Abstract

The online activity of consumers is routinely tracked, not just by visited websites, but also by an increasing number of third-parties who embed their tracking cookies within various websites. This tracked information is sold to advertisers who then use it to identify and target potential new consumers with their ads. In this paper, we build a game theoretic model to show that the presence of such third-party information can not only reduce advertiser profits, but also create a dead-weight loss under certain conditions.

The outcomes depend on two key parameters in our model - the degree of horizontal differentiation among competing advertisers, and the incremental utility a consumer gets from seeing an ad. We find that only when the degree of horizontal differentiation is low or when ads do not increase consumer utility by much, do advertisers benefit from third-party information. In all other cases, advertiser profits are lower than they would have been without third-party information. We describe two mechanisms which cause a reduction in advertiser profits - increased cost of advertising, and ‘irrelevant’ advertising. Thus, we find that more consumer information can, under certain conditions, cause less effective targeting. We also show that such ‘irrelevant’ ads reduce consumer surplus, since consumers miss out on seeing more relevant ads that could have increased their utility. This leads to a dead-weight loss due to third-party tracking. In an extension, we model the externality caused by disabling third-party tracking by some consumers.

Keywords: game theory, targeted advertising, consumer tracking, irrelevant advertising
1 Introduction

The tracking of online browsing behavior of consumers has become ubiquitous in recent years. The number of web tracking cookies embedded in the top hundred websites on the Internet has almost doubled in the last three years, growing from 3,152 in 2012 to 6,280 in 2015 (Altaweel et al. 2016). Web tracking can be of two types - (i) first-party tracking, done by the visited website, and (ii) third-party tracking, done by entities other than the visited website (Mayer & Mitchell, 2012). The proportion of tracking cookies that were from third-parties grew from 85% in 2012 to 94% in 2015 (Altaweel et al. 2016). Such third-party trackers are virtually invisible to consumers.

Third-party tracking agencies collect third-party information (TPI) of consumers by embedding their web-tracking technology within many different websites across the Internet. When a consumer visits one of these websites for the first time, a unique alphanumeric tag is stored in their browser as a tracking cookie by the embedded web tracker. Note that this tracking cookie is specific to the third-party, so an individual may have different unique tracking cookies in their browser from different third-parties. This cookie allows the third-party to track the consumer’s activity on all websites in which they have embedded their tracking technology. This collected information is combined to create a detailed behavioral profile for each consumer, which is then sold to advertisers (Krishnamurthy et al., 2011; Reisman et al., 2014). For example, Figure 1 depicts the number of different trackers embedded in a popular publisher website. Beyond websites, several other entities such as Internet Service Providers (ISPs), social media sites such as Facebook and Twitter, and most mobile Apps routinely collect and share consumer activity data with advertisers. In this paper, we analyze the impact of such third-party tracking data on the various stakeholders in the advertising ecosystem.

Advertisers have long sought to improve the accuracy of their ad targeting. Targeting is believed to reduce wasteful advertising expenditure while reaching more potential consumers (Iyer et al. 2005) and thus improve return-on-investment (ROI). Advertisers usually track the behavior of visitors on their own websites and collect first-party information (FPI) about them. This information is useful in predicting consumer value (Aziz & Telang 2016) and to subsequently target them with relevant ads, a practice commonly referred to in the industry as retargeted advertising. However, for targeting potential consumers who have not yet visited them, firms must rely on third-party trackers.

Such tracking of consumers and their behavioral profiling has received much attention lately
from policy makers. More specifically, there has been a lot of debate about the role of third-party information resellers and their impact on advertisers and consumers. Recent legislation that repealed Federal Communication Commission’s Open Internet Order which had required Internet Service Providers (ISPs) to inform consumers and get their consent about their data collection practices highlighted the arguments on both sides of the debate (Kindy 2017). Proponents of the bill argued that disallowing information sharing “takes away consumer choice” and preclude firms from offering “innovative and cost-saving product offerings” (Flake 2017), while consumer advocates point to evidence that a substantial proportion of U.S. households have concerns about their online security and privacy and this limits their online activity (Goldberg 2016).

In this debate, it is often assumed that more consumer tracking will lead to better targeting accuracy and higher profits for advertisers. However, in this paper, we show that this is not necessarily the case. In fact, under certain conditions, third-party information can actually make targeted advertising less effective by enabling irrelevant advertising. That is, with third-party tracking, some consumers may be targeted in equilibrium by advertisers they are not interested in, which imposes an opportunity cost on them by preventing ads from more relevant advertisers from reaching these consumers. We also find that advertiser profits often reduce with third-party information and that this occurs due to two mechanisms - increasing cost of advertising, as well as irrelevant advertising, which is ineffective. These outcomes depend on the degree of horizontal differentiation between competing advertisers, and how much ads increase consumer utility.
As an extension of our model, in Section 7, we allow some consumers to disable third-party tracking which affects the accuracy of third-party information. When fewer consumers allow tracking, overall accuracy of third-party signal declines, which may impose an externality on other consumers. Hence actions of a few consumers can have large consequences. This is important because disabling tracking, for example, may be technically challenging for some consumers. The externality suggests that even these consumers can benefit from actions by other consumers.

2 Literature Review

Targeted advertising has received considerable attention by researchers and there is a significant literature providing insights about the trade-offs and outcomes in various settings. The literature on the role of information and targeting accuracy has found that targeting generally increases advertiser profits (Iyer et al., 2005; Chen et al., 2001; Johnson, 2013; Marotta et al. 2016) by endogenously increasing differentiation in the market and softening price competition. In our model, we show that this is a special case that occurs when ads have a small or no effect on a consumer’s utility for the advertiser. But if ads have a large enough positive effect on the ad-viewing consumer’s utility, we find that the opposite result holds true. That is, increasing consumer information can increase advertising competition to such an extent, that advertiser profits reduce. We identify two distinct mechanisms by which this reduction in advertiser profits occurs - when third-party information raises the cost of advertising to one’s own visitors, and when the ads served are ‘irrelevant’ in equilibrium.

Bergemann & Bonatti (2011) find that increase in targeting increases the social value of advertising by increasing product-consumer matches. We show that while this is possible, sometimes the reverse may happen, that is, increase in targeting may result in a dead-weight loss. Marotta et al. (2016) explore the welfare effect of advertising when advertisers know a consumer’s relative advertiser preference or relative willingness to pay or both. Yet it is not clear how these two types of information can be distinguished by advertisers or regulated by policymakers. Instead, we classify consumer information by its source - first or third-party, which leads to clear managerial and policy recommendations. Bergemann & Bonatti (2015) develop a model for pricing third-party cookies. Our work addresses an acknowledged limitation of their work by focusing on how access to third-party information affects advertising competition, and how surplus is allocated between the advertisers and consumers. Chen & Stallaert (2014) find that behavioral targeting can increase
publisher revenue but it is not guaranteed, while we find that publisher revenue always increases in the presence of third-party information. Our result explains why publishers have been eager to embrace third-party tracking and why most top websites allow dozens or hundreds of third-party trackers on their website.

The second stream of relevant literature is on customer poaching. Our work is closely related to Esteves & Resende (2016) who describe when advertisers should target consumers who have a strong preference for them compared to their rivals and when they should target their weak segment in the context of location-based targeting. In their setting, the marginal cost of advertising is exogenously set and the resulting equilibrium outcomes depend on this cost. In our setting of display advertising through a second price auction, we completely endogenize the cost of advertising and treat it as an outcome of the equilibrium, instead of an exogeneous parameter. Sayedi et al. (2014) describe customer poaching in sponsored search advertising and characterize the poaching firm’s actions as an attempt to free ride on their rival’s investments in traditional advertising channels. Instead of the free riding mechanism described by them, we describe conditions in which advertisers find it more attractive to poach their rival’s consumers than to persuade their own consumers through ads, which is similar to Shin et al. (2012), who describe how it may be optimal for firms to “fire” some of their high-cost customers.

In the literature on consumer privacy and ad avoidance, Johnson (2013) finds that when consumers have access to ad-avoidance tools, improved targeting generally benefits firms, but does not necessarily benefit consumers. Other studies have found that negative externalities can result when consumers employ ad-avoidance technologies, by either reducing content quality (Anderson & Gans, 2011) or shifting marketing solicitations to other consumers (Hann, 2008). In contrast, we find that choosing to disable third-party tracking can create an externality that is often positive, but maybe negative when irrelevant advertising occurs. D’Annunzio & Russo (2017) describe the effect of consumer’s deciding to block third-party tracking when publisher’s compete, and find too little tracking in equilibrium. Their model differs from ours since they endogenously model the proportion of consumers who block third-party tracking and explicitly include a privacy cost associated with being tracked. In our model, we assume an exogenous proportion of consumers allow third-party tracking and focus on their effect of targeting accuracy.
3 Institutional Details

3.1 Types of Websites

In this paper, we distinguish between two kinds of websites - retailers’ websites and publishers’ websites (Budak et al., 2016). Retailers engage in commercial transactions on their websites with e-commerce and travel websites being common examples of retailers’ websites. On the other hand, publisher’s websites provide content or services to consumers either for free or for a subscription fee. News websites, blogs, social media etc. are examples of publisher’s websites. Often, the dominant revenue stream for publisher websites is advertising. Within their webpages, publishers sell designated ad slots to various buyers. In this paper, retailers are referred to as advertisers and they wish to advertise on publishers’ websites.

3.2 Advertising Allocation

Publishers sell a majority (78 percent) of their ad inventory through what is called ‘programmatic’ advertising, where ad slots are bought and sold through automated algorithms\(^1\). Within programmatic advertising, there are two common mechanisms of allocating ads to advertisers - Real-Time Bidding through open exchanges, and Programmatic Direct.

**Real-Time Bidding:** About 46 percent of programmatic ads were sold through real-time bidding on open exchanges in 2017 (eMarketer, 2017). In this process, publishers sell their ad inventory through real-time auctions on intermediaries called ad exchanges. Ad exchanges are platforms that facilitate the buying and selling of ad slots through second price auctions. Whenever a consumer visits a publisher’s webpage, the ad exchange contracted by the publisher to sell its ad slots extracts the list of cookies stored in the visitor’s browser and matches them with the list of advertisers enrolled with them. If an advertiser has set a cookie on the visitor’s browser, the ad exchange sends that advertiser a request for a bid along with the consumer’s unique ID stored by that advertising in their tracking cookie.

All competing advertisers submit their bids to the ad exchange based on their valuation of the consumer. The advertiser with the highest bid wins the auction and pays the second highest bid as their cost of advertising. For simplicity, we assume in our model that each ad impression displayed is paid for by the advertiser, called cost-per-mille (CPM). Other commonly used mechanisms such as cost-per-click (CPC) or cost-per-action (CPA) do not change the nature of our results since in

\(^1\)https://www.emarketer.com/Article/eMarketer-Releases-New-Programmatic-Advertising-Estimates/1015682
our model, advertisers maximize their profits through purchases induced by advertising.

**Programmatic Direct:** A second and growing mechanism of advertising allocation is Programmatic Direct. In 2017, about 54 percent of programmatic ads were sold through Programmatic Direct (eMarketer, 2017). In this mechanism, publishers negotiate the price of ad slots and the number of impressions directly with selected advertisers. This allows both publishers and advertisers greater control over who advertises where. Recent cases of ads of major brands being displayed on extremist content on YouTube highlights the concern about advertising through real-time bidding on open exchanges\(^2\). Without direct human supervision, programmatic ads may be displayed on sites that are objectionable to the advertiser. This explains the growing popularity of Programmatic Direct over Real-Time Bidding auction mechanism.

We build our model assuming ad slots are allocated through the Real-Time Bidding mechanism. However, our results are applicable, perhaps even more so, in the case of Programmatic Direct ads. Both types of advertising mechanisms involve advertisers estimating the value of advertising to consumers given the information they possess about the consumer. In the real-time bidding scenario, highest bidding advertiser wins the auction and pays the bid of the second highest bidder. In the Programmatic Direct mechanism, publishers negotiate the price of ads with individual advertisers. They would likely enter into contracts with advertisers who are willing to pay the highest amount per ad, and be willing to accept any price higher than the second highest offered price for the same ad inventory. With competition among both publishers and advertisers, we make the assumption that in equilibrium, advertisers pay the second highest offer price for a particular ad inventory even in Programmatic Direct.

A key assumption in our model is that ad slots are rival goods. If one advertiser advertises in a particular ad slot, the competing advertiser cannot advertise in the same ad slot. The Programmatic Direct mechanism strengthens the likelihood of this assumption being valid. If one of two competing advertisers values a publisher’s ad inventory more than the other, it is likely that through Programmatic Direct, only the higher valuation advertiser advertises on the publisher’s site. In Real-Time Bidding, the rival goods assumption is equivalent to assuming that an advertiser’s value of advertising to a consumer is fixed between the consumer’s visits to retailer websites. That is, if one retailer values a consumer more than another, they will win all the ad slots where the two competing retailers bid to advertise on.

\(^2\)https://techcrunch.com/2017/03/21/after-youtube-boycott-google-pulls-ads-from-more-types-of-offensive-content/
3.3 Informational Scenarios

A consumer’s type is characterized by their valuations for both advertisers. An advertiser can learn about a consumer’s valuation for them if the consumer visits them. For non-visitors, advertisers can acquire information about their type from third-parties.

**First-party Information:** First-party information (FPI) is generated whenever a consumer visits an advertiser. By observing consumer behavior on their website, such as time spent, number of pages visited, past transactions, shopping cart contents etc., advertisers can infer whether the consumer has a High ($V_H$) or Low ($V_L$) valuation for them (Aziz & Telang, 2016). Advertisers cannot infer a visiting consumer’s preference for their rival advertiser. This is equivalent to assuming that advertisers cannot distinguish between consumers who visit them exclusively and consumers who also visit their rival. For non-visitors, no first-party information is generated.

**Third-party Information:** Third-party web trackers embed their cookies on various publisher websites which allow them to follow a consumer across the Internet and create an aggregated profile of each individual consumer’s preferences based on their browsing behavior. Thus, the third-party agency can classify consumers as having either High or Low interest in products sold by the two rival advertisers. This third-party information based signal, can be acquired by advertisers who can use it to target ads to non-visitors. Note that the third-party signal is valuable only for non-visitors since advertisers already have first-party information of visitors. We assume third-party information does not provide any additional information over first-party information for a particular consumer.

The third-party signal is assumed to be about a specific product category, but is advertiser-agnostic. For example, third-party information can inform an advertiser if a particular consumer is in the market for sports shoes, but not which brand (advertiser) the consumer has a preference for. In terms of our consumer segment, consumers who have a High valuation for at least one advertiser will be classified by the third-party as being a High type, for the given product category, while consumers who do not have a High valuation for either advertiser will be classified as being Low type consumers. This implies that non-consumers will also be classified as Low type consumers.

4 Model

We model advertising competition between two symmetric advertisers for a segment of consumers who have visited at least one of the two advertiser’s websites. Advertisers compete to serve their
ad in a given ad slot on a publisher’s website that is visited by a potential consumer.

4.1 Timeline

Consumers are assumed to have prior beliefs about their expected valuation for each advertiser, and this determines which advertiser(s) they visit. By visiting an advertiser, consumers learn about the product assortment available from the advertiser and realize their exact valuation for the best-fit product from the advertiser. In turn, the visited advertiser(s) learn about the consumer’s valuation for them based on the consumer’s behavior on their website. Next, when consumers visit publishers’ websites, they are targeted by ads from the advertisers. The advertisers compete in a second price auction to serve their ads. Finally, consumers decide to purchase from either of the two advertisers or choose an outside option. The timeline is depicted in Figure 2.

![Figure 2: Timeline](image)

4.2 Consumer Choice

4.2.1 Consumer Segment

We assume all Internet consumers are divided into two categories - consumers, who have visited at least one of the two advertiser’s websites, and non-consumers, who have not visited either advertiser under consideration. Some of these non-visitors may be interested in the advertisers, but they do not play a role in determining advertising outcomes. This is because such non-visitors can only be targeted with third-party information, in which case each advertiser has the same information about them. This ensures that their bids are equal, which implies that they do not generate any incremental advertiser profits. We normalize the size of the consumer segment to one, while the size of the non-consumer segment is assumed to be very large. Advertising is assumed to have no effect on non-consumers.
4.2.2 Consumer Utility

We assume consumer utility follows the random utility model. For consumer \( i \), advertiser \( j \) and product \( l \), utility is given by

\[
U_{ijl} = U_{ij} + \delta_{ij} \theta + \epsilon_{ijl}
\]

where \( U_{ij} \) is the expected utility for advertiser \( j \) by consumer \( i \); \( \delta_{ij} \) is the ad indicator variable that is 1 if consumer \( i \) was served an ad by advertiser \( j \), and 0 otherwise; \( \theta \) is the incremental utility for a consumer when they see an advertiser’s ad; and \( \epsilon_{ijl} \) is the random utility term for product \( l \) from advertiser \( j \) for consumer \( i \).

The first component of consumer utility \( (U_{ij}) \) is consumer \( i \)’s a priori belief about their expected valuation of advertiser \( j \). We assume this can be - High, \( V_H \) or Low, \( V_L \) for advertiser \( j \in \{A, B\} \); and is 0 for the outside option. We denote the ratio of valuations as \( k = \frac{V_H}{V_L} \), and it represents the degree of horizontal differentiation of the two advertisers. We assume the proportion of consumers who have a Low valuation for a given advertiser is \( \gamma \), \( 0 < \gamma < 1 \).

The second component of consumer utility is the incremental utility of an ad and is captured in our model by adding a term \( \theta \) in consumer utility. \( \delta_{ij} \) is an ad indicator variable which indicates whether consumer \( i \) was served an ad by advertiser \( j \).

\( k \) and \( \theta \) are key parameters of our model that determine advertising bids and outcomes, and we discuss them in further detail in Section 4.3.4.

\( \epsilon_{ijl} \) is the idiosyncratic component of consumer utility. We assume that the actual realization of \( \epsilon_{ijl} \) occurs only when the consumer either visits the advertiser’s website or see their ads. Advertisers remain unaware of the realized value of this term. We assume it is common knowledge that \( \epsilon_{ijl} \) is i.i.d. Extreme Value Type 1 distributed.

4.2.3 Website Visiting Decision

We assume consumers who prefer one advertiser over the other, that is, have a High valuation for one advertiser and a Low valuation for the other advertiser, visit only their preferred advertiser. Consumers who have equal valuations for both advertisers are assumed to visit both. This website visiting behavior can be derived from first principles by comparing the cost of visiting an advertiser’s website and the expected utility gain from doing so. We derive this behavior in detail in Appendix A. Other logically consistent website visiting behaviors are possible as well, and we show in Appendix A that our results do not change when they occur. Figure 3 depicts the consumer segment and the
website visiting behavior.

![Image](image.png)

Figure 3: Consumer Segment, shown with $\gamma = 0.4$. Each region is composed of consumers with the valuation pair $(U_A, U_B)$.

### 4.2.4 Consumer Purchase Decision

Consumers include an advertiser in their consideration set only if they have either visited that advertiser’s website or have viewed ads from the advertiser. Either of these events allow the consumer to discover their random utility term for the best-fit product from the advertiser. Viewing an ad can help consumers realize their $\epsilon_{ijl}$ by either displaying the best-fit product in the ad, or by the consumer clicking on the ad and searching for products on their website. The nature of our results would not change if, instead, we assumed consumers have a probability $p < 1$ of clicking on the ad and discovering the realized value of $\epsilon_{ijl}$.

Consumers are assumed to be rational utility maximizing agents. They choose the option with the maximum utility from among the options in their consideration set.

From the advertiser’s perspective, we denote the component of consumer utility observed by them as $\bar{U}_j = U_j + \delta_j \theta$. If both advertisers are included in a consumer’s consideration set, the purchase probability from advertiser $j$ (recall consumers have an outside option of non-purchase with utility zero) is given as:

$$\Pr(j|\bar{U}_j, \bar{U}_{-j}) = \frac{e^{\bar{U}_j}}{1 + e^{\bar{U}_j + e^{\bar{U}_{-j}}}}$$

(2)
where $\bar{U}_{-j}$ is the utility for the other advertiser.

If consumers have only advertiser $j$ in their consideration set, then their purchase probability from that advertiser, as estimated by the advertiser, is given as:

$$
Pr(j|\bar{U}_j) = \frac{e^{\bar{U}_j}}{1 + e^{\bar{U}_j}}
$$

(3)

4.3 Value of Advertising

4.3.1 For Advertisers

In our model, advertising creates value for advertisers in two ways:

Increasing Consumer Utility: First, advertising can increase a consumer’s utility for the advertiser. This is modeled by the additive term $\theta$ in the consumer utility in Equation (1). This term is essential in explaining the phenomenon of retargeted advertising. If advertising does not change a consumer’s utility, there is little reason for an advertiser to advertise to consumers who have already visited their website, as in Equations (2) and (3). Another possible mechanism for retargeted advertising to be effective would be if such ads create a ‘reminder’ for the consumer. This effect is included in our model if we consider $\theta$ as a transient positive utility shock that decays over time. Thus, when a consumer sees an ad by an advertiser they have already visited, they have a temporary higher utility created by the ad, which leads to a higher probability of purchase.

Modifying Consideration Set: Advertising can affect the consideration set of a consumer by either expanding it, or preventing its expansion. This is only applicable to consumers who have visited only one advertiser. Ads from a non-visited advertiser will add that advertiser to such a consumer’s consideration set. Related to this, ads by the visited advertiser will prevent consumers from seeing ads by their non-visited advertiser and thus prevent it from being added to their consideration set. When a consumer has only visited a single advertiser, and sees ads from the same advertiser, they choose between that advertiser and the outside option, as in Equation (3). But, if the rival non-visited advertiser is able to advertise to such a consumer, the consumer then has a choice between both advertisers and the outside option, as in Equation (2). Thus, we find that the visited advertiser has an incentive to advertise to their exclusive visitors just to prevent them from viewing ads by their rival advertiser. This ‘competition blocking’ effect of advertising has been reported in recent empirical work (Sahni et al., 2016).
4.3.2 For Consumers

Consumers can also benefit from advertising. In our model, viewing ads increases consumer utility directly through the additive utility term $\theta$, but only when they purchase from the advertiser. Consumers may also benefit by expanding their consideration set when they see an ad by a advertiser they have not visited, and hence were not considering. Due to the random utility term $\epsilon$, the non-visited advertiser may offer a product with a higher utility than the visited advertiser, and hence the consumer may benefit from expanding their consideration set.

On the other hand, if consumers have a strong preference for one advertiser, that is $k$ is large, they do not benefit much by viewing ads by their less preferred (disliked) advertiser. This is because ads by a strongly disliked advertiser are not likely to significantly increase the purchase probability from such an advertiser. Instead, consumers miss out on the opportunity to view ads by their strongly preferred advertiser, which would have given them the extra utility $\theta$ when they purchase from them. Thus, as a net effect, consumer surplus may reduce when they are targeted with ‘irrelevant’ ads from their disliked retailer.

4.3.3 Advertising Value Function - with Perfect Information

We define the Advertising Value Function for a consumer as the value of advertising to the consumer by a given advertiser. This value is estimated with perfect information about each consumer’s valuation for each advertiser. Of course, advertisers are unlikely to have perfect information about every consumer. In the next section, we discuss the advertising auction equilibrium when advertisers have first or third-party information.

To keep the focus on advertising and role of information flow, we assume prices to be exogenous to our model and normalize the profit margin to be 1. The value of the ad is then the difference in purchase probabilities when a consumer is exposed to an ad and when they are not. Recall that in our model, a consumer will see an ad from exactly one of the two competing advertisers. The following equation captures this difference in the purchase probability. For advertiser $j$, the Advertising Value Function for a consumer with $\{U_j, U_{-j}\}$ utilities for advertiser $j$ and the rival advertiser ($-j$) is given by

$$V_{\{U_j, U_{-j}\}}(k, \theta) = \Pr(j|k, \theta, \delta_j = 1) - \Pr(j|k, \theta, \delta_j = 0)$$ (4)

The probability of purchase are given by Equations (2) or (3). To simplify our notation, we normalize valuations by setting $V_L = 1$, and therefore, $V_H = k$. Dropping the subscript for
advertiser \( j \), the Advertising Value Function takes the following functional forms for different consumer segments:

\[
V_{H,L}(k, \theta) = \frac{e^{k+\theta}}{1 + e^{k+\theta}} - \frac{e^k}{1 + e^{1+\theta} + e^k} 
\]

\[
V_{L,H}(k, \theta) = \frac{e^{1+\theta}}{1 + e^{1+\theta} + e^k} 
\]

\[
V_{L,L}(k, \theta) = \frac{e^{1+\theta}}{1 + e^{1+\theta} + e} - \frac{e}{1 + e + e^{1+\theta}} 
\]

\[
V_{H,H}(k, \theta) = \frac{e^{k+\theta}}{1 + e^k + e^{k+\theta}} - \frac{e^k}{1 + e^{k+\theta} + e^k} 
\]

In our notation here, the first and second subscripts for \( V \) refer to the consumer’s valuation for the advertising retailer and the rival retailer, respectively. Recall that consumers with a High valuation for one advertiser and Low for the other, visit only the advertiser for whom they have a High valuation; and consumers who have identical valuations for both advertisers visit both.

The first term on the R.H.S. of Equation (5) shows that without seeing ads from the advertiser they have not visited, consumers include only their visited (and preferred) advertiser in their consideration set, as in Equation (3). This is the ‘competition blocking’ effect of advertising, where advertising to one’s own visitors is valuable for an advertiser not just by increasing the consumer’s utility for them, but also because it prevents another advertiser from being included in the consumer’s consideration set. Both these factors increase the consumer’s purchase probability from the advertiser. Also, note that if the visited advertiser does not advertise to their exclusive visitors, the rival (non-visited) advertiser will get to advertise to them, as shown in the second term on the R.H.S. The difference in these two terms in the value of advertising to an exclusive visitor.

Similarly, notice that in Equation (6), unless the non-visited advertiser serves an ad to consumers who have visited only the rival advertiser, the consumers will not include the non-visited advertiser in their consideration set. Thus, there is zero probability of a consumer purchasing from their non-visited advertiser if that advertiser also does not advertise to them. Thus, there is no second term in the Equation (6).

Equations (7) and (8) similarly describe the value of advertising to consumers who have visited both advertisers, and have a Low and High valuation for each, respectively.
4.3.4 Model Parameters

Who advertises to whom, and who benefits from this advertising, are all determined by two key parameters of our model - \( k \), which represents the degree of horizontal differentiation, and \( \theta \), which represents the incremental utility from advertising. We assume \( k > 1 \) and \( \theta \geq 0 \) in our analysis. In other words, ads do not have a negative effect on consumer utility, say, via annoyance or privacy intrusion. In spite of this, we will show that advertising can have a negative effect on consumer surplus.

Lemma 1. Depending on the magnitudes of degree of horizontal differentiation (\( k \)) and the incremental utility of ads (\( \theta \)), there are three possible scenarios:

- Region 1: When \( k \) and/or \( \theta \) are low, we have \( V_{H,H} < V_{H,L} < V_{L,H} \)
- Region 2: When \( k \) and/or \( \theta \) are moderate, we have \( V_{H,H} < V_{L,H} < V_{H,L} \)
- Region 3: When \( k \) and/or \( \theta \) are large, we have \( V_{L,H} < V_{H,L} < V_{H,H} \)

In all Regions: \( V_{L,L} < \min\{V_{H,L}, V_{L,H}, V_{H,H}\} \)

Proof. All proofs are in Appendix C.

The three regions in Lemma 1 are defined precisely in the proof in Appendix C and depicted in Figure 4.
When $\theta$ is small, the incremental utility from advertising is also small. If the ad does not change the consumer's consideration set (such as for consumers who have visited both advertisers), then such ads have little value for the advertiser. Hence, ads are least valuable for consumers who have High valuations for both advertisers in Regions 1 & 2 ($V_{H,H}$). In addition, when $k$ is also small, consumers have similar valuations of the two advertisers. In this case, advertising is especially valuable for the non-visited advertiser since they can add themselves to the consumer's consideration set and significantly increase their purchase probability. Hence, in Region 1, $V_{L,H}$ is largest. When $k$ is large enough, consumers have a strong preference for one advertiser that advertising by the non-visited advertiser is less valuable than by the visited advertiser. Thus, in Regions 2 & 3, $V_{L,H} < V_{H,L}$. Finally, when one or both of $k$ and $\theta$ are large, advertising is most valuable for consumers with High valuations for both advertisers, since they have the highest probability of purchasing from at least one of the two advertisers (due to the large $k$), and an ad can significantly influence their choice (due to the large $\theta$).

If advertisers had complete information about a consumer's valuation for not only themselves but also for their rival advertiser, then bidding the Advertising Value Function for a consumer would be the weakly dominant strategy for the advertisers in second price auctions. However, it is unlikely that such information is accurately available to the advertisers, especially regarding a consumer's valuation for their rival advertiser.

5 Advertising Auction Equilibrium

Consider a simultaneous game where each advertiser decides whether to acquire third-party information and conditional on this decision, makes a bid to advertise to each potential consumer. We can solve for the equilibrium strategy by backward induction, by first calculating the bidding strategies, advertising outcomes and payoffs in each informational scenario. We report the details of second price advertising auction in Appendix B. Here, we note that the weakly dominant strategy for each advertiser is to acquire third-party information.

All through our analysis, we assume that the values of $k$ and $\theta$ are common knowledge among advertisers. They also know that the dominant strategy for each advertiser is to acquire third-party information.

Acquiring third-party information leads to a 'prisoner's dilemma' for advertisers when $\pi_t < \pi_f$. Both advertisers would be better off if neither had third-party information, but when third-party
information is available, each will acquire it to protect themselves from the consequences of the other advertiser acquiring it. We discuss the range of the model parameters when this occurs in the next section.

From our analysis it follows that \( p_{t,max} = \pi_t - \pi_{ft} \) is the maximum an advertiser would be willing to pay for third-party information. Depending upon the competition in the third-party information market, this value will be split between advertisers and the third-party information vendor. If many potential sources of similar third-party information exist, competition among them would lower the price of the third-party information for advertisers. On the other hand, a third-party with a monopoly on proprietary consumer data - such as a social media or search giant, or an ISP - can charge advertisers up to \( p_{t,max} \) and ensure they are in a severe prisoner’s dilemma.

6 Welfare Analysis

In this section, we calculate and compare advertiser profits, consumer surplus, publisher revenue and total surplus in the first and third-party information scenarios. As we have discussed, if third-party information were available, each advertiser would acquire it in equilibrium. Thus, the only way to prevent usage of third-party information in targeting is for a policy maker to prevent third-party information from being tracked and sold to advertisers. This is the counterfactual privacy policy that we will compare against a policy that does not prevent third-party tracking.

All the profits and surpluses referred to in our analysis arise from targeted advertising. Thus, when we calculate the advertiser profit with first-party information, we are referring to the incremental profit the advertiser generates from targeted advertising as compared with the profit they would obtain from selling their products without targeted advertising.

We summarize our notation in Table 1.

6.1 Advertiser Profits

Advertiser profits are given by the difference between the increased purchase probability due to an ad and the cost of serving that ad. In the second price auction setting in our model, the cost of serving an ad is given by the bid of the rival advertiser.

First, we consider consumers who have visited both advertisers. Such consumers have identical valuations for both advertisers - either High, or Low, for both. Advertisers’ bidding strategies are unaffected by whether they have first or third-party information. Thus there is no difference in ad
exposure or bid amounts for such consumers. As discussed in Appendix B, since both advertisers bid their true (and identical) valuation for such consumers, advertiser profits are zero in these consumer segments in either information scenario.

For consumers who have visited only one advertiser, there are multiple scenarios to consider. When no advertiser possesses third-party information, they cannot identify or target advertising to their non-visitors. Thus, an advertiser incurs no cost to advertise to consumers who visit them exclusively, since there are no competing bids\(^3\). So, advertiser’s profit when both have only first-party information is given by

\[
\pi_f = \gamma(1 - \gamma)V_{H,L}
\]  

(9)

When both advertisers have third-party information, we have two scenarios to consider. First, if \(\{k, \theta\} \in R1 \cup R2\), advertisers with third-party information serve ads to their non-visitors, and their profit is given by

\[
\pi^{(R1,R2)} = \gamma(1 - \gamma)(V_{L,H} - V_{H,H})
\]  

(10)

Here, \(V_{L,H}\) is the value of advertising to a non-visitor, while \(V_{H,H}\) is the cost of advertising. \(\gamma(1 - \gamma)\) is the size of the consumer segment.

---

\(^3\)In practice, advertisers will pay a small reservation price set by the publisher.
Second, if \( \{k, \theta\} \in R3 \), advertisers advertise to their own visitors and their profit is given by

\[
\pi_t^{\{R3\}} = \gamma(1 - \gamma)(V_{H,L} - V_{L,H})
\]  

(11)

Notice that now each advertiser incurs a cost of \( V_{L,H} \), since their rivals, equipped with third-party information, bid that amount for these consumers (c.f. Equation (9)).

By comparing profits in the first party case, \( \pi_f \), with profits in the third party case, \( \pi_t^{\{R1,R2\}} \) and \( \pi_t^{\{R3\}} \), we have the following proposition, and is depicted in Figure 5.

**Proposition 1.** In region \( R1 \), for \( k < \hat{k} \) or \( \theta < \hat{\theta} \), advertisers are better off with third-party information, and worse off otherwise, where \( \hat{k} \) and \( \hat{\theta} \) are solutions of \( V_{H,L}(k, \theta) = V_{L,H}(k, \theta) - V_{H,H}(k, \theta) \).

In regions \( R2 \) and \( R3 \), advertisers are always worse off with third-party information.

![Comparison of advertiser profits with third-party and first-party information. Advertiser profits are higher with third-party information in the colored region. Red color represents highest profit while blue represents lowest positive profit. Advertiser profits are lower with third-party information in the white region.](image)

Figure 5: Comparison of advertiser profits with third-party and first-party information. Advertiser profits are higher with third-party information in the colored region. Red color represents highest profit while blue represents lowest positive profit. Advertiser profits are lower with third-party information in the white region.

To compare advertiser profits between the third-party and first-party scenarios, we consider the difference between the additional benefit from advertising to the segment of non-visitors and its cost (third-party information equilibrium), and compare this with the benefit from advertising to one’s own exclusive visitors at almost no cost (first-party information equilibrium). We find that
only when $k < \hat{k}$ or $\theta < \hat{\theta}$ does this comparison result in a net increase in profits for the advertisers with third-party information.

In Region $R_1$, when $k$ or $\theta$ are small, advertisers benefit from third-party information by advertising to their non-visitors. When $k$ is small, consumers have a relatively high valuation for the retailer they have not visited, and is thus not in their consideration set. When they see ads from this non-visited retailer, it gets added to their consideration set which significantly increases their probability of purchasing from them. If, on the other hand, third-party information were not available, such consumers would have seen ads from their visited advertiser, and if $\theta$ is small enough, this would have lead to a lower increase in purchase probability. Thus, advertiser profits increases when $k < \hat{k}$ and $\theta < \hat{\theta}$, and decreases otherwise. Note that $\hat{k}$ is a function of the actual value $\theta$ and $\hat{\theta}$ is a function of the actual value $k$. In other words, a closed form solution for $\hat{k}$ and $\hat{\theta}$ is not possible.

There are two separate mechanisms that reduce advertiser profits in regions $R_2$ and $R_3$, which we discuss below.

**Irrelevant Advertising** In $R_2$, the reduction in surplus is due to the fact that advertising to rival’s exclusive visitors is not very effective in this range of $k$ and $\theta$. Note that $k$ is moderately large in $R_2$, which implies consumers have moderately strong preference for their visited advertiser over the non-visited advertiser. Ads by the non-visited advertiser, while they modify the consumer’s consideration set, only cause a small increase the purchase probability from the non-visited advertiser. Thus, this is a case of targeted, yet ‘irrelevant’ advertising. This is caused by the inability of an advertiser to distinguish between consumers who are their exclusive visitors and those who also visit their rival. Customers who do not visit their rival are more valuable for advertising in region $R_2$, as shown in Lemma 1. But since they cannot be distinguished from consumers who also visit their rival, the Bayes-Nash equilibrium is to bid a single intermediate value for all visitors, which leads ads being displayed by the non-visited advertiser. We call such ads, which are not not very effective, as ‘irrelevant’ ads.

**High cost of advertising:** In region $R_3$, consumers see ads by their visited advertiser, irrespective of whether advertisers have third-party information or not. So, the benefit of advertising does not change. However, with third-party information, advertisers have to pay the bids of the non-visited retailer, whereas this cost would be minimal without third-party information. Thus, the net effect of third-party information in $R_3$ is to reduce advertiser profit by increasing the cost of advertising.
Our key insight here is that we identify a scenario where the inability to distinguish between exclusive and non-exclusive visitors leads to a case where targeted ads, in equilibrium, are ‘irrelevant’. In the next section, we show how such irrelevant ads reduce not just advertiser profits, but also consumer surplus.

### 6.2 Consumer Surplus

Consumer surplus, in our model, depends on how many advertisers are in a consumer’s consideration set and which ads were seen by them. Information possessed by advertisers, whether first-party or third-party, can effect consumer surplus only through changing one or both of these factors.

Note that the expected consumer surplus for a consumer when they choose one among \(N\) options with utilities \(u_i, i = 1, 2, \ldots, N\) is given by (Small & Rosen, 1981)

\[
CS = \log \left( \sum_i e^{u_i} \right)
\]  

(12)

To focus on the effect of advertising, we ignore any cost of privacy or nuisance incurred by consumers if their third-party information has been shared with advertisers.

We are interested in comparing the change in consumer surplus when advertisers have access to third-party information with when they do not. There is no change in the ads viewed by consumers who visit both advertisers and consequently, there is no difference in their surplus, so we ignore them in our comparison. Therefore, we can focus solely on the surplus change for consumers who visit only one advertiser. Such consumers can be shown ads either by the visited advertiser or by the advertiser they have not visited. From the advertising auction equilibria described in Appendix B, we can express the surplus in the two scenarios as follows

\[
CS_{1, visited} = 2\gamma(1 - \gamma) \log(1 + e^{V_H + \theta})
\]  

(13)

\[
CS_{1, non-visited} = 2\gamma(1 - \gamma) \log(1 + e^{V_L + \theta} + e^{V_H})
\]  

(14)

where the first subscript refers to the number of advertisers the consumer has visited and the second subscript refers to whether they see ads by their visited or non-visited advertiser. Note that in Equation (13), a consumer’s consideration set includes only their visited advertiser and the outside option. Thus ads increase consumer utility for the visited (and advertising) retailer and also have a competition-blocking effect by preventing consideration of the non-visited (and non-advertising) retailer. In Equation (14), when consumers see ads by their non-visited (and advertising) retailer,
they add that advertiser to their consideration set. The ads also increases their utility for the non-visited (and advertising) retailer.

Now we can write the consumer surplus with and without third-party information.

With only first-party information, consumers who visit only one advertiser see ads by that advertiser. Hence, consumer surplus can be written as

$$CS_f = CS_{1,visited}$$ \hspace{1cm} (15)

With third-party information, in regions $R_1$ and $R_2$, advertisers advertise to their rivals’ exclusive visitors, whereas in $R_3$, they advertise to their own visitors. So, we have the following consumer surplus expressions

$$CS_{\{R_1,R_2\}} = CS_{1,non-visited}$$ \hspace{1cm} (16)

$$CS_{\{R_3\}} = CS_{1,visited}$$ \hspace{1cm} (17)

Comparing Equation (15) with Equations (16) and (17) gives us an insight into how consumer surplus changes due to third-party tracking and leads to the following proposition.

**Proposition 2.** When $\{k,\theta\} \in R_1$, consumers are better off with third-party tracking. When $\{k,\theta\} \in R_2$, consumers are worse off with third-party tracking due to irrelevant advertising. When $\{k,\theta\} \in R_3$ there is no change in consumer surplus due to third-party tracking.

This is depicted in Figure 6.

Essentially, the trade-off is between whether consumers benefit more from increased utility for the advertiser they have already visited, or from adding their non-visited advertiser to their consideration set. As long as the relative valuation for the non-visited advertiser is high enough (low $k$) or $\theta$ is not too large, the surplus for the consumer increases with third-party tracking. On the other hand, if consumers have a low preference for the non-visited advertiser (moderate $k$) or the incremental consumer utility from ads is strong enough (moderate $\theta$), then third-party tracking may impose an opportunity cost to consumers by exposing them to less relevant ads reducing their surplus. This occurs in region $R_2$. In region $R_3$, the ad equilibrium does not change when advertisers have third-party information and hence there is no change in consumer surplus.

Thus consumers in markets with more less degree of horizontal differentiation between advertisers (such as budget segments in any product category) will likely benefit from such third-party tracking, whereas consumers in markets where they have stronger preferences may be worse off from third-party tracking.
Figure 6: Comparison of consumer surplus with third-party and first-party information. Consumer surplus with third-party information is higher in Region 1, with blue representing the region of highest consumer surplus and yellow the lowest positive consumer surplus. Consumer surplus with third-party information is lower in Region 2, with red region representing the lowest negative consumer surplus. Consumer surplus does not change with third-party information in Region 3, shown in white.

### 6.3 Publisher Revenue

Advertising costs for advertisers can be considered as advertising revenue for the publisher. With only first-party information, advertisers cannot compete for each others exclusive visitors. This leads to a low level of advertising competition generating the least revenue for the publisher. This is given by

\[
Pub_{\text{Rev}}^f = \gamma^2 V_{L,L} + (1 - \gamma)^2 V_{H,H} \quad (18)
\]

With third-party information, advertisers can target their rival’s exclusive visitors. This increases advertising competition and thereby increases publisher revenue. Following Proposition 2, we have the following expressions for publisher revenue when advertisers have third-party information

\[
Pub_{\text{Rev}}^{\{R1,R2\}} = \gamma^2 V_{L,L} + (1 - \gamma^2)V_{H,H} \quad (19)
\]

\[
Pub_{\text{Rev}}^{\{R3\}} = \gamma^2 V_{L,L} + (1 - \gamma)^2 V_{H,H} + 2\gamma(1 - \gamma)V_{L,H} \quad (20)
\]

This leads to the following proposition:

**Proposition 3.** Publisher revenue always increases when advertisers have third-party information.
Publishers, therefore, have an incentive to allow third-party trackers to track visitors on their website. As this information becomes available to advertisers, it increases competition, increasing the publisher’s advertising revenue. This explains why there is such a large number of third-party trackers allowed by publishers, as discussed in the Introduction. This also suggests a role for policy-makers, since publishers have no incentive to check what third-party trackers do with the information collected off their website.

6.4 Total Surplus

Total surplus is the sum of profits of both advertisers, publisher revenue and consumer surplus. Advertising does not merely reallocate total surplus across the participating agents, but may increase or even decrease total surplus depending on whether targeting is relevant or not in equilibrium.

Note that the cost of advertising for advertisers is also the publisher’s revenue. As such, there is reallocation of surplus between these two entities, with the sum remaining constant. Thus, total surplus is determined by what happens to the consumer surplus.

Combining the results from Propositions 1, 2 and 3, we have the following proposition

**Proposition 4.** Total surplus is higher with third-party information when \( \{k, \theta\} \in R1 \), lower when \( \{k, \theta\} \in R2 \), and does not change when \( \{k, \theta\} \in R3 \).

Targeted advertising can create value when degree of horizontal differentiation is relatively low (small \( k \)) by adding a new, well-liked, but non-visited advertiser to a consumer’s consideration set. However, targeted advertising can also create a dead-weight loss, when it is irrelevant for consumers. Consumers miss out on more relevant ads, while advertisers lose by spending money on ineffective ads. This dead-weight loss is caused by the inability to accurately distinguish between one’s own exclusive visitors and those that also visit the rival. When the degree of horizontal differentiation is high or \( \theta \) is large enough, third-party information does not affect total surplus.

7 Extension - Disabling Third-party Tracking

Thus far in our analysis, we have assumed that consumers are passive recipients of advertising. In this section, we allow a fraction of consumers to disable third party tracking and analyze how it affects our results. We let the accuracy of third-party signal be less than perfect, and we make it dependent on the proportion of consumers who allow tracking.
7.1 Do-Not-Track

There are two popular mechanisms by which consumers may protect themselves from being exposed to unwanted advertising. First, consumers can use one of the many available ad-blocking software to ensure that they do not see online ads. The effect of such ad-avoidance mechanisms has been studied in the literature (Johnson 2013). However, most ad-blocking software do not necessarily prevent third-party tracking by default\(^4\). Hence, while ad-blocking software may reduce the annoyance consumers experience from advertising, they do not always protect consumer’s privacy. Indeed, some ad-blocking software allow advertisers to pay them to ‘whitelist’ their ads, while others collect and sell data about the trackers themselves\(^5\).

Second, most popular web browsers allow consumers to block third-party cookies from being set. Figure 7 shows a sample screenshot of settings of the Google Chrome browser. Other major web browsers have similar options. Note that blocking first-party cookies from being set harms the functionality of the website, such as being able to keep a consumer logged in to the website, with the result that few consumers block first-party cookies.

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\(^4\) AdBlock Plus, the most widely used ad-blocking software installed on over 200 million devices, does not block third-party tracking by default. See https://adblockplus.org/en/features#tracking (accessed November 19, 2016). It does, however, allow consumers to disable third-party tracking.

\(^5\) See https://www.wired.com/2016/03/heres-how-that-adblocker-youre-using-makes-money/ (accessed November 19, 2016) for a description of different types of business models of ad-blockers.
Overall, it is estimated that over 10% of Internet consumers activated the do-not-track setting for third-parties on their browsers in March 2017.\(^6\)

We assume that \(\alpha \in [0,1]\) proportion of consumers allow third-party tracking. In our model, this proportion is exogenously fixed and does not depend on consumer type or the model parameters \(k\) and \(\theta\).

### 7.2 Third-party Signal Accuracy

The accuracy of the third-party classification of a consumer as a High or Low interest consumer for a particular product category is likely to be imperfect. We define the accuracy of third-party signal as the probability of a consumer being classified by the third-party as a High type given that they are High type for at least one of the advertisers under consideration.

\[
\text{Accuracy} = \Pr(t = V_H | V_A \text{ or } V_B = V_H) = f(\alpha)
\]

where \(V_A\) and \(V_B\) are the actual valuations of the consumer for advertisers \(A\) and \(B\) respectively, \(V_H\) is the High valuation and \(t\) is the third-party signal.

We assume the accuracy of third-party signal is monotonically increasing in the proportion of consumers that allow third-party tracking. It is a well known result from machine learning literature that the accuracy of a classification algorithm increases with the proportion of the population that is used for training it. If no consumer allows tracking, then the algorithm classification will be uninformative whereas, if all consumers allow tracking, we can expect the algorithm to be more accurate. This is summarized in the following expressions

\[
f'(\alpha) > 0 \quad (22)
\]

\[
0 < f(0) \leq f(\alpha) \leq f(1) \quad (23)
\]

From Proposition 2, if \(\{k, \theta\} \in R1 \cup R2\), advertisers wish to target their rival advertiser’s exclusive visitors. Of all consumers who have visited only one advertiser, only \(\alpha\) proportion allow third-party tracking. Of these consumers, the third-party agency correctly classifies only \(f(\alpha)\) consumers. So the proportion of rival’s exclusive visitors that are targetable by the non-visited advertiser is \(\alpha f(\alpha)\). This advertising outcome is depicted in Figure 8.

Advertiser profits and consumer surplus are a weighted linear combination of first-party and

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third-party profits and surplus respectively, as given by Equations (24) and (25).

\[
\pi_\alpha = \gamma (1-\gamma) [\alpha f(\alpha)(V_{L,H} - V_{H,H}) + (1-\alpha f(\alpha))V_{H,L}]
\]

\[
= \alpha f(\alpha)\pi_t + [1 - \alpha f(\alpha)]\pi_f
\]

\[
CS_\alpha = \alpha f(\alpha)CS_t + [1 - \alpha f(\alpha)]CS_f
\] (25)

### 7.3 Externality of Tracking

A consumer’s decision of whether to allow third-party tracking has two effects - a direct effect on the consumer, and an indirect effect on other consumers. The indirect effect occurs due to the change in the accuracy of the third-party signal.

Differentiating Equation (25) with respect to \(\alpha\), we get

\[
\frac{\partial CS_\alpha}{\partial \alpha} = [f(\alpha) + \alpha f'(\alpha)](CS_t - CS_f)
\] (26)

In Equation (26), \(f(\alpha)(CS_t - CS_f)\) represents the direct change in surplus of the marginal consumer, while \(\alpha f'(\alpha)(CS_t - CS_f)\) represents the change in surplus for other consumers that already allow tracking. This latter term is the externality of the marginal consumer’s decision to allow tracking.
We know that $\alpha f'(\alpha) + f(\alpha) > 0$ from Equations (22) and (23). So, the change in consumer surplus with increase in $\alpha$ is positive if $CS_t - CS_f > 0$ and negative if $CS_t - CS_f < 0$. Therefore, the externality of allowing third-party tracking can be summarized in Proposition 7.

**Proposition 5.** When a consumer enables third-party tracking, then:

- If $\{k, \theta\} \in R1$, there is a positive externality on other consumers.
- If $\{k, \theta\} \in R2$, there is a negative externality on other consumers.
- If $\{k, \theta\} \in R3$, there is no externality on other consumers.

Figure 9 depicts the regions of different types externalities of tracking.

![Figure 9: Parameter ranges for positive, negative and no externality of third-party tracking](image)

In summary, a consumer’s choice of allowing third-party tracking or blocking can create an externality for other consumers. Depending on the values of the model parameters $k$ and $\theta$, this externality might be positive or negative or have no effect.

**8 Discussion**

In this paper, we provide a model of targeting which distinguishes between first-party and third-party information. We describe how advertising can modify a consumer’s consideration set and increase a consumer’s utility for the advertiser. The strength of these two effects leads to several interesting implications for firms, consumers and policy makers.
8.1 Managerial & Policy Implications

Our paper has several managerial and policy implications. First, we find that it can sometimes be more beneficial for firms to target their rival’s visitors than their own. This is true when consumer’s valuation for the two advertisers is comparable or when ads do not increase consumer utility significantly. This can be facilitated by third-party information.

Second, we show that the presence of third-party information can often reduce advertiser profits. This occurs due to increased competition for exclusive visitors or by irrelevant advertising. In spite of this, we show that advertisers must always acquire third-party information even when such third-party information reduces advertiser profit. The possibility that the other advertiser may acquire third-party information makes acquiring it a dominant strategy. This may lead to a ‘prisoner’s dilemma’, where both advertisers could have been better off if neither had third-party information.

In such a situation, a policy directive that bans the tracking of third-party information might be beneficial for advertisers. This suggests that advertisers should not be opposed to some forms of tracking bans, especially when the third-party possessing the information might be a monopoly, such as an ISP, social media or search giant. This is in contrast to the prevalent lobbying efforts of advertisers to oppose any limits on tracking (Rupar, 2017). As shown, if the third-party agency is a monopolist, it can extract all the surplus from advertising and force each advertiser into a severe prisoner’s dilemma.

Next, we find that third-party tracking can enhance consumer welfare in certain conditions. When consumers are less heterogeneous in their preferences, they benefit by being served ads by advertisers other than their preferred advertiser since this allows them to consider new advertisers which they otherwise would have not. This expands the the market size and enables more efficient allocation of ads. However, on the flip side, we find that third-party tracking may sometimes reduce total surplus by making firms compete even while serving irrelevant ads. This may contract the market since consumers miss out on more relevant advertising. This suggests that policymakers need to consider the context in which consumer information is being collected and utilized when devising a data privacy policy that can protect consumers without reducing the relevance of ads.

When third-party tracking effects consumer welfare, policies that effect the proportion of consumers who disable third-party tracking can cause an externality on other consumers. This is due to the decrease in the accuracy of third-party signal as consumers disable third-party tracking. Thus, we show that policies that aim to increase consumer privacy by encouraging practices that
reduce third-party tracking rates may actually reduce consumer welfare.

We also find that publishers always benefit from third-party tracking. According to our model, publishers have no incentive to prevent any third-party tracker from tracking their visitors. This explains the increase in intensity of web tracking on publisher sites over the last few years. However, this also highlights the role of policymakers in regulating privacy policies. Self-regulation by publishers is unlikely to limit the tracking of consumers by third-parties.

Finally, we show that if the third-party agency is a monopoly, such as a social media or search giant, or an Internet Service Provider, it is able to extract most of the advertiser surplus by pricing the third-party information as \( p = \pi_{11} - \pi_{13} \). This ensures a severe case of ‘prisoner’s dilemma’ for the advertisers. To the extent policymakers want to ensure that the allocation of surplus from advertising is distributed to the advertising advertisers as well, it can adopt measures to limit the market power of third-party agencies. Greater concentration of third-party information among a few agencies can harm advertisers.

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Appendix A - Website Visiting Behavior

In this Appendix, we derive the consumer’s choice of which websites to visit based on the search cost of visiting a website. Consumers are assumed to incur a search cost when they visit a website. This represents the time and effort it takes to visit the advertiser’s website and sort through the available assortment to find the best-fit product. We assume the search cost is identical for all consumers.

Note that the expected consumer surplus for a consumer when they choose one among \( N \) options with utilities \( u_i, i = 1, 2, \ldots, N \) is given by (Small & Rosen, 1981)

\[
CS = \log \left( \sum_i e^{u_i} \right)
\] (27)

A consumer decides which website or websites to visit based on their beliefs about their expected utility for the two advertisers as well as the magnitude of the search cost. Consumers visit an advertiser’s website if the expected incremental consumer surplus from visiting the website is greater than the search cost.

We can write this incremental consumer surplus from visiting an advertiser as

\[
\Delta CS_{\text{inc}} = CS_{\text{visit, advertiser}} - CS_{\text{not visit, advertiser}}
\]

We assume the search cost \( \phi \) is low enough that they visit at least one advertiser’s website. The least incremental consumer surplus is gained by consumers who have a Low valuation for both advertisers and are making the decision of whether to visit one of the Low valuation advertisers. This is given by

\[
\Delta CS_{\text{inc, least}} = \log(1 + e^{V_L}) - \log(1) = \log(1 + e^{V_L})
\]

So, to ensure each consumer visits at least one advertiser, we assume \( \phi < \log(1 + e^{V_L}) \). We also assume that when consumers have two different valuations for the two advertisers, they will choose to visit the higher valuation advertiser first.

Having visited one advertiser, the consumer next decides whether to visit the second advertiser. The incremental expected consumer surplus is summarized below:

- (H, L) type consumer: \( \Delta CS_{HL} = \log \left( 1 + \frac{e^{V_L}}{1 + e^{V_H}} \right) \)
- (L, L) type consumer: \( \Delta CS_{LL} = \log \left( 1 + \frac{e^{V_L}}{1 + e^{V_L}} \right) \)
- (H, H) type consumer: \( \Delta CS_{HH} = \log \left( 1 + \frac{e^{V_L}}{1 + e^{V_H}} \right) \)
where the $\Delta CS_{T1,T2}$ refers to the first advertiser being $T1$ type and second advertiser being $T2$ type.

Clearly, $\Delta CS_{HL} < \Delta CS_{LL} < \Delta CS_{HH}$. This shows that the least incremental surplus results when a consumer has visited the High value advertiser first and then visits a Low value advertiser. The largest incremental surplus results when a consumer, having already visited a High value advertiser visits another High value advertiser.

This leads to four regions in which the search cost can lie resulting in four types of website visiting behaviors, as summarized in Table 2:

<table>
<thead>
<tr>
<th>Case</th>
<th>Search Cost</th>
<th>Website Visiting Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\phi &lt; \Delta CS_{HL}$</td>
<td>• Everyone visits both websites</td>
</tr>
</tbody>
</table>
| 2    | $\Delta CS_{HL} < \phi < \Delta CS_{LL}$ | • Consumers who have a preferred advertiser, visit only that advertiser’s website.  
• Other consumers visit both advertiser’s website |
| 3    | $\Delta CS_{LL} < \phi < \Delta CS_{HH}$ | • Visit all High valuation advertiser’s websites  
• Visit only one Low valuation website, if there are no High valuation advertisers |
| 4    | $\phi > \Delta CS_{HH}$ | • Everyone visits only one website |

In Case 1, the search cost is low enough that all consumers visit both advertiser’s websites. In this case, third-party tracking does not play a role since each advertiser has first-party information about every consumer.

In Case 2, which we assume in this paper, search cost is such that consumers who have a preferred advertiser visit only that advertiser. Consumers that are indifferent between the two advertisers (have High or Low preference for both advertisers) visit both. In short, search cost is high enough that consumers do not visit a lower value advertiser after they have already visited a higher value advertiser. However, when both advertisers have identical valuations for a consumer, the search cost is low enough that consumers visit both.

In Case 3, consumers visit all advertisers for whom they have a High valuation. If the consumer has Low valuations for both advertisers, they visit only one of those advertisers chosen with equal probability. Search cost is lower than the expected incremental surplus from visiting a second High
valuation advertiser’s website. However, the search cost is high enough that the consumer would not visit a second Low valuation advertiser if they have already visited one advertiser.

In Case 4, consumers visit only one website since the search cost is too high to justify visiting a second website. If the consumer has a preferred advertiser, they visit its website. If the consumer is indifferent between advertisers, they visit one of the two advertiser’s websites with equal probability.

We focus on Case 2 in our analysis in this paper because it most clearly presents the trade-offs and strategies of the competing advertisers and the resulting outcomes. Below, we briefly describe the equilibrium outcomes for Cases 3 and 4, which are very similar in nature to Case 2.

Case 3

From Table 1, when $\Delta CS_{LL} < \phi < \Delta CS_{HH}$, consumers who have Low valuations for both advertisers visit only one advertiser chosen at random. The website visiting behavior of the other three consumer segments remains identical to what it was in Case 2, which we analyzed in detail in the paper.

Thus, in Case 3, the only difference is the advertising outcome for the consumer segment with Low valuations for both advertisers. Third-party information does not provide any additional information about this segment. So, the conclusions of Case 2, as discussed in the paper still hold in Case 3 as well. Compared to Case 2, we note that advertiser profits are somewhat higher in Case 3, due to the reduced advertising competition for the consumer segment with Low valuations for both advertisers.

Case 4

From Table 1, when $\Delta CS_{HH} < \phi < \log(1 + e^{V_H})$ all consumers visit only one advertiser. If they have identical valuations for both the advertisers, they visit either advertiser with equal probability. If they have a High valuation for one advertiser and Low for the other, they visit the advertiser for which they have a High valuation.

Similar to Case 3, we note that in Case 4 third-party information does not provide any additional targetability for consumers that have identical valuations of the two competing advertisers. As such, the conclusions for Case 2 still hold in this case. advertiser profits are somewhat higher than in Case 3 due to reduced competition for consumers who have High valuations for both advertisers.
Appendix B - Advertising Auction Equilibrium

In this Appendix, we calculate the advertising auction equilibria under different informational scenarios - when both advertisers have only first-party information, when both have third-party information, and when only one advertiser has third-party information while the other has only first-party information.

First-party Information Equilibrium

When a advertiser has access to only first-party information, they are able to identify and bid only for consumers who have already visited them.

Proposition 6. When both advertisers have only first-party information, consumers who have visited only one advertiser see ads by the same advertiser. Consumers who have visited both advertisers have an equal probability of seeing either advertiser’s ad.

Proof. If a consumer who has visited the advertiser’s website is classified as having a Low valuation for them, as inferred from their first-party information, then that consumer must necessarily have a Low valuation for the rival advertiser as well, since they would not have visited the first advertiser’s website otherwise. Thus there is complete information about visitors who are Low type for a advertiser and the weakly dominant strategy is to bid their true value. If a consumer is classified as a High valuation type for a advertiser, then they could either have a High or Low valuation for the rival advertiser. Since advertisers know that their rival does not have third-party information, they know that they can win the advertising auction for their own exclusive visitors with any positive bid and advertise to them at no cost. Hence, they bid $V_{H,H}$ for all consumers who are High type for them.

Thus, the equilibrium bidding strategy by a advertiser when neither advertiser has third-party information as a function of their first-party classified type is:

$$b^*(f = L) = V_{L,L}$$
$$b^*(f = H) = V_{H,H}$$

where $b^*(f = L)$ and $b^*(f = H)$ are equilibrium bids for consumers who have been classified as Low and High valuation type based on first-party information, respectively.

It is easy to see that bidding higher or lower than these values would reduce advertiser surplus. Advertising outcomes follow from the equilibrium bidding strategy. $Q.E.D$
The advertising outcome is depicted in Figure 10.

Among consumers who have a High valuation for a given advertiser, the advertiser cannot identify which of these consumers also have a High valuation for the rival advertiser. Interestingly, in equilibrium, the advertiser’s best strategy is to assume all such consumers have a High valuation for the rival advertiser as well, and bid \( V_{H,H} \) for them. This is because their own exclusive visitors cannot be targeted by their rival advertiser with ads with just first-party information. This implies that a advertiser can serve ads to their own exclusive visitors at no cost by placing any positive bid.

Both advertisers place identical bids for consumers who have visited them both. This bid is also the true value of advertising to such consumers for the advertisers. Hence, such consumers see ads from either advertiser with equal probability, while the advertisers do not make any profit from advertising to such consumers. Consequently, all of a advertiser’s profit is generated entirely from consumers who visit only one advertiser.

**Third-party Information Equilibrium**

Advertisers have no first-party information about consumers who have not visited them and cannot place a bid to target them with ads. However, for these non-visitors, advertisers can acquire information from third-party tracking firms which provide a signal for whether a consumer is a
potential consumer. Thus, with third-party information, advertisers are able to identify consumers who have not visited them but have visited their rival. The third-party would classify them as being a High type consumer. Note that among their own visitors, the advertisers still cannot distinguish between consumers who have also visited their rival advertiser and those who have not.

The information partition for each advertiser is now \{(f = L, t = L), (f = \phi, t = L), (f = \phi, t = H), (f = H, t = H)\}, where \(f\) refers to first-party information, \(t\) refers to third-party information, and they equal \(\phi\) when the advertiser does not have that type of information about the consumer.

We have \((f = L, t = L)\) for consumers who have a Low valuation for both advertisers. \((f = \phi, t = L)\) for non-consumers who are not of interest to either advertiser. \((f = \phi, t = H)\) for consumers who have only visited the rival advertiser. This is the additional segment that can be targeted with third-party information. \((f = H, t = H)\) for visitors with a High valuation for the advertiser.

For now, we assume the third-party signal is perfectly accurate. In the next section, we consider the implications of relaxing this assumption.

**Proposition 7.** When \(\{k, \theta\} \in R1 \cup R2\), consumers who have visited only one advertiser see ads by the other advertiser. When \(\{k, \theta\} \in R3\), consumers who have visited only one advertiser see ads from the same advertiser. For any value of \(k\) and \(\theta\), consumers who have visited both advertisers have an equal probability of seeing either advertiser’s ad.

**Proof.** With third-party information, advertisers have complete information about consumers who have a Low valuation for them. If such consumers have visited them, then they also have a Low valuation for the rival advertiser. If they have not visited them, yet they are signalled as High value by the third-party, then they are exclusive visitors of the rival advertiser. In either case, the weakly dominant strategy for advertisers is to bid their true value for these consumers.

When consumers are classified as having a High type for a advertiser from first-party information, then they can either have a High or Low valuation for the rival advertiser. Since the advertiser cannot distinguish between these two types of consumers, it must bid the same amount for both these segments. We know that the advertising value function for consumers who have a High valuation for both advertisers is \(V_{H,H}\), and it is \(V_{H,L}\) for consumers who have a High valuation for them and a Low valuation for the rival.

In region \(R1\), it is not possible for a advertiser to target their own exclusive visitors since \(V_{H,L} < V_{L,H}\). As such, there is zero cost incurred for own exclusive visitors. Thus, it is a weakly dominant strategy for advertisers to bid \(V_{H,H}\) for consumers who have a High valuation for them.
In region $R3$, each advertiser targets their own exclusive visitors. Bidding $V_{H,H}$ for consumers who have a High valuation for them is a weakly dominant bidding strategy in this region.

In region $R2$, without additional assumptions that narrow the set of feasible bidding strategies, there is no equilibrium to this game due to the discontinuous nature of the profit functions. If one advertiser bids $V_{H,H}$, the other advertiser will bid $V_{L,H} + \zeta$, where $\zeta > 0$ is a small positive number. However, in that case, the first advertiser can bid $V_{L,H} + \zeta/2$ to maximize its own profit. And so on. Clearly, there is no simple equilibrium in this game.

However, we can refine the equilibrium solution concept by imposing the condition that the advertisers do not allow a loss to be made from advertising to any consumer segment. Bidding higher than $V_{H,H}$ creates the possibility of generating a loss from consumers who have a High valuation for both advertisers. This would be a feasible strategy only if advertisers knew the proportion of their High valuation consumers who have a Low valuation for the rival advertiser, that is, only if advertisers knew the value of $\gamma$. In this case, the advertiser could compare the profit from the consumers who have a Low valuation for their rival and the loss from the consumers who have a High valuation for their rival and bid higher than $V_{H,H}$ if this difference is greater than zero. But advertisers do not the value of $\gamma$ since ad auctions occur in real-time with individual bid requests being received sequentially without the advertisers knowing the overall distribution of Low and High valuation consumers. Therefore, without knowing the value of $\gamma$, advertisers will always bid lower than or equal to $V_{H,H}$. By not bidding higher than their valuation for that consumer segment, the advertiser avoids making a loss from that segment. Therefore, with the additional condition that the equilibrium bid is lower than or equal to $V_{H,H}$, we have the equilibrium solution that advertisers bid $V_{H,H}$.

Combining the above strategies, we can write the equilibrium bidding strategy by a advertiser when both advertisers have third-party information as a function of their classified type as:

\[
\begin{align*}
    b^*(f = L) &= V_{L,L} \\
    b^*(f = \emptyset, t = H) &= V_{L,H} \\
    b^*(f = \emptyset, t = L) &= 0 \\
    b^*(t = H) &= V_{H,H}
\end{align*}
\]

where $b^*(f = t_1, t = t_2)$ is the equilibrium bid for consumers who have been classified as $t_1$ and $t_2$ type by first-party and third-party information respectively. $f = \emptyset$ implies there is no first-party information about the consumer.
From Lemma 1, if \( \{k, \theta\} \in R1 \cup R2 \), then \( V_{H,H} < V_{L,H} \). Therefore each advertiser serves ads to their rival’s visitors. If \( \{k, \theta\} \in R3 \), then \( V_{H,H} > V_{L,H} \). Therefore, in region \( R3 \), each advertiser serves ads to their own visitors. Both advertisers bid identical amounts to serve ads to consumers who have visited both advertisers. Thus, consumers have equal probability of being served ads by either advertiser. Q.E.D

With third-party information, advertisers are able to identify and bid for their rival’s exclusive visitors. From Lemma 1, in Region \( R1 \), advertisers find it more valuable to target their rival’s exclusive visitors at the expense of targeting their own exclusive visitors. This is a key result of our paper. This is driven by the informative effect of ads. Advertising to non-visitors is valuable because ads allow such consumers to consider a advertiser they otherwise would have not. \( k \in R1 \) ensures that this newly considered advertiser has only a slightly lesser valuation for non-visitors compared to the visited advertiser, which makes these ads more valuable.

In Region \( R2 \), even though serving ads to one’s own exclusive visitors is more valuable, incomplete information leads to an equilibrium where ads are served to the rival’s exclusive visitors. This is because neither advertiser can identify precisely which of their visitors have also visited their rival. In Region \( R3 \), consumer’s valuation for their preferred advertiser is high enough and/or ads are persuasive enough to make it more valuable for advertisers to serve ads to their own exclusive visitors. The advertising outcomes are depicted in Figure 11.

**Asymmetric Information Equilibrium**

When one advertiser has third-party information, while the other has only first-party information, we refer to it as an asymmetric information case. The equilibrium can be summarized in the following proposition:

When only one advertiser has third-party information, three possible equilibrium outcomes can result depending on which region the parameters \( k \) and \( \theta \) lie in:

**Case 1:** \( \{k, \theta\} \in R1 \)

- Advertising Outcome: Consumers who have visited only one advertiser’s website are served ads by the advertiser with third-party information, irrespective of which advertiser they have visited. Consumers that have visited both advertiser’s websites have an equal probability of seeing either advertiser’s ads.
Figure 11: Advertising Outcome with third-party information

- **Advertiser Surplus:**
  \[
  \pi_{13} = 0 \quad (28)
  \]
  \[
  \pi_{31} = \gamma(1 - \gamma)(V_{H,L} + V_{L,H} - V_{H,H}) \quad (29)
  \]

**Case 2:** \(\{k, \theta\} \in R2\)

- **Advertising Outcome:** Consumers with a High valuation for the advertiser with only first-party information see ads by that advertiser, irrespective of their valuation for the other advertiser. Consumers that have visited only the advertiser with third-party information see ads only by that advertiser. Consumers that have a Low valuation for both advertisers have an equal probability of seeing either advertiser’s ads.

- **Advertiser Surplus:**
  \[
  \pi_{13} = \gamma(1 - \gamma)(V_{H,L} - V_{L,H}) \quad (30)
  \]
  \[
  \pi_{31} = \gamma(1 - \gamma)V_{H,L} \quad (31)
  \]

**Case 3:** \(\{k, \theta\} \in R3\)

- **Advertising Outcome:** Consumers that have visited only one advertiser are served ads by that advertiser. Consumers that have visited both advertisers have an equal probability of being served ads by either advertiser.
Figure 12: Advertising Outcome with asymmetric information

- Advertiser Surplus:

\[ \pi_{13} = \gamma(1 - \gamma)(V_{H,L} - V_{L,H}) \]  
\[ \pi_{31} = \gamma(1 - \gamma)V_{H,L} \]

The advertising outcomes are depicted in Figure 5.

The advertiser with third-party information can identify consumers that have visited only its rival and bid their true value \( V_{L,H} \). Visitors that have been classified as having a Low value for a advertiser are inferred to also have a Low valuation for the rival, since if this were not the case, they would not have visited that advertiser. Irrespective of whether the advertiser has first or third-party information, each advertiser bids the same true value for such consumers \( V_{L,L} \).

For visitors that have been classified by first-party information as having a High value for the advertiser with third-party information, the advertiser knows that the rival cannot identify and bid for their own exclusive visitors. This is because we assume each advertiser knows what information is possessed by the other advertiser. Note that for consumers with a High value classification from the first-party information, the advertiser cannot distinguish between its own exclusive visitors and those that have also visited its rival. It must bid a single value for both types of consumers. Now,
because it knows that the rival does not possess third-party information, it can bid any positive value for its own exclusive visitors and win their ad auction. Therefore, the advertiser with third-party information bids the true value of consumers that have visited both advertisers for all visitors with a High value assigned by first-party information.

Bids by the advertiser with third-party information are as follows:

\[
\begin{align*}
    b_t^*(f = L, t = L|C_L) &= V_{L,L} \\
    b_t^*(f = L, t = H|C_B) &= V_{L,H} \\
    b_t^*(f = H|C_A ∪ C_H) &= V_{H,H}
\end{align*}
\]

For the advertiser with only first-party information, visitors with a Low valuation as classified by first-party information are bid their true value, as discussed earlier \( (V_{L,L}) \). When the first-party classification is High for a consumer, the bid depends on which region \( k \) and \( \theta \) lie in. Using Lemma 1, we have the following bids in equilibrium:

\[
\begin{align*}
    b_f^*(f = L) &= V_{L,L} \\
    b_f^*(f = H) &= \begin{cases} 
        V_{H,H}, & \text{if } \{k, \theta\} \in R_1 \cup R_3 \\
        V_{H,L}, & \text{otherwise}
    \end{cases}
\end{align*}
\]

When \( \{k, \theta\} \in R_1 \), advertiser with first-party information will bid \( V_{H,H} \). This is a weakly dominant strategy.

In region \( R_1 \), consumers value their less-preferred advertiser almost as much as their preferred advertiser, and/or ads are less persuasive. So, the advertiser with third-party information will serve its ads to all consumers that visited only one advertiser, even if they only visited its rival advertiser. Such consumers value their non-visited advertiser almost as much as their visited advertiser but without ads from the non-visited advertiser, would not have included the non-visited advertiser in their consideration set. This makes advertising to rival advertiser’s exclusive visitors more valuable for the advertiser with third-party information.

In region \( R_2 \), consumers value their preferred advertiser somewhat more than their less-preferred advertiser, and/or ads are moderately persuasive. This makes the value of advertising to consumers that have visited only the rival advertiser less attractive to the advertiser with third-party information. In equilibrium, the advertiser with the first-party information actually serves ads to more consumers than the advertiser with third-party information, but gets lower surplus from advertising.
In region $R3$, consumers value their preferred advertiser much more than their less-preferred advertiser, and/or ads are very persuasive. So, even for the advertiser with third-party information, it is not very valuable to serve ads to consumers that have only visited its rival, since such consumers have a Low valuation for them, and have a low probability of switching their purchase decision to them. Thus, in this case, each advertiser serves ads to their own exclusive visitors while consumers that are indifferent between the two advertisers have an equal probability of being served ads from either advertiser.

Advertiser surplus for the advertiser with first-party information is lower due to the fact that the advertiser with third-party information can compete to show ads to their exclusive visitors but not vice versa. In $R1$, the advertiser with first-party information loses this consumer segment to the advertiser with third-party information. In $R2$ and $R3$, while the advertiser with first-party information does serve ads to consumers that have only visited it, it does so at a higher cost due to the competitive bids from the advertiser with third-party information. Both of these effects result in lower advertiser surplus for the advertiser with first-party information compared to the advertiser with third-party information.

Thus, in each case, we have

$$\pi_{13} < \pi_{31} \quad (34)$$

**Third-party Information Acquisition**

In this section, we analyze a advertiser’s decision to acquire third-party information. We model this as a static game where each advertiser has an option of acquiring third-party information. We represent the surplus from advertising for each advertiser in a normal form of this game in Table 3.

<table>
<thead>
<tr>
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<th>FPI</th>
<th>TPI</th>
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<tr>
<td>FPI</td>
<td>$(\pi_f, \pi_f)$</td>
<td>$(\pi_{ft}, \pi_{tf})$</td>
</tr>
<tr>
<td>TPI</td>
<td>$(\pi_{tf}, \pi_{ft})$</td>
<td>$(\pi_t, \pi_t)$</td>
</tr>
</tbody>
</table>

where $\pi_f$ and $\pi_t$ represent the equilibrium advertiser profit when both advertisers have only first-party information and third-party information respectively; and $\pi_{ft}$ ($\pi_{tf}$) represents the advertiser advertising surplus when given advertiser has only first party (third-party) information and the other advertiser has third-party (only first-party) information. We discuss the asymmetric equilib-
rium and the resulting advertiser profits in Appendix C. The relevant result here is that $\pi_{ft} < \pi_{tf}$ always.

**Appendix C - Proofs**

**Proof of Lemma 1:** The boundary between regions $R1$ and $R2$ is given by values of $(k, \theta)$ that solve $V_{H,L}(k, \theta) = V_{L,H}(k, \theta)$. The boundary between regions $R2$ and $R3$ is given by the solution to $V_{H,L}(k, \theta) = V_{H,H}(k, \theta)$. The shape of the regions is shown in Figure 4. There is a fourth region where $V_{L,H} < V_{H,H} < V_{H,L}$, but this region is so narrow that it is not visible in Figure ??1. To keep our analysis simple, we focus on only regions $R1$, $R2$ and $R3$. The equilibrium and results in region $R4$ are identical to those in $R3$.

**Proof of Proposition 1:**

The advertiser’s profit under different information scenarios and regions are given by Equations (9), (10) and (11). Clearly, $\pi_{[R3]} > \pi_{f}$. From Equations (9) and (10), we have

$$\pi_{[R1,R2]} > \pi_{f}$$

$$\iff V_{H,L}(k, \theta) < V_{L,H}(k, \theta) - V_{H,H}(k, \theta)$$

$$\iff k < k^* \& \theta < \theta^*$$

where $(k^*, \theta^*)$ are solutions of $V_{H,L}(k, \theta) = V_{L,H}(k, \theta) - V_{H,H}(k, \theta)$.

**Proof of Proposition 2:**

There is no change in advertising behavior between first and third-party information equilibria in $R3$, and hence there is no change in consumer surplus in $R3$.

Consider a function $f(x) = \frac{x}{1+x}$. Note that $\frac{\partial f}{\partial x} = \frac{1}{(1+x)^2} > 0$. Therefore, $x_1 > x_2 \implies f(x_1) > f(x_2)$.

In $R1$, we have from Lemma 1, $V_{H,L} < V_{L,H}$. This implies

$$e^{V_{H}+\theta} \over 1 + e^{V_{H}+\theta} < e^{V_{L}+\theta} \over 1 + e^{V_{L}+\theta} + e^{V_{H}}$$

$$\implies e^{V_{H}+\theta} > e^{V_{L}+\theta} + e^{V_{H}}$$

$$\implies e^{V_{H}+\theta} < e^{V_{L}+\theta} + e^{V_{H}}$$

$$\implies CS_f < CS_t$$

Therefore, consumer surplus increases with third-party information in region $R1$. Similarly, in region $R2$, we have $V_{L,H} < V_{H,L}$ which implies $CS_t < CS_f$ in region $R2$. 

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**Proof of Proposition 3:** Follows from Equations (18), (19) and (20).

**Proof of Proposition 4:**
Define total surplus as:

$$TS = 2\pi + CS + Pub\_Rev$$  \hspace{1cm} (39)

In region $R1$, from Proposition 2, we have $\Delta CS^{R1} = CS^{R1}_t - CS^{R1}_f > 0$. Thus, we have

$$TS^{R1}_t - TS^{R1}_f = 2[\pi^{R1}_t - \pi^{R1}_f] + [Pub\_Rev^{R1}_t - Pub\_Rev^{R1}_f] + \Delta CS^{R1}$$  \hspace{1cm} (40)

$$= 2\gamma(1 - \gamma)[V_{L,H} - V_{H,L}] + \Delta CS^{R1} > 0$$  \hspace{1cm} (41)

since $V_{L,H} > V_{H,L}$ in $R1$.

In region $R2$, from Proposition 2, we have $\Delta CS^{R2} = CS^{R2}_t - CS^{R2}_f < 0$. Thus, we have

$$TS^{R2}_t - TS^{R2}_f = 2[\pi^{R2}_t - \pi^{R2}_f] + [Pub\_Rev^{R2}_t - Pub\_Rev^{R2}_f] + \Delta CS^{R2}$$  \hspace{1cm} (42)

$$= 2\gamma(1 - \gamma)[V_{L,H} - V_{H,L}] + \Delta CS^{R2} < 0$$  \hspace{1cm} (43)

since $V_{L,H} < V_{H,L}$ in $R2$.

Finally, in region $R2$, from Proposition 2 we have $\Delta CS^{R3} = CS^{R3}_t - CS^{R3}_f = 0$. Thus, we have

$$TS^{R3}_t - TS^{R3}_f = 2[\pi^{R3}_t - \pi^{R3}_f] + [Pub\_Rev^{R3}_t - Pub\_Rev^{R3}_f] + \Delta CS^{R3}$$  \hspace{1cm} (44)

$$= 2\gamma(1 - \gamma)[V_{L,H} - V_{L,H}] + \Delta CS^{R3} = 0$$  \hspace{1cm} (45)

**Proof of Proposition 5:** Follows from Equation (26) and Proposition 2.