Prominent Attributes

Yi Zhu
Assistant Professor of Marketing
Carlson School of Management
University of Minnesota, Twin Cities
yzhu@umn.edu

Anthony Dukes
Associate Professor of Marketing
Marshall School of Business
University of Southern California
dukes@marshall.usc.edu

June 15, 2016

Acknowledgement: We are grateful for comments and discussions with Mark Bergen, Ron Berman, Eric Bradlow, Erman Haruvy, Dmitri Kuksov, Nanda Kumar, Lin Liu, Shijie Lu, Robert Meyer, David Reibstein, Régis Renault, Amin Sayedi, Jeffrey Shulman, Duncan Simester, Vasiliki Skreta, Upender Subramanian, Catherine Tucker, Christophe Van den Bulte, Andre Veiga, Charles Weinberg, Linli Xu, Narine Yegoryan, Pinar Yildirim, Juanjuan Zhang, as well as seminar participants at the 2015 INFORMS Marketing Science Conference, the 8th Workshop on the Economics of Advertising and Marketing at Oxford University, University of British Columbia, M.I.T, University of Pennsylvania, Indian School of Business, University of Washington, London Business School, and University of Texas at Dallas. The first author gratefully acknowledges financial support from the 3M Non-tenured Faculty Grant. The authors are listed in reverse alphabetical order and contributed equally.
Prominent Attributes

Abstract

Evidence shows that marketers can direct consumers’ limited attention to specific product attributes by making them “prominent”. This research asks: How should firms decide which attribute to make prominent in competitive environments? A key feature of this setting is that consumers’ preferences are context-dependent and that a firm’s choice of an attribute affects the evaluation of all products in the category. We develop a model in which firms selectively promote one of two attributes (e.g. image or performance) before competing in price. We find when consumers evaluate both attributes, perceived differentiation within an attribute can become diluted, an effect we call the dilution effect. This implies that making the same attribute prominent can arise in equilibrium. Only if there is a sufficient quality advantage in an attribute do we find equilibria with firms making different attributes prominent. We also show how the dilution effect can be a disincentive for investments in quality improvements.

Keywords: Prominent Attributes, Limited Consumer Attention, Dilution Effect, Context-dependent Preferences, Competitive Strategies, Game Theory.
1. Introduction

Despite the growth in the number of alternatives facing consumers, the amount of time they spend researching and evaluating products has remained constant, at around 30 minutes per day over the past 50 years (Ott 2011). Consequently, consumers may not know about or have the attentive capacity to evaluate all attributes for some purchase decisions. If the consumer’s attention is constrained, then the more attributes she evaluates, the less attention she pays to any particular one. Furthermore, there is a lot of evidence from the psychology literature (reviewed in Taylor & Thompson 1982) that one’s attention can be directed to a certain aspect of a choice environment so that it receives more weight in decision making. Marketers, in particular, can direct consumer’s limited attention and consequently affect which attribute he considers important, or prominent, in purchase situations (e.g. Wright & Rip 1980, Gardner 1983, MacKenzie 1986, and Jiang & Punj 2010). MacKenzie (1986), for instance, shows that manipulations of the prominence of a particular attribute for watches in print ads directly affected the subjects’ attention to that attribute and the subsequent importance they assigned to that attribute. In this research, we ask: Which attribute should the marketer make prominent when communicating to consumers with limited attention?

If a consumer considers only one seller, then deciding which attribute to make prominent is simply a matter of knowing which attribute most accentuates the attractiveness of that product. This decision may not be so simple, however, in a competitive context. To illustrate, consider a consumer deciding which craft beer to buy. She faces a large number of beer options but recalls ads from Brand X that emphasize where it is brewed. As she considers the options, she notices that a different brand, Brand Y, is brewed in her home town, an aspect sufficiently important to her that she is ready to choose it. However, as she sees the label of Brand Y, she notices the
emphasis on its alcohol content, which is quite high and higher than she typically prefers. Since she had not been paying any attention to alcohol content before, she reconsiders Brand X and notices it is light on alcohol. Before she saw Brand Y’s label, her choice was clearly Brand Y. Now, overall, she is relatively indifferent between Brands X and Y, and the beers appear less differentiated than when she only considered the brewing location. This reduction in differentiation is not due to any negative correlation between the two attributes. (In fact, we show that such a reduction occurs independent of the correlational structure.) It is driven purely by the fact that, if a product is better in one attribute (brewing location), then the inclusion of the second attribute (alcohol content) will reduce, on average, its relative attractiveness when the consumer splits her attention.

This beer example illustrates the strategic nuances of choosing which attribute to make prominent in a competitive context because (i) the firm’s decision affects consumer’s preferences for all products, including rivals’, and (ii) each firm can non-cooperatively try to influence the consumer’s attention to the same attribute or distinct attributes. These competitive interactions take on additional significance considering that the marketer typically faces bandwidth constraints, ¹ which limits the number of attributes that can be communicated to consumers. The focus of this research, therefore, is on the strategic interactions among competing marketers in their decisions about which attribute to make prominent.

The beer illustration also conveys the fundamental mechanism behind our findings: When consumers have limited attention, any perceived differentiation within a single attribute can become diluted across multiple attributes, an effect we call the dilution effect. We find that a firm can have an incentive to emphasize the same attribute as its rival to avoid the dilution effect.

¹ For instance, firms are often limited to a commercial of 30 seconds, a single page ad in a magazine, or a side-banners on a webpage.
and maintain product differentiation. In fact, listing craft beers by name and brewing location only, as is done at many bars, can be a way to avoid the dilution effect. (See Figure 1.)

![Figure 1: Typical Craft Beer Menu (From Le Garage, Baltimore)](image)

The dilution effect is fundamentally linked to limitations in consumer’s attention. When consumers have limited attention, they may not always evaluate and compare alternatives on all attributes (Russo & Dosher 1983, Bettman, Luce, & Payne 1998). For new products or low involvement purchases consumers might not even be initially aware of all the relevant attributes. Furthermore, the attributes the consumer pays attention to can be affected by the market environment (Johnson et al. 1988, Tversky et al. 1988, Shavitt & Fazio 1991). As mentioned above, marketers may try influence the market environment through advertisements, packaging, or branding so that a consumer’s attention is drawn to a specific attribute. If firms emphasize the same attribute, then a consumer evaluates products only on that attribute. If, however, firms emphasize different attributes, limited attention implies the consumer splits her attention across multiple attributes. In this latter case, a consumer’s attention is diluted. As we show, this effect
has implications for how a consumer compares available alternatives, which affects the competitive intensity among firms. Consequently, firms can make strategic choices on the prominent attributes to avoid heavier price competition.

The consumer psychology literature, most notably, has explored the mechanism by which attribute prominence affects the consumer decision process with limited attention. That research provides experimental evidence that the manner in which an attribute is presented has an impact on how a consumer chooses among alternatives (e.g. Tversky, Sattath, & Slovic 1988, Payne, Bettman, & Johnson 1993). Specifically, a consumer, who has seen an advertisement emphasizing a certain product attribute, tends to consider that attribute more important when evaluating available alternatives (Wright & Rip 1980, Gardner 1983, and MacKenzie 1986). In light of this evidence, we consider consumers’ preferences to be context-dependent such that the relative preference for products depends on the market context and is affected by firms’ marketing choices.

Our findings are consistent with the principle of product differentiation – competing firms prefer that consumers perceive products as maximally differentiated. However, one might intuit from this classic principle that competing firms should always emphasize the distinct attribute in which they excel. An implication of limited attention is that perceived differentiation may be largest when the consumer considers fewer attributes. In fact, even if one firm has a quality advantage in one attribute, it may be better to emphasize the same attribute with its competitor to avoid the risk of reducing perceived product differentiation. Only if a firm’s quality advantage in a particular attribute is sufficient, does it want to emphasize a different attribute than its competitor.
These results arise from a model in which competitive firms selectively choose an attribute to make prominent. We define an attribute as any aspect of a product category that the consumer regards as relevant for determining the level of utility. An attribute needs not be a physical feature of the products in that category. Rather, we assume it is a mutually accepted component of the consumer’s utility. For example, a consumer may assess the expected enjoyment of an untried beer based on the attribute of brewing location, or alcohol content, or both attributes. We further assume that any attribute has a horizontal component. In the beer example, the brewing location and alcohol content each has idiosyncratic aspects on which only the consumer herself knows her preference. Furthermore, an attribute may have a vertical component that all consumers value in the same way. Continuing with the beer example, consumers may prefer beer from a location that is famous for its high brewing standards (such as beer from Germany).

There are two firms, each with two attributes (e.g. brewing location and alcohol content). We assume each firm announces exactly one attribute as its prominent attribute. For example, firms will highlight their chosen attribute via advertising, promotion materials, or packaging. Consumers evaluate products by examining random fit parameters for the prominent attributes. We first study the case of symmetric firms when attributes are only horizontally differentiated. In this case, firms choose the same attribute to make prominent in equilibrium. If firms emphasize the same attribute, they ensure that consumers regard that single attribute as the most important one. Given idiosyncratic tastes on that attribute, the consumer purchases the product that is most satisfying on that attribute. However, if firms emphasize different attributes, the consumer evaluates products on multiple attributes. If the consumer’s attention is limited, then he is forced to split his attention across multiple attributes, which attenuates his relative appreciation of any
single attribute. With significant likelihood, the consumer will find that a firm who dominates on one attribute will dominate to a lesser degree on both. On average, the consumer will view the products as relatively less differentiated. Under the dilution effect, firms are induced to price more competitively.

We then consider the case in which each firm has a quality advantage in exactly one attribute. In this case, firms will make the same attribute prominent in equilibrium if the quality advantage is not too large. That is, one of the firms actually makes its inferior attribute prominent in order to avoid the dilution effect. Only if the quality advantage is sufficiently large will firms benefit from making their best attribute prominent. This situation is considerably different when one firm has the quality advantage in both of its attributes. The better firm may want consumers to evaluate both attributes in order to capture the true differential value of its product whereas the worse firm prefers that consumers evaluate on a single attribute to avoid the dilution effect. These differing incentives induce a mixed-strategy equilibrium in which the better firm randomly picks an attribute to be prominent in order to keep the other firm from selecting the same attribute. We call such an outcome a “cat & mouse” equilibrium to reflect the idea that the low quality firm is trying to catch the better firm’s choice of attribute.

For the above results, we assume that firms decide about prominent attributes for exogenous quality advantages. But, in practice, firms may want to build a quality advantage in an attribute after establishing it as prominent. For instance, the Volvo brand of automobiles, having been known for its safe performance, continues to invest in safety and maintain its competitive advantage in performance. Conversely, a firm’s decision to invest in quality may be affected by subsequent competition in attribute prominence. In light of the interplay between quality investment and prominent attributes, we extend the model to the case of endogenous quality.
Regardless of the timing of quality investments, we find that limited attention and the implied dilution effect can deter investment in quality.

This paper connects to two broad literatures devoted to consumers’ context-dependent preferences. First, as noted above, the consumer psychology literature has long suggested that consumers tend to put more attention on an attribute when that attribute is emphasized in a choice context (Tversky et al. 1988 and Shavitt and Fazio 1991), which can be affected by the marketer (Wright & Rip 1980, Gardner 1983, and MacKenzie 1986). A more recent stream of work in economics formalizes the relationship between attention and choice when consumers have context dependent preferences (Köszegi and Szeidl 2013, Bordolo, Gennaioli, and Shleifer 2013). None of this work, however, formally considers the equilibrium interactions with firms when consumers’ limited attention is directed by marketing. Second, there is a literature from economics and from marketing that examines the firm-side implications of context-dependent preferences in an equilibrium framework (Wernerfelt 1995, Kamenica 2008, and Guo & Zhang 2012). Our work, unlike all of the above mentioned research, has two important distinctions. First, we concentrate on the case of competitive firms, each of whom contributes to the consumer’s choice context. A firm needs to anticipate the other firm’s attribute strategy when deciding the optimal prominent attribute, even though it maybe suboptimal in monopoly. Second, we model a consumer’s context-dependent preferences at the product category level instead of individual products. Specifically, when one firm makes an attribute prominent, the attribute is prominent for the consumer on all products in that category. This implies interesting strategic interactions, which has not yet been examined to the best of our knowledge.

There is a number of related works that studies firms’ disclosure policies and advertising content strategies. For instance, Anderson & Renault (2006), Johnson & Myatt (2006), Bar-
Isaac, Caruana, & Cuñat (2012b) and Branco, Sun, & Villas-Boas (2015) examine a firm’s decision about how much “match” information to reveal to potential customers. That prior work considers content for consumers that is specifically related to the product(s) it sells. Firms in our setting can be interpreted along the same lines – revealing match values to consumers on one attribute. The key distinction, however, is that in our model firm’s disclosure applies to the attribute match values for all products in the category, not the disclosing firm only. This implies that a firm’s advertising strategy affects how consumers evaluate competitive firms as well as its own.2

A related stream of research studies firms’ communication strategies under bandwidth constraints. Like our paper, Bhardwaj, Chen, & Godes (2008) and Mayzlin & Shin (2011) start with the premise that a firm cannot communicate all aspects about its product to consumers, as is typically assumed in models of informative advertising (Butters 1977, Grossman & Shapiro 1984). Bhardwaj et al. (2008) compares “seller-initiated” versus “buyer-initiated” information revelation regarding which subset of attributes the buyer can learn about. Mayzlin & Shin (2011) study a firm’s decision about which, if any, product attribute to present to consumers. In both papers, there is asymmetric information about the product’s overall quality and the firm’s decision about what information to reveal is governed by its impact on consumer beliefs about quality (via signaling). In contrast, in our work there is no uncertainty about quality and the firm’s decision about which attribute to announce is determined by competitive interactions when consumer’s attention is limited.

---

2 In this way, our paper connects to Anderson & Renault (2009), which studies a model of comparative advertising in which a firm can reveal to consumers horizontal match information about competitive products. Advertising in our model can be interpreted as revealing partial match information about both products.
The remainder of the paper is structured as follows. In the next section, we lay out the basic framework of consumer preferences under limited attention and the corresponding purchase rule. Then, in section 3, we examine a number of scenarios to understand the choices regarding which attribute firms make prominent in equilibrium. Next, in section 4, we extend our model to allow firms to endogenously decide their quality. Section 5 concludes with an overview and discussion of future research. Proofs of technical results are all relegated to the Appendix. An online Supplemental Appendix contains additional technical details.

2. The Model

There are two firms, indexed as $j = 1,2$, each of which produces a product defined by two attributes $k = A, B$. Each attribute has a firm-specific quality component $q_{jk}$. Any firm can have a quality advantage in one or both attributes. For simplicity we assume that there exists only two quality levels in each attribute that firms may produce: low quality $q^L$ and high quality $q^H > q^L$.

We also normalize the marginal production cost for both products to be zero. The mass of consumers is normalized to one and each consumer has unitary demand. We first assume that the quality level is exogenously given, to allow us to abstract away from the firms’ quality decision. In section 4, we expand our analysis of prominent attributes when firms make endogenous quality decision.

The timing of the model is as follows: first, each firm decides its prominent attribute. Second, after observing each other’s prominent attribute choice, firms choose their product prices. Third, consumers form their preferences on prominent attribute(s), which are determined by firm’s announcement strategies. Finally, consumers evaluate products and purchase one product from a firm.
2.1 Consumer Utility from Evaluation

Each consumer has an idiosyncratic match value for each attribute of each product. If the consumer knows of one attribute she evaluates all products on that one attribute only. If both attributes are prominent, the consumer evaluates all products on both. In this way, we do not explicitly model the consumer’s search process or the decision on which attributes or products to evaluate, which allows us to focus on firms’ competitive strategies.

Suppose a consumer has evaluated only attribute $k$ from both products. Her utility of product $j$ based on inspecting attribute $k$ is given by,

$$u_{jk} = (V - p_j) + \theta_k (q_{jk} + \nu_k \varepsilon_{jk}),$$  \hspace{1cm} (1)

where $V$ is the intrinsic value of the product, $q_{jk}$ is the quality of firm $j$ in attribute $k$, and $\varepsilon_{jk} \sim N(0,1)$, which captures randomness in consumers’ tastes among products on attribute $k$. The parameter $\varepsilon_{jk} \sim N(0,1)$ is often referred to as the fit or match value that the consumer draws for product $j$ on attribute $k$. Match values that are normally distributed have a convenient additive property, which is useful in defining preferences when the consumer evaluates both attributes. We assume the distributions of random match value from two attributes are independent. The parameter $\nu_k$ captures the degree of heterogeneity of consumer tastes in attribute $k$. The parameters $V$, $p_j$, $q_{jk}$, and $\nu_k$ are common to all consumers whereas the component $\varepsilon_{jk}$ is idiosyncratic to the individual consumer. The factor $\theta_k$ is the weight parameter for attribute $k$ depending on firms’ prominent attribute decisions.

---

3 Independence is not necessary for our results and is made for analytical convenience. Section 3.3 offers a discussion of the impact of correlation in attribute match values and the Supplemental Appendix provides additional analysis.
When consumers evaluate both attributes, we assume the total utility is additive on the attribute-dependent utility components. In particular, for a consumer evaluating product \( j \) on both attributes, her utility is given by

\[
u_{j_{AB}} = (V - p_j) + \alpha_A \theta_A q_{j_A} + (1 - \alpha_A) \theta_B q_{j_B} + \nu_{AB} \epsilon_{j_{AB}},
\]

where \( \nu_{AB} \epsilon_{j_{AB}} = \theta_A \nu_A \epsilon_{j_A} + \theta_B \nu_B \epsilon_{j_B} \). Since \( \nu_k \epsilon_{j_k} \sim N(0, \nu_k^2) \), \( k = A, B \) and independent, the random fit term \( \nu_{AB} \epsilon_{j_{AB}} \sim N(0, \nu_{AB}^2) \), where \( \epsilon_{j_{AB}} \sim N(0,1) \) and \( \nu_{AB} = \sqrt{\theta_A^2 \nu_A^2 + \theta_B^2 \nu_B^2} \).

We now discuss the influence of firms’ decisions on the value of the weight parameters, \( \theta_A \) and \( \theta_B \). These weight parameters reflect the amount of attention the consumer pays to an attribute.\(^4\) Our approach to modeling consumer’s attention is akin to Köszegi and Szeidl (2013) and Bordolo et al. (2013). Firms’ announcements do not affect consumers’ total attention level, which we normalize to be 1, without loss of generality. We also assume the weight parameter is completely decided by firms’ prominent attribute strategies. Another way to interpret this assumption is that consumers are initially unaware of an attribute, and therefore have no preferences about it, until a firm draws attention to it. We make this assumption to capture the influence of firms’ marketing activities on consumers’ preferences on attributes, which is the key focus of this paper. If both firms make attribute \( k \) prominent, consumers treat that attribute as the only important attribute and will devote all their attention to it. Hence in this case, the weight parameter \( \theta_k = 1 \). On the other hand, when firms announce different attributes, then consumers treat both attributes as equally important and split their attention equally across the two attributes. Therefore, \( \theta_A = \theta_B = \frac{1}{2} \): less attention leads to lower appreciation of any single

\(^4\) The consumer psychology literature connects the amount of attention paid to an attribute to its importance in decision making. For example, Fishbein & Aizen (1975) argue “attributes that are important are typically evaluated more positively or negatively (i.e. are more polarized) than attributes that are unimportant” (p.228) as cited in MacKenzie (1986). Thus, the more important attribute \( k \in \{A, B, AB\} \) is considered, the greater the polarity, or variance, of the term \( \theta_k \nu_k \).
attribute. The symmetric split of attention may, at first, seem to be a restrictive assumption. For instance, consumers may have asymmetric preferences over the two attributes, putting more importance on one over the other. Our main result holds, however, as long as consumers do not fully ignore an attribute and correspondingly allocate some positive attention to both attributes \((0 < \theta_A = 1 − \theta_B < 1)\). Therefore, we assume equal weight on each attribute for simplicity and without loss of generality.

There is no incomplete information in product evaluation and consumers learn the available match values at no cost. However, the weight parameter, \(\theta\) is endogenously decided by firms’ strategies in choosing prominent attributes, which is the key feature of the context-dependent preferences in this model. And, in order to focus on firm interactions, we do not consider the consumer’s economic decision of how to allocate attention.\(^5\)

2.2 Consumer Purchase Decisions and Firms’ Market Share

The consumer decides only the product to purchase, which depends on whether she’s evaluated one or two of the products’ attributes. Specifically, if attribute(s) \(k \in \{A, B, AB\}\) is evaluated, then her chosen product is \(j^*(k) = \text{argmax}_j u_{jk}\). Because error terms \(v_{jk}\), \(\epsilon_{jk}\), \(j = 1,2\), are normally distributed, closed-form choice probabilities are not possible. In this case, equilibrium results are available only by numerical simulation and do not lend themselves to conveying the model’s intuition.\(^6\) To surmount this difficulty without compromising any result, we approximate

\(^5\) In this model we capture the effect of firms’ announcements on consumer preferences on category level. This feature has an alternative interpretation of persuasive advertising in which a firm’s announcement affects consumer utility, but for all products in a category. On the other hand, one might envision a traditional product-specific persuasion element for this model. However, incorporating this into the model would not affect consumers’ decisions in equilibrium since the competitive effect of such persuasion will cancel out in the symmetric setting.

\(^6\) See the Supplemental Appendix for details.
\( \nu_k \epsilon_{jk} \approx_D \mu_k \sigma_{jk} \), where \( \sigma_{jk} \) is distributed as type-I extreme value and \( \mu_k \propto \nu_k \). This permits us to formulate the purchase probability for each firm as a discrete choice logit demand\(^8\) (Anderson et al. 1992), which is derived in the following lemma.

**Lemma 1** If consumers evaluate only one attribute, \( k \in \{A, B\} \), then the probability of purchasing from firm \( j = 1,2 \) is

\[
S_{1k} = 1 - S_{2k} \approx \frac{e^{(q_{1k} - p_1)/\mu_k}}{e^{(q_{1k} - p_1)/\mu_k} + e^{(q_{2k} - p_2)/\mu_k}},
\]

where \( \mu_k \propto \nu_k \). (The constant of proportionality is defined in the Appendix.)

If consumers evaluate both attributes \( A \) and \( B \), then the probability of purchasing from firm \( j = 1,2 \) is

\[
S_{1AB} = 1 - S_{2AB} \approx \frac{e^{(\theta_A q_{1A} + \theta_B q_{1B} - p_1)/\mu_{AB}}}{e^{(\theta_A q_{1A} + \theta_B q_{1B} - p_1)/\mu_{AB}} + e^{(\theta_A q_{2A} + \theta_B q_{2B} - p_2)/\mu_{AB}}},
\]

where \( \mu_{AB} = \frac{1}{2} \sqrt{\mu_A^2 + \mu_B^2} \).

Since there is no closed form solution of a choice model when the match value is normally distributed, Lemma 1 is a useful technical result. The purchase probabilities expressed in Lemma 1 exhibit the usual properties of the logit demand. In particular, greater variance in the consumer’s maximized value of \( \nu_k \epsilon_{jk} \), as measured by \( \mu_k \) and proportional to \( \nu_k \), implies lower sensitivity to differences in quality and price. Because of the proportional relationship between

---

\(^7\) This approximation borrows from Extreme Value Theory and has the following interpretation. Suppose each firm sells \( n > 1 \) products, each defined by the attribute-specific match value \( \epsilon_{ijk}, i = 1, \ldots, n \). The consumer’s choice among firms equates to a choice across \( u_{jk} = V - p_j + \theta_A q_{ja} + \theta_B q_{jb} + \nu_k \max\{\epsilon_{ijk}\} \), where attribute(s) \( k \in \{A, B, AB\} \) are evaluated. As \( n \) becomes large, \( \nu_k \max\{\epsilon_{ijk}\} \approx_D \mu_k \sigma_{jk} \), where \( \mu_k \sigma_{jk} \) is distributed type-I extreme value. See the Appendix for more details of this approximation.

\(^8\) In the current model we assume the market is fully covered by two firms and there is no outside options. Our results are quantitatively unchanged if consumers have an outside option. See the Supplemental Appendix for details.
\(\mu_k\) and \(v_k\), we henceforth refer only to the parameter \(\mu_k\) when discussing the degree of horizontal differentiation within attribute \(k\). With closed-form expressions for choice probabilities at each firm, we can find the equilibrium of the game played by the two firms.

3. The Prominent Attribute Decision in Equilibrium

In this section, we solve for the prominent attribute decisions of firms in equilibrium.\(^9\) We assume \(\mu_A = \mu_B = \mu\), for symmetry.\(^{10}\) Let \(\Delta_k \equiv q_{1k} - q_{2k}\) be the quality advantage for firm 1 in attribute \(k = A, B\). We start with the benchmark case in which there is no quality differentiation among firms in any attribute, \(\Delta_k = 0\). In this case, as demonstrated in Proposition 1 firms make the same attribute prominent in equilibrium. This is a helpful benchmark because it isolates the key mechanism in all of our results regarding firms’ prominent attributes decisions.

**Proposition 1** If \(\Delta_k = 0\), \(\forall k\), then firms choose the same attribute to make prominent in equilibrium.

To see why announcing the same attribute is an equilibrium, consider a deviation by one firm to announce the other attribute. Doing so implies that the consumer evaluates both products on both attributes. Because a consumer’s attention is fixed, she splits her attention across both attributes and therefore places less weight on any particular attribute. The variance of the consumer’s random match value is reduced as a result: \(\text{var}(\mu_{AB}e_{jAB}) = \frac{\mu^2}{2} < \mu^2 = \text{var}(\mu_{j}e_k)\).

---

\(^9\) Because of our interest in competitive interactions in the prominence decision, we do not present the case of monopoly. The monopolist’s problem regarding attribute prominence can be mapped to the modeling framework of Johnson & Myatt (2006).

\(^{10}\) One may wonder about the case of asymmetrically differentiated attributes \((\mu_A \neq \mu_B)\). Generally, if \(\mu_A/\mu_B > 1\) then choosing attribute \(A\) is more attractive than \(B\) for both firms. See footnote 11 for the basic implication of this asymmetry. However, because this case leads to fairly predictable results, we have otherwise maintained \(\mu_A = \mu_B\) throughout.
This implies that equilibrium prices, and corresponding profits, fall for both firms when announcing different attributes: \( p_{AB} = \sqrt{2} \mu < 2 \mu = p_A = p_B \) and \( \pi_{AB} = \frac{\sqrt{2}}{2} \mu < \mu = \pi_A = \pi_B \).

We call this the *dilution effect*: on average, a reduction in the variance of the match value dilutes perceived firm differentiation by the consumer. To see this another way, suppose a consumer evaluates only attribute \( A \) and prefers firm 1 (\( u_{1A} > u_{2A} \)). Then, it is likely that the strength of her preference for firm 1 will shrink when consumers evaluate both attributes.\(^{11}\) The dilution effect, therefore, has a downward impact on equilibrium prices. Thus, it is not profitable for a firm to announce an attribute different than its rival.\(^{12}\)

In the remainder of this section we present the main results pertaining to prominent attribute decisions with exogenous qualities. We first examine the case of symmetric quality advantages in which each firm has an advantage in one attribute. Finally, we study the case in which one firm has quality advantage on both attributes.

### 3.1 Firms with Symmetric Quality Advantage

In this section, we extend the above setting and consider firms with a quality advantage in one attribute. Define \( \Delta_k = q_{1k} - q_{2k} \) be firm 1’s quality advantage in attribute \( k \). Without loss of generality, assume \( q_{1A} = q_{2B} = q^H \), and \( q_{1B} = q_{2A} = q^L \), with \( q^H > q^L \) so that firm 1 has the quality advantage on attribute \( A \) and firm 2 on attribute \( B \). Lemma 2 shows that if only one attribute is prominent, then firms have opposing preferences for which one it is.

---

\(^{11}\) Mathematically, \( E[u_{1AB} - u_{2AB}|u_{1A} > u_{2A}] < E[u_{1A} - u_{2A}|u_{1A} > u_{2A}] \).

\(^{12}\) If \( \mu_A/\mu_B > 1 \) then profits for both firms are larger when they coordinate on attribute \( A \) than on attribute \( B \). The two equilibria of Proposition 1 remain if \( \mu_A/\mu_B \) is not too large. Only if \( \mu_A/\mu_B \) is sufficiently large, does it rule out both firms choosing attribute \( B \) in equilibrium.
**Lemma 2**: Let $\Delta = \Delta_A = -\Delta_B = q^H - q^L > 0$. If both firms make attribute $A$($B$) prominent, then firm 1(2)’s corresponding equilibrium price, market share, and profit are higher than firm 2(1).

When firms announce the same prominent attribute, consumers evaluate products only on that attribute. Therefore, one firm appears to have an absolute quality advantage over the other. Naturally, the better quality firm enjoys a larger market share and a higher price. Lemma 2 does not say, however, whether such a quality advantage is sufficient to overturn the mutual benefit of avoiding the dilution effect. Suppose, for example, firm 1 announces attribute $A$. Is it better for firm 2 to announce attribute $B$ in order to neutralize the quality advantage? Or should firm 2 announce attribute $A$ to avoid the dilution effect? Proposition 2 shows that the answer depends on the relative strength of the quality advantage.

**Proposition 2** There exists a threshold $\Delta > 0$ such that if $\Delta < \bar{\Delta}$, then both firms make the same prominent attribute in equilibrium; otherwise they make different attributes prominent.

The main intuition of Proposition 2 ($\Delta > 0$) extends that of Proposition 1 ($\Delta = 0$). Regardless of the size of $\Delta$, announcing different attributes dilutes consumer attention into two attributes. If $\Delta < \bar{\Delta}$ is small, therefore, the quality differentiation is not large enough to offset advantages in quality. That is, even though firm 2 with, say, a quality advantage in attribute $B$, it may prefer to
join firm 1 in emphasizing attribute \( A \) to avoid the dilution effect.\(^{13}\) Only when \( \Delta > \bar{\Delta} \) is large, does firm 2 prefer to accentuate his quality by announcing attribute \( B \).

### 3.2 Firms with Asymmetric Quality Advantages

In this section, we consider the case when one firm has a uniform quality advantage in both attributes. It is first helpful to prove a general result regarding the shape of firms’ profits as a function of the degree of horizontal differentiation when firms differ in their relative qualities. Lemma 3 indicates that a firm who is perceived as high quality does not always benefit from greater horizontal differentiation.

**Lemma 3** Suppose that attribute \( k \in \{A, B, AB\} \) is prominent and that firm 1 has a quality advantage in that attribute: \( \Delta = \Delta_k > 0 \). There exists a unique threshold \( \bar{\mu}_\Delta > 0 \) such that the high quality firm’s (1’s) profit decreases with \( \mu \) when \( \mu < \bar{\mu}_\Delta \) and increases with \( \mu \) when \( \mu > \bar{\mu}_\Delta \). The low quality firm’s (2’s) profit always increases with \( \mu \).

Lemma 3 applies to both the symmetric and asymmetric quality advantage cases. It says that, for the firm with the quality advantage in the prominent attribute(s) (the high quality firm), its profit is non-monotonic in \( \mu \) (or \( \mu_{AB} \)). While the low quality firm always benefits from a higher variance \( \mu \), the high quality firm’s profit first decreases in \( \mu \) and then increases, achieving its lowest at \( \mu = \bar{\mu}_\Delta \). High variance increases the horizontal differentiation between two firms and therefore benefits the low quality firm unambiguously. However, if \( \mu < \bar{\mu}_\Delta \), higher

\(^{13}\) Though we focus on pure-strategy equilibria, there also exists a proper mixed strategy equilibrium under the conditions of Propositions 1 and 2. When firms have no quality advantage, announcing each attribute with probability \( \frac{1}{2} \) is an equilibrium. If \( \Delta_A = -\Delta_B < \bar{\Delta} \), then the symmetric mixed-strategy equilibrium has firm 1 announcing attribute \( A \) with probability greater than \( \frac{1}{2} \) and equal to firm 2 announcing attribute \( B \). The reasoning in this latter setting maps to the classic 2x2 game of Battle of the Sexes.
horizontal differentiation diminishes the high quality firm’s quality advantage. To see this, consider a case when the variance $\mu \approx 0$ and $\Delta$ is high. Because there is no horizontal differentiation, the high quality firm can charge a price at $\Delta - \epsilon$ and has all the consumers. But once the variance increases, the low quality firm gains demand due to the random matching. The added demand at firm 2 corresponds to a loss in demand at firm 1. Thus, an increase in the variance may reduce the high quality firm’s profit. Only if $\mu > \overline{\mu}_\Delta$, does the high quality firm gain from added horizontal differentiation. This logic further implies that the threshold, $\overline{\mu}_\Delta$, is increasing in $\Delta$. Figure 2 graphically illustrates the firms’ profit across different $\mu$ as derived in Lemma 3.

![Figure 2: Firms’ Profits as Functions of $\mu$](image)
Now assume that $q_{1A} = q_{1B} = q^H$, and $q_{2A} = q_{2B} = q^L$, so that the quality advantage for firm 1 is the same for both attributes $A$ and $B$. Firm 2 has no quality advantage, regardless of the announcement strategy. For the case of asymmetric quality advantage, $\Delta_A = \Delta_B > 0$, Lemma 3 implies that firm 2 prefers the outcome which has the most variation in the match value. In contrast, firm 1 prefers less variance whenever $\mu < \bar{\mu}_\Delta$. This divergence in preferences implies an equilibrium in mixed strategies. As before, both firms are subject to the dilution effect even when one firm has the overall quality advantage. Firm 2’s best outcome, therefore, is for consumers to evaluate products on only a single attribute. Firm 1’s trade-off, however, can be different than firm 2’s. As shown in Lemma 3, firm 1’s profit can actually decrease in $\mu$. In this case, firm 1’s best outcome, therefore is for consumers to evaluate both attributes. This leads to a “cat & mouse” equilibrium which is summarized in Proposition 3.

**Proposition 3** Let $\Delta = \Delta_A = \Delta_B > 0$, with the corresponding cutoff point $\bar{\mu}_\Delta$ defined in Lemma 3.

(a) If $\mu < \bar{\mu}_\Delta$, then the high quality firm (1) wants to announce different attribute while the low quality firm (2) prefers to announce the same attribute. In equilibrium, firms play mixed strategies choosing each attribute with equal probabilities.

(b) If $\mu > \bar{\mu}_\Delta$ then both firms announce the same attribute.

Announcing the same attribute is always good for the low quality firm since it increases the variation of the match value; on the other hand, as Lemma 3 indicates, higher variation in the match value may hurt the high quality firm’s quality advantage. This leads to the first result in which firm 2 (cat) tries to match the attribute choice of firm 1 (mouse), while firm 1 tries to
avoid a match with firm 2. When the \( \mu \) is large enough, the higher variation (jointly implemented by announcing the same attribute) can compensate for the loss of the quality differentiation. Thus, firms announce the same attribute.

### 3.3 Market Requirements for the Dilution Effect

In this section, we review the market requirements, or necessary conditions, for the dilution effect. These key requirements are listed below along with a discussion of the boundary conditions for each.

1. **Limited Attention**: When consumers are attention constrained, they spread their attention across the attributes they know about. This implies that any perceived differentiation within a single attribute becomes diluted across multiple attributes. This is a necessary condition for the dilution effect. If consumers’ attention is unlimited, it is possible that consumers do not reduce the weight on each attribute when firms announce different prominent attributes. In this case, considering an additional attribute would not diminish the perceived differentiation from those attributes already considered. The dilution effect, therefore, would not exist. One important question, which we do not address here, is: how do consumers allocate their limited attention? We assume instead that for some purchase decisions, consumers devote a fixed amount of attention. Such purchase decisions might include low involvement purchases in, for example, CPG categories.

2. **Context-dependent preferences**: A key assumption in our model is that the marketing context affects consumer’s preferences over available products. In particular, consumers evaluate an attribute only if emphasized by a firm. And, equally important for our model, is the fact that the firm’s marketing decisions affect consumer’s evaluation of all products in the category. This latter point is at the heart of the strategic interactions among
competing firms in our model and is distinct from earlier models of context-dependency and firm behavior (Werenerfelt 1995, Kamenica 2008, and Guo & Zhang 2012). Context dependency in our setting strictly regards attribute awareness and attention. Therefore, we expect the incentives surrounding the dilution effect to be relevant for product attributes which consumers have not yet experienced and may not be fully aware of their existence. As illustrated in the example of the introduction, casual drinkers may not pay attention to or be aware of all attributes of beer.

3. **Relative quality advantages**: As shown in Propositions 2 and 3, the dilution effect is a dominant strategic consideration in firms’ attribute choices if quality distinctions between firms are either small or asymmetric. If, in contrast, each firm has a major strength in a distinct attribute, then firms focus simply on what they do best despite the presence of the dilution effect. Thus, if each firm excels in a unique attribute, then the novel incentives we identify in our model may matter to a lesser extent for marketers.

It is also helpful to understand the role of several technical assumptions made for analytical convenience, which do not play an essential role in our results. (1) The normal distribution on match value terms is assumed to capture idiosyncratic consumer tastes, which are not known to the firm. Normality is helpful for adding the match values in the utility $u_{AB}$, but is otherwise not essential. Similarly, our approximation to the type-I extreme value distribution is a convenience device that permits closed-form demand functions, which in turn allows us to derive equilibrium results. As such, the exact form of the match value distributions do not play an important role in generating our results. (2) We have also assumed that consumers have perfectly symmetric preferences over attributes ($\theta_A = \theta_B = 1/2$). This assumption helps isolate the incentives of firms in the prominent decision, which is based on firm differentiation rather than
on attribute differentiation. Allowing asymmetric preference does not affect our main results as long as some consumers do not completely ignore any attribute. (3) Finally, we assumed that attribute-specific match values were independent ($\epsilon_{jA} \perp \epsilon_{jB}$), which may not necessarily be the case in many purchase situations. Positive correlation tends to weaken the dilution effect because a high draw of a match value on one attribute is less likely to come with a lower draw on the second attribute. For example, suppose the sleek styling of an automobile relates positively to its sporty performance. Then a strong preference toward a car’s sleekness is less likely to be diminished by the discovery of its sporty performance. Conversely, negative correlation among attributes exacerbates the dilution effect. In either case, however, perfect independence of match values is not essential for our results. In a Supplemental Appendix, we provide detailed arguments showing that relaxing any of these three assumptions do not compromise our results.

4. Prominent Attribute Strategies under Endogenous Quality

In the previous section, we assumed quality differences were exogenously specified. However, as motivated in the introduction, firms may target certain attributes for quality improvements. We consider two different timings. In section 4.1 we consider the timing in which the investment decision is made after the attribute decision. This setting attempts to capture the situation in which positioning an attribute as prominent is harder to establish and modify relative to an incremental quality improvement. In section 4.2, we reverse this timing so that quality investment precedes attribute choice. This case captures situations in which attribute prominence can be affected more readily than quality investment. Our focus, in both situations, is to determine whether limited attention, and the implied dilution effect, can deter investment.
4.1 Quality Investment After Prominence Decision

In this section we study the decision of prominent attributes when firms can subsequently invest in quality. While the dilution effect served to intensify price competition, as we show in the case of endogenous quality, the dilution effect may help firms avoid intense competition on quality.

We consider a multi-stage game to accommodate the prominent attribute-quality decisions made by firms. In the first stage, firms simultaneously choose their prominent attributes. In the second stage, after both firms’ choices of prominent attributes are known, firms choose whether to invest in the quality of their prominent attribute. Firms then set their product prices in stage three. Finally, consumers form their preferences and make their purchase decisions as in section 3. This multi-stage timing reflects settings in which firms make quality decisions after attributes have been well-established as prominent (e.g. the Volvo example of the introduction).

Consumer decisions are the same as the benchmark model given in section 2. Therefore, we only discuss the quality investment decision stage. Each firm is endowed with the same quality level, \( q^L \), in all attributes, but they can spend a cost \( c > 0 \) in stage 2 to improve the product quality in one attribute to \( q^H \). For simplicity, we assume the binary quality outcomes \( (H \) or \( L) \), but our results are quantitatively unchanged with continuous quality. Finally, we assume that it is too costly for firms to invest on both attributes so that each firm can only invest on one of them. This allows us to focus on the connection between the choice of the prominent attribute and the decision on quality investment.

We first derive the equilibrium quality decisions for each prominent attribute outcome and then the equilibrium decisions on prominent attributes. A key aspect of this model is that, regardless of the prominent attribute decision in stage one, a firm enjoys a quality advantage only
if it invests in quality and the rival firm does not. If both firms invest in quality, then the advantage is competed away in prices. Hence, firms are always worse off when they both invest relative to when they both do not invest. Nevertheless, if investment costs are not too high, then a prisoner’s dilemma ensues and investing in quality is a dominant strategy. We are particularly interested in situations in which firms can avoid a prisoner’s dilemma by their choice of prominent attributes in stage one. Lemma 4 raises this possibility that firms choose different attributes.

**Lemma 4**

(a) If firms make the same attribute prominent, then there exists \( c > 0 \) such that both firms invest in quality when \( c < \bar{c} \).

(b) If firms make the different attribute prominent, then there exists \( \bar{c} > 0 \) such that both firms do not invest in quality when \( c > \bar{c} \).

If firms make the same attribute prominent, then according to Lemma 4(a), both firms’ dominant strategy is to invest in quality as long as \( c \) is not too large. Under this scenario firms overinvest on quality due to competitive pressure. If firms choose different attributes, so that consumers evaluate products on both attributes, then as long as \( c \) is sufficiently large, firms can avoid the prisoner’s dilemma. Overall, Lemma 4 implies that firms may choose to emphasize different attributes as a commitment to avoid competition on quality, but only if \( \bar{c} < \bar{c} \).

**Lemma 5** For any \( \mu > 0 \), there exists a threshold \( \hat{\Delta}(\mu) > 0 \) such that if \( \Delta < \hat{\Delta}(\mu) \), then \( \bar{c} < \bar{c} \).
Lemma 5 establishes that when Δ is not too high, there exists a well-defined range of investment costs $c \in \left( \underline{c}, \bar{c} \right)$ such that, if firms make the same attribute prominent, then they will invest in quality and if firms make different attributes prominent, they will not invest in quality. For both firms to make different attributes prominent in equilibrium it must be mutually beneficial to avoid the prisoner’s dilemma. Recall from Proposition 2 that the dilution effect is stronger than the quality advantage effect when Δ is not too large. As discussed in section 3, this was due to intensified price competition from a lower variance in $\mu_{AB} \epsilon_{ij}$. Thus, firms would prefer to be in the prisoner’s dilemma with quality competition over suffering from a severe dilution effect if $c < \left( 1 - \frac{\sqrt{2}}{2} \right) \mu$.

**Proposition 4** When $\Delta < \tilde{\Delta}(\mu)$, firms make different attributes prominent for any $c \in \left( \max\{\underline{c}, (1 - \frac{\sqrt{2}}{2})\mu\}, \bar{c} \right)$.

We are assured the conditions for Proposition 4 occur in our model by Lemmas 4 and 5. The proposition states that, under these conditions, firms announce different attributes in equilibrium. They do this despite and because of the adverse consequences of the dilution effect. While the dilution effect has a downward impact on profits due to more intense price competition, the dilution effect also lowers the incentive to invest in quality when $\Delta$ is not too large. This extension shows that, in contrast to Proposition 2, the dilution effect can discourage firms from competing on quality. That is, firms can dilute consumer attention by making both attributes prominent so that a unilateral investment in quality in one attribute does not bring a sufficient advantage to induce a prisoner’s dilemma.
4.2 Quality Investment Before Prominence Decision

In this section, we study a situation in which the prominence decision can be interpreted as more flexible than investment in quality. For instance, an innovation in an attribute may require a long term investment after which the firm can choose whether to make it a prominent attribute. The goal is to understand whether the dilution effect in the second stage can undermine firms’ incentive to invest in the first stage. In order to study this, we compare two different scenarios. The first one is a benchmark case in which consumers are restricted to evaluation on exactly one attribute. This is the simplest way to study the model when the dilution effect cannot affect the investment decision. We compare the benchmark case with a second scenario – the full model with the possibility firms announce different attributes, so the dilution effect is a strategic possibility. Through this comparison, we establish that the dilution effect alone can deter investment in quality, when that investment would otherwise occur.

We assume a multi-stage game similar to that in section 4.1, except that the simultaneous investment decision occurs before the attribute decision. First, in Lemma 6, we establish the thresholds for the benchmark case and the full model.

Lemma 6

(a) (One Attribute Only) Suppose firms are restricted to making the same attribute prominent. Then there exists $\hat{c} > 0$ such that both firms invest in quality if $c < \hat{c}$.

(b) (Two Attributes) If firms can invest either attribute and then choose either attribute to be prominent, then for any $\mu > 0$, there exists thresholds $\bar{\Delta}(\mu) > 0$ and $\hat{c} > 0$ such that when $\Delta > \bar{\Delta}(\mu)$, neither firm invests in quality when $c > \hat{c}$. 
The intuition of Lemma 6(a) is similar to that of Lemma 4(a). In the one attribute case, we have a prisoner’s dilemma when \( c < \hat{c} \). A firm suffers from lost market share when it does not invest but the rival does. Therefore, each firm invests in quality when it is not too costly and firms are less profitable in equilibrium than if they both had not invested. Lemma 6(b) identifies conditions of no investment when both firms choose among the two attributes in which to invest and which to subsequently make prominent. Since the timing of the game is reversed from the model in Section 4.1, firms’ decision on investment is not only endogenously decided, but also influenced by the choice of prominent attribute in stage 2. Lemma 6(b) shows that when \( \Delta \) and \( c \) are large enough, neither firm invests in quality in the full model. In fact, under these conditions, investing in quality is a dominated strategy. When \( \Delta > \bar{\Delta}(\mu) \), the non-investing firm always makes different attribute prominent in order to counter a disadvantage in quality. Hence the incentive of investing in quality is undermined by the possible threat of making different attribute prominent by the non-investing firm. When \( c \) is significantly large, neither mutually investing in different attributes nor a unilateral investment is profitable for both firms.

**Proposition 5** Suppose \( \Delta > \bar{\Delta}(\mu) \). There exists a well-defined range of costs (i.e. \( \bar{c} < \hat{c} \)) such that for any \( c \in (\bar{c}, \hat{c}) \) the following hold.

(a) *If firms are restricted to making the same attribute prominent, then both firms invest in the quality of that attribute.*

(b) *If firms can make any attribute prominent, then no firm invests in the quality of either attribute.*

Lemma 6(a) shows that, when there is no threat of the dilution effect, an investment in quality is the equilibrium outcome if \( c < \hat{c} \). However, Lemma 6(b) indicates when \( c > \bar{c} \), the
investment is deterred when firms can make different attribute prominent. Proposition 5 assures us that the conditions of Lemma 6 (a) and (b) can simultaneously occur since $\dot{c} < \dot{\hat{c}}$. Thus, this extension establishes that the dilution effect is the ingredient that can undermine quality investment.

4.3 The Dilution Effect & Quality Investment – Summary

In summary, regardless of the timing of the investment choice relative to the attribute prominence choice, we have shown that the dilution effect can have a deterrent effect on quality investment. However, the outcome and the mechanism in the two settings are different. When the prominent attribute is chosen first, firms emphasize different attributes. By emphasizing different attributes, they commit to a more competitive situation, which undermines the incentive to invest. In the reverse timing, firms are able to coordinate on the same attribute after choosing not to invest. When a firm considers investing in an attribute in the first stage, it knows there is a credible threat of the other firm not to coordinate on attribute announcements in the second stage: When $\Delta$ is large, the firm who does not invest in the first stage can threaten to announce a different prominent attribute in the second stage. This threat is credible and, therefore, facilitates the coordination of firms to not invest in quality in the beginning.

5. Conclusion

The premise of this research is that firms face limitations when communicating to consumers: limited attention and limited bandwidth. These limitations imply that a firm cannot provide a full description of each of its product’s attributes and that consumers do not necessarily devote all of their attention to all product attributes before purchase. Consequently, a consumer’s preference over available products depends on which attributes marketers choose to highlight, or make
prominent. In competitive scenarios, one firm’s decision on a prominent attribute affects how a consumer evaluates other firms’ products. Our model examined the corresponding strategic interactions between firms deciding which attribute to make prominent. The model points to several results.

First, symmetric firms, which are horizontally differentiated, choose the same attribute to make prominent. With limited consumer attention, firms jointly prefer consumers to devote their attention to few attributes. When consumers evaluate on multiple attributes they perceive less horizontal differentiation because they split their attention across the attributes – a notion we termed the dilution effect. This result was derived from a model of two firms selling products with two attributes. When each firm has a unique quality advantage in exactly one attribute, our model finds that firms may still emphasize the same single attribute. Only if the quality advantages are sufficiently large, will firms emphasize different attributes in equilibrium.

Next we considered the case in which one firm dominates in both attributes. In this case, the two firms’ preferences over prominent strategies can diverge. The dominant firm may prefer that consumers evaluate products on both attributes so as to accentuate its entire quality advantages. The other firm, in contrast, prefers that consumers consider only one attribute so as not to be subject to the dilution effect. This leads to firms playing mixed strategies, with the dominant firm trying to make its choice of prominent attribute unpredictable for the other firm.

Finally, we considered the case in which firms could strategically invest in the quality of their prominent attribute. We showed that the dilution effect can be used as a threat to quality investments. When firms are permitted to make incremental quality improvements after making the same attribute prominent, there is a prisoner’s dilemma in which firms invest in quality but compete away any additional profits. To avoid that prisoner’s dilemma, firms choose different
attributes prominent to ensure the dilution effect erodes an unilateral benefit from investing in quality. When firms must make investment decisions before committing to a prominent attribute, the threat of the dilution effect by the non-investing firm in the second stage keeps the unilateral benefit of investing sufficiently low to prevent investment in the first stage.

Our research points to a few areas for future inquiry. Perhaps most obviously is to examine the question of how consumers optimally allocate their attention when it is limited. The notion that consumers pay differential attention across attributes when making a choice has only recently received attention in the economics literature (Közegi and Seidl 2013, Bordolo et al. 2013) but is otherwise mum on the manner in which consumers allocate their attention. Examining consumers’ attention allocation problem could have important implications for marketers when communicating product information.

In reality consumers may not always be able to evaluate the product’s quality before purchase, especially when consumers’ attention is limited. In such situations, a firm’s attribute choice not only discloses the importance of the attribute, but may also “signal” the quality of the product in this attribute. Incorporating the signaling role of choosing a prominent attribute may be a useful direction for future research.

The literature on economics of search, while large, typically focuses on product evaluation at a holistic level and has only recently begun to examine the case in which consumers search at the attribute-level. None of this work, however, has studied the case in which attributes differ in terms of the differential attention that consumers pay to certain attributes. Developing an alternative modeling of consumers, with more micro-foundations of consumer learning and attention allocation would be a fruitful line of inquiry.
In our competitive model, we assumed that firms moved simultaneously in their announcements of attributes. But consumers encounter some firms before others and even some firms have a timing advantage in moving more quickly than rivals. It is reasonable to anticipate that sequential exposure might have differential context biases on consumer’s attention. For instance, if recency effects are significant, then moving last would lend that marketer an advantage in directing consumer’s attention where it wants. In contrast, primacy effects would do the opposite. The presence of either form of bias would have implications on how firms strategically time their announcement of a prominent attribute.

References


Appendix

This appendix contains the proofs of all results in the main text.

Before proving Lemma 1, we start with a formalization of the approximation used in all of the results. The approximation can be interpreted as the choice framework in section 2 except that each firm carries \( n \geq 1 \) products, with identical quality within each firm. Consider a sequence of i.i.d. random variables \( \{v_k \epsilon_{ijk}\}_{i=1}^{n} \), where \( \epsilon_{ijk} \sim N(0,1) \) and \( v_k > 0 \). Define \( M_{jk} \equiv \max_{1 \leq i \leq n} \{v_k \epsilon_{ijk}\} \) as the maximum random-match value obtained when evaluating all \( n \) products from firm \( j \) on attribute(s) \( k \in \{A, B, AB\} \). (Note that \( i^*(j, k) = \arg\max_i \{v_k \epsilon_{ijk}\} \) is the consumer’s most preferred of the \( n \) products offered by firm \( j \).) Fix \( j = 1, 2 \) and, with a slight abuse of notation, \( k = A, B, AB \). Consider the sequence \( \{v_k \epsilon_{ijk}\}_{i=1}^{n} \), which is an i.i.d. sample of size \( n \) from \( N(0, v_k^2) \), the normal distribution with zero mean and variance \( v_k^2 \). Haan & Ferreira (2006) show that the Fisher-Tippet-Gnedenko Theorem implies that the distribution of \( M_{jk} \) approximates that of a type-I, extreme-value random variable with a cdf:

\[
\Pr[M_{jk} < x] \approx e^{-e^{-n \Phi'(\alpha_n)x/v_k}},
\]

where \( \alpha_n = \Phi^{-1}\left(1 - \frac{1}{n}\right) \). This approximation reflects the convergence as \( n \to \infty \) of \( v_k \max_\{\epsilon_{ijk}\} \to_D \mu_k \sigma_jk \), where \( \sigma_jk \) is a standard type-I extreme value random variable and \( \mu_k = v_k \frac{1}{n \Phi'(\alpha_n)} \). Thus, for any \( k \), the consumer’s choice of a firm approximates a logit choice framework.
Proof of Lemma 1

Using the above approximation, suppose a consumer evaluates products on one attribute, \( k = A \) or \( B \), only. Then her choice of firm depends only on the best product from each firm:

\[
 u_{jk} = \max\{u_{ijk}\} = V + q_{jk} - p_j + \max\{v_k \varepsilon_{ijk}\}.
\]

In this case, the consumer chooses firm 1 if and only if \( u_{1k} > u_{2k} \), or equivalently:

\[
 M_{1k} > \frac{(q_{2k} - q_{1k}) - (p_2 - p_1)}{\mu_k} + M_{2k}, \quad k = A, B, \quad \mu_k = \nu_k \left[ \frac{a_n}{n \Phi'(a_n)} \right]
\]

which occurs with probability \( s_{1k} = 1 - s_{2k} \), as shown in Anderson et al. (1992), and is expressed in the first part of the lemma’s statement. Suppose a consumer evaluates products on both attributes so that \( \nu_{AB} \varepsilon_{AB} \approx D \mu_{AB} \sigma_{AB} \), where \( \mu_{AB} = \frac{1}{2} \sqrt{\mu_A^2 + \mu_B^2} \) and \( \sigma_{AB} \) is a random variable with standard, type-1 extreme value distribution. For \( \mu_A = \mu_B = \mu, \mu_{AB} = \frac{\sqrt{2}}{2} \mu \). The consumer chooses firm 1 if and only if \( u_{1AB} > u_{2AB} \), which is equivalent to:

\[
 M_{1AB} > \frac{(q_{2AB} - q_{1AB}) - (p_2 - p_1)}{2 \mu / \sqrt{2}} + M_{2AB},
\]

which occurs with probability \( s_{1AB} = 1 - s_{2AB} \) and is expressed in the second part of the lemma.

\[\Box\]

Proof of Proposition 1

Based on Lemma 1, we know that firm \( j \)’s profit when consumers evaluate attribute(s) \( k (k = A, B, AB) \) is equal to \( \pi_{jk} = p_j s_{jk} \) where \( s_{jk} \) is given in Lemma 1. The first order condition gives us

\[
 \frac{\partial \pi_{jk}}{\partial p_j} = s_{jk} + p_j \frac{\partial s_{jk}}{\partial p_j} = s_{jk} - \frac{1}{\mu_k} s_{jk} s_{-jk} p_j = 0, \quad \text{where} \quad -j \neq j.
\]

Therefore we know \( p_j = \mu_k s_{-jk}^{-1} \), and the profit is equal to \( \pi_{jk} = \mu_k s_{jk}^{s_{jk}} \). The condition \( \Delta = 0 \) directly implies that in equilibrium \( s_{jk} = s_{-jk} = \frac{1}{2} \). Therefore \( \pi_{jk} = \pi_{-jk} = \mu_k \). Since \( \mu_A = \mu_B > \mu_{AB} \), firms will prefer to announce the same attribute. \[\Box\]
Proof of Lemma 2
The consumer’s utility from firm $j$ when evaluating only attribute $k$ is $u_{jk} = V - p_j + q_{jk} + \mu_k \max\{\epsilon_{ijk}\}$, where $\max\{\epsilon_{ijk}\}$ is distributed as a type I extreme value distribution by Lemma 1.

Anderson et al (1992) established that there exists a unique price equilibrium in a same setting. Moreover, Anderson et al (1992) show that as the quality difference $\Delta$ increases, the equilibrium price of the high quality firm increases as well. Hence, $\Delta_k > 0$ leads to $p_1 > p_2$. From firms’ first order conditions, $p_j = \mu_j / s_{-jk}$, we further deduce that $s_{1k} > s_{2k}$. Correspondingly, the profit for firm 1 is larger than for firm 2. ■

Proof of Proposition 2
When firms announce different attributes, the consumer’s utility when buying from firm $j$ is $u_{jAB} = (V - p_j) + \frac{1}{2}q_{jA} + \frac{1}{2}q_{jB} + \mu_{AB}\epsilon_{jAB}$. In the symmetric equilibrium both firms have equal market share, prices, $p_1 = p_2 = 2\mu_{AB}$ and profit, $\pi_{1AB} = \pi_{2AB} = \mu_{AB}$. Suppose, without loss of generality, firm 1 makes attribute $A$ prominent. Let $\pi_{1A}$ and $\pi_{2A}$ be firms’ profits when both make attribute $A$ the prominent attribute. From Lemma 2 we know that $\pi_{1A} > \pi_{2A}$ and that

$$\frac{\partial \pi_{2A}}{\partial \Delta} < 0 \text{ for } \Delta > 0.$$ 

We also know that $\lim_{\Delta \to 0} \pi_{2A} \to \mu_A > \mu_{AB}$ (from Proposition 1). Furthermore, $\lim_{\Delta \to \infty} s_{2k} = 0$ (from Lemma 1), so that $\lim_{\Delta \to \infty} \pi_{2A} \to 0$. Because $\pi_{2A}$ is strictly monotonically decreasing in $\Delta$, there exists a $\Delta > 0$ such that $\pi_{2A} > \pi_{2AB}$ whenever $\Delta < \Delta$. Under this condition, it is beneficial for firm 2 to make attribute $A$ prominent as well, rather than attribute $B$. But when $\Delta > \Delta$, we have $\pi_{2A} < \pi_{2AB}$ so that firm 2 will choose to make attribute $B$ prominent. Similarly, $\pi_{1AB} > \pi_{1B}$, which means firm 1 prefers to make attribute $A$ prominent when firm 2 makes $B$ prominent. Thus, firms make different attributes prominent in equilibrium whenever $\Delta > \Delta$. ■
Proof of Lemma 3

Suppose both firms make attribute $A$ prominent. We establish the result by signing the derivatives of firms’ profit functions with respect to $\mu$. From Lemma 1 we can derive the following derivatives:

$$\frac{\partial s_1}{\partial p_1} = - \frac{1}{\mu} e^{(q^H-p_1)/\mu} \frac{q^H-p_1}{\mu} + \frac{1}{\mu} \left( e^{q^H-p_1/\mu} \right)^2 \frac{q^H-p_1}{\mu^2} = -\frac{1}{\mu} s_1 (1 - s_1) = -\frac{1}{\mu} s_1 s_2;$$

$$\frac{\partial s_1}{\partial p_2} = \frac{1}{\mu} e^{(q^H-p_1)/\mu} \frac{q^H-p_1}{\mu^2} \frac{1}{\mu} s_1 s_2;$$

$$\frac{\partial s_1}{\partial \mu} = -\frac{1}{\mu^2} \left[ e^{(q^H-p_1)/\mu} \right] \frac{q^H-p_1}{\mu^2} \frac{1}{\mu} s_1 s_2 + \frac{1}{\mu^2} \left[ e^{q^H-p_1/\mu} \right] \frac{q^H-p_1}{\mu^2} \frac{1}{\mu} s_1 s_2;$$

$$= \frac{1}{\mu^2} s_1 s_2 \left[ (q^H - p_1) + q^L - p_2 \right] < 0;$$

$$\frac{\partial s_2}{\partial p_1} = \frac{1}{\mu} s_1 s_2; \quad \frac{\partial s_2}{\partial p_2} = -\frac{1}{\mu} s_1 s_2; \quad \text{and} \quad \frac{\partial s_2}{\partial \mu} = \frac{1}{\mu^2} s_1 s_2 \left[ (q^H - p_1) - (q^L - p_2) \right] > 0;$$

The firm 1’s profit is given by $\pi_1 = p_1 s_1$. Therefore,

$$\frac{d \pi_1}{d \mu} = \frac{\partial \pi_1}{\partial p_1} \frac{dp_1}{d \mu} + \frac{\partial \pi_1}{\partial p_2} \frac{dp_2}{d \mu} + \frac{\partial \pi_1}{\partial \mu};$$

We know $\frac{\partial \pi_1}{\partial p_2} = p_1 \frac{\partial s_1}{\partial p_2} = p_1 \frac{1}{\mu} s_1 s_2$, and similarly, $\frac{d \pi_1}{d \mu} = p_1 \frac{\partial s_1}{\partial \mu}$. In addition, we know the first order conditions for profit maximization in firms’ prices $p^*_1, p^*_2$, which simultaneously solve the following equations.

$$F_1(p_1, p_2; \mu) \equiv \frac{\partial \pi_1}{\partial p_1} = s_1 + p_1 \frac{\partial s_1}{\partial p_1} = s_1 - p_1 \frac{1}{\mu} s_1 s_2 = \mu - p_1 s_2 = 0, \quad (A1)$$
In this same setting, Anderson et al. (1992) showed that as quality increases, the equilibrium price increases as well so that \( p_1 > p_2 \). Furthermore, from the first order condition we have \( p_1 > p_2 \Leftrightarrow s_1 > s_2 \). Then \( f(\mu) \equiv (q^H_{1A} - p_1) - (q^L_{2A} - p_2) > 0 \)

We can assess the signs of \( \frac{\partial \pi_j}{\partial \mu} \), for \( j = 1,2 \), by taking the total derivatives:

\[
\frac{\partial F_1}{\partial p_1} \frac{dp_1}{d\mu} + \frac{\partial F_1}{\partial p_2} \frac{dp_2}{d\mu} + \frac{\partial F_1}{\partial \mu} = 0 \quad \text{and} \quad \frac{\partial F_2}{\partial p_1} \frac{dp_1}{d\mu} + \frac{\partial F_2}{\partial p_2} \frac{dp_2}{d\mu} + \frac{\partial F_2}{\partial \mu} = 0. 
\]

Rearranging we have

\[
\frac{dp_1}{d\mu} = \frac{-\frac{\partial F_2}{\partial p_1} \frac{\partial F_1}{\partial p_2} \frac{\partial F_2}{\partial \mu} - \frac{\partial F_1}{\partial p_1} \frac{\partial F_2}{\partial p_2} \frac{\partial F_2}{\partial \mu}}{\partial F_1 \frac{\partial F_1}{\partial p_2} - \frac{\partial F_2}{\partial p_2} \frac{\partial F_2}{\partial \mu}}, \quad \text{and} \quad \frac{dp_2}{d\mu} = \frac{\frac{\partial F_2}{\partial p_1} \frac{\partial F_1}{\partial p_2} \frac{\partial F_2}{\partial \mu} - \frac{\partial F_1}{\partial p_1} \frac{\partial F_2}{\partial p_2} \frac{\partial F_2}{\partial \mu}}{\partial F_1 \frac{\partial F_1}{\partial p_2} - \frac{\partial F_2}{\partial p_2} \frac{\partial F_2}{\partial \mu}}. 
\]

We know from (A1) that

\[
\frac{\partial F_1}{\partial p_1} = -s_2 - p_1 \frac{\partial s_2}{\partial p_1} = -s_2 - p_1 s_1 s_2 = -1, \\
\frac{\partial F_1}{\partial p_2} = -p_1 \frac{\partial s_2}{\partial p_2} = p_1 s_1 s_2 = s_1, \\
\frac{\partial F_1}{\partial \mu} = 1 - p_1 \frac{\partial s_1}{\partial \mu} = 1 + p_1 s_1 = 1 - \frac{1}{\mu} s_1 f(\mu). 
\]

Similarly, from (A2) we have

\[
\frac{\partial F_2}{\partial p_1} = s_2, \quad \frac{\partial F_2}{\partial p_2} = -1, \quad \text{and} \quad \frac{\partial F_2}{\partial \mu} = 1 - p_2 \frac{\partial s_1}{\partial \mu} = 1 + \frac{1}{\mu} s_2 f(\mu). 
\]

It can be verified that \( \frac{\partial F_1}{\partial p_1} \frac{\partial F_2}{\partial p_2} - \frac{\partial F_1}{\partial p_2} \frac{\partial F_2}{\partial p_1} = 1 - s_1 s_2 > 0 \), since \( s_j < 1/2, j = 1,2 \). Hence,
\[
\frac{dp_1}{\mu} = \frac{\partial F_2 \partial F_1 + \partial F_1 \partial F_2}{\partial \mu \partial \mu} \frac{\partial F_2}{\partial \mu} \frac{1}{\mu} - \frac{1}{\mu} s_1 f(\mu) + s_1 (1 + \frac{1}{\mu} s_2 f(\mu))
\]
\[
\frac{dp_2}{\mu} = \frac{\partial F_2 \partial F_1 + \partial F_1 \partial F_2}{\partial \mu \partial \mu} \frac{\partial F_2}{\partial \mu} \frac{1}{\mu} = \frac{s_2 (1 + \frac{1}{\mu} s_1 f(\mu) + (1 + \frac{1}{\mu} s_2 f(\mu))}{1-s_1 s_2}
\]

We first prove an intermediate result. We show that \( g(\mu) = \frac{f(\mu)}{\mu} \) is strictly decreasing in \( \mu \). To see this, we know that
\[
\frac{d(g(\mu))}{d\mu} = \frac{df(\mu)/d\mu}{\mu^2} - \frac{f(\mu)}{\mu^2} = \frac{dp_1}{d\mu} + \frac{dp_2}{d\mu} = -\frac{\partial F_1 + \partial F_2}{\partial \mu} \frac{1}{1-s_1 s_2}
\]
\[
\frac{s_2}{1-s_1 s_2} = \frac{-s_1 (1 - \frac{1}{\mu} s_1 f(\mu)) + s_2 (1 + \frac{1}{\mu} s_2 f(\mu))}{1-s_1 s_2}
\]
Therefore:
\[
\frac{d(f(\mu)/\mu)}{d\mu} = \frac{s_2 - s_1 + \frac{1}{\mu} f(\mu)(s_1^2 + s_2^2)}{1-s_1 s_2} \mu - f(\mu) = -\mu(s_1 - s_2) + f(\mu)(s_1^2 + s_2^2 - 1 + s_1 s_2) = -\mu(s_1 - s_2) + s_1 s_2 f(\mu) < 0
\]

Therefore we can see that \( g(\mu) = \frac{f(\mu)}{\mu} \) is strictly decreasing in \( \mu \), and for any given \( \mu \), there only exists a unique \( g(\mu) \).

We can thus write,
\[
\frac{d\pi_1}{d\mu} = \frac{\partial \pi_1}{\partial \mu} \frac{dp_2}{d\mu} + \frac{\partial \pi_1}{\partial \mu} + p_1 \frac{\partial s_1}{\partial \mu} \frac{\partial s_2}{\partial \mu} \frac{\partial F_2}{\partial \mu} \frac{1}{1-s_1 s_2} + p_1 \frac{\partial s_1}{\partial \mu} = p_1 \frac{s_1 s_2}{1-s_1 s_2} \frac{s_2 (1 - \frac{1}{\mu} s_1 f(\mu) + (1 + \frac{1}{\mu} s_2 f(\mu))}{1-s_1 s_2} +
\]
\[
p_1 \frac{\partial s_1}{\partial \mu}
\]
\[
= \frac{p_1}{1-s_1 s_2} \left[ 1 + s_2 - s_2 - s_2 \frac{1}{\mu} s_1 f(\mu) + \frac{1}{\mu} s_2 f(\mu) - \frac{1}{\mu^2} s_1 s_2 f(\mu) \right]
\]
\[
= \frac{p_1}{1-s_1 s_2} \left[ 1 + s_2 + \frac{1}{\mu} s_2 f(\mu) - \frac{1}{\mu} f(\mu)(1 - s_1 s_2) \right]
\]

42
\[ p_1 \frac{1-s_1 s_2}{1-s_1 s_2} \left[ 1 + s_2 - \frac{1}{\mu} s_1 f(\mu) \right] \]

We can see that \( s_1 = \frac{e^\mu}{1 + e^\mu} = \frac{e^{\theta(\mu)}}{1 + e^{\theta(\mu)}} \) and \( s_2 = \frac{1}{1 + e^{\theta(\mu)}} \). Next we show that there exists a unique value of \( \mu \) that makes the squared-bracketed expression zero.

For \( 1 + s_2 - \frac{1}{\mu} s_1 f(\mu) = 0 \iff \frac{1 + s_2}{s_1} = \frac{1}{\mu} f(\mu) \iff 1 + \frac{2}{e^{\theta(\mu)}} = g(\mu) \iff \frac{1}{e^{\theta(\mu)}} = \frac{1}{2} (g(\mu) - 1) \). It can be seen that the functions \( \frac{1}{e^{\theta(\mu)}} \) and \( \frac{1}{2} (g(\mu) - 1) \) always have an unique intersection at \( g(\mu) > 0 \). And since \( g(\mu) \) is strictly decreasing in \( \mu \). Therefore, we can define \( \overline{\mu}_\Delta \) as the unique value of \( \mu \) that makes the squared-bracketed expression zero. And we can see when \( \mu < \overline{\mu}_\Delta \),

\[ \frac{1}{e^{\theta(\mu)}} < \frac{1}{2} (g(\mu) - 1) , \text{ therefore } \frac{d\pi_1}{d\mu} < 0 ; \text{ otherwise when } \mu > \overline{\mu}_\Delta , \frac{1}{e^{\theta(\mu)}} > \frac{1}{2} (g(\mu) - 1) , \text{ therefore } \frac{d\pi_1}{d\mu} > 0 . \]

A similar derivation can show that \( d\pi_2 / d\mu > 0 \) for all \( \mu \). ■

**Proof of Proposition 3**

We first show when \( \mu > \overline{\mu}_\Delta \), both firm announce the same attribute in equilibrium. It follows from Lemma 3 that when \( \mu > \overline{\mu}_\Delta \), both firms’ profits are monotonically increasing in \( \mu \). Making different attributes prominent, however, strictly lowers \( \mu \) since \( \mu_{AB} = \frac{\sqrt{2}}{2} \mu \). Hence no firm has the incentive to deviate to a different attribute.

When \( \mu \leq \overline{\mu}_\Delta \), firm 1’s profit is monotonically decreasing in \( \mu \), while firm 2’s profit is increasing in \( \mu \). Therefore firm 1 prefers to have both attributes prominent, while firm 2 prefers to have the same prominent attribute. We can prove by contradiction that there does not exist a pure strategy equilibrium in this situation. Assuming firm 1 makes attribute \( A \) prominent in equilibrium, then the best strategy for firm 2 is to make attribute \( A \) prominent as well. Given firm 2 picks attribute \( A \), the best strategy for firm 1 is to make attribute \( B \) prominent. Therefore there
does not exist any pure strategy equilibrium. To characterize the mixed strategy equilibrium, we refer to the payoff matrix in Figure A1.

<table>
<thead>
<tr>
<th>Attribute Choice</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(\pi_{1A}, \pi_{2A})</td>
<td>(\pi_{1AB}, \pi_{2AB})</td>
</tr>
<tr>
<td>B</td>
<td>(\pi_{1AB}, \pi_{2AB})</td>
<td>(\pi_{1B}, \pi_{2B})</td>
</tr>
</tbody>
</table>

**Figure A1**: Payoff Matrix when \(\Delta = \Delta_A = \Delta_B > 0\)

We notice \(\pi_{1A} = \pi_{1B} > \pi_{2A} = \pi_{2B}\), and \(\pi_{1AB} > \pi_{1A}\) and \(\pi_{2AB} < \pi_{2A}\). In this case the mixed strategy for both firms is choosing each attribute with equal probabilities. □

**Proof of Lemma 4**

For (a), when both firms make attribute \(k\) prominent, each firm’s payoff when deciding on the investment is given by the 2 by 2 matrix of Figure A2.

<table>
<thead>
<tr>
<th>Invest on Quality</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>(\pi_{1k}^{HH} - c, \pi_{2k}^{HH} - c)</td>
<td>(\pi_{1k}^{HL} - c, \pi_{2k}^{HL})</td>
</tr>
<tr>
<td>N</td>
<td>(\pi_{1k}^{LK}, \pi_{2k}^{LK} - c)</td>
<td>(\pi_{1k}^{LL}, \pi_{2k}^{LL})</td>
</tr>
</tbody>
</table>

**Figure A2**: Payoff Matrix When Firms Make Attribute \(k\) Prominent

In this payoff matrix, \(\pi_{1k}^{HH}\) represents firm 1’s profit when the prominent attribute is \(k\) and firm 1 invests in quality (so the quality is \(q^H\)) and firm 2 does not invest (so the quality remains at \(q^L\)).

We can see that when both firms improve the quality, they have the same quality \(q^H\) and therefore share the market equally with payoff \(\pi_{1k}^{HH} = \pi_{2k}^{HH} = \mu\). Similarly we know that \(\pi_{1k}^{LL} = \pi_{2k}^{LL} = \mu\). We can also see \(\pi_{1k}^{LH} = \pi_{1k}^{HL} = \pi_k > \mu > \pi_{1k}^{HH} = \pi_{2k}^{HH} = \pi_k\) based on Lemma 2. Define
\( \bar{c} = \min\{|\bar{\pi}_k - \mu|; |\bar{\pi}_k - \mu|\} > 0 \). Then when \( c < \bar{c}, \mu - c > \bar{\pi}_k \), and \( \bar{\pi}_k - c > \mu \). Firm 1’s dominant strategy is to invest on quality regardless firm 2’s strategy, and vice versa for firm 2. Therefore, the equilibrium has both firms investing on quality.

The proof for (b) is quite similar to (a), when both firms make different attributes prominent, each firm’s payoff when deciding on the investment is given by the 2 by 2 matrix in Figure A3.

![Figure A3: Payoff Matrix When Firms Make Different Attributes Prominent](image-url)

We know the payoff \( \pi_{1AB}^{HH} = \pi_{2AB}^{HH} = \frac{\sqrt{2}}{2} \mu \). Similarly we know that \( \pi_{1AB}^{LL} = \pi_{2AB}^{LL} = \frac{\sqrt{2}}{2} \mu \). We can also see \( \pi_{2AB}^{LM} = \pi_{1AB}^{LM} = \bar{\pi}_{AB} > \frac{\sqrt{2}}{2} \mu > \pi_{1k}^{LM} = \pi_{2k}^{LM} = \pi_{AB} \) based on Lemma 2. Let’s define \( \underline{c} = \max\{|\bar{\pi}_{AB} - \frac{\sqrt{2}}{2} \mu|; |\bar{\pi}_{AB} - \frac{\sqrt{2}}{2} \mu|\} > 0 \). We can see that when \( c > \underline{c} \) firm 1’s dominant strategy is to not invest on quality, and vice versa for firm 2. The equilibrium has neither firm investing in quality. ■

**Proof of Lemma 5**

We prove this lemma by first showing that \( \bar{c} = \min\{|\bar{\pi}_k - \mu|; |\bar{\pi}_k - \mu|\} = \mu - \bar{\pi}_k \), and \( \underline{c} = \max\{|\bar{\pi}_{AB} - \frac{\sqrt{2}}{2} \mu|; |\bar{\pi}_{AB} - \frac{\sqrt{2}}{2} \mu|\} = (\bar{\pi}_{AB} - \frac{\sqrt{2}}{2} \mu) \). Intuitively it means the marginal increase in profit for the firm who improves the quality is larger than the corresponding decrease in profit for the low quality firm. Then we show there always exist a \( \Delta(\mu) > 0 \) such that when \( \Delta < \Delta(\mu) \) we have \( \underline{c} < \bar{c} \).
By assumption, when both firms do not invest, they have the same quality \( q^L \). So \( \Delta_k = 0 \), \( \forall k = A,B \). Now assume firm 1 invests to increase its product quality to \( q^H \) in attribute \( k \) so that \( \Delta \equiv \Delta_k > 0 \). We know that \( \pi_{1k}^H = \pi_k > \mu > \pi_{2k}^H = \pi_k \). What we want to show is \( \pi_{1k}^H - \mu > \mu - \pi_{2k}^H \), or equivalently \( \frac{d\pi_1}{d\Delta} + \frac{d\pi_2}{d\Delta} > 0 \). To show this, we again refer to the firm’s first order condition that:

\[
F_1(p_1, p_2; \Delta) \equiv \mu - p_1 s_2 = 0, \tag{A3}
\]

\[
F_2(p_2, p_1; \Delta) \equiv \mu - p_2 s_1 = 0. \tag{A4}
\]

Taking the total derivatives and rearranging we have:

\[
\frac{dp_1}{d\Delta} = \frac{-\partial F_2 \partial F_1}{\partial p_1 \partial p_2} + \frac{\partial F_1 \partial F_2}{\partial p_1 \partial p_2} \frac{dp_2}{d\Delta} = \frac{s_1^2}{1-s_1s_2}, \quad \text{and} \quad \frac{dp_2}{d\Delta} = \frac{\partial F_2 \partial F_1}{\partial p_1 \partial p_2} + \frac{\partial F_1 \partial F_2}{\partial p_1 \partial p_2} \frac{dp_1}{d\Delta} = \frac{-s_2^2}{1-s_1s_2}.
\]

We know from (A3) and (A4) that

\[
\frac{\partial F_1}{\partial p_1} = -1, \quad \frac{\partial F_1}{\partial p_2} = s_1, \quad \text{and} \quad \frac{\partial F_1}{\partial \Delta} = -p_1 \frac{\partial s_2}{\partial \Delta} = p_1 \frac{1}{\mu} s_1 s_2 = s_1;
\]

\[
\frac{\partial F_2}{\partial p_1} = s_2, \quad \frac{\partial F_2}{\partial p_2} = -1, \quad \text{and} \quad \frac{\partial F_2}{\partial \Delta} = -s_2.
\]

Therefore,

\[
\frac{dp_1}{d\Delta} = \frac{-\partial F_2 \partial F_1}{\partial p_1 \partial p_2} + \frac{\partial F_1 \partial F_2}{\partial p_1 \partial p_2} \frac{dp_2}{d\Delta} = \frac{s_1^2}{1-s_1s_2}, \quad \text{and} \quad \frac{dp_2}{d\Delta} = \frac{\partial F_2 \partial F_1}{\partial p_1 \partial p_2} + \frac{\partial F_1 \partial F_2}{\partial p_1 \partial p_2} \frac{dp_1}{d\Delta} = \frac{-s_2^2}{1-s_1s_2}.
\]

We know that

\[
\frac{d\pi_1}{d\Delta} = \frac{\partial \pi_1}{\partial p_2} \frac{dp_2}{d\Delta} + \frac{\partial \pi_1}{\partial p_1} \frac{dp_1}{d\Delta} = p_1 \frac{ds_2}{d\Delta} \frac{dp_2}{d\Delta} + p_1 \frac{\partial s_2}{\partial \Delta} = p_1 \frac{1}{\mu} s_1 s_2 \left( \frac{-s_2^2}{1-s_1s_2} + 1 \right) = p_1 \frac{1}{\mu} s_1 s_2 \frac{s_1}{1-s_1s_2} = \frac{s_1^2}{1-s_1s_2} > 0;
\]

\[
\frac{d\pi_2}{d\Delta} = \frac{\partial \pi_2}{\partial p_1} \frac{dp_1}{d\Delta} + \frac{\partial \pi_2}{\partial p_2} \frac{dp_2}{d\Delta} = p_2 \frac{ds_1}{d\Delta} \frac{dp_1}{d\Delta} + p_2 \frac{\partial s_2}{\partial \Delta} = p_2 \frac{1}{\mu} s_1 s_2 \left( \frac{s_1^2}{1-s_1s_2} - 1 \right) = \frac{-s_2^2}{1-s_1s_2} < 0;
\]
Since $p_1 > p_2$ when $\Delta > 0$, we can see that $\frac{d\pi_1}{d\Delta} + \frac{d\pi_2}{d\Delta} > 0$. Therefore we know that $\pi_{1k} - \mu > \mu - \pi_{2k}$, which gives us $\bar{c} = \mu - \pi_k$. The proof for $c = (\bar{\pi}_{AB} - \frac{\sqrt{2}}{2}\mu)$ is very similar and therefore omitted.

Next we show that there always exists a $\hat{\Delta}(\mu) > 0$ for any given $\mu$ such that when $\Delta < \hat{\Delta}(\mu)$, we have $c < \bar{c}$. $c < \bar{c} \iff \bar{\pi}_{AB} - \frac{\sqrt{2}}{2}\mu < \mu - \bar{\pi}_k \iff \bar{\pi}_{AB} + \bar{\pi}_k < \mu + \frac{\sqrt{2}}{2}\mu$. We can see when $\Delta = 0$, $\bar{\pi}_{AB} + \bar{\pi}_k = \mu + \frac{\sqrt{2}}{2}\mu$. Next we check the sign of $\frac{d(\bar{\pi}_{AB} + \bar{\pi}_k)}{d\Delta}$. We know that $\frac{d\pi_k}{d\Delta} = \frac{-s^2_{2k}}{1-s_1ks^2_{2k}}$, and $\frac{d\pi_{AB}}{d\Delta} = \frac{s^2_{1AB}}{1-s_{1AB}s^2_{2AB}}$. However, when both attributes are prominent, the quality advantage is diluted and the variance becomes small. To see this, consider the following example: if firm 1 invests on quality in attribute A, and firm 2 does not match the investment. The consumer’s utility of buying from firm 1 is $u_{1AB} = V - p_1 + \frac{1}{2}q^H + \frac{1}{2}q^L + \frac{\sqrt{2}}{2}\mu \max\{\epsilon_{i1AB}\}$, and from firm 2 is $u_{2AB} = V - p_1 + q^L + \frac{\sqrt{2}}{2}\mu \max\{\epsilon_{i2AB}\}$. The overall vertical difference due to the quality different becomes $\frac{1}{2}\Delta$, which is lower than the difference $\Delta$ when firms make the same attribute prominent. Because of that, $\frac{d(\bar{\pi}_{AB} + \bar{\pi}_k)}{d\Delta} = \frac{d\pi_k}{d\Delta} + \frac{1}{2} \frac{d\pi_{AB}}{d\Delta} = \frac{-s^2_{2k}}{1-s_1ks^2_{2k}} + \frac{1}{2} \frac{s^2_{1AB}}{1-s_{1AB}s^2_{2AB}}$. Therefore, $\frac{d(\bar{\pi}_{AB} + \bar{\pi}_k)}{d\Delta} \bigg|_{\Delta \to 0} = \frac{-s^2_{2k}}{1-s_1ks^2_{2k}} + \frac{1}{2} \frac{s^2_{1AB}}{1-s_{1AB}s^2_{2AB}} \bigg|_{\Delta \to 0} = -\frac{1}{6}$ (since $s_{2k}$ and $s_{1AB}$ are equal to $\frac{1}{2}$ when $\Delta \to 0$). Hence we show that $\bar{\pi}_{AB} + \bar{\pi}_k < \mu + \frac{\sqrt{2}}{2}\mu$ when $\Delta \to 0$. Because of the continuity of profit functions, there must exist a threshold $\hat{\Delta}(\mu) > 0$ such that when $\Delta < \hat{\Delta}(\mu)$, we have $c < \bar{c}$. ■

**Proof of Proposition 4**

When $\Delta < \hat{\Delta}(\mu)$, from Lemma 5 we know that $c < \bar{c}$, which leads to the payoff for firms to be $\frac{\sqrt{2}}{2}\mu$ if making both attributes prominent, and $\mu - c$ if making a single attribute $k$ prominent.

Hence when $c > \mu - \frac{\sqrt{2}}{2}\mu$ we have $\frac{\sqrt{2}}{2}\mu > \mu - c$, which gives us the required conditions for firms
to make different attributes prominent in equilibrium. Figure A4 graphically illustrates $c$, $\bar{c}$ and $(1 - \sqrt{2}/2)\mu$ by simulation.

Figure A4: Illustration of $c$, $\bar{c}$ and $(1 - \sqrt{2}/2)\mu$ when $\mu = 10$ and $\Delta \in [0,35]$
Proof of Lemma 6

Part (a) is proved by examining the payoff matrix, which is given in Figure A5.

<table>
<thead>
<tr>
<th>Invest on Quality</th>
<th>Y</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>$\mu - c, \mu - c$</td>
<td>$\bar{\pi}_k - c, \pi_k$</td>
</tr>
<tr>
<td>N</td>
<td>$\bar{\pi}_k, \pi_k - c, \mu, \mu$</td>
<td></td>
</tr>
</tbody>
</table>

**Figure A5**: Payoff Matrix When Firms Make Attribute $k$ Prominent

Define $\hat{c} \equiv \max\{|\bar{\pi}_k - \mu|, |\pi_k - \mu|\}$. Recall that in Lemma 5 we proved $\bar{c} = \min\{|\bar{\pi}_k - \mu|, |\pi_k - \mu|\} = \mu - \bar{\pi}_k$. Therefore, $\hat{c} = \bar{\pi}_k - \mu$. Thus, for $c > \hat{c}$, $\mu - c < \bar{\pi}_k$ and $\bar{\pi}_k - c < \mu$.

So not investing is a dominant strategy in this game when $c > \hat{c}$. To prove part (b), consider the full model in which both firms choose among the two attributes in which to invest and to make prominent. There are at most four types of outcomes:

1. Both firms invest in the same attribute;
2. Firms invest in different attributes;
3. Only one firm invests in an attribute; and
4. No firm invest in an attribute.

Cases (1), (2) and (4) are straight forward from the earlier results in Section 3. For (1) and (4), based on the Proposition 1 ($\Delta = 0$), we know both firms prefer to announce the same attribute. The payoff for each firm is $\mu - c$. For (2), we can apply the intuition from Proposition 2. If $\Delta < \bar{\Delta}$, both firms make the same attribute as prominent despite one firm having a quality disadvantage. When $\Delta \geq \bar{\Delta}$, however, firms announce different attributes and earn payoffs of $\sqrt{2} \mu - c$. Therefore, when $\Delta \geq \bar{\Delta}$, either firm would be better off investing in the same attribute as
her rival. So we can eliminate case (2) since it would never be part of any equilibrium of this game.

Now consider case (3) by assuming that firm 1 invests in attribute $A$ and 2 invests in no attributes. When both firms make attribute $A$ prominent, denote by $\pi_A$ the profits to firm 1 and $\pi_A$ ($\pi_A > \pi_A$) the profits to firm 2. Similarly, denote the profits (net of investment) when firms choose different attributes as $\pi_{AB} > \pi_{AB}$. Firms earn $\mu$ (net of investment) when both announce $B$. The 2x2 payoff matrix in Figure A6 depicts the subgame of case (3). Immediately we have that $\pi_A > \pi_{AB}$. And, from Proposition 2, we argue that, as long as the quality advantage is sufficiently large, $\Delta > \bar{\Delta}$, then firm 1 prefers to emphasize its higher quality attribute $A$ despite the dilution effect: $\pi_{AB} > \mu$. Hence, choosing attribute $A$ is a dominant strategy when $\Delta > \bar{\Delta}$.

Consider firm 2’s decision on which attribute to choose given that $A$ is the dominant strategy for firm 1. Define $\bar{\Delta}$ as the level of quality such that $\pi_A - \pi_{AB} = 0$ so that firm 2 is indifferent between announcing $A$ and $B$, respectively. The difference $\pi_A - \pi_{AB}$ is strictly decreasing in $\Delta$ so that if $\Delta > \bar{\Delta}$, then avoiding the dilution effect, by announcing $B$, is not worth the severe quality disadvantage suffered when announcing $A$. We have established that, for $\Delta > \bar{\Delta}$, for some $\bar{\Delta} > 0$, $A-B$ is the equilibrium of the subgame of case (3).

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attribute Choice</td>
<td>$A$</td>
</tr>
<tr>
<td>$A$</td>
<td>$\pi_A - c, \pi_A$</td>
</tr>
<tr>
<td>$B$</td>
<td>$\pi_{AB} - c, \pi_{AB}$</td>
</tr>
</tbody>
</table>

**Figure A6**: Subgame Payoff Matrix for Case (3): $\Delta_A = \Delta > 0$ and $\Delta_B = 0$
We now analyze the stage 1 investment game given the corresponding equilibria in subgames (1), (3), and (4), under the condition that $\Delta > \bar{\Delta}(\mu) \equiv \max\{\bar{\Delta}, \bar{\Delta}\}$. Figure A7 depicts the payoff matrix. First, if $c > \mu - \pi_{AB}$, then investing is also not optimal when the rival does not invest. Second, if $c > \bar{\pi}_{AB} - \mu$, then investing is not optimal when the rival invests. Define $\hat{c} \equiv \max\{\mu - \pi_{AB}, \bar{\pi}_{AB} - \mu\}$. Then investing in quality is a dominated strategy for $c > \hat{c}$.

<table>
<thead>
<tr>
<th>Firm 1 Invest on Quality</th>
<th>Firm 2 Y</th>
<th>firm 2 N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>$(1): \mu - c, \mu - c$</td>
<td>$(3): \bar{\pi}<em>{AB} - c, \pi</em>{AB}$</td>
</tr>
<tr>
<td>N</td>
<td>$(3): \pi_{AB}, \bar{\pi}_{AB} - c$</td>
<td>$(4): \mu, \mu$</td>
</tr>
</tbody>
</table>

**Figure A7**: Payoff Matrix for Stage 1 Investment Decision with $\Delta > \max\{\bar{\Delta}, \bar{\Delta}\}$

**Proof of Proposition 5**

We establish that $\hat{c} < \check{c}$ by showing $\check{c} = \bar{\pi}_k - \mu > \max\{\mu - \pi_{AB}, \bar{\pi}_{AB} - \mu\} = \check{c}$. First suppose $\check{c} = \bar{\pi}_{AB} - \mu$. Since $\bar{\pi}_k > \bar{\pi}_{AB}$, for $k = A, B$, we immediately have $\check{c} = \bar{\pi}_k - \mu > \bar{\pi}_{AB} - \mu$. Now suppose $\check{c} = \mu - \pi_{AB}$. From Lemma 6 we know that $\bar{\pi}_k - \mu > \mu - \pi_k$ and that under the condition $\Delta > \bar{\Delta}(\mu), \pi_{AB} > \pi_k$. Therefore, we know $\check{c} = \bar{\pi}_k - \mu > \mu - \pi_k > \mu - \pi_{AB}$. Hence $\check{c} < \hat{c}$. The claims in (a) and (b) follow directly from Lemma 6. ■