frac{c}{2}$, and the marginal consumer in the second period if the firm has low cost is $\hat{v}_{2,L} = \frac{2\hat{p}_1 - ac}{2(2 - \mu)}$. Therefore, when both types of firms have to charge $\hat{p}_1$ in the first period, consumer surplus is $\tilde{CS}_i = \int_{\hat{v}_1}^{1}(s - \hat{p}_1)ds + \int_{\hat{v}_{2,i}}^{\hat{v}_1}(\mu s - \hat{p}_{2,i}^*)ds$ if the firm has cost $c_i$. Since $\hat{p}_1 < p_{pool,1}^*, \hat{p}_{2,H} < p_{H,pool,2}^*$.
\[ \hat{p}_{2,L} < p^*_{L,\text{pool},2}, \hat{v}_1 < v^*_{\text{pool},1}, \hat{v}_{H,2} < v^*_{H,\text{pool},2}, \text{ and } \hat{v}_{L,2} < v^*_{L,\text{pool},2}, CS_i > CS^*_{\text{pool}}, \forall i \in \{H,L\}, \]
therefore \( E[CS] > E[CS^*_{\text{pool}}] \).

The value of \( E[CS] \) bridges the comparison of \( E[CS] \) and \( E[CS^*_{\text{pool}}] \). One can show that

\[ E[CS] = E[CS] = \frac{48-4c(4-3\mu)(20-(30-11\mu)\mu)-\mu(326-32111\mu)}{2(4-3\mu)^3} \]

after simple algebra.

Therefore, \( E[CS] > E[CS^*_{\text{pool}}] \). Thus, when \( \sigma^* \) is pooling, cost transparency increases the expected consumer surplus. ■

PROOF OF LEMMA 3 AND 4. See Technical Appendix

ROBUSTNESS CHECK OF PROPOSITION 4. We consider two relaxations of our assumptions.

(1) Assume that in the second period, each entrant incurs a fixed cost \( F \) to enter the market.

In the second period, the incumbent firm and all entrants involve in a Cournot competition, in which each firm decides its output \( \bar{q} \), and the equilibrium price clears the market. First consider the high-cost firm. The marginal consumer who purchase in the first period is \( v^*_{u,1} + v^*_{l,2} \). In the second period, the demand is \( D(\bar{p}_{2,H}; \bar{p}_{1,H}, \bar{p}_{2,H}) = \frac{\bar{p}_{H,1}-\bar{p}_{H,2}}{1-\mu} - \frac{\bar{p}_{H,2}}{\mu} \), and the inverse demand function is \( \bar{p}_{H,2} = \frac{1-\mu}{\mu} (\bar{p}_{H,1} - \bar{p}_{H,2}) - \mu D \). Suppose there are \( n \) firms in the market in the second period. The \( j \)-th firm’s second period profit is \( \bar{\Pi}_j = (\bar{p}_{H,2} - c)q_j = \left[ \frac{1-\mu}{\mu} (\bar{p}_{H,1} - \bar{p}_{H,2}) - \mu D - c \right] q_j \). Because in equilibrium the market clears, \( D = \sum_{k=1}^{n} \bar{q}_k \). Substituting it into \( \bar{\Pi}_j \) and
construct the first order condition with respect to \( q_j \), we have

\[
\bar{q}_j^* = \frac{\bar{p}_{H,1} - \bar{p}_{H,2} - c}{1 - \mu} \cdot \frac{\sum_{k=1}^{n} q^k - \frac{c}{\mu}}{2}.
\]

Combining the first order condition for all \( n \) firms together, we have

\[
\bar{q}_j^* = \frac{\mu}{1 - \mu} \cdot \frac{\bar{p}_{H,1} - \bar{p}_{H,2} - c}{n + 1}, \quad \forall j = 1, 2, \ldots, n.
\]

The equilibrium price in the second period is \( \bar{p}_{H,2} = \frac{\bar{p}_{H,1} - \bar{p}_{H,2} + nc}{1 - \mu} \). Therefore, \( \bar{\Pi}_j^* = \frac{1}{\mu} \left( \frac{\mu}{n + 1} \right)^2 \cdot \frac{1}{\mu} \left( \frac{\mu}{n + 1} \right)^2 \).

In equilibrium, no entrants outside the market will enter and no firms inside the market will exit, which implies that \( \bar{\Pi}_j^* = F \). Thus in equilibrium, there are \( n^* = \frac{\mu}{1 - \mu} \cdot \frac{1 - \mu + \sqrt{\mu F}}{2} - 1 \) firms in the market. Substituting this into \( \bar{p}_{H,2}^* \), we have \( \bar{p}_{H,2}^* = c + \sqrt{\mu F} \). Because consumers have rational expectation of \( \bar{p}_{H,2}^* \), \( \bar{p}_{H,2}^* = c + \sqrt{\mu F} \).

The incumbent firm’s first period profit is \( \bar{\pi}_{H,1}^* = (\bar{p}_{H,1} - c) \left( 1 - \frac{\bar{p}_{H,1} - c - \sqrt{\mu F}}{1 - \mu} \right) \). The optimal first period price is therefore \( \bar{p}_{H,1}^* = c + \frac{1 - \mu + \sqrt{\mu F}}{2} \), and the first period profit is

\[
\bar{\pi}_{H,1}^* = \frac{(1 - \mu + \sqrt{\mu F})^2}{4(1 - \mu)}.
\]

The firm’s total profit is \( \bar{\pi}_H = \bar{\pi}_{H,1}^* + \bar{\Pi}_j^* - F = \frac{(1 - \mu + \sqrt{\mu F})^2}{4(1 - \mu)} \), which is independent to \( c \).

The analysis of the low-cost firm is analogous, and it can be shown that the firm’s total profit is \( \bar{\pi}_L^* = \frac{(1 - \mu + \sqrt{\mu F})^2}{4(1 - \mu)} \), the same as the high-cost firm’s profit.

(2) Assume that the cumulative distribution function of \( \nu \) has a general form \( \mathcal{F}(\nu) \).
Suppose the firm has high cost. The marginal consumer who will purchase the product in the first period is $\tilde{\nu}_{H,1} = \frac{\tilde{p}_{H,1} - c}{1 - \mu}$. Thus the first-period demand is $1 - \mathcal{F}\left(\frac{\tilde{p}_{H,1} - c}{1 - \mu}\right)$. The firm’s profit all comes from the first period, which is $\pi_{H,1} = (1 - \mathcal{F}\left(\frac{\tilde{p}_{H,1} - c}{1 - \mu}\right))(\tilde{p}_{H,1} - c)$. Moreover, let $p^{0*} = \arg\max_{p \geq 0} (1 - \mathcal{F}\left(\frac{p}{1 - \mu}\right))p$ (we assume that $p^{0*}$ uniquely exists). Note that $\tilde{p}_{H,1}$ and $c$ appear only in the term $\tilde{p}_{H,1} - c$ together, the optimal $\tilde{p}^{*}_{H,1}$ that maximizes $\tilde{\pi}_{H,1}$ subject to $\tilde{p}_{H,1} > c$ must be $\tilde{p}^{*}_{H,1} = p^{0*} + c$. Thus $\tilde{\pi}^{*}_{H,1} = \left(1 - \mathcal{F}\left(\frac{p^{0*}}{1 - \mu}\right)\right) \cdot p^{0*}$, which is independent of $c$. Further note that $p^{0*}$ is the optimal price for the low-cost firm, thus $\tilde{\pi}^{*}_{L,1} = \left(1 - \mathcal{F}\left(\frac{p^{0*}}{1 - \mu}\right)\right) \cdot p^{0*} = \tilde{\pi}^{*}_{H,1}$. ■
Reference


Technical Appendix

PROOF OF LEMMA 1 AND 2: To find the LMSE, we first focus on all separating equilibria and find $\sigma^*_{sep}$, the separating equilibrium which gives the high-type firm the highest payoff. Next we focus on all pooling equilibria and find $\sigma^*_{pool}$, the pooling equilibrium which gives the high-type firm the highest payoff. Finally we compare the high-type firm’s profit in $\sigma^*_{sep}$ and $\sigma^*_{pool}$ to find the PBE giving the high-type firm the highest payoff.

Before starting to solve the game without cost-information transparency, we first solve the sub-game perfect equilibrium of the game with cost-information transparency by backward induction. Suppose the firm is high-type. Because we require that there are some consumers who purchase the product in the second period, $u_1 > u_2$ implies $u_1 > 0$. The marginal consumer who purchases in the first period is $\tilde{\varpi}^*_{H,1} = \frac{\tilde{p}_{H,1} - \tilde{p}_{H,2}}{1 - \mu}$, and the marginal consumer who purchase the product in the second period is $\tilde{\varpi}^*_{H,2} = \frac{\tilde{p}_{H,2}}{\mu}$. In the second period, the firm maximizes its second period profit $\pi^0_{H,2} = \left(\tilde{p}_{H,1} - \tilde{p}_{H,2}\right)\left(\tilde{p}_{H,2} - c\right)$, which gives the optimal $p_2$ is $\tilde{p}^*_{H,2} = \frac{\mu(\tilde{p}_{H,1} - \tilde{p}_{H,2}) + (1 - \mu)c}{2(1 - \mu)}$. Because consumers have rational expectation, i.e. $\tilde{p}^*_{H,2} = \tilde{p}_{H,2}$, one can show that $\tilde{p}^*_{H,2} = \frac{(1 - \mu)c + \mu \tilde{p}_{H,1}}{2 - \mu}$. In the first period, the firm maximizes its total profit $\pi_{H} =$

\[
\left(1 - \frac{\tilde{p}_{H,1} - (1 - \mu)c + \mu \tilde{p}_{H,1}}{2 - \mu}\right)\left(\tilde{p}_{H,1} - c\right) + \left(\tilde{p}_{H,1} - \frac{(1 - \mu)c + \mu \tilde{p}_{H,1}}{2 - \mu}\right)\left(\frac{(1 - \mu)c + \mu \tilde{p}_{H,1}}{2 - \mu} - c\right),
\]

which yields $\tilde{p}^*_{H,1} = \frac{(2 - \mu)^2}{2(4 - 3\mu)} + \frac{c}{2}$, $\tilde{p}^*_{H,2} = \frac{\mu(2 - \mu)}{2(4 - 3\mu)} + \frac{c}{2}$ and $\tilde{\varpi}^*_{H} = \frac{(1 - \mu)^2}{4 - 3\mu} + \frac{(\mu - c)^2}{4\mu}$. Finally, we have to
guarantee that there are some consumers who purchase the product in the second period, i.e. 
\[ \hat{v}_{H,1} > \hat{v}_{H,2} \text{ and } \hat{p}_{H,2} > c, \text{ which yields } c < \frac{(2-\mu)^2}{4-3\mu}. \]

The derivation of the low-type’s optimal price and profit is analogous. The low-type firm’s optimal first-period price and profit are 
\[ \hat{p}_{L,1}^* = \frac{(2-\mu)^2}{2(4-3\mu)}, \hat{p}_{L,2}^* = \frac{\mu(2-\mu)}{2(4-3\mu)} \text{ and } \hat{n}_L^* = \frac{(1-\mu)^2}{4-3\mu} + \frac{\mu}{4}. \]

Next we solve the game when cost-information transparency is absent.

**Step 1:** Consider separating equilibria. Denote a separating equilibrium \( \sigma_{sep} = \)
\[ \{p_{H,sep,t}\}_{t=1,2}, \{p_{L,sep,t}\}_{t=1,2}, G(v, \phi), \text{ where } p_{i,sep,t} \text{ is the } i\text{-type firm’s price in period } t, \]
\( G: [0,1] \rightarrow \{\text{buy, not buy}\} \) describes whether consumer \( v \) purchase the product in the first period, and \( \phi = \phi(i|p) \) is consumers’ belief of the probability that the firm is \( i\)-type conditional seeing the first-period price \( p \). We assume that \( \phi \) is consistent wherever Bayes rule can be applied. One can see that for all PBE that are sequential, \( G(v) \) maximizes consumer \( v \)’s expected utility, so the value of \( G(v) \) can be pinned down given \( \{p_{H,sep,t}\}_{t=1,2}, \{p_{L,sep,t}\}_{t=1,2} \) and \( \phi \) except for the marginal consumer who has zero mass on interval \( [0,1] \). Moreover, because in the second period, both the high-type firm and the low-type firm will charge their perfect-information price conditional on \( p_{H,sep,1}, p_{L,sep,1} \) and \( \phi \), the values of \( p_{H,sep,2} \) and \( p_{L,sep,2} \) are functions of \( p_{H,sep,1}, p_{L,sep,1} \) and \( \phi \). Therefore, it is convenient to denote a separating equilibrium \( \sigma_{sep} = \)
\( (p_{H,sep,1}, p_{L,sep,1}, \phi) \) without loss of generality.

Note that in a separating equilibrium the high-type firm has no incentive to mimic the low-type firm and consumers can correctly infer the firm’s type. Thus, the low-type firm will set its
prices the same as in the perfect information case, i.e. \( p_{L,sep,1}^* = \frac{(2-\mu)^2}{2(4-3\mu)} \) and \( p_{L,sep,2}^* = \frac{(2-\mu)\mu}{2(4-3\mu)}. \)

Low-type firm’s profit is \( \pi_{L,sep}^* = \frac{(2-\mu)^2}{4(4-3\mu)}. \)

The high-type firm’s profit is \( \pi_{H,sep}^* = \left( 1 - \frac{p_1 \frac{(1-\mu)c + \mu p_{H,sep,1}^*}{2-\mu}}{1-\mu} \right) (p_1 - c) + \left( \frac{p_1 \frac{(1-\mu)c + \mu p_{H,sep,1}^*}{2-\mu}}{1-\mu} - \frac{(1-\mu)c + \mu p_{H,sep,1}^*}{2-\mu} \right) \left( \frac{1}{2-\mu} - c \right) \). In a separating equilibrium, we need to guarantee the low-type firm has no incentive to mimic the high-type firm. If the low-type firm mimics the high-type firm, its profit is \( \pi_L(p_{H,sep,1} | \phi(H|p_{H,sep,1}) = 1) \) and \( \bar{p}_2 = \frac{(1-\mu)c + \mu p_{H,sep,1}^*}{2-\mu}. \)

Therefore,
\[
\pi_L(p_{H,sep,1}, p_{L,2} | \phi(H|p_{H,sep,1}) = 1) = \left( 1 - \frac{p_{H,sep,1} \frac{\phi(H|p_{H,sep,1})}{2-\mu}}{1-\mu} \right) p_{H,sep,1} + \left( \frac{\phi(H|p_{H,sep,1})}{2-\mu} - \frac{p_{L,2}}{2-\mu} \right) p_{L,2}. \]

The FOC gives \( p_{L,2}^* = \frac{\mu(p_{H,sep,1} \frac{\phi(H|p_{H,sep,1})}{2-\mu})}{2(1-\mu)}, \) so
\[
\pi_L(p_{H,sep,1}, p_{L,2} | \phi(H|p_{H,sep,1}) = 1) = \left( 1 - \frac{p_{H,sep,1} \frac{\phi(H|p_{H,sep,1})}{2-\mu}}{1-\mu} \right) p_{H,sep,1} + \frac{\mu(p_{H,sep,1} \frac{\phi(H|p_{H,sep,1})}{2-\mu})^2}{4(1-\mu)^2}. \]

Therefore, to guarantee the low-type firm has no incentive to mimic the high-type firm, we need to have \( \pi_{L,sep}^* \geq \pi_L(p_{H,sep,1}, p_{L,2} | \phi(H|p_{H,sep,1}) = 1) \), which requires \( p_{H,sep,1} \in (0, \frac{(2-\mu)^2 + 2c(1-\mu) - (2-\mu)\sqrt{c(4+c-4\mu)}}{2(4-3\mu)} \cup \left( \frac{(2-\mu)^2 + 2c(1-\mu) + (2-\mu)\sqrt{c(c+c(1-\mu))}}{2(4-3\mu)} \right), 1 - \mu + c) \). One can show that under this condition, \( \pi_{H,sep} \) is maximized at \( p_{H,sep,1}^* = \frac{(2-\mu)^2 + 2c(1-\mu) + (2-\mu)\sqrt{c(c+c(1-\mu))}}{2(4-3\mu)}. \) Thus, in the separating equilibrium which gives the high-type firm the highest profit, \( p_{H,sep,1}^* = \frac{(2-\mu)^2 + 2c(1-\mu) + (2-\mu)\sqrt{c(c+c(1-\mu))}}{2(4-3\mu)}, p_{H,sep,2}^* = \frac{(2-\mu)^2 + 2c(1-\mu) + (2-\mu)\sqrt{c(c+c(1-\mu))}}{2(4-3\mu)} \).
Step 2: Consider pooling equilibria. Let \( \sigma_{pool} = (p_{pool,1}, \phi) \) be a pooling equilibrium, where \( p_{pool,1} \) is the price charged by both the high-type firm and the low-type firm in the first period, and \( \phi = \phi(i|p) \) is consumers’ belief of the probability that the firm is i-type conditional seeing the first-period price \( p \). In equilibrium, consumers believe that the firm is high-type with probability \( \alpha \), and that the firm is low-type with probability \( 1 - \alpha \). If a consumer \( v \) purchases the product in the first period, her utility is \( u_{pool,1} = v - p_{pool,1} \). If she purchases the product in the second period, her utility is \( u_{pool,2} = \alpha \cdot \max\{\mu v - p_{H, pool,2}, 0\} + (1 - \alpha) \cdot \max\{\mu v - p_{L, pool,2}, 0\} \).

To make the analysis interesting, we assume that in the second period there are some consumers who purchase the product even if the firm turns out to be high-type in the second period, which requires that the marginal consumer in the first period \( v_1^* \) satisfies \( \mu > \mu v_1^* > p_{H, pool,2} \). Therefore, \( v_1^* - p_{pool,1} = \alpha\left(\mu v_1^* - \bar{p}_{H, pool,2}\right) + (1 - \alpha)\left(\mu v_1^* - \bar{p}_{L, pool,2}\right) \), where \( \bar{p}_{L, pool,2} \) is the belief of the second period price in the pooling equilibrium if the firm is type-i.

Simple algebra yields \( v_1^* = \frac{p_{pool1} - \alpha \bar{p}_{H, pool,2} - (1 - \alpha) \bar{p}_{L, pool,2}}{1 - \mu} \). The high-type firm’s second period profit is \( \pi_{H, pool,2} = \left(v_1^* - \frac{p_{H, pool,2}}{\mu}\right)(p_{H, pool,2} - c) \), and the low-type firm’s second period profit is \( \pi_{L, pool,2} = \left(v_1^* - \frac{p_{L, pool,2}}{\mu}\right)p_{L, pool,2} \). First order condition yields \( p_{H, pool,2}^* = \frac{\mu (p_{pool1} - \alpha \bar{p}_{H, pool,2} - (1 - \alpha) \bar{p}_{L, pool,2}) + (1 - \mu)c}{2(1 - \mu)} \) and \( p_{L, pool,2}^* = \frac{\mu (p_{pool1} - \alpha \bar{p}_{H, pool,2} - (1 - \alpha) \bar{p}_{L, pool,2})}{2(1 - \mu)} \). Because
the consumer has rational belief, $\bar{p}_{H,pool,2} = p_{H,pool,2}^*$ and $\bar{p}_{L,pool,2} = p_{L,pool,2}^*$, which yields

$$p_{H,pool,2}^* = \frac{\mu(2p_{pool,1} - ac)}{2(2-\mu)} + \frac{c}{2}, p_{L,pool,2}^* = \frac{\mu(2p_{pool,1} - ac)}{2(2-\mu)},$$

and $v_1^* = \frac{2p_{pool,1} - ac}{2-\mu}$. Therefore the high-type firm’s profit in a pooling equilibrium is $\pi_{H,pool} = (1 - v_1^*)\left(p_{pool,1} - c\right) + \left(v_1^* - \frac{p_{H,pool,2}^*}{\mu}\right)(p_{H,pool,2}^* - c)$ =

$$\frac{-4\mu(4-3\mu)p_{pool,1}^2 + 4\mu\left(2-\mu\right)^2 + 2(2+c(2-2(1-\mu)) - 4(2-\mu)^2)\mu}{4\mu(2-\mu)^2},$$

and the low-type firm’s profit in a pooling equilibrium is $\pi_{L,pool} = (1 - v_1^*)p_{pool,1} + \left(v_1^* - \frac{p_{L,pool,2}^*}{\mu}\right)p_{L,pool,2}^* = \frac{-4(4-3\mu)p_{pool,1}^2 + 4\left(2(2-2(1-\mu)) + 2(1+\alpha)^2\right)p_{pool,1} + \alpha^2c^2\mu}{4(2-\mu)^2}$.

Let $M(p_{pool,1}) = \{\phi | \phi(H|p_{pool,1}) = \alpha\}$ be the set of beliefs which are consistent with the pooling equilibrium (i.e., $\phi(H|p_1 = p_{pool,1}) = \alpha$). We proceed to show that $(p_{pool,1}, \phi)$ is a PBE for some $\phi \in M(p_{pool})$ if and only if neither the high-type nor low-type firm has an incentive to deviate to another price $p'$ when consumers have a belief $\phi \in M(p_{pool})$ with $\phi(H|p_1 = p') = 0, \forall p' \neq p_{pool,1}$ (i.e., consumers believe that the firm is low-type with probability 1 when $p' \neq p_{pool,1}$). This is because on the one hand, if this condition is satisfied, then $(p_{pool,1}, \phi)$ is a PBE. On the other hand, if this condition is violated, then for any other $\phi \in M(p_{pool,1})$, $(p_{pool,1}, \phi)$ cannot be a PBE since deviating to $p'$ will give the firm even higher profit. Consequently, this condition is a necessary and sufficient condition for $(p_{pool,1}, \phi)$ to be a PBE for some $\phi \in M(p_{pool,1})$. Hence hereafter we only consider the belief $\phi$ such that $\phi(H|p_1 = p_{pool,1}) = \alpha$ and $\phi(H|p_1 \neq p_{pool,1}) = 0$. 
The highest possible profit a low-type firm can get by deviating is its perfect-information profit $\pi^*_{L,\text{pool},\text{dev}} = \pi_{L,\text{sep}} = \frac{(2-\mu)^2}{4(4-3\mu)}$. For the high-type firm, suppose it deviates by charging $p_{H,\text{pool},\text{dev}}$ in the first period. In the second period, its profit is $\pi_{H,\text{pool},\text{dev}} = \frac{\mu p_{H,\text{pool},\text{dev}} - \mu p_{H,\text{pool},\text{dev}_2}}{2-\mu}(p_{H,\text{pool},\text{dev}_2} - c)$, where $\bar{p}_2 = \frac{\mu p_{H,\text{pool},\text{dev}}}{2-\mu}$.

because $\phi(H|p_1 = p_{H,\text{pool},\text{dev},1}) = 0$. When $p_{H,\text{pool},\text{dev},1} < \frac{(2-\mu)c}{2\mu}$, $\pi_{H,\text{pool},\text{dev},2} < 0$. When $p_{H,\text{pool},\text{dev},1} \geq \frac{(2-\mu)c}{2\mu}$, the high-type firm’s optimal second-period price and profit are

$$p_{H,\text{pool},\text{dev},2}^* = \frac{\mu}{2-\mu} p_{H,\text{pool},\text{dev},1} + \frac{c}{2} \quad \text{and} \quad \pi_{H,\text{pool},\text{dev},2}^* = \frac{(2\mu p_{H,\text{pool},\text{dev},1} - c(2-\mu))^2}{4\mu(2-\mu)^2}.$$

The high-type firm’s total profit by deviating is:

$$\pi_{H,\text{pool},\text{dev}} = \begin{cases} 
\left(1 - \frac{p_{H,\text{pool},\text{dev},1} - \bar{p}_2}{1-\mu}\right) \left(p_{H,\text{pool},\text{dev},1} - c\right) + \left(\frac{p_{H,\text{pool},\text{dev},1} - \bar{p}_2}{1-\mu} - \frac{p_{H,\text{pool},\text{dev},2}}{\mu}\right) \left(p_{H,\text{pool},\text{dev},2} - c\right), & \text{if } p_{H,\text{pool},\text{dev},1} > \frac{(2-\mu)c}{2\mu} \\
(1 - p_{H,\text{pool},\text{dev},1})(p_{H,\text{pool},\text{dev},1} - c), & \text{if } p_{H,\text{pool},\text{dev},1} \leq \frac{(2-\mu)c}{2\mu} 
\end{cases}.$$

Next we will find the $p_{H,\text{pool},\text{dev},1}$ which maximizes $\pi_{H,\text{pool},\text{dev}}$. (1) When $p_{H,\text{pool},\text{dev},1} \leq \frac{(2-\mu)c}{2\mu}$, $\pi_{H,\text{pool},\text{dev}}$ increases with $p_{H,\text{pool},\text{dev},1}$ if $p_{H,\text{pool},\text{dev},1} \leq \frac{1+c}{2}$ and decreases if $p_{H,\text{pool},\text{dev},1} > \frac{1+c}{2}$. Because we have assumed $c \leq (2-\mu)^2/4(4-3\mu)$, $\frac{1+c}{2} \geq \frac{(2-\mu)c}{2\mu}$ is always true.

Therefore, when $p_{H,\text{pool},\text{dev},1} \leq \frac{(2-\mu)c}{2\mu}$, $\pi_{H,\text{pool},\text{dev}}$ is maximized when $p_{H,\text{pool},\text{dev},1} = \frac{(2-\mu)c}{2\mu}$ with $\pi_{H,\text{pool},\text{dev}} = \frac{(2\mu - (2-\mu)c)(2-3\mu)c}{4\mu^2}$. (2) When $p_{H,\text{pool},\text{dev},1} > \frac{(2-\mu)c}{2\mu}$, $\pi_{H,\text{pool},\text{dev}}$ increases with $p_{H,\text{pool},\text{dev},1}$ when $p_{H,\text{pool},\text{dev},1} \leq \frac{(2+c-\mu)(2-\mu)}{2(4-3\mu)}$, and decreases when $p_{H,\text{pool},\text{dev},1} \geq \frac{(2+c-\mu)(2-\mu)}{2(4-3\mu)}$. 

47
When \( c \leq \frac{(2-\mu)^2}{(2+\mu)(4-3\mu)} \), \( \frac{(2+c-\mu)(2-\mu)}{2} \geq \frac{(2-\mu)c}{2\mu} \) is always true. Therefore, when \( p_{H, pool, dev, 1} > \frac{(2-\mu)c}{2\mu} \), \( \pi_{H, pool, dev} \) is maximized when \( p_{H, pool, dev, 1} = \frac{(2+c-\mu)(2-\mu)}{2(4-3\mu)} \) with \( \pi_{H, pool, dev} = \frac{2(2-\mu)c^2 + \mu(2-\mu)^2 - 2c\mu(6-5\mu)}{4\mu(4-3\mu)} \). One can show that when \( c \leq \frac{(2-\mu)^2}{(2+\mu)(4-3\mu)} \), \( \frac{2(2-\mu)c^2 + \mu(2-\mu)^2 - 2c\mu(6-5\mu)}{4\mu(4-3\mu)} > \frac{(2-\mu)(2-\mu)c(2-3\mu)c}{4\mu^2} \). As a result, \( \pi_{H, pool, dev} \) is maximized at \( p_{H, pool, dev, 1} = \frac{(2+c-\mu)(2-\mu)}{2(4-3\mu)} \) with \( \pi^*_{H, pool, dev} = \frac{2(2-\mu)c^2 + \mu(2-\mu)^2 - 2c\mu(6-5\mu)}{4\mu(4-3\mu)} \).

In a pooling equilibrium with the first-period price \( p_{pool, 1} \), neither the high-type nor the low-type firm has an incentive to deviate, which requires \( \pi_{H, pool} \geq \pi^*_{H, pool, dev} \) and \( \pi_{L, pool} \geq \pi^*_{L, pool, dev} \). Condition \( \pi_{H, pool} \geq \pi^*_{H, pool, dev} \) is equivalent to

\[
-4\mu(4-3\mu)p_{pool, 1}^2 + 4\mu(2-\mu)^2 + c(2+2\alpha(1-\mu)) \geq \frac{2(2-\mu)c^2 + \mu(2-\mu)^2 - 2c\mu(6-5\mu)}{4\mu(4-3\mu)},
\]

which can be equivalently expressed as \( p_{pool, 1} \in \left[ \frac{2(2-\mu)^2 + c(2-\mu+2\alpha(1-\mu))}{2(4-3\mu)} - \frac{2(2-\mu)\sqrt{4\alpha(4\alpha-3\alpha-\mu)-4(4-3\mu)-(2-\alpha)^2}}{2(4-3\mu)} \right] \). We define \( \mathcal{P}_1 = \left[ \frac{2(2-\mu)^2 + c(2-\mu+2\alpha(1-\mu))}{2(4-3\mu)} - \frac{2(2-\mu)\sqrt{4\alpha(4\alpha-3\alpha-\mu)-4(4-3\mu)-(2-\alpha)^2}}{2(4-3\mu)} \right] \). The condition \( \pi_{L, pool} \geq \pi^*_{L, pool, dev} \) implies that

\[
-4(4-3\mu)p_{pool, 1}^2 + 4\left( \frac{(2-\mu)^2 + 2\alpha(1-\mu)}{2(4-3\mu)} \right) \geq \frac{(2-\mu)^2}{4(4-3\mu)},
\]

which is equivalent to \( p_{pool, 1} \in \left[ \frac{(2-\mu)^2 + 2\alpha(1-\mu)}{2(4-3\mu)} - \frac{2(2-\mu)\sqrt{4\alpha(4\alpha-3\alpha-\mu)+\alpha\alpha\alpha}}{2(4-3\mu)} + \frac{(2-\mu)\sqrt{4\alpha(4\alpha-3\alpha-\mu)+\alpha\alpha\alpha}}{2(4-3\mu)} \right] \). We define \( \mathcal{P}_2 = \left[ \frac{(2-\mu)^2 + 2\alpha(1-\mu)}{2(4-3\mu)} - \frac{2(2-\mu)\sqrt{4\alpha(4\alpha-3\alpha-\mu)+\alpha\alpha\alpha}}{2(4-3\mu)} + \frac{(2-\mu)\sqrt{4\alpha(4\alpha-3\alpha-\mu)+\alpha\alpha\alpha}}{2(4-3\mu)} \right] \).
\[
\left[ \frac{(2-\mu)^2 + 2ac(1-\mu)}{2(4-3\mu)} - \frac{(2-\mu)\sqrt{ac(4(1-\mu)+ac)}}{2(4-3\mu)}, \frac{(2-\mu)^2 + 2ac(1-\mu)}{2(4-3\mu)} + \frac{(2-\mu)\sqrt{ac(4(1-\mu)+ac)}}{2(4-3\mu)} \right].
\]

Therefore, there exists a belief \( \phi \in M(p_{pool,1}) \) such that \( (p_{pool,1}, \phi) \) is a PBE if and only if \( p_{pool,1} \in \mathcal{P}_1 \cap \mathcal{P}_2 \).

The LMSE gives the high-type firm the highest profit among all PBE. One can show that without the restriction of \( p_{pool,1} \in \mathcal{P}_1 \cap \mathcal{P}_2 \), \( \pi_{H,pool} \) is maximized when \( p_{pool,1} = \frac{(2-\mu)^2 + c(2-\mu + 2a(1-\mu))}{2(4-3\mu)} \). It is obvious that \( \frac{(2-\mu)^2 + c(2-\mu + 2a(1-\mu))}{2(4-3\mu)} \in \mathcal{P}_1 \) when \( \alpha \geq \frac{\sqrt{4(1-\mu)^2 + 2c^2 - 2(1-\mu)}}{c} \), \( \frac{(2-\mu)^2 + c(2-\mu + 2a(1-\mu))}{2(4-3\mu)} \in \mathcal{P}_2 \), so the pooling PBE outcome which gives the high-type firm the highest profit are \( \pi_{pool,1}^* = \frac{(2-\mu)^2 + c(2-\mu + 2a(1-\mu))}{2(4-3\mu)} \) and \( \pi_{H,pool}^* = \frac{\mu(2-\mu)^2 + c^2(4-(2-2a)x) \mu - 2c\mu(6-5\mu - 2a(1-\mu))}{4\mu(4-3\mu)} \). When \( \alpha < \frac{\sqrt{4(1-\mu)^2 + 2c^2 - 2(1-\mu)}}{c} \), \( \frac{(2-\mu)^2 + c(2-\mu + 2a(1-\mu))}{2(4-3\mu)} \) is higher than the upper bound of \( \mathcal{P}_2 \), \( \frac{(2-\mu)^2 + 2ac(1-\mu)}{2(4-3\mu)} \). Therefore, \( \pi_{H,pool} \) is an increasing function of \( p_{1,pool} \) on \( \mathcal{P}_1 \cap \mathcal{P}_2 \). As a result, the pooling PBE outcome which gives the high-type firm the highest profit are \( \pi_{pool,1}^* = \frac{(2-\mu)^2 + 2ac(1-\mu)}{2(4-3\mu)} + \frac{(2-\mu)\sqrt{ac(4(1-\mu)+ac)}}{2(4-3\mu)} \) and \( \pi_{H,pool}^* = \frac{\mu(2-\mu)^2 + c^2(4-(3+2a)\mu) - 2\mu(6-5\mu - \sqrt{ac(4(1-\mu)+ac)})}{4\mu(4-3\mu)} \).

**Step 3:** We compare \( \pi_{H,pool}^* \) and \( \pi_{sep,H}^* \) to choose the LMSE. One can see that \( \pi_{H,sep}^* = \frac{(2-\mu)^2 + c^2(4-5\mu) + 2c(5\mu + \sqrt{c(c+4(1-\mu))-6})}{4\mu(4-3\mu)} \) is independent of \( \alpha \). When \( \alpha \geq \frac{\sqrt{4(1-\mu)^2 + c^2 - 2(1-\mu)}}{c} \), \( \pi_{H,pool}^* = \frac{\mu(2-\mu)^2 + c^2(4-(2-2a)x) \mu - 2c\mu(6-5\mu - 2a(1-\mu))}{4\mu(4-3\mu)} \). When \( \alpha < \frac{\sqrt{4(1-\mu)^2 + c^2 - 2(1-\mu)}}{c} \), \( \pi_{H,pool}^* = \frac{\mu(2-\mu)^2 + c^2(4-(3+2a)\mu) - 2\mu(6-5\mu - \sqrt{ac(4(1-\mu)+ac)})}{4\mu(4-3\mu)} \). Next we show that \( \pi_{H,pool}^* \) is an increasing
function of $\alpha$. When $\alpha < \frac{\sqrt{4(1-\mu)^2+c^2-2(1-\mu)}}{c}$, it is obvious that
\[
\frac{\mu(2-\mu)^2+c^2(4-3+2\alpha)\mu-2c\mu(6-5\mu-\sqrt{ac(4(1-\mu)+ac)})}{4\mu(4-3\mu)}
\]
increases with $\alpha$. When $\alpha \geq \frac{\sqrt{4(1-\mu)^2+c^2-2(1-\mu)}}{c}$,
\[
\pi_{H,pool}^* = \frac{\mu(2-\mu)^2+c^2(4-2+(2-\alpha)\mu)-2c\mu(6-5\mu-2\alpha(1-\mu))}{4\mu(4-3\mu)}.
\]
Because $\nu_1^* = \frac{2p_{pool,1}^* ac}{2-\mu} < 1$,
\[
2(1-\mu) - c(1-\alpha) > 0.
\]
As a result, $\frac{\partial \pi_{H,pool}^*}{\partial \alpha} = \frac{c[2(1-\mu)-c(1-\alpha)]}{2(4-3\mu)} > 0$. Moreover, one can show that $\pi_{H,pool}^*$ is a continuous function of $\alpha$. Therefore, $\pi_{H,pool}^*$ is a strictly increasing function of $\alpha$ when $\alpha \in (0,1)$.

When $\alpha \geq \frac{\sqrt{4(1-\mu)^2+c^2-2(1-\mu)}}{c}$, $\pi_{H,pool}^* \geq \pi_{H,sep}^*$, implies that
\[
\frac{\mu(2-\mu)^2+c^2(4-2+(2-\alpha)\mu)-2c\mu(6-5\mu-2\alpha(1-\mu))}{4\mu(4-3\mu)} \geq \frac{(2-\mu)^2+c^2(4-5\mu)+2c\mu(5\mu+c\sqrt{c+4(1-\mu)-6})}{4\mu(4-3\mu)}, \text{ i.e.,}
\]
\[
\alpha \geq \frac{\sqrt{4-\frac{8c}{1-\mu}-2\left(\frac{c}{1-\mu}\right)^2 + \frac{4c}{1-\mu}\sqrt{\frac{c}{1-\mu}(2+\frac{c}{1-\mu})-2\left(\frac{1-c}{1-\mu}\right)}}}{\frac{2c}{1-\mu}}.
\]
One can show that
\[
\frac{\sqrt{4-\frac{8c}{1-\mu}-2\left(\frac{c}{1-\mu}\right)^2 + \frac{4c}{1-\mu}\sqrt{\frac{c}{1-\mu}(2+\frac{c}{1-\mu})-2\left(\frac{1-c}{1-\mu}\right)}}}{\frac{2c}{1-\mu}} \geq \frac{\sqrt{4(1-\mu)^2+c^2-2(1-\mu)}}{c}.
\]
This result, together with the fact that $\pi_{H,sep}^* = \frac{(2-\mu)^2+c^2(4-5\mu)+2c\mu\left(5\mu+c\sqrt{c+4(1-\mu)-6}\right)}{4\mu(4-3\mu)}$ is independent of $\alpha$ and $\pi_{H,pool}^*$ is a strictly increasing function of $\alpha$ when $\alpha \in (0,1)$, implies that $\pi_{H,pool}^* \geq \pi_{H,sep}^*$ if and only if $\alpha \geq \frac{\sqrt{4-\frac{8c}{1-\mu}-2\left(\frac{c}{1-\mu}\right)^2 + \frac{4c}{1-\mu}\sqrt{\frac{c}{1-\mu}(2+\frac{c}{1-\mu})-2\left(\frac{1-c}{1-\mu}\right)}}}{\frac{2c}{1-\mu}} \equiv \alpha^*$. In conclusion, when consumers do not know the firm’s cost, when $\alpha < \alpha^*$, the equilibrium is separating, $\pi_{H,sep}^* = \frac{(2-\mu)^2+c^2(4-5\mu)+2c\mu\left(5\mu+c\sqrt{c+4(1-\mu)-6}\right)}{4\mu(4-3\mu)}$ and $\pi_{L,sep}^* = \frac{(2-\mu)^2}{4(4-3\mu)}$. When $\alpha \geq \frac{\sqrt{4(1-\mu)^2+c^2-2(1-\mu)}}{c}$,
\( \alpha^* \), the equilibrium is pooling, \( \pi_{H, \text{pool}}^* = \frac{\mu(2-\mu)^2 + c^2(4-(2(2-\alpha)\mu)-2c(6-5\mu-2\alpha(1-\mu)))}{4\mu(4-3\mu)} \) and

\[
\pi_{L, \text{pool}}^* = \frac{(2-\mu)^2 + 4(1-\mu)\alpha c - (1-\alpha)^2 c^2}{4(4-3\mu)}. \]

When consumers know the firm’s cost, \( \hat{\pi}_H = \frac{(1-\mu)^2}{4-3\mu} + \frac{(\mu-c)^2}{4\mu} \), and \( \hat{\pi}_L = \frac{(1-\mu)^2}{4-3\mu} + \frac{\mu}{4} \).

**Proof of Lemma 3 and 4.** In this proof, we adopt the notations in the proof of Lemma 1 and 2 wherever possible. We start by solving the sub-game perfect equilibrium of the game with cost-information transparency. Suppose the firm has high cost. Because the second period price will be \( c \), the marginal consumer who purchases in the first period is \( \hat{\theta}_{H,1} = \frac{\hat{p}_{H,1}-c}{1-\mu} \), and the marginal consumer who purchases the product in the second period is \( \hat{\theta}_{H,2} = \frac{c}{\mu} \). Because the market becomes perfectly competitive in the second period, the firm earns all its profit from the first period. Hence \( \hat{\pi}_H = \left( 1 - \frac{\hat{p}_{H,1} - c}{1-\mu} \right) (\hat{p}_{H,1} - c) \). The optimal \( \hat{p}_{H,1} \) is \( \hat{p}_{H,1}^* = \frac{1-\mu}{2} + c \), and the maximum profit is \( \hat{\pi}_H^* = \frac{1-\mu}{4} \). Finally we have to guarantee that \( \hat{\theta}_{H,1}^* > \hat{\theta}_{H,2}^* \), which yields \( c < \frac{\mu}{2} \).

Similarly, for the low-cost firm, \( \hat{p}_{L,1}^* = \frac{1-\mu}{2} \) and \( \hat{\pi}_L^* = \frac{1-\mu}{4} \).

Next we solve the game when cost-information transparency is absent.

**Step 1:** Consider separating equilibria. Note that in a separating equilibrium the high-cost firm has no incentive to mimic the low-cost firm and that consumers can correctly infer the firm’s cost type. Thus, the low-cost firm will set its price the same as the perfect information case, i.e. \( p_{L, \text{sep},1}^* = \frac{1-\mu}{2} \) and its profit is \( \pi_{L, \text{sep}}^* = \frac{1-\mu}{4} \).
The high-cost firm’s profit is \( \pi_{H,sep} = \left(1 - \frac{p_1 - c}{1 - \mu}\right)(p_1 - c) \). In a separating equilibrium, the low-cost firm has no incentive to mimic the high-cost firm by charging the high-cost firm’s price in the first period. If the low-cost firm mimics the high-cost firm by charging \( p_1 = p_{H,sep,1} \), its profit is \( \pi_L(p_{H,sep,1} \mid \phi(H \mid p_{H,sep,1}) = 1) = \left(1 - \frac{p_{H,sep,1} - c}{1 - \mu}\right)p_{H,sep,1} \cdot \pi^*_L,sep \geq \pi_L(p_{H,sep,1} \mid \phi(H \mid p_{H,sep,1}) = 1) \) yields \( p_{H,sep,1} \in (0, \frac{1 - \mu + c - \sqrt{c(c + 2(1 - \mu))}}{2}, 1 - \mu) \). Moreover, any \( p_{H,sep,1} \) in this region can form a PBE \( (p_{H,sep,1}, \frac{1 - \mu}{2}, \phi) \), where \( \phi(H \mid p) = 1 \) if \( p = p_{H,sep} \) and \( \phi(H \mid p) = 0 \) if otherwise.

One can show that under the condition \( p_{H,sep,1} \in (0, \frac{1 - \mu + c - \sqrt{c(c + 2(1 - \mu))}}{2}, 1 - \mu) \), \( \pi_{H,sep} \) is maximized at \( p_{H,sep,1}^* = \frac{1 - \mu + c + \sqrt{c(c + 2(1 - \mu))}}{2} \), and \( \pi_{H,sep}^* = \frac{(1 - \mu)^2 - 2c(c + 1 - \mu - \sqrt{c(c + 2(1 - \mu))})}{4(1 - \mu)} \).

**Step 2:** Consider pooling equilibria. In such equilibria, consumers believe that the firm is high-type with probability \( \alpha \), and that the firm is low-type with probability \( 1 - \alpha \). If a consumer \( v \) purchases the product in the first period, her utility is \( u_{pool,1} = v - p_{pool,1} \). If she purchases the product in the second period, her utility is \( u_{pool,2} = \alpha \cdot \max\{\mu v - c, 0\} + (1 - \alpha) \cdot \mu v \).

To make the analysis interesting, we assume that in the second period there are some consumers who purchase the product even if the firm turns out to be high-type in the second period, which requires that the marginal consumer in the first period \( v_1^* \) satisfies \( \mu > \mu v_1^* > c \).

Therefore, \( v_1^* = p_{pool,1} = \alpha (\mu v_1^* - c) + (1 - \alpha) \mu v_1^* \). Simple algebra yields \( v_1^* = \frac{p_{pool,1} - \alpha c}{1 - \mu} \).
Therefore, the high-cost firm’s profit is 
\[ \pi_{H,pool} = \left(1 - \frac{p_{pool,1} - ac}{1 - \mu}\right)(p_{pool,1} - c), \]
and the low-cost
firm’s profit is 
\[ \pi_{L,pool} = \left(1 - \frac{p_{pool,1} - ac}{1 - \mu}\right)p_{pool,1}. \]

Analogous to the proof of Lemma 1 and 2, it is without loss of generality for us to only consider the belief \( \phi \) such that 
\[ \phi(H|p_1 = p_{pool,1}) = \alpha \]
and 
\[ \phi(H|p_1 \neq p_{pool,1}) = 0. \]
The highest possible profit a low-cost firm can get by deviating is its perfect-information profit 
\[ \pi^*_{L,pool,dev} = \hat{\pi}_L = \frac{1 - \mu}{4}. \]
For the high-cost firm, if it deviates by charging \( p_{H,pool,dev,1} = p_{pool,1} \), its profit will be 
\[ \pi_{H,pool,dev} = \left(1 - \frac{p_{H,pool,dev,1}}{1 - \mu}\right)(p_{H,pool,dev,1} - c). \]
Therefore 
\[ \pi^*_{H,pool,dev} = \frac{(1 - \mu - c)^2}{4(1 - \mu)}. \]

In a pooling equilibrium, neither the high-cost firm nor the low-cost firm has incentive to deviate. This requires 
\[ \pi_{H,pool} = \left(1 - \frac{p_{pool,1} - ac}{1 - \mu}\right)(p_{pool,1} - c) \geq \pi^*_{H,pool,dev} = \frac{(1 - \mu - c)^2}{4(1 - \mu)} \]
and
\[ \pi_{L,pool} = \left(1 - \frac{p_{pool,1} - ac}{1 - \mu}\right)p_{pool,1} \geq \pi^*_{L,pool,dev} = \frac{1 - \mu}{4}. \]
The first condition is equivalent to
\[ p_{pool,1} \in \left[\frac{1 - \mu + (1 + \alpha)c - \sqrt{ac(ac - 2c - 2\mu + 2)}}{2}, \frac{1 - \mu + (1 + \alpha)c + \sqrt{ac(ac - 2c - 2\mu + 2)}}{2}\right]. \]
Let \( P_1 = \left[\frac{1 - \mu + (1 + \alpha)c - \sqrt{ac(ac - 2c - 2\mu + 2)}}{2}, \frac{1 - \mu + (1 + \alpha)c + \sqrt{ac(ac - 2c - 2\mu + 2)}}{2}\right]. \)
The second condition is equivalent to
\[ p_{pool,1} \in \left[\frac{1 - \mu + ac - \sqrt{ac(ac + 2(1 - \mu))}}{2}, \frac{1 - \mu + ac + \sqrt{ac(ac + 2(1 - \mu))}}{2}\right]. \]
Let \( P_2 = \left[\frac{1 - \mu + ac - \sqrt{ac(ac + 2(1 - \mu))}}{2}, \frac{1 - \mu + ac + \sqrt{ac(ac + 2(1 - \mu))}}{2}\right]. \)
Therefore, there exists a belief \( \phi \in M(p_{pool,1}) \) such that \((p_{pool,1}, \phi)\) is a PBE if and only if \( p_{pool,1} \in P_1 \cap P_2 \).

The LMSE gives the high-type firm the highest profit among all PBE. One can show that without the restriction of \( p_{pool,1} \in P_1 \cap P_2 \), \( \pi_{H,pool} \) is maximized when 
\[ p_{pool,1} = \frac{1 - \mu + (\alpha + 1)c}{2}. \] It is obvious that 
\[ \frac{1 - \mu + (\alpha + 1)c}{2} \in P_1. \]
When 
\[ \alpha \geq \frac{\sqrt{(1 - \mu)^2 + c^2} - (1 - \mu)}{c}, \frac{1 - \mu + (\alpha + 1)c}{2} \in P_2, \] so the pooling PBE
outcome which gives the high-type firm the highest profit is \( p_{pool,1}^* = \frac{1-\mu+(\alpha+1)c}{2} \), and \( \pi_{H, pool}^* = \frac{[1-\mu-(1-\alpha)c]^2}{4(1-\mu)} \). When \( \alpha < \frac{\sqrt{(1-\mu)^2+c^2-(1-\mu)} - (1-\mu) + (\alpha+1)c}{c} \), \( \frac{[1-\mu-(1-\alpha)c]^2}{4(1-\mu)} \) is higher than the upper bound of \( \mathcal{P}_2 \).

\[
1-\mu+\alpha c+\frac{\sqrt{ac(ac+2(1-\mu))}}{2} \] Therefore, \( \pi_{H, pool}^* \) is an increasing function of \( p_{pool,1} \) on \( \mathcal{P}_1 \cap \mathcal{P}_2 \). As a result, the pooling PBE outcome which gives the high-cost firm the highest profit is \( p_{pool,1}^* = \frac{1-\mu+\alpha c+\sqrt{ac(ac+2(1-\mu))}}{2} \) and \( \pi_{H, pool}^* = \frac{(1-\mu)^2-2ac^2-2c\left(1-\mu-\sqrt{ac(ac+2(1-\mu))}\right)}{4(1-\mu)} \).

**Step 3:** We compare \( \pi_{H, pool}^* \) and \( \pi_{sep,H}^* \) to choose the LMSE. One can see that \( \pi_{H, sep}^* = \frac{(1-\mu)^2-2c(c+1-\mu-\sqrt{c(c+2(1-\mu))})}{4(1-\mu)} \) is independent of \( \alpha \). When \( \alpha \geq \frac{\sqrt{(1-\mu)^2+c^2-(1-\mu)}}{c} \), \( \pi_{H, pool}^* = \frac{[1-\mu-(1-\alpha)c]^2}{4(1-\mu)} \) also increases with \( \alpha \).

Next we show that \( \pi_{H, pool}^* \) is an increasing function of \( \alpha \). When \( \alpha < \frac{\sqrt{(1-\mu)^2+c^2-(1-\mu)}}{c} \), rearranging \( \pi_{H, pool}^* \) yields \( \pi_{H, pool}^* = C + \frac{c}{2(1-\mu)}\left[\sqrt{(\alpha c+(1-\mu))^2-(1-\mu)^2} - \alpha c\right] \), where \( C \) is independent of \( \alpha \). It is easy to see that \( \pi_{H, pool}^* \) strictly increases with \( \alpha \) when \( \alpha < \frac{\sqrt{(1-\mu)^2+c^2-(1-\mu)}}{c} \). When \( \alpha \geq \frac{\sqrt{(1-\mu)^2+c^2-(1-\mu)}}{c} \), \( \pi_{H, pool}^* = \frac{[1-\mu-(1-\alpha)c]^2}{4(1-\mu)} \) also increases with \( \alpha \).

Moreover, one can show that \( \pi_{H, pool}^* \) is a continuous function of \( \alpha \). Therefore, \( \pi_{H, pool}^* \) is a strictly increasing function of \( \alpha \) when \( \alpha \in (0,1) \).

When \( \alpha > 1 \), \( \pi_{H, pool}^* \geq \pi_{H, sep}^* \), implies that \( \frac{[1-\mu-(1-\alpha)c]^2}{4(1-\mu)} > \frac{(1-\mu)^2-2c(c+1-\mu-\sqrt{c(c+2(1-\mu))})}{4(1-\mu)} \), i.e. \( \alpha > 1 - \frac{1-\sqrt{\frac{c(1-\mu)^2}{1-\mu}}}{\frac{c}{1-\mu}} \). One can also show that
\[ 1 - \sqrt{1 - \frac{1}{\sqrt{\frac{1}{1-\mu} \left( \frac{c}{1-\mu} (1+\mu) - \frac{c}{1-\mu} \right)^2}} - \frac{\sqrt{1-\mu}^2 + \frac{c}{1-\mu}^2 - (1-\mu)}{c}} \geq \frac{\sqrt{1-\mu}^2 + \frac{c}{1-\mu}^2 - (1-\mu)}{c}. \] This result, together with the fact that \( \pi_{H,sep}^* = \frac{(1-\mu)^2 - 2c(1+c-\mu-\sqrt{c(c+2(1-\mu)))}}{4(1-\mu)} \) is independent of \( \alpha \) and \( \pi_{H, pool}^* \) is a strictly increasing function of \( \alpha \) when \( \alpha \in (0,1) \), implies that \( \pi_{H, pool}^* \geq \pi_{H, sep}^* \) if and only if \( \alpha \geq 1 - \frac{1}{\sqrt{1 - \frac{1}{\sqrt{\frac{1}{1-\mu} \left( \frac{c}{1-\mu} (1+\mu) - \frac{c}{1-\mu} \right)^2}} - \frac{\sqrt{1-\mu}^2 + \frac{c}{1-\mu}^2 - (1-\mu)}{c}}} \equiv \alpha_{comp}^* \).

In conclusion, when consumers do not know the firm’s cost, when \( \alpha < \alpha_{comp}^* \), the equilibrium is separating, \( \pi_{H, sep}^* = \frac{1-\mu-2c(1+c-\mu-\sqrt{c(c+2(1-\mu)))}}{4(1-\mu)} \) and \( \pi_{L, sep}^* = \frac{1-\mu}{4} \). When \( \alpha \geq \alpha_{comp}^* \), the equilibrium is pooling, \( \pi_{H, pool}^* = \frac{(1-\mu-(1-\alpha)c)^2}{4(1-\mu)} \) and \( \pi_{L, pool}^* = \frac{(ac+1-\mu)^2-c^2}{4(1-\mu)} \). When consumers know the firm’s cost, \( \hat{\pi}_{H}^* = \hat{\pi}_{L}^* = \frac{1-\mu}{4} \). ■