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Abstract

We consider multiple activities with imperfectly correlated cash flows and zero operational synergies. These activities may be separated financially (through separate incorporation or the use of special purpose entities), allowing each to optimize its financial structure, or merged into an entity with a single optimal financial structure. In the presence of corporate taxes and default costs, mergers can realize purely financial synergies due to reduced risk and the potential for greater leverage, as suggested by Lewellen (1971). But his conclusion that mergers always create positive financial synergies is incorrect: the loss of separate debt/equity choices may exceed the gain from risk reduction. Characteristics of activities that gain from combination or separation are identified, and the magnitudes of gains and their distribution between stockholders and bondholders are examined. The results have direct application to mergers, spin-offs, and the use of structured finance techniques such as project finance and asset securitization.

JEL Classification Codes: G32, G34

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1. Introduction

Decisions that alter the scope of the firm are amongst the most important faced by management, and amongst the most studied by academics. Mergers and spin-offs are classic examples of such decisions. More recently, equity carve-outs, joint ventures, and structured finance have been used alter the boundaries of the firm.\footnote{Structured finance techniques include project finance and asset securitization, often based on special purpose entities (SPEs). Project finance is defined in Kleimeir and Megginson (1999) as “limited or non-recourse financing of a new project through the establishment of a vehicle company (separate incorporation).” Special purpose vehicles or entities (SPVs/SPEs) are typically trusts or other legal structures that have separate, limited-recourse financing secured by the cash flows and assets of a specific activity, and are the building blocks for various structured finance methods. Skarabot (2002) examines structured finance techniques such as asset securitization.}

Positive or negative \textit{operational synergies} are often cited as a prime motivation for decisions that change the scope of the firm. A rich literature addresses the roles of economies of scope and scale, market power, incomplete contracting, property rights, and agency costs in determining the optimal boundaries of the firm.\footnote{Classic studies include Coase (1937), Williamson (1975), and Grossman and Hart (1986). See also Holmstrom and Roberts (1998) and the references cited therein.}

But operational synergies are not the focus of this paper. Rather, the analysis considers purely \textit{financial synergies}. To facilitate this focus, it is assumed that the operational cash flow of the combined activities is simply the sum of the operational cash flows of the separate activities, at the time the decision on scope is made. If operational synergies exist, their effect will be \textit{incremental} to the financial synergies examined here.\footnote{The assumption that operational cash flows are additive rules out “agency” costs related to changes in firm scope, and parallels the Modigliani- Miller (1958) assumption that cash flows are invariant to changes in capital structure. While much of subsequent capital structure theory has dealt with relaxing this M-M assumption, it still stands as the base from which extensions are made.}

In a Modigliani-Miller [1958] world without taxes, bankruptcy costs, informational asymmetries, or agency costs, there will be no purely financial synergies. Capital structure is irrelevant to total firm value. But in a world with taxes and bankruptcy costs, capital structure matters. And changes in the scope of the firm, which will affect optimal capital structure, will also create purely financial synergies. Financial synergies will influence the optimal boundaries of startup firms as well as extant firms: should an entrepreneur with two or more projects structure them as a single firm, or as separate firms?
Financial separation of activities—whether through separate incorporation or special purpose entity—allows each activity to have its appropriate capital structure, with an optimal amount of debt and equity. Each firm or special purpose entity has limited liability. And its debt has recourse only to its own cash flow and assets. If activities are notably different in terms of risks, default costs, and taxes, their optimal leverage ratios may vary significantly. In contrast, a firm merging multiple activities must have a single leverage ratio.\(^4\) And equity in the merged firm loses the multiple bankruptcy shelters of separated activities.

Countering these financial advantages to separate incorporation is the fact that, when activities’ cash flows are imperfectly correlated, risk can be lowered via a merger or initial consolidation. Lower risk will reduce expected default costs and interest rates on extant debt. Leverage can potentially be increased, with greater tax benefits, as first suggested by Lewellen (1971). However, Lewellen asserts that the purely financial benefits of mergers \textit{always} are positive, a widely-cited contention that is shown below to be incorrect.

This paper examines four questions:

(i) How do the financial advantages of merging versus separating activities depend upon the riskiness and correlation of cash flows of the activities, as well as upon payout rates, tax rates, default costs, and relative size?

(ii) How important are the \textit{magnitudes} of potential financial synergies?

(iii) How does the existence of outstanding debt affect the ability of current equity holders to capture the gains of mergers or spinoffs?

(iv) How does the opportunity to hedge risks affect the desirability of merging activities?

We derive formulas that provide quantitative estimates of financial synergies, as well as identifying the capital structures that maximize these synergies. While operational synergies may exceed financial synergies in many mergers or spin-offs, financial synergies can be sizable in situations that are identified in this paper. And financial synergies are often the \textit{principal} reason cited for the use of structured financing techniques, both in the

\(^4\) The analysis considers a single class of debt for each separate firm. Multiple seniorities of debt within a firm will not affect the results as long as all classes have potential recourse to the firm’s total assets.
“true-sale” or the synthetic form.⁵ We illustrate these points with examples of project finance and asset securitization.

2. Previous Work

A number of theoretical and empirical studies have considered the effects of conglomerate mergers and diversification. Lewellen (1971) first considers purely financial synergies. He correctly argues that combining imperfectly correlated assets, while not value-enhancing per se, has a coinsurance effect: it reduces the risk of default, and thereby increases a measure of debt capacity. He then asserts that higher debt capacity (by his measure) will lead to greater optimal leverage and will create greater value for the merged firm.⁶ In Section 6, we quantify the coinsurance effect, and show that it does not always overcome the disadvantage of forcing a single financial structure onto multiple activities. When the latter dominates, separation rather than merger creates greater total value.

Higgins and Schall (1975), Kim and McConnell (1977), and Stapleton (1982) consider the distribution of merger gains between extant bondholders and stockholders, and note that bondholders may gain at shareholders’ expense. Like Lewellen’s work, these papers do not have an explicit model of optimal capital structure before and after mergers. Therefore their conclusions about the sign and distribution of financial synergies are incomplete. We develop a model with optimal capital structure in Sections 1 - 4, and consider distributional issues in Section 7.

Many papers have considered the potential impact of firm scope on operational synergies. Flannery, Houston, and Venkataraman (1993) consider investors who issue external debt and equity to invest in risky projects, and must decide upon separate or joint incorporation of the projects. Like us, they find that joint incorporation is more valuable when project returns have similar volatility and lower correlation. But

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⁵ “True-sale” (cash) structured finance methods are based on sale of the assets to an SPE and primarily serve funding purposes. Synthetic methods involve portfolio swaps to transfer the risk to an SPE. In both cases there is a financing motivation for the (true or synthetic) separation of the assets. In principle, funded or unfunded separation can achieve the same results.

⁶ Lewellen (1971, p. 533). Stapleton (1982) uses a slightly different definition of debt capacity but also states that mergers always have a positive effect on total firm value.
operational rather than financial synergies are key to their conclusions: investment and therefore the cash flows of the merged firm will be different than the sum of investments and cash flows of the separate firms. John (1993) uses a related approach to analyze spin-offs, while Chammanur and John (1996) use managerial ability and control issues to explain project finance and the scope of the firm.

Inderst and Müller (2003) consider optimal firm scope using an optimal contracting approach. Unlike our work, they assume nonverifiable cash flows and investment decisions. Separation of activities may be financially desirable, but only if there are increasing returns to scale for second-period investment. While cash flows are additive in the first period, investment levels and therefore operational cash flows in the second period depend upon whether activities are merged (centralized) or separated (decentralized).

Several papers have focused on negative operational synergies arising from managerial agency costs, including the misallocation of internal capital, that lead to a “conglomerate discount.” There is a rich but still inconclusive empirical literature testing whether such a discount exists.

Without denying the potential importance of managerial agency costs and other possible sources of operational synergies, we focus on the purely financial effects resulting from alternative boundaries of the firm. To obtain quantitative as well as qualitative results, we extend the continuous-time models of Leland (1994) and Goldstein, Ju, and Leland (2001). These models provide relatively simple closed-form solutions for valuation and optimal capital structure, and in contrast with two-period models, allow for (potentially) infinite firm life, random time to default, and endogenous default decisions.

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7 Inderest and Müller (2003, p. 1048).
9 Martin and Sayrak (2003) provide a useful summary of this research. Berger and Ofek (1995) suggest conglomerate discounts on the order of 15%. This and related results have been challenged on the basis that diversified firms trade at a discount prior to diversifying: see Lang and Stulz (1994), Campa and Kedia (2002), and Graham, Lemmon, and Wolf (2002). Mansi and Reeb (2002) conclude that the conglomerate discount of total firm value is insignificantly different from zero, when debt is priced at market rather than book value.
10 Continuous time models also can be extended to include dynamic adjustments of leverage see Fischer, Heinkel, and Zeckner (1989), Leland (1998), and Goldstein, Ju, and Leland (2001). However, we do not consider dynamic leverage here. Both positive and negative financial synergies are likely to be somewhat magnified in such dynamic debt models, and optimal leverage levels are smaller. Morellec (2002) uses a continuous time model to consider mergers as exchange options. Positive operational synergies are assumed but are not explicitly examined; the focus is on the dynamic evolution of firm values and the timing of mergers.
3. The Model

3.1 Individual Activities’ Cash Flows and Values

Consider two separate activities that generate cash flows (“EBIT”) \( X_i \), \( i = \{1,2\} \), each following a lognormal dynamic process

\[
dX_i = \mu_i X_idt + \sigma_i X_idZ_i,
\]

where \( \mu_i \) is the (constant) instantaneous risk-adjusted expected growth rate of the cash flow, i.e. the drift rate under the risk neutral measure\(^{11} \), \( \sigma_i \) is the (constant) volatility of the growth rate, and \( dZ_i \) is the increment to a joint Brownian motion with (constant) correlation \( \rho \), i.e.

\[
E(dZ_1dZ_2) = \rho dt.
\]

Extending this notation beyond two dimensions is straightforward.

We assume a riskless asset exists with a constant rate of return \( r \). Under these assumptions, Goldstein, Ju, and Leland (2001) show that the total value to all claimants (including the government) of the operational cash flows at time \( t \) will be

\[
F_i(X_i(t)) = \frac{X_i(t)}{r - \mu_i}.\(^{12}\)
\]

The value of the unlevered activity or firm, after paying corporate income taxes, will be given by

\[
V_i(X_i(t)) = \frac{(1 - T_i)X_i(t)}{r - \mu_i},
\]

where \( T_i \) is the effective tax rate for the separate activity \( i \).\(^{13}\) Observe that the payout rate (dividends and coupons, assuming the entire after-tax cash flow is distributed) on this market value will be

\(^{11}\) Alternatively, \( \mu_i \) is the expected growth rate of cash flows in an economy where all investors are risk neutral.
\(^{12}\) This is simply the “Gordon growth formula”, with \( \mu_i \) the risk-adjusted growth rate rather than the actual growth rate.
\(^{13}\) We assume a constant corporate tax rate applied to all levels of \( X_i(t) \geq 0 \), when there is no corporate debt. More complex modeling would allow for nonlinear tax schedules, e.g. as in Leland (1994, Section VIA).
The dynamics of value $V_i$ under the risk-neutral measure are given by Ito’s Lemma, with

$$dV_i = \mu_{V_i} V_i dt + \sigma_{V_i} V_i dZ_i$$

where $\mu_{V_i} = r - \delta_i = \mu_i$ and $\sigma_{V_i} = \sigma_i$. Inclusive of the payout rate $\delta_i$, the total (risk-adjusted) expected rate of return to owning shares with value $V_i$ is $\delta_i + \mu_{V_i} = r$, as it must be under the risk-neutral measure.

### 3.2 The Purely-Additive Merged Firm

When the activities of each separate firm or activity are combined into a single firm with no operating synergies at time $t$, the cash flow $X_M$ of the merged firm will satisfy

$$X_M(\tau) = X_1(\tau) + X_2(\tau), \quad \text{for all } \tau \geq t.$$  

It follows from (7) that there is no change in the value of operational cash flows at the time of merger:

$$F_M(X_M(t)) = F_1(X_1(t)) + F_2(X_2(t)).$$

Note, however, that $X_M(\tau) = X_1(\tau) + X_2(\tau)$ is not a lognormal dynamic process. From Ito’s lemma,

$$\frac{dX_M(\tau)}{X_M(\tau)} = h_1(\tau) \frac{dX_1(\tau)}{X_1(\tau)} + h_2(\tau) \frac{dX_2(\tau)}{X_2(\tau)},$$

where the weights

$$h_i(\tau) = \frac{X_i(\tau)}{X_1(\tau) + X_2(\tau)}, \quad i = \{1,2\}$$

on the individual return processes are stochastic and equal to the relative cash flows for $\tau \geq t$. Thus the drift and volatility of the return process (9) are stochastic, and the valuation formula (3) does not hold for $i = M$. This
reflects the fact that the sum of two lognormal return processes is not itself a lognormal return process. \(^{14}\) The analysis of Leland (1994) no longer applies, and optimizing capital structure would require numerical techniques.

### 3.3 An Alternative Dynamic Process for Merged Firm Cash Flows

Consider now a return process for the post-merger cash flows that evolves as a constant rather than stochastic mix of the individual return processes, with constant weights \(w_i\):

\[
\frac{dX_M(\tau)}{X_M(\tau)} = w_1 \frac{dX_1(\tau)}{X_1(\tau)} + w_2 \frac{dX_2(\tau)}{X_2(\tau)} \quad \tau \geq t.
\]

This implies that future merged cash flows follow a lognormal dynamic process, with

\[
\frac{dX_M(\tau)}{X_M(\tau)} = \mu_M dt + \sigma_M dZ_M, \quad \tau \geq t
\]

\[
\mu_M = w_1 \mu_1 + w_2 \mu_2;
\]

\[
\sigma_M = (w_1^2 \sigma_1^2 + 2 \rho \sigma_1 \sigma_2 w_1 w_2 + w_2^2 \sigma_2^2)^{1/2}.
\]

\[
X_M(t) = X_1(t) + X_2(t).
\]

Note that (15), in contrast with (7), holds at the time of merger \(t\) only. After the merger, the cash flow process follows (11) rather than (9). \(^{15}\)

The weights \(w_i\) that determine the merged firm’s dynamics (11) still need to be determined. Equations (7) and (8) imply that the merger creates neither operational cash flow synergy nor operational value change when the merger occurs. Similarly, in addition to the absence of cash flow synergy (15), there should be zero

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\(^{14}\) An exception to this assertion is the special case where \(X_2\) is a constant multiple of \(X_1\). In this case, the weights \(h_i(\tau)\) are constant, the processes’ rates of return have equal drift and volatility, and the correlation is 1.

\(^{15}\) There are two ways to consider process (11)-(14). First, it can be viewed as an approximation to the process (9) - (10). This is examined in Appendix B. Alternatively, it can be viewed as the exact process that results from a (self-financing) post-merger management that continuously re-allocates resources between divisions to keep them in constant relative value weights \(w_i\) as specified in (16) below. The process (11)-(14) is similar to the return process of a portfolio of two assets, each following a lognormal dynamic process, which is continuously rebalanced to keep asset proportions constant. A portfolio managed in this way will be self-financing, and will follow a lognormal process given by equations (11)-(14), where \(w_i\) is the value proportion of asset \(i\) in the portfolio. Note that, in assuming individual firms’ cash flows (which most likely combine the cash flows of different sub-activities) are themselves lognormal, we may have (implicitly) already made this assumption at the individual firm level.
operational value change at the time of the merger, i.e. equation (8) must also hold at \( t \) for the process (11). This condition uniquely determines the weights \( w_i \):

**Proposition 1: (Relative Value Weighting)**

Given the dynamic specifications (12)–(14), equation (8) will hold and there will be a zero operational value change at the time of merger if and only if the weights \( w_i \) are proportional to the relative values of the individual activities’ operational cash flows at the time of the merger:

\[
(16) \quad w_i = \frac{F_i(X_i(t))}{F_i(X_1(t)) + F_2(X_2(t))}, \quad i = \{1, 2\}.
\]

**Proof:** In Appendix A.

Proposition 1 shows that the constant weights \( w_i \) that determine the drift \( \mu_M \) and volatility \( \sigma_M \) of the post-merger cash flows in (11) depend on the relative values \( F_i \) at the time of the merger \( t \), and are different from the random weights \( h_i(\tau) \) in equation (9) that are based on relative cash flows at \( \tau \geq t \). Finally, recall from (4) – (6) that the drift \( \mu_M \) of value \( V_M \) is the same as the drift \( \mu_V \) of combined cash flow \( X_M \), and the payout rate is \( \delta_M = r - \mu_M \).

**Appendix B** compares the distribution of future unlevered merged-firm operational cash flows (or values) that results from the process (11) with weights (16), with the distribution of future cash flows (or values) that results from the purely-additive process (9) with weights (10). It can be seen in Figures 7A and 8A that the probability distributions of the future cash flows or values are quite similar, for the parameters in Base Cases 1 and 2 specified in Tables 1 and 4 below, respectively.

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16 If \( \mu_1 = \mu_2 \), \( w_i = h_i(t) \) and the relative value and cash flow weights will be the same at the time of merger (although \( h_i(\tau) \) will subsequently move randomly). If \( \mu_1 \neq \mu_2 \), the expected future growth rate of combined cash flows will exceed the growth rate based on current cash flow weights \( h_i(t) \), since more-quickly growing activities will have a greater expected cash-flow weight in the future. Thus it is appropriate to use current value weights \( w_i(t) \), which through (3) will accord greater weights to activities with higher growth rates.

17 An alternative approach would be to specify a joint normal (rather than lognormal) dynamic process for the evolution of cash flows \( X_i \), in which case the future cash flows of the additive merged firm would also have a normal distribution. However, such a specification would allow unboundedly negative cash flows and values, and the authors are unaware of closed-form solutions for optimal capital structure in this case.
Proposition 2: (No After-tax Value Synergies)

Given the dynamic process (12)-(14) for merged cash flows, with weights as in equation (16), there will be a zero after-tax operational value change at the time of merger, i.e.

\[ V_M(X_M(t)) = V_1(X_1(t)) + V_2(X_2(t)), \]

iff the corporate income tax rate on the merged firm’s cash flows, \( T_M \), satisfies

\[ T_M = w_1T_1 + w_2T_2 \]

for arbitrary tax rates \( T_i \) on the individual activities’ cash flows.

Proof: The proof is immediate by substitution, using (4), (12), and (18).

The tax rate for the merged activities given by (18) will be termed the value-neutral tax rate. Higher (lower) tax rates on the merged firm’s cash flows will produce lesser (greater) financial synergies than those reported in subsequent tables, which are based on value-neutral tax rates.

4. Optimal Capital Structure for Separate and Merged Activities

Because after-tax operating values follow equation (6), our framework is consistent with the environment assumed by Leland (1994), where firms can issue debt with long (infinite, in fact) maturity. The separate firms (indexed \( i = 1, 2 \)) and merged firm (\( i = M \)) are described by the following variables:

\[
\begin{align*}
V_i(\tau) & \quad \text{value of after-tax operational cash flows at time } \tau, \text{ given by equation (4)} \\
\delta_i & \quad \text{(constant) payout rate as fraction of after-tax asset value } V_i \\
\mu_i & \quad \text{(constant) risk-adjusted drift of } \frac{dV_i(\tau)}{V_i(\tau)}, \quad \tau \geq t \\
\sigma_i & \quad \text{(constant) volatility of } \frac{dV_i(\tau)}{V_i(\tau)}, \quad \tau \geq t \\
\alpha_i & \quad \text{fractional costs to after-tax value } V_i \text{ in the event of default,}
\end{align*}
\]

---

18 Absent agency costs, Leland and Toft (1996) show that firms will prefer to issue debt of maximal maturity. Dynamic extensions of the debt model here are considered by Fischer, Heinkel, and Zechner (1989), Leland (1998), and Goldstein, Ju, and Leland (2001). Such models generally create somewhat higher benefits to leverage, and somewhat lower (initial) leverage. Both positive and negative financial synergies are likely to be somewhat magnified in such models.

19 Recall from equation (6) and following \( \mu_{Vi} = \mu_i = r - \delta_i \), and \( \sigma_{Vi} = \sigma_i \).
\( P_i \) principal value of long-term debt issued by firm,
\( C_i \) coupon flow rate to long-term (infinite life) debt, chosen initially so the debt sells at par,
\( R_i \) rate of interest on debt issued by firm \( (C_i/P_i) \),
\( T_i \) tax rate on operational cash flows less coupon payments,
\( v_i(\tau) \) total value of the levered firm at time \( \tau \),
\( D_i(\tau) \) value of the firm’s debt at time \( \tau \),
\( E_i(\tau) \) value of equity at time \( \tau \),
\( V_{Bi} \) (constant) value which, if reached, the firm optimally defaults on debt.

We proceed under the assumption that firms optimize their financial structure. The coupon will equate market and principal value of debt at time \( t \). Leland (1994) derives closed-form expressions for the optimized (starred) variables at \( \tau = t \), when \( V_{Bi} \) is endogenously chosen. These include

\[
C_i^* = V_i(1 + x_i)h_i \right)^{1/x_i}
\]
\[
D_i^* = C_i^* \left(1 - k_i \right) \left[\left(1 + x_i \right)h_i\right]^{-1} / r \quad ( = P_i^*)
\]
\[
v_i^* = V_i \left[1 + (T_i / r) \left(1 + x_i \right)h_i\right]^{-1/x_i} \left(x_i / \left(1 + x_i \right)\right)
\]
\[
E_i^* = v_i^* - D_i^*
\]
\[
V_{Bi}^* = V_i \left(m_i / h_i\right)^{\delta_i}
\]

where
\[
h_i = \left(1 + x_i + \alpha_i \left(1 - T_i\right)x_i / T_i\right)m_i
\]
\[
k_i = \left(1 + x_i - (1 - \alpha_i) \left(1 - T_i\right)x_i\right)m_i
\]
\[
m_i = \left[\left(1 - T_i\right)x_i / r \left(1 + x_i\right)\right]^{x_i} \left(1 + x_i\right)^{-1}
\]
\[
x_i = \left(r - \delta_i - 0.5\sigma_i^2 \right) + \left((r - \delta_i - 0.5\sigma_i^2)^2 + 2\sigma_i^2 r \right)^{1/2} / \sigma_i^2.
\]

\[\text{---}^{20}\text{Alternative assumptions are possible to analyze, e.g. “firms issue half the optimal amount of debt.” No single alternative seems readily defensible, however, and we do not pursue such analysis here.}\]

\[\text{---}^{21}\text{\(V_{Bi}\) is chosen to maximize equity value and satisfies a smooth pasting condition. We also assume that absolute priority holds: bondholders receive all remaining value in default, after default costs. Leland (1994, Section VI.C) allows this assumption to be relaxed.}\]
Purely financial synergies at the time of merger $t$ will be positive if and only if $v_M^* > v_1^* + v_2^*$, i.e. the optimally leveraged merged firm has greater value than the sum of values of the optimally-leveraged separate firms.

5. Measuring Financial Synergies

Financial synergies can be measured in a number of ways. An obvious measure of merger benefits is

**Measure 1. The percentage increase (or decrease) in total firm value:**

$$\frac{(v_M^* - (v_1^* + v_2^*))}{(v_1^* + v_2^*)}.$$ 

Competition may induce an acquiring firm to bid an amount that reflects total synergies, including financial synergies.\(^22\) We adopt the convention that Firm 1 is the acquiring firm, realizing none of the benefits, and Firm 2 is the acquired or target firm, realizing all the benefits of the merger. This suggests that an alternative measure of financial synergies is the percentage premium over current value of Firm 2 that Firm 1 could pay:

**Measure 2. The percentage increase in the value of the acquired firm:**

$$\frac{(v_M^* - (v_1^* + v_2^*))}{v_2^*}.$$ 

A final yardstick, Measure 3, considers the case of an optimally-leveraged target firm. It is assumed that all benefits will accrue to the stockholders of the target firm.\(^23\)

**Measure 3. The percentage increase in the equity value of the acquired firm:**

$$\frac{(v_M^* - (v_1^* + v_2^*))}{E_2^*}.$$ 

Note Measure 1 $< $ Measure 2 $< $ Measure 3 when these measures are positive.

Separation of activities is optimal when $v_M^* < v_1^* + v_2^*$. Measure 1 (or its negative) also measures the benefit of separation, as a percentage of the total value after separation. Measure 2 (or its negative) reflects the

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\(^22\) Numerous studies (e.g. Andrade, Stafford, and Mitchell (2001)) suggest that acquiring firms realize little or no increases in their market value. Firms being acquired, however, realize substantial value premiums.

\(^23\) The distribution of merger benefits between stockholders and bondholders is considered in Section 7.
benefit as a fraction of the value of the assets divested. An alternative measure of separation benefits would be their fraction of equity value prior to separation, \( \frac{-(v_M^* - (v_1^* + v_2^*))}{E_M^*} \).

6. Identifying the Sources and Magnitudes of Financial Synergies

6.1 Base Case 1: Symmetric Firms

*Base Case 1* assumes two symmetric firms, identical in size and in the nature of their cash flows, but imperfectly correlated. The objective is to determine the magnitude of financial synergies for a merger of equals, and then see how these synergies depend on the firms’ correlation, payout and tax rates, volatilities, and default costs. *Table 1* lists parameters for Base Case 1. Firm risk \( \sigma \) of 25% and payout (including to debt holders) rate \( \delta \) of 5% of firm value are chosen to approximate a “typical” mature firm. Default costs \( \alpha \) of 20% of default value fall within a range of recent empirical estimates.\(^{24}\) The correlation between firms’ cash flows is 0.20, and the net tax rate of 20% reflects both the corporate income tax rate and the tax advantage of equity.\(^{25}\) Tax benefits for the optimally-leveraged firms are consistent with recent studies, as are recovery rates and yield spreads.\(^{26}\)

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<th>TABLE 1: BASE CASE 1 PARAMETERS</th>
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<td><strong>Firm 1</strong></td>
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<td>Cash flow (EBIT)</td>
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<td>Correlation</td>
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<td>Risk free rate</td>
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\(^{24}\) Anderson and Betker (1995) estimate a median \( \alpha = 34.7\% \), and Gilson (1997) estimates \( \alpha = 45.3\% \). Andrade and Kaplan (1998) suggest a range of default costs from 10% to 20%, based on studies of firms undergoing highly leveraged takeovers (HLTs). However, firms subject to HLTs are likely to have lower-than-average default costs, since high leverage is more likely to be optimal for firms with this characteristic.


\(^{26}\) Graham [2000] estimates that leveraging the firm optimally increases value by about 8.2 percent. Our model implies maximal leverage benefits of about 8.4 percent in Base Case 1. Yield spreads and recovery rates are consistent with bond ratings in the BB-BBB range: See Elton et al. (2001, Tables I and III).
Table 2 calculates financial effects of mergers using the formulas listed in Section 4.

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<thead>
<tr>
<th>TABLE 2: SEPARATE AND MERGED FIRM VARIABLES</th>
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<tr>
<td>Post-tax Operational Value</td>
</tr>
<tr>
<td>V</td>
</tr>
<tr>
<td>80.00</td>
</tr>
<tr>
<td>80.00</td>
</tr>
<tr>
<td>160.00</td>
</tr>
<tr>
<td>Post-tax Cash Flow</td>
</tr>
<tr>
<td>(1-T)X</td>
</tr>
<tr>
<td>4.00</td>
</tr>
<tr>
<td>4.00</td>
</tr>
<tr>
<td>8.00</td>
</tr>
<tr>
<td>Optimally-levered Value</td>
</tr>
<tr>
<td>v*</td>
</tr>
<tr>
<td>86.74</td>
</tr>
<tr>
<td>86.74</td>
</tr>
<tr>
<td>175.37</td>
</tr>
<tr>
<td>Equity</td>
</tr>
<tr>
<td>E*</td>
</tr>
<tr>
<td>33.99</td>
</tr>
<tr>
<td>33.99</td>
</tr>
<tr>
<td>63.64</td>
</tr>
<tr>
<td>Coupon</td>
</tr>
<tr>
<td>C*</td>
</tr>
<tr>
<td>3.64</td>
</tr>
<tr>
<td>3.64</td>
</tr>
<tr>
<td>7.03</td>
</tr>
<tr>
<td>Debt</td>
</tr>
<tr>
<td>D*</td>
</tr>
<tr>
<td>52.75</td>
</tr>
<tr>
<td>52.75</td>
</tr>
<tr>
<td>111.74</td>
</tr>
<tr>
<td>Leverage Ratio</td>
</tr>
<tr>
<td>L*</td>
</tr>
<tr>
<td>60.8%</td>
</tr>
<tr>
<td>60.8%</td>
</tr>
<tr>
<td>63.7%</td>
</tr>
<tr>
<td>Interest Rate on Debt</td>
</tr>
<tr>
<td>R*</td>
</tr>
<tr>
<td>6.91%</td>
</tr>
<tr>
<td>6.91%</td>
</tr>
<tr>
<td>6.29%</td>
</tr>
<tr>
<td>Value Triggering Default</td>
</tr>
<tr>
<td>VB*/V*</td>
</tr>
<tr>
<td>33.7%</td>
</tr>
<tr>
<td>33.7%</td>
</tr>
<tr>
<td>38.4%</td>
</tr>
<tr>
<td>Implied Recovery Rate</td>
</tr>
<tr>
<td>RR*</td>
</tr>
<tr>
<td>40.9%</td>
</tr>
<tr>
<td>40.9%</td>
</tr>
<tr>
<td>44.0%</td>
</tr>
<tr>
<td>SYNERGIES</td>
</tr>
<tr>
<td>Measure 1</td>
</tr>
<tr>
<td>1.09%</td>
</tr>
<tr>
<td>Measure 2</td>
</tr>
<tr>
<td>2.18%</td>
</tr>
<tr>
<td>Measure 3</td>
</tr>
<tr>
<td>5.56%</td>
</tr>
</tbody>
</table>

Financial synergies are positive, and of modest magnitude. Measure 3, which always yields the highest value, indicates that Firm 1 could make an offer for Firm 2’s equity at a premium exceeding 5.5 percent. As measured relative to the total value (debt and equity) of the two firms, Measure 1, the value increase from purely financial synergies is 1.09 percent.

Table 2 indicates that there are two sources of financial synergy in this case, both related to lower operational risk after the merger. Optimal leverage is slightly increased after the merger: from 61% to 64%. And the merged firm can issue its optimal debt amount at a lower credit spread than was required by the separate firms: 129 basis points, versus 191 basis points. Note that, despite the increase in the debt amount (and therefore in leverage), the annual coupon C* paid by the optimally-levered merged firm has actually declined relative to the total coupons paid when the firms are separate. This reflects the lower yield spread of the less-risky merged debt.

We now consider the sensitivity of the three benefit measures to changes in the parameters, still assuming the merged firms are symmetric. Figure 1 examines the financial benefits to merger as a function of the firms’ cash flow correlation $p$. Observe that the financial benefits decline from about 1.5% for Measure 1 (7.7% for Measure 3) to zero, as correlation increases from zero to one. (Benefits would be higher if cash flows were negatively correlated).
Table 3 considers the effect on merger benefits of altering the parameters for both the separate firms. The middle row in each cell is the base case parameter. Financial benefits are never negative in these cases, because the separate firms are by assumption symmetric, and there is no advantage to having different leverage ratios through separate incorporation. For the symmetric case, Lewellen’s (1971) assertion that there will be positive purely financial synergies to mergers is correct. The magnitudes of such synergies are modest but not inconsequential. In the “extreme” case at the bottom of Table 3, purely financial benefits are 3.27% by Measure 1, and 38.1% by Measure 3.

**TABLE 3: COMPARATIVE STATICS OF MERGER BENEFITS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Measure 1</th>
<th>Measure 2</th>
<th>Measure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Payout Rate δ</strong></td>
<td>2.00%</td>
<td>1.68%</td>
<td>3.37%</td>
<td>9.35%</td>
</tr>
<tr>
<td></td>
<td>5.00%</td>
<td>1.09%</td>
<td>2.18%</td>
<td>5.56%</td>
</tr>
<tr>
<td></td>
<td>8.00%</td>
<td>0.64%</td>
<td>1.27%</td>
<td>3.06%</td>
</tr>
<tr>
<td><strong>Volatility σ</strong></td>
<td>10.00%</td>
<td>1.17%</td>
<td>2.34%</td>
<td>8.24%</td>
</tr>
<tr>
<td></td>
<td>25.00%</td>
<td>1.09%</td>
<td>2.18%</td>
<td>5.56%</td>
</tr>
<tr>
<td></td>
<td>40.00%</td>
<td>0.86%</td>
<td>1.72%</td>
<td>3.91%</td>
</tr>
<tr>
<td><strong>Default Cost α</strong></td>
<td>10.00%</td>
<td>0.95%</td>
<td>1.89%</td>
<td>5.99%</td>
</tr>
<tr>
<td></td>
<td>20.00%</td>
<td>1.09%</td>
<td>2.18%</td>
<td>5.56%</td>
</tr>
<tr>
<td></td>
<td>30.00%</td>
<td>1.19%</td>
<td>2.37%</td>
<td>5.21%</td>
</tr>
<tr>
<td><strong>Tax Rate T</strong></td>
<td>10.00%</td>
<td>0.58%</td>
<td>1.17%</td>
<td>2.13%</td>
</tr>
<tr>
<td></td>
<td>20.00%</td>
<td>1.09%</td>
<td>2.18%</td>
<td>5.56%</td>
</tr>
<tr>
<td></td>
<td>30.00%</td>
<td>1.57%</td>
<td>3.14%</td>
<td>10.14%</td>
</tr>
<tr>
<td><strong>Most Extreme:</strong></td>
<td>3.27%</td>
<td>6.53%</td>
<td>38.13%</td>
<td></td>
</tr>
<tr>
<td>$\rho = 0$, $\delta = 2%$, $\sigma = 10%$, $\alpha = 30%$, $T = 30%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Low correlation, low payout rates (implying high risk-adjusted growth rates), low volatility, and high tax rates are consistent with greater financial benefits to merger. These environments are generally consistent with higher initial optimal leverage ratios. But an exception to this principle is default costs. The risk reduction from diversification particularly benefits high-default cost firms and increases Measure 1 benefits, although Measure 3 may be reduced because the optimal leverage ratio is less, implying a larger $E_2$. 

15
6.2. Base Case 2: Asymmetric Firms

Differences between firms will also affect potential financial synergies. Recall that if firms (or activities) are dissimilar, it may be advantageous to separate them financially. This is because capital structures can be optimized separately. Base Case 2 considers two firms that differ in optimal leverage when separate. Firm 1 is assumed to be a “mature” firm, with modest growth, P/E ratio, risk, and default costs. Firm 2 is a smaller, with higher growth rate (implying low payouts), P/E ratio, risk, and default costs.\textsuperscript{27} Parameters are given in Table 4.

<table>
<thead>
<tr>
<th>TABLE 4: BASE CASE 2 PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
</tr>
<tr>
<td>EBIT Value $F$</td>
</tr>
<tr>
<td>Cash flow (EBIT) $X$</td>
</tr>
<tr>
<td>Payout $\delta$</td>
</tr>
<tr>
<td>Drift $\mu$</td>
</tr>
<tr>
<td>Volatility $\sigma$</td>
</tr>
<tr>
<td>Tax Rate $T$</td>
</tr>
<tr>
<td>Default Costs $\alpha$</td>
</tr>
<tr>
<td>Correlation $\rho$</td>
</tr>
<tr>
<td>Risk free rate</td>
</tr>
</tbody>
</table>

As with drift, volatility, and tax rates, the default cost fraction of the merged firm is assumed to be the value-weighted combination of the individual default cost fractions. Table 5 computes key variables for Base Case 2.

<table>
<thead>
<tr>
<th>TABLE 5: BASE CASE 2 SEPARATE AND MERGED FIRM VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
</tr>
<tr>
<td>Post-tax Operational Value $V$</td>
</tr>
<tr>
<td>Post-tax Cash Flow $(1-T)X$</td>
</tr>
<tr>
<td>Optimally-levered Value $v^*$</td>
</tr>
<tr>
<td>Equity $E^*$</td>
</tr>
<tr>
<td>Coupon $C^*$</td>
</tr>
<tr>
<td>Debt $D^*$</td>
</tr>
<tr>
<td>Leverage Ratio $L^*$</td>
</tr>
<tr>
<td>Interest Rate on Debt $R^*$</td>
</tr>
<tr>
<td>Value Triggering Default $VB^*/V$</td>
</tr>
<tr>
<td>Implied Recovery Rate $RR^*$</td>
</tr>
</tbody>
</table>

SYNERGIES

Measure 1 | 0.56% |
Measure 2 | 6.69% |
Measure 3 | 9.22% |

\textsuperscript{27} Andrade, Mitchell, and Stafford (2001) estimate the median target firm was about 11.7% the size of the acquiring firm, based on a sample of mergers over the period 1973-1998. We assume 10% for Base Case 2.
Notice the substantial difference in the initial capital structures of the two firms. The optimal leverage and implied recovery rate for Firm 1 are considerably higher than for Firm 2, which has both high volatility and high default costs. Despite these differences, there are still positive financial synergies to merging the firms, due to the lower risk created through diversification.

But the synergies become negative when the correlation increases. Figure 2 indicates that separation is preferable to merger when $\rho > 0.60$. A conglomerate firm that combines the two activities would benefit from a spin-off in this case. This serves as a counter-example to Lewellen’s (1971) contention that the purely financial synergies of mergers are always positive.

Benefits of merger by Measure 1 are smaller here than those of Base Case 1. But the benefits relative to the size of the acquired Firm 2 are quite substantial. Measures 2 and 3 are more than twice those in Base Case 1. The acquiring Firm 1 could bid a substantial premium (9.22%) above the current equity value of the target Firm 2 based on financial synergies alone. Andrade, Stafford, and Mitchell (2001) find that, over the period 1973-1998, the 3-day abnormal returns to acquired firms is 16% of equity value (24% over a longer window that includes close of the merger). They also find that the return to acquiring firms is slightly negative but insignificantly different from zero, consistent with using Measure 3. Measure 3 in Base Case 2 suggests that financial synergies could explain a substantial proportion –between 35% and 55% –of realized merger gains.

Tables 6 and 7 provide comparative statics in Base Case 2, for both Firm 1 (the acquiring firm) and Firm 2 (the acquired or target firm). The middle row of each cell is the base case parameter value.

---

28 If the target firm is 50% of the size of the acquiring firm, synergies are negative if the correlation exceeds 0.30.
29 A counter-example can also be created that does not use the approximation of the true process (9)-(10) by the process (11)-(16). Consider the case when Firm 2’s operational cash flows are a constant multiple of Firm 1’s. The cash flows will have identical growth (and payout) rates with equal variance and a correlation $\rho = 1$, implying the sum of the cash flows will be exactly lognormal (and the weights $h(\tau) = w$, for all $\tau \geq t$). In this case, a difference in the separate optimal leverage ratios resulting from a difference in default costs $\alpha$, or tax rates $T$, will create strictly negative financial synergies to merger, with no offsetting advantage to diversification (since the correlation is one). By continuity of the merged firm’s risk and therefore value function in $\rho$, financial synergies will be negative with correlations close to (but strictly less than) one.
### TABLE 6: COMPARATIVE STATICS OF MERGER BENEFITS - BASE CASE 2

**ACQUIRING FIRM (1)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Measure 1</th>
<th>Measure 2</th>
<th>Measure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payout Rate $\delta$</td>
<td>2.00%</td>
<td>0.57%</td>
<td>7.00%</td>
<td>9.66%</td>
</tr>
<tr>
<td></td>
<td>5.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>8.00%</td>
<td>0.36%</td>
<td>4.24%</td>
<td>5.84%</td>
</tr>
<tr>
<td>Volatility $\sigma$</td>
<td>5.00%</td>
<td>0.32%</td>
<td>3.97%</td>
<td>5.47%</td>
</tr>
<tr>
<td></td>
<td>10.00%</td>
<td>0.36%</td>
<td>4.24%</td>
<td>5.84%</td>
</tr>
<tr>
<td></td>
<td>40.00%</td>
<td>-0.09%</td>
<td>-1.07%</td>
<td>-1.48%</td>
</tr>
<tr>
<td>Default Cost $\alpha$</td>
<td>10.00%</td>
<td>0.42%</td>
<td>5.12%</td>
<td>7.06%</td>
</tr>
<tr>
<td></td>
<td>20.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>30.00%</td>
<td>0.64%</td>
<td>7.60%</td>
<td>10.48%</td>
</tr>
<tr>
<td>Tax Rate $T$</td>
<td>10.00%</td>
<td>0.61%</td>
<td>7.57%</td>
<td>10.43%</td>
</tr>
<tr>
<td></td>
<td>20.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>30.00%</td>
<td>0.51%</td>
<td>5.90%</td>
<td>8.13%</td>
</tr>
</tbody>
</table>

### TABLE 7: COMPARATIVE STATICS OF MERGER BENEFITS - BASE CASE 2

**TARGET FIRM (2)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Measure 1</th>
<th>Measure 2</th>
<th>Measure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payout Rate $\delta$</td>
<td>0.00%</td>
<td>0.76%</td>
<td>9.05%</td>
<td>12.88%</td>
</tr>
<tr>
<td></td>
<td>2.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>4.00%</td>
<td>0.35%</td>
<td>4.26%</td>
<td>5.71%</td>
</tr>
<tr>
<td>Volatility $\sigma$</td>
<td>20.00%</td>
<td>0.59%</td>
<td>6.74%</td>
<td>13.39%</td>
</tr>
<tr>
<td></td>
<td>40.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>60.00%</td>
<td>0.16%</td>
<td>1.94%</td>
<td>2.38%</td>
</tr>
<tr>
<td>Default Cost $\alpha$</td>
<td>40.00%</td>
<td>0.64%</td>
<td>7.56%</td>
<td>13.44%</td>
</tr>
<tr>
<td></td>
<td>80.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>100.00%</td>
<td>0.48%</td>
<td>5.82%</td>
<td>7.51%</td>
</tr>
<tr>
<td>Tax Rate $T$</td>
<td>10.00%</td>
<td>0.09%</td>
<td>0.95%</td>
<td>1.07%</td>
</tr>
<tr>
<td></td>
<td>20.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>30.00%</td>
<td>0.91%</td>
<td>11.86%</td>
<td>20.42%</td>
</tr>
</tbody>
</table>
Beyond the negative relationship between correlation and merger benefits seen in Figure 2, observe that

(i) Lower payout rates $\delta$ for both the acquiring and target firms favor mergers. This is because there are greater benefits to risk reduction via merger when optimal leverage ratios are relatively high, which in turn is the case when payout rates are low. For the same reason, higher payout activities are more likely to benefit from remaining separate.

(ii) Relatively high or very low ratio of volatilities between the acquiring and target firm tends to disfavor merger. Differing volatility, *ceteris paribus*, leads to differing optimal leverage ratios for the separate activities, which in turn favors keeping them separate.\(^{30}\)

*Figure 3* demonstrates this result for very high or very low volatilities of the target Firm 2.

(iii) Higher default costs of Firm 1, and lower default costs of Firm 2, tend to favor merger. This is because the two default costs started at very different levels in Base Case 2. Greater similarity of default costs implies closer initial leverage ratios for the separate activities, which in turn enhances the benefits to merger.

(iv) Lower tax rates to Firm 1 and higher tax rates to Firm 2 would offset the initial disparities in optimal leverage ratios, again increasing the benefits of merger.

The optimal leverage of the merged activities typically is greater than the value-weighted average of the optimal leverages of the separate activities, when mergers are favorable.\(^{31}\) The theory therefore suggests that financial leverage will modestly increase following mergers, consistent with empirical results.\(^{32}\)

The financial synergies in the preceding tables are based on the assumption that the tax rate of the merged firm is a value-weighted combination of the tax rates of the separate activities (see Proposition 2). Additional benefits (costs) to merging occur if tax rates for the merged firm are lower (higher) than the rates assumed.

---

\(^{30}\) The reader may have noticed from Table 6 that synergies are negative when the volatility of Firm 1 equals the volatility of Firm 2, yet positive when $\sigma_1 < \sigma_2$. But in fact the optimal leverage ratios of the separate firms will be closer when the volatilities differ, since the volatility difference offsets the assumed difference in default costs $\alpha$.

\(^{31}\) While true in the majority of cases, there are minor exceptions. And financial benefits (Measure 1) are not monotonically related to the change in leverage resulting from a merger.

\(^{32}\) See, for example, Kim and McConnell (1977) and Ghosh and Jain (2000).
Figure 4 demonstrates that relative size matters. Size affects both the diversification achieved, and the cost of losing separate financial structures. Both the magnitude and sign of the measures of financial synergy may depend on size. For the parameters of Base Case 2, Measure 1 synergies increase as the size of the target rises to about 20% of the acquiring firm—the “ideal” target size for merger in this case. As the target becomes larger, synergies decrease and become negative when the target size exceeds 55% of the potential acquirer. Thus relative size, in addition to the characteristics of the separate firms, affects merger desirability.

7. Who Realizes the Benefits of Mergers?

If equity holders are to capture the full financial benefits of merger, bondholders must be prevented from participating in the gains. This poses no problem for start-up firms, where there is no initial debt. The entrepreneur decides whether to incorporate activities jointly or separately, and subsequently levers the firm(s) optimally by issuing debt at a price that fairly reflects firm risk(s). If the separate firms have extant debt that is callable at par, or otherwise can be retired at a price that reflects pre-merger risks, again the separate firms’ bondholders will not participate in windfall gains from a merger.

Potential problems arise when the extant debt of the separate firms is noncallable and is assumed by the post-merger firm. In Base Case 1, extant bond prices will rise dramatically after the merger. The optimally-levered merged firm value will rise by $1.89 (1.09% by Measure 1) relative to the optimally-levered separate firms. But the extant debt of the separate firms will jump in value by $9.07 after the merger, reflecting the lower risk of the merged firm. Shareholders will suffer a substantial loss of value in this case, 4.0% by Measure 1, and 10.6% of the initial equity values of the separate firms. Value-enhancing mergers may be rejected by shareholders, similar to the underinvestment created by the “debt overhang” problem discussed by Myers (1977). Even in the extreme case considered in Table 3, the total benefits of the merger ($5.91), barely exceed bondholder

---

33 Higgins and Schall (1977), Kim and McConnell (1977) and Stapleton (1982) consider transfers to bondholders when total leverage does not change. They do not consider optimal financial structure explicitly.

34 This is the increase in value of the bonds continuing to pay the same total coupons as paid before merger ($C_1^* + C_2^*$). Some debt may subsequently be retired to reduce the coupon to $C_M^*$ (see Table 2). This could benefit extant bondholders even more than $9.07, if the debt buy-back is at a price that reflects the lower optimal leverage.
gains ($5.87). Base Case 2 also leads to bond price increases ($1.10) that exceed merger benefits ($0.55), and this holds for all parameter values in Tables 6 and 7. 35

These results underscore the reasons for callable debt. Such debt enables corporations to reduce or eliminate windfall gains to bondholders that could prevent the firm from pursuing valuable opportunities. Alternatively, the “leakage” of value to bondholders can be reduced when the firm uses short-term debt, since subsequent debt rollovers will be issued at a price that reflects post-merger risks. But short-term debt limits the potential gains from leverage: Leland and Toft (1996) show that optimal leverage ratios, and consequent tax benefits, are considerably reduced by the use of short-term debt.

8. Structured Finance

Firms’ increasing use of project finance, asset securitization, and other structured financing tools raises the question: why? Structured financing is claimed beneficial both for activities with high-risk cash flows (e.g., from major investment projects) and activities with low-risk cash flows (e.g., from assets that are securitized). Purely financial reasons are often cited for using off-balance-sheet financing for both types of activities. But little formal analysis has examined the validity of such claims. 36

In our framework, structured finance represents an off-balance-sheet financial separation of new or extant activities from the Parent Firm. The tools developed previously can be used to examine the types of activities that may benefit from structured finance. Below we examine two particular forms of structured finance: project finance, and asset securitization.

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35 These results are generally consistent with the findings of Mansi and Reeb (2002), who conclude that although the value of equity in conglomerate mergers drops, there is a significant increase in the value of debt above par (book value). They find that the net change in total value is insignificantly different from zero.

36 Chemmanur and John (1996) examine several of the issues considered here. But their focus is not on the cited purely financial synergies, but rather on operational synergies resulting from differential managerial abilities across projects, and different benefits of control. Flannery et al. (1993) consider operational synergies related to underinvestment.
8.1 Project Finance

Project finance is characterized as the formation of a separate corporation (or SPE) to finance and manage a specific new investment project, rather than financing the project within the sponsoring (or “Parent”) firm.\textsuperscript{37} The Parent typically retains a substantial fraction (sometimes all) of the equity in the corporation formed to undertake the project. The formation of a separate financial entity ensures that debt financing for the project has recourse only to the project’s cash flows and assets. Large and risky investments with high cash flows such as oilfield development, power plants, or transportation infrastructure are regularly financed by this technique.\textsuperscript{38} Esty (2003) notes that investments using project finance are also characterized by high payout ratios and relatively low default costs.

Commonly-cited justifications for project finance include greater total financing ability, cheaper financing for assets that remain in the firm, and preserving core firm assets from bankruptcy risk. Berkovitch and Kim (1990) and Espy (2003) also mention the possible benefits of project finance in reducing the opportunity costs of underinvestment in large and risky projects. These benefits would be additional to the purely financial synergies that we consider here.

Our project finance example considers a Parent Firm (“Firm 1”) that is identical to Firm 1 in Base Case 2. The Investment Project (“Firm 2”) is large (50% of the size of the Parent Firm), has a substantial payout rate on operational value (8%), and similar default costs and corporate income tax rate (both 20%). Its cash flow has correlation of 0.20 with the Parent firm’s cash flow. The Parent Firm may finance the new project internally, or through project finance. By assumption, operational cash flows of the project are independent of how it is financed. Figure 5 plots the financial advantage of using project finance vs. internal finance, as the volatility of the project increases. It can be observed that project finance is preferred whenever the standard deviation of the project’s annual cash flow exceeds 24%. Higher risk brings greater benefits to project finance. When the annual standard deviation of the project’s cash flow is 50%, the benefits are significant: 4.2% of project value (Measure

\textsuperscript{37} Kleimeier and Megginson (1999), Esty (2002), and Esty (2003) provide further descriptions of project finance.\textsuperscript{38} See Esty (2002). Project finance has also been used for low risk projects, such as power plants with long-term fixed-price contracts. These cases can be explained by the results in Section 8.2 below.
or 6.9% of the Parent Firm’s pre-project equity value. The benefits of high-risk project finance are also increased if the project correlation, payout rate, and risk increases, or if the project tax rate or default costs fall.\textsuperscript{39}

Many of the commonly-cited reasons justifying project finance are valid for high-risk investments. Compared with internal financing, the use of project finance allows greater additional debt financing (54% of project value vs. 42%), and keeps the credit spread of the Parent firm at 50 bps (vs. 150 bps if the Parent firm uses internal finance). With project finance, the high risk and leverage (optimally) incurs a high credit spread on the debt issued (710 bps).

### 8.2 Asset Securitization

In recent years, financial and industrial firms have transferred trillions of dollars of mortgages, commercial loans, accounts receivables, corporate debt obligations, and other cash flow sources to special purpose vehicles or entities. These vehicles are bankruptcy-remote from the originating (or “Parent”) firm. Debt (typically in tranches with differing seniority) and equity (often termed the “residual tranche”) are then issued by the SPE, with the proceeds used to compensate the Parent firm for the assets transferred. As such, a bankruptcy-remote SPE with limited-recourse financing has the key features of a separate firm from our analytical perspective.\textsuperscript{40} We retain the zero operational synergy assumption: pre-tax cash flows from the assets transferred from parent to an SPE are not affected by the transfer itself. We also assume that securitized assets are financed by the issuance of a single class of debt, plus residual equity.

Asset securitization is frequently justified by the assertion that separate, low volatility assets offer lower cost financing and can be more effectively leveraged. Other reasons cited for securitization, which are not explored here, include flexibility in issuance of multiple debt classes (“tranching”), relaxation of capital constraints (for financial institutions), and reduced informational asymmetry, agency costs, taxes, and default costs.\textsuperscript{41}

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\textsuperscript{39} These changes will further decrease the optimal leverage of the project, relative to the optimal leverage of the Parent firm without the project.

\textsuperscript{40} Guarantees from a Parent firm to an SPE would reduce this parallel, and our analysis assumes such guarantees are not present. We also assume that there is no cross-default trigger between the Parent and the SPE.

\textsuperscript{41} See, for example, Greenbaum and Thakor (1987), Rosenthal and Ocampo (1988), and Lockwood, Rutherford, and Herrera (1996).
Securitized assets such as accounts receivable typically have relatively low cash flow volatility (compared with the firms’ other activities) and utilize high leverage. As an example of asset securitization (e.g., securitization of accounts receivable), we use Base Case 1 parameters for the Parent firm prior to securitization. We assume that the securitized assets have one-quarter the value of the Parent firm. The SPE containing the securitized assets has average payouts ($\delta_2 = 5\%$), relatively low default costs ($\alpha_2 = 5\%$), and a correlation with the Parent firm’s other cash flows of 0.50.

*Figure 6* plots the benefits of asset securitization versus keeping the assets inside the firm, as a function of the volatility ($\sigma_2$) of the securitized assets. The solid (green) line indicates the benefits of securitization as a percent of the value of the assets secured. This is simply the negative of Measure 2, as defined in Section 5. For annual cash flow volatilities of the secured assets that are less than 16%, securitization is desirable. At a 5% (2%) annual cash flow volatility, benefits amount to approximately 330 (560) basis points of the value of assets secured. If the Parent firm’s cash flows have a 30% annual volatility before securitization, the benefits rise to 410 (650) basis points.

These results validate the first justification cited above for asset securitization. When securitized cash flow has annual volatility of 5%, the SPE optimally uses 83% leverage and pays a credit spread on its borrowings of only 23 bps. After securitization, the Parent firm optimally reduces leverage from 61 percent to 54 percent, but pays a higher credit spread (280 bps vs. 190 bps) reflecting the higher risk of the remaining Parent activities. Securitization leads to a slightly greater total leverage (62% vs. 61% of value).

Both project finance and asset securitization can be desirable, despite their very different risk characteristics, precisely because the risks (and perhaps default costs) of the separated activities are significantly different from the risks of their Parent firms’ other activities. Separation is beneficial, once again because it allows substantial differences in the capital structure appropriate for the separate entities.
8.3 Who Realizes the Benefits of Structured Finance?

Parallel to our discussion of mergers, all value gains will flow to shareholders of the Parent Firm if extant debt is called (or otherwise retired) at its fair market value prior to project finance or asset securitization, and the firm is optimally re-levered thereafter. If the original Parent firm’s debt remains outstanding, the situation becomes more complex. The issues are different for project finance and asset securitization.

New activities are undertaken with project finance. Typically, most of the required capital investment is raised externally and the debt value of the Parent firm will not be significantly affected. In the example considered in Section 8.1, Parent firm debt has a yield spread of 50 bps both before and after project finance.

If the new project is undertaken internally, the change in extant debt value depends upon the seniority of newly-issued debt. If extant debt has the same seniority as new debt, its value will fall substantially. Yield spread on the Parent firm’s debt will rise from 50 bps to 150 bps and the market value of extant debt will drop by more than 15%. If extant debt is senior to new financing, its value may increase, exacerbating the “debt overhang” problem. As noted by Esty (2003), project finance can reduce the debt overhang problem.

Asset securitization, on the other hand, serves to reduce the investments directly held by the firm. Since the assets transferred are typically low risk, the remaining investments will tend to have higher risk. Even when some extant bonds of the Parent firm are retired to reduce debt to its new optimal amount, yield spreads will rise after securitization. For the example in Section 8.2, when securitized assets have 5% volatility, yield spreads on Parent firm bonds rise from 190 bps to 280 bps. Asset securitization therefore may reduce extant bondholder value, meaning that stock holders can realize benefits that exceed the purely financial benefits calculated above.

Because distributional issues depend on the seniority structure of debt, which is not part of our model, they are not examined further in this paper. Nonetheless, the distribution of gains does not affect the total size of the purely financial gains calculated previously—although it may affect shareholder willingness to undertake structured finance.

Thus far the analysis has not addressed the impact of hedging opportunities on the relative benefits of merger vs. separation. But because the risks of the separate and merged firms importantly condition our conclusions, it is reasonable to speculate that the opportunities to hedge may affect the results derived above. Here, some preliminary observations are offered, under the conditions that

(i) hedges are fairly priced, implying the net present value of hedge returns less hedge price is zero; and

(ii) the hedged cash flows of the separate and merged firms continue to follow a logarithmic dynamic process, as in equations (1) and (12).42

Under these two conditions, the analysis developed previously applies when $\sigma_1$ and $\sigma_2$ are the after-hedging risks of the two activities, $\rho$ is the correlation of the two hedged cash flows, and the hedging done after the merger is the same as the hedging done before the merger.43

Base Case 2 can be used to illustrate the impact of hedging. Assume a zero-cost hedging instrument is available to Firm 2 that reduces operational risk but does not change correlation. We consider two situations: where the hedging instrument can reduce $\sigma_2$ from 40% to 25%, and where it can reduce $\sigma_2$ from 40% to 5%.44

In the first situation, an independent Firm 2 can increase value by increasing leverage from 27.5% to 41.9% when the hedge is in place. But the merger of the hedged Firm 2 with Firm 1 will be even more valuable than before, creating an increase in value of 7.6% (by Measure 2) compared with 6.7% in the case without hedging. This can be observed in Figure 3. With hedging, the optimal leverage of Firm 2 is now closer to the optimal leverage of Firm 1. Independent capital structures are less useful, and the value increment to merging

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42 If risks were reduced proportionately by hedging, this condition would hold.

43 In the environments studied, the optimal zero-cost hedge will be chosen to reduce cash flow volatility by the maximum, since (ceteris paribus) the value of the levered firm will be greatest when $\sigma$ is minimized. (Froot and Stein’s (1998) Proposition 1 formalizes this result in a somewhat different context.) For fixed correlation $\rho$, the risk of the merged firm $\sigma_M$ will be minimized by minimizing the separate firm risks. Thus the hedging after the merger will remain unchanged. Our analysis could be extended to consider hedging instruments that also change the correlation between the hedged cash flows. Such considerations are beyond the scope of this paper, however.

44 Like Froot and Stein (1998), we assume that some risks remain after hedging. Although the cost of distress in their model is related to the potential need for costly outside financing rather than default costs, the motivation of their optimal capital structure is similar to ours.
rises. After the merger, the combined firm will wish to continue the hedge—if it did not, the combined firm’s optimally-levered value would fall by 0.33%.

In the second (admittedly extreme) situation, a separate Firm 2 will be able to increase its leverage to 88.5% if it hedges. Its optimally leveraged value will increase substantially. But now the merger will not be beneficial. Although the value of the merged firms rises relative to the non-hedged case, the value when the two firms are kept separate rises more. The extreme differences in initial leverage (resulting from the hedging opportunities of Firm 2) imply that the firms should be kept separate. Again this can be observed in Figure 3. The benefits to merger become negative when \( \sigma_2 < 8\% \). When \( \sigma_1 = 25\% \) (rather than 10%), the firms should remain separate whenever hedging can reduce Firm 2 volatility to less than 23%.

Thus the potential to hedge will affect the extent of purely financial synergies, and the optimal scope of the firm. Results are reasonably intuitive when hedging can reduce risks without changing correlation. Our approach is sufficiently general to explore the effects of hedges that also change correlation. But we do not pursue such an extension here.

10. Conclusions

We have developed quantitative measures of the purely financial synergies that result from combining (or separating) multiple activities. The model abstracts from operational synergies, without denying their potential importance. There is a value-maximizing financial scope to the firm in a world with taxes and default costs, and we have identified the factors that determine the optimal scope.

Contrary to Lewellen’s (1971) contention, purely financial synergies from mergers can be negative as well as positive. Combining activities into a single firm offers the advantage of risk-reduction from diversification. Keeping activities separate offers the advantage of different optimal capital structures and separate limited liability. A complex interaction of activities’ cash flow risks, correlation, payout rates, tax rates,
default costs, and the relative size determines the value-maximizing scope of the firm. Because hedging can modify risks, the advantages to merger or separation can be altered by risk-management opportunities.

Despite the complexity of the factors that determine optimal scope, some general principles can be ascertained. Activities whose separate optimal leverage ratios are quite similar, and whose cash flows have low correlation, are likely to realize financial benefits from merger. As a general (but not exact) rule, benefits will be positive if the optimal leverage of the merged firm exceeds the value-weighted average of the optimal leverages of separate firms. Financial synergies from mergers are generally quite modest. But they are nonetheless capable of explaining a substantial fraction of recent empirical estimates of merger gains, even if they may not serve as the primary motivation.

Activities whose separate optimal leverages are quite different, and whose cash flows are more positively correlated, will typically realize financial benefits from separation. This may occur through separate initial incorporation, through spin-offs, or structured finance. Purely financial considerations may well be the primary motivation for structured financing decisions, where operating synergies are less likely to be a prime motivating factor. Our theory predicts that large investments with high risk relative to the Parent firm will benefit from project finance—and empirical evidence suggests that many projects using this type of financing are large and risky. Similarly, the theory predicts that asset securitization will be desirable for assets with stable cash flows and low default costs relative to the Parent firm—again consistent with observed practice.

While our results identify the impact of changing firm scope on total value, the distribution of value changes remains an important topic for further examination. Our results, based on a single class of debt, suggest that extant debt holders may benefit more than the total value gains from a merger, leaving shareholders worse off. In other situations such as asset securitization, debt holders may suffer a loss in benefits. Debt models that explicitly consider seniority are needed to study more fully the distribution of financial benefits from altering firm scope.
FIGURE 3: Percentage Financial Benefit to Merger Case 2

FIGURE 4: Percentage Financial Benefit to Merger Case 2
FIGURE 5: PROJECT FINANCE
Percentage Financial Benefit to Project Finance

FIGURE 6: SECURITIZATION
Percentage Financial Benefit to Asset Securitization
References


APPENDIX A: Proof of Proposition 1:

**Sufficiency:**

\[
F_M(X_M(t)) = \frac{X_M(t)}{r - \mu_M} = \frac{X_1(t) + X_2(t)}{r - \mu_M} = \frac{X_1(t)}{r - \mu_1} + \frac{X_2(t)}{r - \mu_2} = F_1(X_1(t)) + F_2(X_2(t)),
\]

where the first line follows from equation (3) and equation (11), the second line follows from equation (14), the third line from equations (12) and (16) (with simplification), and the last line from equation (3).

**Necessity:** follows from the fact that the third line (and therefore the fourth line) will not equal the second line (and therefore the first line) whenever \(\mu_1 \neq \mu_2\) and (16) is violated.

\[\blacksquare\]
APPENDIX B. Comparison of the risk-neutral probability density function (p.d.f.) of the value of the merged firm with the p.d.f. of the sum of the separate firm values.

Figures 7A and 8A below compare the (risk-neutral) probability density functions (p.d.f.s) of firm value \( V \) 20 years\(^{45} \) after the time of merger (when initial value is normalized to 1), for

(a) the merged firm whose value \( V_M \) follows the lognormal process in equations (12) - (14) with weights (16) (the blue dotted line);

(b) the sum of the values of the two separate firms \( (V_1 + V_2) \), each of which follows a lognormal process (the solid red line).

The p.d.f. for case (a) is derived directly from the lognormal distribution. The p.d.f. for case (b) is computed numerically, and is the density function of the sum of two lognormal random variables. (Note from equation (4) that the p.d.f. for future cash flows is identical (when initial cash flow is normalized to 1) to the distribution of future value.)

We first examine Base Case 1 described in Tables 1 and 2. The default value for the merged firms is \( V_B = .384 \)

Figure 7A gives the risk-neutral p.d.f.s (a) and (b) for this case:

FIGURE 7A: Base Case 1: PDFs of Separate and Merged Firms After 20 Years

Note the p.d.f.s look quite similar. However, for debt value and optimal leverage purposes, it is the left tail (up to \( V_B \)) of the risk-neutral p.d.f. that is crucial. Note that the red p.d.f. lies slightly above the blue p.d.f. for low firm values.

\(^{45}\) 20 years (= 1/r) is the duration of (riskless) infinite-life debt, given a riskfree rate of 5%. Even though the duration of risky debt is shorter than 20 years, we err on the side of conservatism in using this longer horizon, as the p.d.f.s will be more similar at shorter horizons.
The numerically-calculated risk-neutral (cumulative) probability that the sum of values (solid red line) is less than the default value $V_B = .384$ after twenty years (27.6%) is somewhat higher than the corresponding risk-neutral probability (25.1%) for the lognormal merged value process (12) (dotted blue line). That is, the benefits of leverage determined in Base Case 1 and based on the merged firm’s process (a), reflect a slightly lower risk of default than would be the case if the merged firm’s cash flow were actually the sum of the two separate firms’ cash flows (b). This could lead to an overestimate of optimal leverage and the potential benefits of merger, when compared with the process (b).

A way to correct for this bias is to choose a slightly higher volatility than given by (14) for the merged firm’s assumed lognormal stochastic process (a), such that the risk-neutral probability of default (i.e. $V \leq V_B$) is the same as computed for the sum of the separate firms (27.6%). This is achieved by increasing the annual volatility of the merged process to 20.4%, rather than the 19.4% using weights (16). (Such an increase could be effected by using (16), but increasing the correlation between cash flows from 0.20 to 0.33). Since the drift term remains the same, it follows from equation (4) that the value of the firm at the time of merger is not affected by the assumed change in risk.

Figure 7B re-plots the p.d.f. of the merged firm (blue dotted line), with the higher volatility that gives equal risk-neutral probabilities of default. A slight leftward-shift can be observed, relative to that in Figure 7A. And the distributional fit is extremely good.

FIGURE 7B: Base Case 1: PDFs of Separate and Merged Firms After 20 Years

With the higher risk, the optimal amount of leverage is reduced from 63.7% to 63.1%, and Measure 1 benefits are slightly lower, 0.87% rather than the initially estimated 1.09%. Note that this adjustment is for a long time horizon—20 years. The volatility-matching adjustment will be smaller when shorter horizons are considered.

Figure 8A gives the p.d.f.s equivalent to Figure 7A, for the Base Case 2 considered in Tables 4 and 5. The default point $V_B = .507$. The p.d.f.s of the merged firms still conform quite closely, although there are considerable differences in the firms being merged.
FIGURE 8A: Base Case 2: PDFs of Separate and Merged Firms After 20 Years

Again, the left tail is crucial. The risk-neutral cumulative probability that $V \leq V_B = .507$ is slightly higher (10.1%) for the sum of values of the separate firms than the probability computed for the merged firm (9.1%).

Increasing the volatility for the merged firm to $\sigma_M = 10.8\%$ from 10.4% (as would result, for example, by raising the correlation of cash flows from 0.20 to 0.31) gives an equal risk-neutral probability (10.1%) that $V \leq V_B = .507$. With the higher volatility, optimal leverage for the merged firm falls from 70.0% to 69.5%, and the Measure 1 benefits to merger fall from 0.56% to 0.38%. Again the required volatility adjustment would be smaller if we matched default probabilities at a horizon less than 20 years.

Figure 8B shows the effect of the slight increase in volatility on the p.d.f. of the merged firm.

FIGURE 8B: Base Case 2: PDFs of Separate and Merged Firms After 20 Years
These examples suggest that the calculated financial benefits of mergers, based on the lognormal dynamic process (12) – (14) using weights (16), provide a close approximation to the benefits that would be derived under the value process of the (non-lognormal) sum of the individual firms’ values. Our approximation allows closed-form estimates of these financial benefits.