Aggregate Issuance and Savings Waves *

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Abstract

We use firms’ decisions in the cross-section about their sources and uses of funds in order to make inferences about the aggregate cost of external finance. The basic intuition is as follows: Firms which raise costly external finance can invest the issuance proceeds in productive capital assets, or in liquid financial assets with a low physical rate of return. If firms raise costly external finance and allocate some of the funds to liquid assets, either the cost of external finance is relatively low, or the total return to liquidity accumulation, including its value as a hedging asset, is particularly high. We construct and estimate a quantitative, dynamic model of firms’ financing and savings decisions. We then use the model’s predictions for variation in firm policies and implied cross sectional moments, along with empirical moments from Compustat, to infer the average cost of external finance per dollar raised in the US time series 1980-2010.

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I. Introduction

We propose and implement a method for using data on firms’ decisions in the cross-section about their sources and uses of funds in order to make inferences about the aggregate cost of external finance. The basic intuition is as follows: Firms which raise costly external finance can invest the issuance proceeds in productive capital assets, or in liquid financial assets with a low physical rate of return. If firms raise costly external finance and allocate some of the funds to liquid assets, either the cost of external finance is relatively low at that time, or the total return to liquidity accumulation, including its value as a hedging asset, is particularly high.

We explore this intuition theoretically and empirically by constructing a dynamic, quantitative model of firms’ financing and savings decisions in which both aggregate productivity, and the aggregate cost of external finance, vary over time. In the model, as in the data, firms typically finance investment with operating cash flows. However, when the cost of external finance is low, firms in our model raise external finance, invest some of the proceeds, and save the remainder of proceeds in liquid assets. As a result, when the aggregate cost of external finance is low, firms are more likely to both raise external finance, and to accumulate liquid assets. Consistent with this idea, we show that in the model and in the data, cross sectional moments describing the incidence of firms raising external finance, and importantly, the co-incidence of firms raising external finance and saving the proceeds, are informative about the aggregate cost of external finance. Aggregate issuance and savings waves coincide with a high correlation in the cross section between issuance and saving, and tend to occur when traditional empirical proxies indicate that the aggregate cost of external finance is low. Our study is aimed at providing an estimate of a “revealed preference” measure of the aggregate cost of external finance in the US time series from 1980-2010 based on this intuition and results from our quantitative model.

We begin by documenting three new stylized facts. The first is the strong positive correlation between issuance and savings at the aggregate level. For all but the very largest firms, the aggregate correlation between external finance raised and liquidity accumulation is 0.6. This high correlation is not due to some firms raising, and other firms saving. Conditioning on firms that raise external finance, the aggregate correlation increases to 0.74. The second stylized fact we develop is the strong relationship between firms’ issuance and savings decisions, and traditional proxies for the cost of external finance. We show that the cross sectional correlation between external finance raised and liquidity accumulated, $\rho_{is,ei}^{xs}$, tends to be low when the default spread, the tightness of lending standards, and consumer sentiment, indicate that external finance is particularly costly, and vice versa. We argue, then, using the results and intuition from our model, and the empirical relationship between $\rho_{is,ei}^{xs}$ and traditional proxies for the cost of external
finance, that firms’ behavior in the cross section contains useful information about the aggregate cost of external finance. Consistent with this, the third stylized fact we develop is that the difference in investment between high and low productivity firms (controlling for average productivity) is larger when \( \rho_{i,e}^{z_s} \) is high. This “difference in differences” result provides further support for \( \rho_{i,e}^{z_s} \) as a measure of the level of the cost of external finance. It also highlights the connection between our research strategy, and the large literature which uses differences in differences to identify the effects of supply shocks on corporate policies. As in that literature, we exploit cross sectional heterogeneity in our identification scheme. We further build on these prior results by using a structural model to enable us to extend our results to a broad sample and to aggregate implications.

Our structural model consists of a panel of firms which face idiosyncratic and aggregate productivity shocks, as well as aggregate shocks to the cost of external finance, and choose their desired stock of physical capital and liquid assets in order to maximize the present discounted value of payouts net of issuance costs. Firms in our model are risk neutral, but behave as if they are averse to the risk of having their sources of funds fall below their desired level of fund usage. Gross payouts are defined as after tax operating profits plus interest on liquid assets less investment in physical capital and liquid assets, and less investment adjustment costs. In our analysis, we pay careful attention to the role of capital adjustment costs, as well as external financing costs, in driving firm issuance, investment, and savings behavior. Our model features a convex cost of investment adjustment, a constant fixed cost of external finance, and a convex cost of external finance, the level of which is determined by an aggregate shock.

Aggregate issuance and savings waves arise when the aggregate cost shock is low. When the aggregate cost of external finance is low, firms which have high enough productivity issue external finance in order to take advantage of the relatively high net return to investment. However, the convex investment adjustment cost incentivizes firms to smooth investment over time. As a result, issuing firms save some of their proceeds in liquid assets in order to both smooth investment, and to avoid repeatedly paying the fixed cost of external finance. When the aggregate cost of external finance is high, high productivity firms will also want to invest, however because the return to investment net of financing costs is lower they will invest less. Moreover, since productivity is persistent over time, these firms will tend to be able to fund this lower level of investment using operating cash flows. And, finally, because investment adjustment costs are smaller at lower investment rates, these firms will be able to achieve a smooth enough investment policy without the use of liquid assets.

After developing our structural model and its intuition, we turn to estimating the main
parameters of interest using Simulated Method of Moments. With our estimated parameters in hand, we compare the model moments to the data. We show that the model does a good job of matching the main aggregate moments we document. The model generates aggregate issuance and savings waves in line with the data. The correlation between liquidity accumulation and external finance is 0.60 in the data. The model also generates a strong positive correlation between $\rho^{l,s}_{i,t,e}$ and the level of the cost of external finance, consistent with the empirically high correlation between $\rho^{l,s}_{i,t,e}$ and proxies such as the default spread, lending standards, and consumer sentiment. Finally, the model is able to replicate the difference in differences in investment between high and low productivity firms conditional on proxies for the level of the cost of external finance. In addition, the model predicts that this difference in differences will be larger for small firms, a fact we confirm empirically.

We also test our model against its main alternative, a model with constant costs of external finance in which aggregate issuance and savings waves can potentially arise from productivity shocks alone. This model is a nested version of the model with a stochastic cost of external finance, and is strongly rejected by the data using a formal $J$-test. Intuitively, this test shows that the benefit in terms of model fit more than justifies the additional parameters introduced by the full model. The restricted model also fails to generate aggregate issuance and savings waves at the estimated parameter values; the data force the model to compromise on the aggregate correlation between issuance and savings in order to minimize as much as possible the large overall model errors.

Because we cannot feasibly test our model against every alternative, we instead provide substantial evidence that the model we estimate is consistent with empirical moments that we do not target specifically, but that vary with firms’ key issuance and savings policies in the way our model predicts. First, the model replicates the larger difference in investment between high and low productivity firms when the cross sectional correlation between liquidity accumulation and external finance is high. Second, the moments we show are correlated in the model with the state variable for the level of the cost of external finance are correlated with traditional empirical proxies for the cost of external finance (such as the index for lending standards), but are only weakly correlated with TFP. Finally, we also note that our model with stochastic costs is consistent with the findings from the large literature in empirical corporate finance upon which we build which uses difference in differences to show that credit supply shocks impact corporate policies.

After developing and estimating our model, and comparing the implied model moments to their empirical counterparts, we turn to using the intuition we develop, along with the estimated
version of our structural model, to construct a time series of the average per dollar cost of external finance raised for the US from 1980 to 2010. Our strategy is to use the policy functions from the structural model, along with the estimated parameters, to identify moments from the cross section which vary substantially with the aggregate state. When applied to the data, this variation in cross sectional moments then offers identification of an aggregate hidden state variable.

We first construct a continuous series for this average cost using a regression based “cost of external finance index”. Specifically, we construct the index weights by running a regression in the model of the cost of external finance on the cross sectional moments describing firms’ issuance and savings decisions on that date. We then use the model implied weights, along with the empirical moments from Compustat data to construct an estimate of the empirical cost of external finance in US data. We argue that our index measure of the aggregate cost of external finance, which exploits firms’ revealed preferences implied by their financing and savings decisions, is a useful complement to existing measures. For example, the widely used default spread only measures the cost of debt finance, and much of the default spread may be due to a fair return adjustment for risk. Indeed, we show that our index implied cost contains new information relative to the default spread. For example, the index implied cost predicts that external finance was less costly in 1986 and more costly in 2001 than the default spread seems to imply.

Next, we use Simulated Method of Moments (SMM) to construct a binary measure of the cost of external finance. Specifically, we at each date we find the value of the aggregate state for the level of the cost of external finance which sets the model moments describing issuance and savings behavior in the cross section closest to their empirical counterparts. Thus, we propose two methods for using cross sectional moments, along with a calibrated model, to make inferences about a hidden aggregate state. The advantage of the SMM method is that it takes advantage of more of the structure of the model, however the cost is that it is constrained to choose states that are part of the estimated process for the cost. Accordingly, the advantages of the regression index method is that it is more flexible, and the resulting estimate is a continuous measure. We show that the results from both methods are consistent, and in particular indicate high costs in the early 1980’s, followed by lower costs in the mid 1980’s, high costs in the early 1990’s, very low costs in the mid 1990’s through 2000, high costs in 2001 and low costs thereafter until the onset of the financial crisis.
II. Related Literature

This paper contributes to the growing literature at the intersection of finance and macroeconomics which studies the interaction between firm financing, savings, and investment decisions, and the macroeconomy. Two recent prominent papers document the cyclical behavior of firm financing. Jermann and Quadrini (forthcoming), and Covas and Den Haan (2011a) both document that debt issuances are highly procyclical, and Covas and Den Haan also report procyclical equity issuances.\(^1\) We are the first to incorporate data on firms’ liquidity accumulation, as well as their investment, into a business cycle model aimed at considering the role of pure financing shocks vs. shocks to productivity in explaining firm level and aggregate investment and financing activities.\(^2\) We argue that looking at the joint dynamics of liquidity accumulation and external finance is useful for examining the role of shocks to the cost of external finance, since how firms use funds may help to disentangle financing shocks from shocks that drive investment opportunities. Therefore while previous studies have focused on how external funds are raised, whether by debt or equity financing, our paper shows that how external funds are used is also useful in understanding the cost of external finance.

Several recent papers develop theoretical models which use a shock which originates in the financial sector to better match business cycle facts.\(^3\) Jermann and Quadrini (forthcoming) show how a model with an endogenous credit limit and a shock to capital liquidity can generate realistic business cycles as well as match the procyclical debt issuance and countercyclical equity issuance which they document using US Flow of Funds data. Covas and Den Haan (2011b) develops a model in which countercyclical equity issuance costs are useful for generating both procyclical equity issuance and a countercyclical default rate. Khan and Thomas (2011) build a quantitative business cycle model in which credit shocks drive aggregate productivity down by inhibiting productive investment reallocation across firms. This effect shows up in our model as well, and we show that estimated TFP is below actual TFP when external finance is costly. Hugonnier et al. (2011) build a search theory of external finance and show how idiosyncratic external finance risk affects corporate savings, investment, and payout policy. Bolton et al. (2011) develop a dynamic

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\(^1\) Choe et al. (1993), and Korajczyk and Levy (2003) also study issuances over the business cycle. Both find that equity issuance is procyclical. Korajczyk and Levy (2003) report countercyclical debt issuance. Huang and Ritter (2009) provides evidence that active issuance decisions are driven by the relative cost of equity vs. debt.

\(^2\) Eisfeldt and Rampini (2009) builds an aggregate model of internal and external finance to study the implications of corporate liquidity demand for the observed low return on liquid assets, but does not consider shocks to the cost of external finance. Covas and Den Haan (2011a) focus on debt and equity issuances, but they do note that, empirically, firms tend to both accumulate financial assets and invest when they issue external finance.

\(^3\) These papers build on the seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997) on the role of financial market conditions on firm investment and business cycle dynamics.
theory of firm finance and risk management with stochastic financing costs, and show analytically that such costs can increase savings and can delink external finance from investment at the firm level in a model with constant investment opportunities. Our model confirms these effects in a calibrated, quantitative model with stochastic investment opportunities, and, importantly we also document their empirical relevance.

We also use our model to estimate the cost of external finance in the US time series. Thus, our paper is most closely related to Jermann and Quadrini (forthcoming), with two key differences. First, Jermann and Quadrini (forthcoming) focus on the distinction between debt vs. equity in their estimation, and estimate a debt financing cost shock, whereas we do not distinguish between sources of external finance and instead incorporate information regarding how all external funds are used into our estimation strategy. Second, Jermann and Quadrini (forthcoming) use an assumed binding constraint to identify their shock. While we cannot solve our model for the cost of external finance shock in closed form, we think that the use of cross sectional moments to identify a hidden aggregate state is a complementary methodology with other potential uses.

Despite this renewed interest, the fact that financial constraints, or shocks originating in the financial sector, are important for either firm level investment, or business cycle dynamics, is not a foregone conclusion amongst economists. While Ivashina and Scharfstein (2010), Duchin et al. (2010), Campello et al. (2010), Matvos and Seru (2011), Almeida et al. (2009), and Chodorow-Reich (2014), provide evidence that the financial crisis hindered external finance, investment, and employment activity at the firm level, Paravisini et al. (2011) find only small effects of credit supply shocks on trade. Gomes et al. (2006) point out that the shadow cost of external finance is procyclical in a standard business cycle model with agency costs of external finance. Finally, Chari et al. (2008) argue that aggregate data do not support the occurrence of a credit crunch and question the appropriateness of government interventions aimed at improving access to external finance. In contrast to these papers, our paper uses corporate policies from our structural model along with US data to extract information about the level of financing frictions in the US time series.

Our paper is also related to papers which develop dynamic models of corporate saving. The main difference is in focus; these papers are focused on understanding firm level dynamics or making inferences about firm level of financial constraints. In contrast, our paper, which is focused on understanding the dynamics and the effects of the aggregate component of the cost of

\[ \text{Likewise, Chari et al. (2007) use business cycle accounting to argue that shocks to the cost of installing capital, or to the return on capital, are only of tertiary importance for explaining the US fluctuations output, investment, and employment. However, papers such as Justiniano et al. (2010), Christiano et al. (2010), Hall (2011), Shourideh and Zetlin-Jones (2012), and Gilchrist and Zakrajsek (2012b), find that such shocks explain a large fraction of business cycle fluctuations.} \]
external finance connects ideas from this literature to the macro finance literature which studies business cycles with financial frictions. Kim et al. (1998) develop a three date model and show that cash accumulation is increasing in the cost of external finance, the variance of future cash flows, and the return on future investment opportunities, but decreasing in the return differential between physical capital and cash. Almeida et al. (2004) study the cash flow sensitivity of cash and empirically document a link between the propensity to save out of cash flow and financial constraints. Riddick and Whited (2009) construct a fully dynamic model of corporate savings and emphasize the importance of uncertainty for determining corporate savings, and argue that in such a model, the propensity to save is not an accurate measure of financial constraints. Thus, the firm level link between financial constraints and investment in financial assets is also under debate. A contemporaneous paper with a related focus to ours, but again directed at understanding firm level behavior, is Warusawitharana and Whited (2011), which uses simulated method of moments to show that equity misvaluation shocks can help explain firm level corporate issuance and savings policies. Note that because both Riddick and Whited (2009), and Warusawitharana and Whited (2011) are focused on firm level moments, the parameter estimates they do not incorporate any information in aggregate moments. However, our calibration focusing on aggregate moments is not too dissimilar, and supports the applicability of the basic Riddick and Whited (2009) framework for aggregate studies.

Finally, our paper is related to dynamic models of capital structure. The fact that firms tend to simultaneously raise external finance and accumulate liquidity is at odds with standard static pecking order intuition. Static pecking order theories based on Myers (1984) predict that firms will first draw down cash balances and only once these are exhausted will they seek external finance. Thus, such theories predict a counterfactually negative correlation between liquidity accumulation and external finance. Our dynamic model features a pecking order in the sense that internal funds are less costly than external funds, and generates the observed positive correlation between liquidity accumulation and external finance. This result is similar to the implications of the models in Hennessy and Whited (2005) and Strebulaev (2007) for the trade off theory of capital structure. Those papers show that data which appear to be inconsistent with static trade-off theories of capital structure can be generated by dynamic models in which firms’ objectives are based precisely on the trade-off between the tax benefits and distress costs of debt.

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5 For a model which instead focuses on the value of the flexibility of cash for adjusting net leverage, see Gamba and Triantis (2008).  
6 See also Faulkender and Wang (2006) for evidence that cash is more valuable when held by financially constrained firms. Harford et al. (2011) argue that firms save to insure against refinancing risk and document an inverse relationship between debt maturity and cash holdings which is stronger when credit market conditions are tighter.
III. Stylized Facts

A. Data Description

Our main data set consists of annual firm level data from Compustat from 1980-2010. We focus on Compustat data since we are able to analyze firm level, as well as aggregate, facts. Thus, our sample selection criterion closely follows that in Covas and Den Haan (2011a). We also show that using Flow of Funds data to construct aggregate issuance and savings moments yields similar results. The Data Appendix gives a detailed description of the construction of our data.

We use firm level cash flow statements to track corporate flows. We define liquidity accumulation as changes in cash and cash equivalents. We define net external finance raised as the negative of the sum of net flows to debt and net flows to equity. We define flows to debt as debt reduction plus changes in current debt plus interest paid, less debt issuances, and flows to equity as purchase of common stock plus dividends less sale of common stock. Following Covas and Den Haan (2011a), and Fama and French (2005), we also consider using the negative of the change in total liabilities as flows to debt and negative changes in book equity as flows to equity. We find similar results using these stock measures. We focus on the flow measures in the interest of brevity, and since our model does not feature issuances which are not truly “external” like those related to mergers or employee compensation which are emphasized in Fama and French (2005). Finally, we have also verified that the results are similar if we just focus on issuances of debt and equity, rather than the total net flows from these claim holders. We define investment (in physical capital) as capital expenditures. We do not include acquisitions in our investment measure. Firm level acquisitions are very lumpy, which can bias the correlations we compute. Including acquisitions does not change our aggregate results, since the aggregate series smooths out individual firm lumpiness.

When computing most aggregate and firm level moments, we normalize firm level variables by current total book assets. When computing aggregate correlations, we instead normalize by the lag of book assets, to avoid inducing spurious correlations. However, book assets are slow moving and fairly acyclical and thus shouldn’t induce any cyclical variation. Our results are robust to alternative normalizations, such as aggregate output or aggregate gross-value added from the corporate sector. We use the Hodrick and Prescott (1997) filter to remove any remaining series trends when computing aggregate correlations, since, for example cash holdings have trended

We do not use the balance sheet measure of cash since the stock measure is affected by acquisitions. Covas and Den Haan (2011a) instead remove firms involved in mergers which increase sales by more than 50%. We have checked that our findings are similar using stock measures and the non-merger sample. All non-reported robustness checks are available from the authors upon request.
upwards as a share of assets over our sample \cite{Bates2009}. The filter ensures that the empirical series are stationary, which is consistent with the stationary model we study. Thus, our focus is on the business cycle dynamics of the cost of external finance.

As in \cite{CovasDenHaan2011a}, our main analysis drops the top 10% of firms by asset size. There are several reasons to do this. First, the very largest firms present unique measurement problems. More of the investment for these firms falls under the accounting category “other investments”. These other investments are typically long term receivables to unconsolidated subsidiaries. Thus, a large firm may raise funds on behalf of a smaller subsidiary, which in turn may use the funds to build a new factory, or may store the funds as liquid assets. Since we are not able to measure these funds’ ultimate use, we are not able to identify accumulated liquidity vs. physical investment, the main goal of this paper. Second, the largest firms tend to have a much larger share of foreign earnings. Cash accumulation for firms with large foreign earnings may be influenced by tax motives and repatriation timing. Third, as \cite{CovasDenHaan2011a} point out, external finance for the largest firms is not representative of the rest of the sample. They show in particular that one incidence of AT&T raising equity during a recession in 1983 has implications for the cyclicity of aggregate equity issuance. They advocate dropping the top firms because they have an unusually large influence on the aggregate series. Fourth, it is possible that the very largest firms face little or no financial constraints. Finally, we note that in the type of stationary model we study, the distribution of firm sizes will be much less skewed than that in the data. As a result, aggregate model data will not be as heavily driven by the activities of a few large firms and is more readily comparable to our sample which excludes these extremely large firms.

For the Flow of Funds data, we normalize each series by the HP filter implied trend in gross-value added of the corporate sector.\footnote{If we very narrowly define the accumulation of liquid assets as the net acquisition of financial assets minus trade receivables minus miscellaneous assets, the flow of funds data display a counterfactual decrease over time in this series for liquid assets held within the corporate sector. Thus, the Flow of Funds data do not do a good job of identifying and classifying all corporate investment in marketable securities. There is a large, and growing, category “miscellaneous assets,” which contains both marketable and non-marketable assets. To account for this, we also include 1/3 of miscellaneous other assets as liquid.}{8} If we very narrowly define the accumulation of liquid assets as the net acquisition of financial assets minus trade receivables minus miscellaneous assets, the flow of funds data display a counterfactual decrease over time in this series for liquid assets held within the corporate sector.\footnote{Results using total GDP are similar.}{9} Thus, the Flow of Funds data do not do a good job of identifying and classifying all corporate investment in marketable securities. There is a large, and growing, category “miscellaneous assets,” which contains both marketable and non-marketable assets. To account for this, we also include 1/3 of miscellaneous other assets as liquid.\footnote{The decision to use 1/3 of other miscellaneous assets was based on personal communication with staff at the Board of Governors. Their rough estimate using recent IRS data is that about 1/3 of miscellaneous other assets were marketable securities.}
B. Main Facts

We document three new stylized facts describing aggregate issuance and savings waves. First, the aggregate time series correlation between external finance raised and liquidity accumulation is strongly positive. Second, the cross sectional correlation between issuance and saving tends to be low when traditional empirical proxies indicate that the external finance is high. Third, and building on these results, we show that the difference in investment between high productivity firms and low productivity firms is higher when the cross sectional correlation between liquidity accumulation and external finance is low, indicating a low current cost of external finance.

Our first fact is that firms tend to issue and save in aggregate waves. For all but the top 10% of Compustat firms, the aggregate correlation between liquidity accumulation and external finance is 0.60 and is statistically significant at the 5% level. Figure 1 plots the cyclical component of aggregate net liquidity accumulation vs. the cyclical component of aggregate net external finance and clearly illustrates our first stylized fact. This aggregate correlation is higher (0.74) if one conditions on firms that are currently raising external finance. Thus, the positive aggregate correlation does not seem to be driven by some firms saving, and other firms issuing external finance, nor is it driven by the behavior of payouts.

The aggregate correlation is also higher when one excludes more of the largest firms. For the top half of firms, the correlation between aggregated external finance raised and liquidity accumulated is 0.84. This is in contrast to conditioning on other measures of financial constraints, such as whether a firm pays no dividends, or has no credit rating, in which case we find correlations close to that for the larger sample (0.68 and 0.56 respectively). This could be due to the importance of fixed costs in accessing external financial markets, or it could be that size is simply a better proxy for financial constraints. Finally, we also find a positive correlation using flow of funds data. If we very narrowly define liquid assets as the net acquisition of financial assets minus trade receivables minus miscellaneous assets, we find a correlation between liquidity accumulation and external finance of 0.33. Including 1/3 of miscellaneous other assets as liquid helps align the flow of funds data with the fact that the net accumulation of liquid assets within the financial sector has been positive over recent history. Using this measure, we find a correlation of 0.38 which is statistically significant. Table I displays our main aggregate issuance and savings stylized facts.

Table V displays the correlations between liquidity and investment with debt vs. equity separately. While we see that the correlation with liquidity accumulation is stronger for equity (0.69) than debt (0.16), both are positive. Conditional on firms raising external finance, we see both correlations increase to 0.77 and 0.33, respectively, and both are statistically significant.
We also note that investment is more correlated with debt (0.60) than equity (-0.15). This fact has been pointed out by DeAngelo et al. (2010) who argue that debt might be used more frequently for investment. Also, we note that debt drives most of the variation in external finance, with a correlation with external finance of 0.77 vs 0.43 for equity. For parsimony, and to match our model, we focus on the overall correlation with external finance and abstract from debt vs equity. Studying total external finance allows us to focus on what is new in our work; whereas other studies have focused on variation in firms’ sources of funds, our study focuses on firms’ uses of the external finance they raise.

The second main new fact that we document is that in the cross section, firms are more likely to raise external finance and save the proceeds when the proxies for the cost of external finance are low. We proxy for the cost of external finance with the default spread, index of tightening lending standards, and consumer sentiment. This is consistent with the intuition we illustrate theoretically that when financing costs are high, firms are unlikely to raise costly external finance only to save the proceeds in low-return, liquid, assets. This finding is closely related to the evidence provided by McLean (2010), who shows that share issuance-cash savings are inversely related to microstructure measures of stock market liquidity. However, a key difference between our empirical analysis and that in McLean (2010), in addition to his focus on equity issuance alone, is that we specifically attempt to distinguish productivity and financing cost shocks using our structural model, whereas McLean (2010) uses moments of GDP growth to proxy for financing conditions. At each date, we compute the cross sectional correlation between aggregate net external finance raised and liquidity accumulation (each normalized by lagged book assets), and construct a time series of this cross sectional correlation, which we call $\rho_{xs}^{zl}$. We then show that the correlation between $\rho_{zl}^{ze}$ and the negative of the Baa-Aaa default spread is 0.64. Similarly, the correlation between $\rho_{zl}^{ze}$ and the negative of the fraction of banks reporting tighter lending standards is 0.58. Both correlations are statistically significant at the 5% level. Figure 2 illustrates the strong relationship between $\rho_{zl}^{ze}$, the percentage of firms raising external finance, the default spread, the index of consumer sentiment, and lending standards by plotting the time series for $\rho_{zl}^{ze}$ and the percent of firms raising external finance (top panel) along with the negative of the default spread, lending standards, and consumer sentiment (bottom panel). Although all the series are highly correlated, there is independent information in $\rho_{zl}^{ze}$. For example, the high $\rho_{zl}^{ze}$ indicates a low cost of external finance in the boom of 1986, however the default spread was not particularly low then. The tech bust of 2001 is also more apparent in the drop in $\rho_{zl}^{ze}$ than it is in the relatively small increase in the default spread, potentially suggesting that this was largely an increase in the cost of equity issuance not
captured by the default spread. Finally, we show that, by contrast, $\rho_{i,t,e}$ is less correlated with TFP (0.48).

The final main fact we document regarding aggregate issuance and savings waves is a fact about “differences in differences”. In particular, we show that when the cross sectional correlation between liquidity accumulation and external finance is high, indicating that the the cost of external finance is low, the difference in investment by high productivity firms vs. low productivity firms is higher than when external finance is more costly by our measure. This makes sense since costly external finance inhibits investment precisely by those firms with good investment opportunities, and thus this fact provides additional support for $\rho_{i,t,e}$ as a proxy for the level of the cost of external finance.

IV. Model

A. Two Date Model

We develop and analytically characterize the relationship between the cost of external finance, the amount of external finance raised, and investment in capital and liquid assets in a simple two date model, in order to provide some basic intuition for our main results. In particular, we illustrate the reason why firms’ use of the external finance they raise provides information about the cost of those funds. The intuition in both the simple and the full models is as follows: The marginal benefit of investing in physical capital is high at low levels, but decreases with the level of investment. Liquidity accumulation displays less decreasing returns to scale, but has a marginal benefit that is on average lower than the marginal benefit of investment in physical capital. As a result, firms will only invest in liquid assets once they have invested enough in physical capital to push the marginal return below that on liquid assets, but will typically invest a positive amount in physical capital. Importantly, the lower the cost of external finance is, the more likely it is that the marginal return to physical capital investment will be pushed down below the marginal return to cash because a lower cost of funds increases investment.

In the simple two date model, the decreasing marginal return to physical capital is due to decreasing returns to scale, and we fix the marginal return on liquid assets. Clearly, for positive liquidity accumulation to occur, this fixed return must exceed the discount rate, and for positive capital accumulation it must be lower than the marginal return on investment below some level of investment. In the two date model, we simply assume a fixed return that exceeds the discount rate, but is lower than the marginal return on capital for low levels of investment. In our dynamic model, physical capital investment has decreasing marginal returns due to both
decreasing returns to scale, as well as to convex adjustment costs. The physical return to liquid assets is fixed at a rate lower than the discount rate, and thus positive liquidity accumulation only arises when the total return is endogenously pushed above the discount rate due to the benefit of liquid assets’ use as future internal funds for investment.

In this section, we study a firm which maximizes the present value of cash flows over two dates, zero and one. For simplicity, we set the interest rate to zero. At date zero, the firm receives an endowment of liquid assets, and internal funds from operating cash flows, both of which we normalize to zero without (qualitative) loss of generality. The firm then chooses how much to invest in both physical capital \((i_k)\) and liquid assets \((i_l)\). At date one, the firm receives cash flows from its productive physical capital and from its liquid assets. Liquid assets produce \(r_l > 1\) at date one. Physical capital produces output according to \(zi_k^\theta\) and does not depreciate. We define payouts gross of financing costs, \(e\), as internal funds minus investment in physical capital and liquid assets. If \(e < 0\), the firm is raising external finance, and pays a cost \(\xi e^2\), where \(\xi\) is interpreted as the current “level” of the cost of external finance.

The firm’s objective over date zero and date one cash flows, respectively, is then:

\[
\max_{i_k, i_l} \left\{ \left[ e - 1_{\{e < 0\}} \frac{\xi e^2}{2} \right] + \left[ (i_k + z i_k^\theta) + (i_l + i_t) r_l \right] \right\},
\]

subject to:

\[
e = -i_l - i_k,
\]

\[
i_l \geq 0.
\]

where \(1_{\{e < 0\}}\) is an indicator equal to one if the firm is raising external finance. With operating cash flows normalized to zero this indicator will always equal one due to the inada condition on the production function. We use \(\psi_l\) to denote the multiplier on the constraint \(i_l \geq 0\). The first order condition with respect to investment in liquid assets, \(i_l\), is:

\[
r_l - 1 + \psi_l = -\xi (-i_l - i_k).
\]

The first order condition with respect to capital investment, \(i_k\), is:

\[
\theta z (i_k)^{\theta - 1} = -\xi (y - i_l - i_k).
\]

There are two cases depending on whether the firm’s constraint on negative liquidity accumulation is binding or not. In the case that the firm both invests and accumulates liquidity, \(\psi_l = 0\) and the first order conditions equate the marginal product of capital, the return on liquid assets,
and the marginal cost of raising external finance. In the case that $i_l = 0$, the marginal return on capital is set equal to the marginal cost of external finance, but exceeds the marginal return on liquidity accumulation with $\psi_l$ capturing the wedge between the two.

These first order conditions imply the following optimal financing and investment policies:

$\dot{i}_l = \frac{r_l - 1}{\xi} - \left(\frac{r_l - 1 + \psi_l}{\theta z}\right)^{\frac{1}{\theta - 1}}$

$\dot{i}_k = \left(\frac{r_l - 1 + \psi_l}{\theta z}\right)^{\frac{1}{\theta - 1}}$

$-e = \frac{r_l - 1 + \psi_l}{\xi}$

Figure 3 illustrates the firm’s investment and financing decisions graphically by plotting the net marginal benefit of capital investment and investment in liquid assets, along with the marginal cost of external finance for three levels of $\xi$, high, medium, and low. Consider the middle panel, which depicts the case for a medium level of $\xi$, $\xi_M$, and start from zero external finance raised, zero investment, and zero liquidity accumulation. Because the production function satisfies an inada condition, and the zero marginal cost of external finance at zero, the firm will raise a positive amount of external finance, and spend the first funds it raises on investment. The firm will then raise external finance and invest the funds, moving down the marginal benefit of investment curve and up the marginal cost of external finance curve, until the marginal return on an additional dollar of investment is either driven below the marginal cost of an additional dollar of external finance, or below the constant marginal return of a dollar invested in liquid assets, whichever happens first. As long as $\xi$ is low enough, so that the marginal cost of external finance does not increase too quickly, there will be positive liquidity accumulation as depicted in the middle panel. The left panel of Figure 3 graphs the case in which $i_l = 0$ because the marginal cost curve is steep enough so that the firm chooses not to drive the marginal return to investment down to the marginal return on liquidity accumulation. In this case, $\psi_l$ is positive and drives a wedge between the marginal returns on the two assets. The right hand panel depicts the case for a low level of $\xi$, $\xi_L$. One can see that as $\xi$ decreases from $\xi_M$ to $\xi_L$, the firm raises additional external finance $-e$, and accumulates each unit of additional funds in the liquid asset.

With these firm policies conditional on $z$ in hand, we can consider how the level of $\xi$ affects the correlation between liquidity accumulation and external finance in a panel of firms described by the problem in equation (1) by considering a cross section of firms with heterogeneous $z_i$. 

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Figure 4 graphs a panel of three firms with heterogeneous levels of productivity $z_i$, along with the three levels of $\xi$ from Figure 3. For the highest level of $\xi$, the with the steepest marginal cost, only the firm with the lowest level of productivity accumulates liquidity. As $\xi$ decreases, first the middle productivity firm, and then the high productivity firm, accumulate liquidity. Moreover, each dollar of liquidity accumulated comes from funds raised externally. Although the specifics of this example rely on the assumption that internal funds are zero, because funds are spent first on investment, the qualitative intuition generalizes to the case in which firms have positive internal funds. For this panel of firms, the cross sectional correlation between liquidity accumulation and external finance is:

$$
\rho_{x,e,i}^s(t) = \text{corr} \left( \frac{r_l - 1 + \psi_{lit}}{\xi}, \frac{r_l - 1 + \psi_{lit}}{\xi} - \left( \frac{r_l}{\theta z_i} \right)^{\frac{1}{\psi - 1}} \right). \tag{2}
$$

Clearly, this correlation is zero when $\xi$ is high enough such that $i_l = 0$ for all firms. As $\xi$ decreases, more firms will save some of the proceeds from the external finance they raise. Moreover, equation (2) illustrates that the lower the level of $\xi$, the more the firms’ choices for liquidity accumulation and external finance will both be dominated by the shared term with $\xi$ in the denominator. Thus, as a result of both an extensive and an intensive margin for liquidity accumulation, as $\xi$ decreases, these two flows will be more correlated.

Finally, we construct the investment returns for physical capital and liquid assets. We present these returns for this simple model in order to connect the intuition developed in this section to that for the dynamic model, for which we provide analogous returns. Each return is the physical return times an external finance discount factor. The external finance discount factor is the ratio of a firm’s marginal value of funds tomorrow relative to today. In this model, all else equal, the external finance discount factor is high when $\xi$ is large, and this high cost reduces the return to investment and liquidity accumulation. Specifically:

$$
R_k = \frac{1}{1 - \xi e} \frac{\theta z_i^{\psi - 1} + 1}{1},
$$

We can interpret the first term as the external finance discount factor and the second term as the physical return. A high current cost of external finance reduces the marginal return to investment. For the return on liquidity accumulation, we have:

$$
R_l = \frac{1}{1 - \xi e} \frac{r_l + \psi_{il}}{1}.
$$

At the optimum, all returns are equated, and equal to one since there is no discounting. If
liquidity accumulation is positive at the optimum, then \( \psi_l = 0 \) and \( \frac{\psi_l}{1 - \xi} \) is both the return to liquidity accumulation and the return to an additional dollar of external finance, since that dollar will be invested in liquid assets.

### B. Dynamic Model

We study a continuum of risk neutral firms which maximize the expected present discounted value of their net payouts, taking the interest rate as given. All firms have access to the same production and financing technologies, and are subject to common aggregate shocks. However, firms differ in terms of their idiosyncratic productivity realizations and as a result choose heterogeneous stocks for their physical and liquid capital stocks. Thus, we study a panel of heterogeneous firms which experience both idiosyncratic and aggregate shocks. We begin by describing the model, and the implied investment returns, and then turn to its estimation and policy function analysis.

Firms produce output or cash flows using physical capital \( k \) according to:

\[
y = z k^\theta
\]

where \( z \) is the level of the firm’s productivity and \( \theta \in (0, 1) \). Each firm’s productivity \( z \) is the product of an idiosyncratic shock \( z_i \), and an aggregate shock \( z_{agg} \). The aggregate productivity level, and each idiosyncratic productivity level, follow AR(1) processes in logs with identical persistence parameters, however, we allow for the idiosyncratic and aggregate processes to have different volatilities. These assumptions allow us to construct each firm’s productivity shock as follows:

\[
z = z_i z_{agg} \tag{3}
\]

\[
\ln(z_i') = \rho \ln(z_i) + \epsilon_i' \tag{4}
\]

\[
\ln(z_{agg}) = \rho \ln(z_{agg}) + \epsilon_{agg}' \tag{5}
\]

\[
\ln(z') = \rho \ln(z) + \epsilon_i' + \epsilon_{agg}'. \tag{6}
\]

Capital evolves according to the standard law of motion:

\[
k' = (1 - \delta)k + i_k, \tag{7}
\]

where \( i_k \) is investment and \( \delta \in (0, 1) \) is the depreciation rate. Investment in physical capital is
subject to convex adjustment costs \( \phi_i(i_k, k) \) given by:

\[
\phi_i(i_k, k) = \frac{a}{2} \left( \frac{i_k}{k} \right)^2 k.
\]  

(8)

where \( a \) determines the slope of the marginal adjustment cost. Liquid assets \( l \) evolve according to:

\[
l' = (l + i_l)(1 + r(1 - \tau))
\]  

(9)

where \( i_l \) is investment in liquid assets and \( r \) is the risk free rate. Thus, corporate payouts are motivated by a tax wedge, \( \tau > 0 \), which as in Riddick and Whited (2009). We note, however, that in practice payout policy is also likely to be driven by agency and information considerations.

Because financing costs will be paid only if payouts gross of financing costs are negative, it is convenient to define a firm’s pre-financing payout \( e \) as internal cash flows minus investment in physical capital and liquidity accumulation, less investment adjustment costs.

\[
e = zk^\theta(1 - \tau) - i_l - i_k - \phi_i(i_k, k),
\]  

(10)

If \( e > 0 \) the firm is paying out funds and if \( e < 0 \) the firm is raising external finance. Intuitively, the firm raises external finance if after tax operating profits do not cover the firms’ total investment in physical and liquid assets, net of physical adjustment costs. Firms maximize this payout, net of financing costs. Following Gomes (2001), Hennessy and Whited (2005), Hennessy and Whited (2007), and Riddick and Whited (2009), and in order to facilitate estimation, we parameterize the cost of external finance exogenously as follows:

\[
\phi_e(e, \xi) = \mathbb{1}_{\{e < 0\}} (\lambda_1 + \frac{\xi}{2} \lambda_2 e^2),
\]  

(11)

where \( \lambda_1, \lambda_2 > 0 \), \( \mathbb{1}_{\{e < 0\}} \) is an indicator that takes the value 1 when \( e < 0 \) and 0 otherwise, and \( \xi > 0 \) denotes the aggregate level of external financing costs. The aggregate state of external financing costs, \( \xi \), follows an AR(1) in logs.

\[
\ln(\xi') = c + \gamma \ln(\xi) + \eta'
\]  

(12)

We choose the value \( c \) so that \( \xi \), is on average 1 and then the average level of the marginal cost of external finance is given by \( \lambda_2 \).

To keep things as parsimonious as possible, we employ a standard model of dynamic corporate finance with the minimal elements we found to be quantitatively important in earlier versions.
of the model. We discuss the role of each element in our results in our discussion of the model’s policy functions below. Relative to Riddick and Whited (2009), for example, we have added an aggregate productivity shock and an aggregate shock to the cost of external finance, but stripped out the parts of the investment and external financing costs which we found not to be quantitatively relevant in earlier iterations of the model.\footnote{Specifically, we do not have fixed investment costs or a linear external finance cost as in Riddick and Whited (2009). As in other studies, we found little role for the linear cost of external finance. Similarly, at the firm level, and in the presence of a fixed cost of external finance, we did not find a quantitatively important role for the fixed cost of investment.} To facilitate empirical analysis, we parameterize the dynamics of the cost exogenously. Microfoundations of a time varying cost of external finance have been considered in the context of an agency friction that varies over time, along the lines of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), as collateral constraints as in Kiyotaki and Moore (1997), or as an endogenously time varying adverse selection problem as in Eisfeldt (2004), Kurlat (2011), and Bigio (2013). Strong empirical evidence for the important role of fixed financing costs is provided by Welch (2004) and Leary and Roberts (2005).

We denote the firm’s value function by $V$ and write the firm’s problem in recursive form, using the constraints and laws of motion given in equations (3)-(12). Each firm’s state is given by its size, $k$, its liquid asset balance, $l$, its productivity $z$ (the product of an aggregate and idiosyncratic shock), and the aggregate level of the marginal cost of external finance, $\xi$. We define $s \equiv \{k, l, z, \xi\}$ to summarize an individual firm’s state vector. We use the standard “prime” notation to denote next period values for the state, and a subscript $s$ to denote expectations conditional on the current state.

Each firm then solves the Bellman Equation:

$$V(s) = \max_{k', l' \in [0, \infty), e' \in [0, \infty)} \left\{ e - 1_{\{e < 0\}}(\lambda_1 + \frac{\xi}{2} \lambda_2 e^2) + \frac{1}{1 + r} E_s [V(s')] \right\} \quad (13)$$

subject to:

$$e = (1 - \tau)zk^\theta - \frac{\lambda'}{1 + (1 - \tau)r} + l - k' + (1 - \delta)k - \frac{a}{2} \left( \frac{k' - (1 - \delta)k}{k} \right)^2 k$$

C. Investment Returns and Policy Functions

We first illustrate the main intuition for a firm’s policy functions by studying the returns to investing in physical capital and in liquid assets, extending the production returns from Cochrane (1991) and Cochrane (1996) to include the effects of financial constraints and to account for firms’
investment in more than one asset type. We highlight the role of an “external finance discount factor”, $\mathcal{F}$. Although firms in our economy are risk neutral, $\mathcal{F}$ twists the physical probability distribution to weigh more heavily those states in which internal funds are more valuable to the firm. Internal funds are most valuable when the firm would otherwise be raising costly external finance, therefore the firm would like to hedge precisely these states when the marginal value of internal funds is high.

We begin by stating the first order conditions and envelope conditions for the program in (13), and then combine these to form investment return Euler equations. We denote the multiplier on the constraint that $l' \geq 0$ by $\psi_l \geq 0$, and note that the constraint that $k' \geq 0$ will not bind due to an inada condition. In order to account for the fixed cost of external finance, we add a “pseudo constraint” to the dynamic program, requiring that $e \geq 0$. We denote the multiplier on this pseudo constraint $\psi_e$, and require that $\psi_e > 0$ iff $e = 0$, and $\psi_e = 0$ otherwise. For expositional purposes, we define a new multiplier $\hat{\psi}_e$ which incorporates either the binding multiplier on the pseudo constraint that $e \geq 0$, or the marginal cost of external finance if the firm is raising funds, or zero if the firm is making payouts as follows:

$$\hat{\psi}_e \equiv \begin{cases} 
0, & \text{if } e > 0 \\
\psi_e, & \text{if } e = 0 \\
-\lambda_2 \xi e, & \text{if } e < 0
\end{cases}$$

Using subscripts to denote partial derivatives of the value function, the first order condition for $k'$ is:

$$\left(1 + \hat{\psi}_e\right) \left(1 + \frac{a i_k}{k}\right) = \frac{1}{1 + r} E_s[V_{k'}(s')] .$$

There are three cases, determined by the firm’s choice for external finance. If $e > 0$, then $\hat{\psi}_e = 0$. If $e = 0$, then $\hat{\psi}_e = \psi_e > 0$. Finally, if $e < 0$, then $\hat{\psi}_e = 0$. The right hand side of this first order condition is the marginal value of an additional unit of capital tomorrow. If the firm is paying funds out, the marginal value equals the marginal cost of one plus the adjustment cost. If the firm is raising external finance, the marginal value equals the marginal cost including the marginal cost of an additional unit of external finance, $\lambda_2 \xi e$. Finally, if the firm is neither paying out nor raising funds, the marginal value of an additional unit of capital exceeds the marginal cost, but not by enough to make it worthwhile for the firm to pay the fixed cost of external finance, and this wedge is captured by $\psi_e$. 

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The envelope condition at the optimal choice for physical capital is:

$$V_k(s) = \left(1 + \hat{\psi}_e\right)\left(\theta(1 - \tau)z k^{\theta - 1} + (1 - \delta) + \frac{ai_k}{k} \left(\frac{1}{2} k^{\delta} + (1 - \delta)\right)\right)$$  \hspace{1cm} (16)$$

where we again have three cases for $e$. Combining the first order condition with the envelope condition in the subsequent period, we form the Euler equation which sets the discounted expected return on investment in physical capital equal to the value of a dollar payout today as follows:

$$1 = \frac{1}{1 + r} E_s \left[\frac{\left(1 + \hat{\psi}_e'\right)\left(\theta(1 - \tau)z' k'^{\theta - 1} + (1 - \delta) + \frac{ai'_k}{k'} \left(\frac{1}{2} k'^{\delta} + (1 - \delta)\right)\right)}{\left(1 + \hat{\psi}_e\right)\left(1 + \frac{ai_k}{k}\right)}\right]$$  \hspace{1cm} (17)$$

Defining the “external finance discount factor”, $F$, which governs firms’ state pricing as

$$F = \frac{1 + \hat{\psi}_e'}{1 + \hat{\psi}_e}$$  \hspace{1cm} (18)$$

and using $\hat{R}_k$ to denote the physical return to investing in physical capital, we can write the Euler equation for the return to investing in physical capital as:

$$1 = \frac{1}{1 + r} E_s \left[F \hat{R}_k\right]$$  \hspace{1cm} (19)$$

The return on capital describes the marginal benefit of increasing capital one unit today relative to the marginal cost. The physical marginal benefit to increasing capital by one unit is: a marginal increase in output, the value of the additional depreciated capital, and a lower convex cost of investment in the following period. The physical marginal cost of increasing capital by one unit is a dollar, plus adjustment costs. The ratio of the physical marginal benefit to the physical marginal cost is the physical return to capital, $\hat{R}_k$. The total return is then the physical return multiplied by the shadow value of how much a dollar will be worth inside the firm tomorrow relative to today, $F$. If the firm is raising funds in both periods, this is equal to the marginal cost of funds in the following period relative to the current period. In general, additional capital is more valuable if internal funds are expected to be scarce in the future, i.e. if the firm will be, or will desire to be, raising funds externally. Since the conditional expected return on capital is constant, quantities must adjust for this asset pricing Euler equation to hold. The return to capital is decreasing in investment and increasing in productivity. Thus, the firm will increase investment in high productivity states until this optimality condition holds.
The first order condition for $l'$ is:

$$\left(1 + \hat{\psi}_e\right) \left(\frac{1}{1 + (1 - \tau)r}\right) = \frac{1}{1 + r} E_s[V_l(s')] + \psi_l$$

(20)

where we now have six cases. There are three cases for $e$ as in the first order condition for $k'$. There are two cases for the choice of future liquid assets. If $l' > 0$ then $\psi_l = 0$ and if $l' = 0$ then $\psi_l > 0$. The envelope condition at the optimal choice of liquid asset balances is:

$$V_l(s) = 1 + \hat{\psi}_e,$$

(21)

where again we have the three cases for $e$. Combining the first order condition with the envelope condition in the subsequent period, we form the return on investment in liquid assets as follows:

$$1 = \frac{1}{1 + r} E_s \left[\frac{1 + \hat{\psi}'_e + \psi_l}{1 + \hat{\psi}_e} \left(1 + (1 - \tau)r\right)\right].$$

(22)

Defining the “external finance discount factor”, $F$, as above, and defining the normalized multiplier $\hat{\psi}_l$

$$\hat{\psi}_l = \frac{\psi_l}{1 + \hat{\psi}_e}$$

we can write the return to investing in liquid assets as:

$$1 = \frac{1}{1 + r} E_s \left[(F + \hat{\psi}_l) \hat{R}_l\right].$$

(23)

where $\hat{R}_l$ is the physical return on liquid assets $(1 + (1 - \tau)r)$. There are eighteen cases for the investment return for liquid assets, depending on the choices for $e$ and $l'$, and the choices for $e'$, but the general intuition is straightforward. If the firm chooses to make positive payouts in the current and subsequent period, and chooses to carry forward a positive liquid balance, then $F = 1$, $\psi_l = 0$, and the return to liquid assets equals the pecuniary return. If $\psi_l > 0$ then the true return to investing in liquid assets is less than one, and the multiplier adjusts to set the return equal to one. Finally, the external finance discount factor $F$ accounts for the relative value of additional internal funds in the subsequent, relative to the current, period. This discount factor equals one if the firm places equal shadow values on funds in both periods. If the firm expects to be raising more finance in the subsequent period (the marginal cost of an additional dollar of external finance, $\lambda_{e < 0} \lambda_{\xi' e'}$, is large) or expects the pseudo multiplier $\hat{\psi}'_e$ to be large, the effect through $F$ will be to generate a higher return to investing in liquid assets. The opposite is true if the firm places a relatively high shadow value on internal funds in the
current, relative to the subsequent period, and the effect will be to drive the return to investing in liquid assets down.

**Lemma 1.** For each level of capital, liquid assets, and level of the cost of external finance, there is a cutoff for productivity \( \hat{z} \) above which the firm finds it optimal to pay the fixed cost and raise external finance.

**Proof.** By standard arguments, the fixed cost of external finance leads to inaction regions, and a threshold rule for \( e \). We use a variational argument to show that there is a threshold \( \hat{z} \) above which the firm will raise external finance. Consider a firm with productivity \( z \) which is deciding whether to raise external finance and invest. Suppose the firm increases \( k \) to \( k^* \) by raising external finance today, but leaves capital unchanged at all other dates. The firm will get \( \frac{1}{1+r} E[z'k^*\theta] - \frac{1}{1+r} E[z'k\theta] = \rho \frac{1}{1+r} z(k^*\theta - k\theta) \) in terms of present discounted productivity gains and would have \( (k^* - k)(1 - \delta) \) extra depreciated capital. However, the firm would pay today investment adjustment costs and external finance costs of \( a \left( \frac{i^*2 - i^2}{k} \right) \) + \( \lambda_1 + \lambda_2 \frac{e^2}{2} \). The firm would also have lower investment adjustment costs next period because we assume it leaves the future path of capital unchanged \( \frac{a}{2} \left( \frac{i^*2 - i^2}{k} \right) \). It is also possible that the policy for raising external finance next period will change. However, since the firm has extra capital, the pseudo constraint on internal funds associated with \( \psi_e \) will be more slack and/or the costs of external finance will be lower. Importantly, only the gain in output next period depends on \( z \). Moreover, because the output gain increases monotonically in \( z \), for given parameters and initial levels of capital and liquid assets, there is always a \( z \) for which the above strategy is positive NPV. Thus, there exists a \( \hat{z} \), conditional on the firm’s other state variables, such that the firm will raise external finance and invest whenever \( z > \hat{z} \).

Because the fixed cost implies a threshold productivity *above* which firms will raise external finance, it reverses the result from the two date model in which liquidity accumulation is higher for low productivity firms that exhaust their investment opportunities at lower levels of investment. In the dynamic model, it is firms with high productivity which raise external finance and save some of the proceeds. A firm which enters the period with high productivity has the highest marginal return to investment in physical capital. As shown in Lemma 1, this leads high productivity firms to pay the fixed cost, raise external finance, and invest some of the proceeds. As the firm invests, however, the convex cost of investment, and decreasing marginal returns from the concave production function, drive the returns to investment in capital down. Moreover, since productivity is persistent, a high \( z \) today makes a high future \( z \) more likely. Thus, the returns to future internal funds increase with \( z \), which raises the return to liquidity accumulation. These liquid assets are then used by high productivity firms to smooth investment over time.
We describe the policy functions graphically, using the numerical solution to the model at estimated parameters in Figure 5. The top panel of Figure 5 plots investment for high and low productivity firms, defined as having above and below the median level of productivity at each date, conditional on the realization of the stochastic cost. High productivity firms always invest more than low productivity firms, but this difference is larger when the cost of external finance is low. The difference in investment is also highest for small firms, due to decreasing returns to scale and the higher marginal product of capital at a smaller scale. The bottom panel of Figure 5 graphs liquidity accumulation and payouts (e) or external finance (−e) for the high and low cost states. Most of the time, the firm is making payouts, and is not accumulating liquidity. However, if the firm is small enough, and the cost of external finance is low enough, the firm will both raise external finance and accumulate liquidity. In the companion online appendix to this paper we present complete graphs of the policy functions. To present firm policies in two dimensions, we are averaging over some of the heterogeneity which leads to the variation in the cross section we exploit empirically, however Figure 5 shows enough variation to explain the basic intuition for firms’ policies. Firms in this model tend not to raise external finance when the cost is high. When the cost is low, firms with high enough returns to capital accumulation will raise external finance, invest some of the proceeds, and save the remainder. Decreasing returns to investment, due to decreasing returns to scale and convex adjustment cost, reduce the return to capital accumulation below that of liquidity accumulation. The return to liquidity accumulation is high for these firms because productivity is persistent, and they desire to smooth investment, and to avoid repeatedly paying fixed financing costs.

V. Estimation and Model Solution

A. Estimation Procedure

We estimate as many of the key structural parameters of the model as is computationally feasible using simulated method of moments (SMM), and calibrate the remainder. We use standard discrete state space dynamic programming techniques to solve the model for given parameter values. Specifically, we use the value function iteration method in Ljungqvist and Sargent (2004). Given the policy functions implied by our model solution, along with values for the model state variables and the stochastic processes for the exogenous states, we simulate a panel of firms, and analyze the data from that panel as in Gomes (2001). Specifically, we simulate 1,000 idiosyncratic productivity processes, one aggregate productivity process, and one aggregate stochastic cost process following the persistence and volatility given in Table II. We then create 1,000 total firm
productivity shocks by summing each firm specific and aggregate productivity series and taking the exponential. We simulate 600 years of data, throwing away the first 100 years to avoid any dependence on initial values. We then aggregate across firms to form aggregate corporate flows, analogous to our procedure in Compustat. The SMM procedure chooses parameters which minimize the weighted sum of squared errors between the model moments and their empirical counterparts. We use the efficient weighting matrix, which places more weight on moments which are more precisely estimated.\footnote{See \cite{Hansen1982}, \cite{Hansen1982}, and \cite{Lee1991}.}

To discretize the state space, we approximate the realization of the productivity and stochastic cost of external finance shocks using standard Gauss-Hermite quadrature techniques (see \cite{Tauchen1991}). There are six productivity states, which govern both idiosyncratic and aggregate productivity, and two aggregate states for $\xi$. For capital and liquid assets, we choose a large enough grid such that the stationary probabilities of being at the upper bound of the grid are negligible, something we verify ex-post.

Table II displays our calibration and compares our parameter choices to those in the literature. The second to last column offers a comparison to the parameters in \cite{Riddick2009}, (RW). Since our focus is on aggregate moments, where available we also report the parameters used in the real business cycle (RBC) literature as reported in \cite{Cooley1995}

We begin by describing the calibrated parameters. We set the risk free rate $r = 4\%$. We set $\tau$, the tax rate to 10\%. In our model, liquid assets are only accumulated in order to ultimately hedge investment opportunities in physical capital. Moreover, firms can simply over-accumulate physical capital and hedge via the additional cash flows that capital produces. Thus, if the tax rate is too high, liquidity can become a dominated asset. In practice, most firms do not pay the 34\% statutory rate. \cite{Leland2004} uses an estimate of 13\% to incorporate both personal and corporate taxation and we choose a number close to this value. We use a standard RBC value for depreciation, 8\%. For the production function curvature parameter, we specify 0.65, which is consistent with evidence in \cite{Cooper2006}. Higher curvature parameters, i.e. production functions closer to linear, imply too large investment volatilities and disinvestment frequencies. We calibrate the persistence of both idiosyncratic and aggregate productivity shocks to be 0.66, which allows us to consolidate idiosyncratic and aggregate productivity into a single state variable. This value is equal to that used by RW for the firm level. \cite{Khan2008}, page 407, contains a detailed discussion of the disagreement in the literature about this parameter, however based on our reading 0.66 is a modal value from prior studies. Moreover,
we also found the average industry level persistence in the data from Basu et al. (2006) to be close to this value (0.65). Note that the higher curvature of firms’ production function in our model implies that firms will be smaller than in RW, and thus smaller values for cost parameters can imply higher costs relative to firm size. RW uses the estimates from Hennessy and Whited (2007) for the costs of external finance, however they do not incorporate aggregate moments as we do. Finally, using two trend breaks, as advocated in Fernald (2007), we find an aggregate persistence of about 0.62 using the data from Fernald (2009). We set the total volatility of firm level productivity equal to the value in RW, which is also near the value used by Khan and Thomas (2011). We then specify that aggregate volatility is about one fourth of total volatility as in Khan and Thomas (2008) and Cooper and Haltiwanger (2006). We set the risk free rate to 0.04 as in the RBC literature, and in RW.

Given these calibrated parameters, we estimate the remaining, key, parameters. Specifically, we estimate: the fixed cost of raising external finance $\lambda_1$, the quadratic cost $\lambda_2$, the persistence of the stochastic cost $\gamma$, the volatility of the cost shocks $\sigma_\eta$, and the marginal investment adjustment cost parameter $a$ via Simulated Method of Moments (SMM). We seek to minimize $g(b)^tWg(b)$ where $g(b)$ is a vector of model errors (the difference between the model moments and data moments), $b = [\lambda_1, \lambda_2, \gamma, \sigma_\eta, a]^t$ is our vector of parameters, and $W$ is a weighting matrix. We set the weighting matrix to be the inverse variance-covariance matrix of the moments. We use the following seven moments to estimate the vector of parameters $b$: the pairwise correlations and the volatilities of investment, liquid asset accumulation, and external finance, and the average level of liquid assets to total assets. The correlations are chosen as our main focus is the co-movement between these variables. Including the volatilities ensures the magnitudes are fairly close and by targeting both correlations and volatilities we are ensuring co-variances are close to the data. Identification of our estimation parameters requires that the moments respond to changes in parameter values. In the companion appendix to this paper, we discuss which moments are primarily affected by which parameters by performing comparative statics on the model moments for changes in each of the estimated parameters. We find that the fixed cost of external finance is important in matching the level of cash holdings, and the quadratic cost is helpful in matching the volatility of external finance. The persistence and volatility of the stochastic cost help match the correlations between liquidity accumulation and external finance as well as between investment and external finance. Without the stochastic cost, the correlation between external finance and investment is very large because external finance only occurs when there are good productivity shocks and hence good investment opportunities. The stochastic cost breaks this high correlation and simultaneously generates the strong correlation between
external finance and liquid assets through market timing by firms. We report the moments in
the data and their counterparts in the model in Table III. We also report a large number of
additional moments which our estimation procedure was not explicitly targeted to match.

We report parameter estimates and standard errors in the middle panel of Table II. The stan-
dard errors show that all parameter values are significant. Intuitively, the fairly small standard
errors mean that the moments chosen do indeed respond greatly to changes in parameter values
so that the moments could not easily be generated by significantly different parameter values.
Our estimation implies an average percentage issuance cost of 3%, which is well within empirical
estimates. For example, Asquith and Mullins (1986) find that abnormal stock returns around
secondary equity offerings are about 3% and Hennessy and Whited (2007) estimate that firms
face an issuance cost of 8.3% on the first million dollars raised. Relative to RW, our estimated
parameters feature a higher fixed cost of raising external funds and lower linear and quadratic
components. The higher fixed cost is consistent with the importance of the variation in the
extensive margin of external finance over the business cycle, as well as with the evidence in

\cite{Bazdresch:2005} and \cite{Cummins:2004} which emphasize the importance of lumpy
external finance.

We estimate the persistence of the stochastic cost at 0.36. We note this is very close to
the annual persistence of the Baa-Aaa default spread in the data which we estimate at 0.4. We
assume the stochastic cost is uncorrelated with the aggregate productivity shock, an assumption
consistent with our empirical estimates. The empirical correlation between innovations in the
default spread and TFP shocks is -0.2 and is not statistically significantly different from zero.
Similarly, the empirical correlation between innovations to the expected default adjusted excess
bond spread from \cite{Gilchrist:2012} and TFP shocks is -0.1 and is again not statistically significantly different from zero. The autocorrelation of innovations in this series
based on annualized quarterly data is 0.35, so this is also very close to our persistence parameter
for \( \xi \) of 0.36, though our estimation was not targeted to match this persistence. We estimate the
volatility of the \( \xi \) shock to be around 2. Finally, it is important to note that given this volatility,
we choose the mean of the \( \xi \) shock \( \left( \exp \left( \mu - \frac{\sigma^2}{2} \right) \right) \) to generate an average credit shock of one.
This is without loss of generality, but makes the cost parameters \( \lambda_2 \) more easily comparable
to other models. More specifically, there is a one to one mapping between estimating \( \lambda_2 \) and
estimating the mean cost shock, but \( \lambda_2 \) is more easily interpretable.
B. Aggregate Analysis

Table III presents a comparison of aggregate issuance, savings, and investment moments in the data and in the model. Because we use the efficient weighting matrix in our estimation, the weights are based on statistical precision with which the empirical moments are estimated. These weights may not always correspond to the weights one would assign based on economic importance within the context of the current study; however, one benefit is that the overall fit of the model is better than when weights are chosen “by hand”. The model roughly matches the correlation between liquidity accumulation and external finance. The correlation between liquidity accumulation and external finance is higher, as it is in the data, when one conditions on firms raising external finance. The model generates fairly realistic correlations between liquidity accumulation, external finance, and investment with GDP.

Most importantly, the model generates the observed, highly positive, correlations between \( \xi \) and \( \rho_{l,e} \) and the percentage of firms raising external finance. This is for the same reasons that firms issue and save more in the two-date model when \( \xi \) is low. Only when the cost of external finance is low do firms find it optimal to issue costly external finance and save the proceeds. We also note that the correlation between TFP and these cross-sectional moments describing financing and savings by firms are lower than the correlations with \( \xi \), as is true empirically. When computing TFP in the model, we estimate TFP with a log linear production function, treating financial and investment costs as deadweight costs, which matches the empirical estimation of TFP used in the data. In the model, the correlation between \( xsrho_{l,e} \) and TFP is slightly positive due to the positive effect of better resource allocation when \( \xi \) is low.

C. Out of Sample Test: Firm Level Analysis

Although our focus is on aggregate moments, the firm level moments provide an additional check on the model. We use the optimal policy functions for a given firm to compare the model generated firm level moments to those in the data. We report the relevant moments in Table VI. Most correlations increase with aggregation, both in the data and in the model. This is not surprising, given that all firms are subject to independent idiosyncratic shocks, and common aggregate shocks. For firm level issuance and savings activity, we find a correlation between external finance and liquidity accumulation of 0.52 in the model, and 0.18 in the data. Thus, the model overshoots on this correlation at the firm level. We note that the lower correlation at the firm level in Compustat data may be due to noise, or accounting statement timing issues.

We find a slightly negative correlation between liquidity accumulation and investment in the model (-0.13) as well as in the data (-0.06). At the firm level, as in the aggregate, the
correlation between external finance and investment is too high in the model (0.77) relative to the data (0.20). Liquid asset balances in the model are fairly close to the firm level average in the data at 0.19 vs 0.15. The (unconditional) probability of raising funds is about 11% in the model. This probability is much lower than in the data (43%), However, we note that the high empirical probability of issuance may be due to a high incidence of small issuances with low costs, such as drawdowns on lines of credit. Consistent with this, Bazdresch (2005) provides evidence that a small fraction of large issuances account for most of firms’ external financing activity. Likewise, the average amount of external finance raised is smaller in than in the data. The model does reasonably well in matching the average level of investment and its volatility, and on the standard deviation of external finance. The model produces too low autocorrelation of investment at the firm level, but again this correlation increases with aggregation. Overall, given that our estimation targets aggregate moments, and that our study is focused on understanding aggregate moments, the performance of the model at the firm level seems satisfactory.

D. Are Stochastic Costs Necessary?

In this section, we provide two further strands of evidence in support of our estimated model. First, we show that a restricted version with constant costs, which is main alternative to our model, is rejected in favor of the full model with stochastic costs. Second, we show that key additional predictions of the model, moments that were not targeted in the estimation, are supported by empirical evidence.

First, we show that stochastic costs are statistically justified by the improvement in model fit, relative to a nested alternative model in which the cost of external finance is constant and the only shock firms face is to their (aggregate and idiosyncratic) productivity. Specifically, we test the improvement that the full model offers over a restricted version in which the volatility of the stochastic cost shock is restricted to be zero (at this point the persistence parameter is no longer identified). The difference in the minimized objective function for the unrestricted version of the model will clearly be smaller, but the J-test evaluates whether the difference in model errors more than justifies the addition of the cost persistence and volatility parameters. The difference between the objective functions, \( J_{restr} - J_{unrestr} \), is distributed \( \chi^2 \) with degrees of freedom equal to 2 (the number of restrictions). We are able to reject the the null hypothesis that the improvement in fit by the full model, relative to the restricted model, does not justify the addition of two additional parameters, at well below the 1% level. Thus, the data strongly prefer the full model with stochastic costs. Table IV presents the results of this test, along with a full comparison of the parameter estimates and implied moments from the full and restricted
models. Finally, we also note that the estimate of the volatility of the shock to the cost of external finance in the full model is highly significant.

We can also analyze the individual moments from the nested restricted model in order to understand economically its failures. Table [V] shows that a model with constant costs has several important shortcomings. The first is that it has difficulty generating cash balances with liquid assets only making up 2% of total assets. The second is that the only time the firm will want to raise external finance is when it also has a good shock to investment opportunities. Thus the correlation between investment and external finance in this model is close to 1, whereas it is 0.46 in the data. Finally, and importantly, at estimated parameter values the constant cost model does not generate aggregate issuance and savings waves. The estimation procedure, along with the set of target moments from the data, forces a compromise on the correlation between liquidity accumulation and external finance, and the resulting correlation is only weakly positive.

The \( J - \text{test} \) formally confirms that the full model fits the data better than what we consider to be its main alternative. Rather than rule out all other possible alternative explanations, we instead seek to show that important additional implications of the model are supported by the data. These include: (1) The predictions about the relationship between the cross sectional moments \( \rho_{it,e}^{rs} \) and %raise, and traditional empirical proxies for the cost of external finance, and (2) The larger difference in investment between high and low productivity firms when \( \rho_{it,e}^{rs} \) is high.

Consider the policy function in Figure 5. An implication of the model with a stochastic cost of external finance is that a firm with identical levels of productivity, liquid assets, and capital may have different levels of investment depending on the cost of external finance. Moreover, this difference in investment will be larger when investment opportunities are better. The first two panels of the Figure plot the optimal choice of investment for a firm with high productivity when the cost of external finance is low vs when the cost is high. This is a firm that has a strong desire to invest due to the high level of productivity and would likely raise external finance to do so (this depends somewhat on the stock of liquid assets but is true at average values. Therefore, this firms’ investment will be particularly sensitive to the cost of external finance. A very high cost of external finance will substantially constrain investment. Loosely speaking, the marginal product of capital will be set to the marginal cost of external finance, which will mean firms will optimally choose much lower investment when this cost is high. In short, the firm would like to invest but is constrained by the high cost of funds. In contrast, compare this to a firm with low current productivity. This firm does not have good investment opportunities and would therefore likely not want to raise external finance regardless of how costly it is. There is thus
very little difference in investment between high and low cost of external finance states for firms with low productivity. The testable implication is that the difference in investment between high and low productivity firms should depend largely on the cost of external finance. When the cost is high, differences in investment will tend to be much smaller even when the dispersion in productivity remains large. Figure 6 shows this result using the full investment policy functions for the highest and lowest productivity realizations, and for the high and low cost of external finance states.

Our difference in differences exercise is aimed at testing this key result empirically. Specifically, we define firm level productivity as the residual from regressing log operating income on the log capital stock. We then define an indicator for a firm as high or low productivity in each period based on whether they are above or below median productivity. Next, we use our $\rho_{i,t,e}^{s}$ variable to proxy for the cost of external finance at each date and create a dummy for when this variable is above or below its median value. The coefficient on the interaction term between these two dummies captures the differential effect of the cost of external finance on investment for high and low productivity firms as shown for model policy functions in Figure 5.

Table VII shows the results from the regression where we include a number of additional controls. We find a negative, statistically significant coefficient on the interaction term of $-0.24$. This is consistent with our intuition and implies that the difference in investment rates between high productivity firms and low productivity firms is $0.24\%$ lower on average during times when the cost of external finance is high compared to when it is low. A back of the envelope calculation implies that if investment by low productivity firms is invariant to changes in the cost of external finance, then the investment rate of high productivity firms is $0.24\%$ lower when the cost of external finance is high than when it is low. The regression is consistent with the fact that investment is more constrained by the cost of external finance for high productivity firms who have good investment opportunities. According to the Figure 5, we should also see this effect being stronger for small firms both because they are relatively more constrained (due to the fixed cost) and because they respond more strongly to productivity shocks because of their steeper marginal product of capital. We see this in the data as well. Accordingly, conditioning on the smallest 50% of firms, the difference in differences interaction coefficient increases to $-0.36\%$ and conditioning on the smallest 25% of firms it increases to $-0.68\%$. This is qualitatively in line with the model predictions. These results also support the idea that our measure of the cost of external finance based on $\rho_{i,t,e}^{s}$ is economically meaningful, since variation in $\rho_{i,t,e}^{s}$ has an empirically significant impact on investment.

These findings are complementary to, but different from the existing literature that relates
shocks to the cost of external finance to investment. The existing literature focuses on a shock
to the financial sector that affects some firms more than others (i.e., a common approach in
the recent crisis was bank connectedness to Lehman as a proxy for lender health). The idea
is that credit tightens more for certain firms and therefore these firms will reduce investment
and employment by more. The identification scheme in these studies is aimed at ensuring that
treated and untreated firms have similar investment opportunities. In contrast, in our design,
we assume that all firms are hit by an aggregate shock to cost of external finance, but that the
shock has a much greater impact on high productivity firms who have a greater incentive to
invest. Thus, identification comes from the fact that firms are affected differently by common
shocks, rather than because the shock is different across firms. This key difference is what allows
us to do aggregate inference using cross sectional variation in firm policies.

This diff in diff exercise provides unique support for our model with stochastic costs of ex-
ternal finance. The model with constant costs, for example, could not explain these results. In
that model the only state variables are capital, stock of liquid assets, and productivity. Control-
ing for these, investment should not vary. That alternative model is certainly a simplification
relative to the data, however, it is not obvious in what more elaborate alternative model $\rho_{i,t,e}^{s}$
would capture differing investment policies along the lines we see in the data.

VI. Estimating The Cost of External Finance

Intuitively, if at a given date firms are simultaneously raising external funds and saving them, it
is likely that costs of raising external finance are low. Empirically we find correlations between
the cross sectional correlation between liquidity accumulation and external finance, $\rho_{i,t,e}^{s}$, and the
default spread and lending standards of -0.64 and -0.58, respectively. We also show empirically
that the percentage of firms raising external finance at any given date, $%\text{raise}_t$, is highly corre-
related with these measures as well. Analytically, we show in a two-date model that $\rho_{i,t,e}^{s}$ will be
higher when $\xi$ is lower, and we show that this intuition also holds in the calibrated infinite hori-
zon model. In data generated by the estimated version of that model, the correlation between
$\rho_{i,t,e}^{s}$ and $\xi$ is 0.86. Thus, we argue that focusing on times when firms simultaneously raise funds
and save them (as measured by $\rho_{i,t,e}^{s}$) can be informative about aggregate credit conditions.

Building on these insights, we use the relationship in the model between financing and savings
activity in the cross section, and the aggregate cost of external finance, to estimate the average

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13See Ivashina and Scharfstein (2010), Duchin et al. (2010), Campello et al. (2010), Almeida et al. (2009). See also
Matvos and Seru (2011) for evidence of financing shocks from their estimation of a structural model comparing resource
allocation by diversified and undiversified firms during the financial crisis.
cost of external finance per dollar raised in the US time series 1980-2010. Specifically, we construct an aggregate external finance index in the model, in which the cost is predicted to be a weighted average of cross sectional moments describing firms’ issuance and savings. We then use the coefficients from this index, along with the empirical cross sectional moments at each date, to construct our estimates for the US time series.

In the model, we can express the average cost of a unit of external finance per dollar raised as \( \frac{E[\phi_e(e, \xi)]}{E[e|e<0]} \). This is the average cost of external finance paid across firms at any given date, divided by the average amount raised. We compute this expression at each point in time in the model and then use this series as the dependent variable in the following time series regression estimated on model data:

\[
E[\phi_e(e, \xi)] = \alpha + \beta_1 \%\text{raise}_t + \beta_2 \rho_{ss, e,t} + \varepsilon_t
\]

We normalize the independent variables, \%\text{raise}_t and \( \rho_{ss, e,t} \), to have mean zero and unit standard deviation. We report the regression results in Table VIII. We find that both the coefficients, \( \beta_1, \beta_2 \) are negative, as expected, and around -0.01 in magnitude. This implies that a standard deviation increase in either variable is associated with a 1% decrease in the average cost paid per dollar raised. The constant term, \( \alpha \), is 3% and represents the average cost of external finance paid per dollar raised in the model. Thus, each of these two cross sectional moments is economically important for explaining variation in external financing costs in the model data. Moreover, the \( R^2 \) in this regression is 71%, and thus these two variables alone do a very good job explaining variation in the cost paid by firms in the model economy. We also show the explanatory power of each variable individually, and finally, we show that these variables also measure \( \xi_t \), the stochastic cost of external finance “level” shock series, in the model. In both cases, the explanatory power of these variables jointly exceeds using each one individually, showing that each contains additional information to uncover the cost of external finance. This makes sense given the fact that including both moments helps sort out the effects of TFP shocks vs. \( \xi \) shocks, as discussed above.

We then apply the estimated regression coefficients, \( \hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2 \), to the analogous empirical moments to form a time series of estimates of the cost of external finance paid per dollar raised at each point in time. We plot this series in the top panel of Figure 7. Our estimated cost of external finance series has very intuitively appealing properties. The estimated cost is high in the early 1980’s, late 1980’s, around the 2001 dot com crash, and in the recent financial crisis. Moreover, each spike in this cost is associated with a recession. Economically, the cost per dollar raised varies from essentially zero in the mid to late 1990’s, up to 4.5% during the recent
financial crisis, which is a full 50% larger than the overall average cost of 3%. This value seems economically large if we consider the impact on the required return for a firm debating whether to take on an investment project for which it must raise external finance.

We next turn to a more structural estimation of the cost of external finance using the model. As in the regression method, we use our calibrated model, along with cross sectional moments to make inferences about the aggregate cost of external finance. In particular, at each date in time, we use a version of Simulated Method of Moments (SMM) to infer the value of the stochastic cost that generates simulated cross sectional moments as close as possible to the data. The difference between our estimation exercise and a typical SMM estimation is that we are looking to uncover a hidden state instead of estimating a parameter. This distinction matters since state variables, unlike parameters, influence the model’s transition dynamics over time, not just through policy functions, but also directly.\(^{14}\) However, note that while our estimates in later periods for the state will depend on earlier estimated values, we will still get a consistent and unbiased estimate of the state.

As moments, we choose the correlation between liquidity accumulation and external finance as well as the percentage of firms raising external finance at each date, as these are each informative about this cost in the model. More specifically, we define the vector \(M_t\) as follows:

\[
M_t = \begin{bmatrix}
\rho_{N,\text{mod}} \left( \frac{i_n}{i_n + e} \left( \xi_t \right); \frac{i_n}{i_n + e} \left( \xi_t \right) \right) - \rho_{N,\text{data}} \left( \frac{i_n}{i_n + e} \left( \xi_t \right) \right) \\
E_{N,\text{mod}} [1_{\epsilon<0} (\xi_t)] - E_{N,\text{data}} [1_{\epsilon<0}]
\end{bmatrix}
\]

where \(\rho_{N,\text{mod}}(x, y)\) represents the cross sectional correlation between \(x\) and \(y\) in the model, which is a function of \(\xi_t\), and \(\rho_{N,\text{data}}(x, y)\) represents the empirical counterpart. We use \(N\) to emphasize that these are cross sectional, rather than time-series moments.\(^{15}\)

At every date \(t\) we choose the value \(\xi_t\) that minimizes deviations of the cross sectional model implied moments and empirical moments. Specifically, we choose \(\xi_t\) to minimize the following objective function

\[
\min_{\{\xi_t\}} M_t'WM_t
\]

where we set \(W = I_{2x2}\) as the identity matrix which weights all moments equally. We choose equal weights for parsimony, because the regression coefficients in the index are almost exactly

\(^{14}\)An early example of using a model along with empirical moments to make inferences about a hidden aggregate state is [Eisfeldt and Rampini (2006)](Eisfeldt2006), which uses a calibrated model along with moments describing capital reallocation to infer the business cycle dynamics of capital liquidity.

\(^{15}\)We have also estimated the series adding two additional moments: \((\xi_t - \mu_\xi) - \rho (\xi_{t-1} - \mu_\xi)\) and \(((\xi_t - \mu_\xi) - \rho (\xi_{t-1} - \mu_\xi))^2 - \sigma_\varepsilon^2\) which help ensure that the series \(\xi_t\) follows an AR(1) with the parameters we calibrated. The results are similar.
equal, and because we do not see a reason for this exercise to weight by the statistical precision with which the two moments are estimated.

We initialize the series by first starting capital and liquidity at their steady state values, and then feeding in the observed aggregate TFP and the default spread series beginning in 1951. We do this to ensure that the model distribution over capital and liquidity stocks reflects the history of TFP shocks in the US given calibrated $\xi_t$ shocks. We use the empirical realizations of these series to proxy for $z_{t,agg}$ and $\xi_t$ until 1980 when our Compustat sample begins. Thereafter, we estimate $\xi_t$ date-by-date by setting the value of aggregate productivity to equal to our observed TFP series and choosing $\xi_t$ to minimize our objective function. This allows us to estimate a rolling time series for the stochastic cost of external finance.

The results from the SMM estimation procedure are displayed in the bottom panel of Figure 7, where high cost dates are indicated in red. Comparing these dates to the regression implied estimates, one sees that the estimates from both methods are very similar; the SMM method picks a high $\xi$ when the regression method implies a high average cost. Both methods also estimate high costs during the recessions in our sample, though they are not simply recession indicators. The main limitation of the SMM methodology is that the procedure must choose one of the two states specified in the numerical solution of the model, whereas the regression index method estimates form a continuous time series.

In sum, both of our estimation procedures appear to pick up events that our priors suggest might be associated with costly external finance, such as the recession of the early 80’s, the crash of the tech boom in 2001, and the recent financial crisis. We take these findings to be additional evidence which supports using moments implied by our model to identify times when the cost of external finance is high or low. Our model and moment implied cost estimates further provide a “revealed preference” measure of the cost of external finance which is complementary to the existing list of empirical proxies.

VII. Conclusion

We document the empirical regularity of aggregate issuance and savings waves, and show that cross sectional moments describing firms’s issuance and savings behavior are informative about the aggregate cost of external finance. We document that, empirically, the time series of the cross sectional correlation between external finance and liquidity accumulation, and the time series of the percentage of firms raising external finance, are highly correlated with standard measures of the aggregate state of the cost of external finance, such as the default spread and
the fraction of banks tightening lending standards.

We argue that both the observed realization of the correlation between external finance and liquidity accumulation in the cross section, $\rho_{il,e}^{xs}$, and our model implied estimate of the level of the cost of external finance in the US time series 1980-2010, are useful measures of the state of the aggregate level of the cost of external finance. Using firms’ actual decisions about how much external finance to raise and how they use the proceeds from external finance is a revealed preference method of making inferences about the true cost of external finance. Such a measure seems useful for measurement and for policy guidance.

Understanding the role of a potentially time varying cost of external finance is important for several reasons. First, studying whether shocks to the cost of external finance are important for firm financing, liquidity accumulation, and investment dynamics may help to further understand the role of the financial sector in business cycles. From a theoretical standpoint, many models featuring endogenous variation in the cost of external finance in business cycles feature a tight link between variation in fundamentals such as productivity and variation in the informational frictions which make external finance costly. In contrast, shocks to the cost of external finance in recent DSGE models are often only partially correlated with other fundamental shocks. By using our dynamic model as a filter on the US time series for firm financing, liquidity accumulation, and investment, we provide implied estimates of the cost of external finance at each date.

References


**VIII. Data Appendix**

Data Appendix

Our data construction closely follows Covas and Den Haan (2011a). Our primary source of data is the Compustat fundamentals annual file. Our main results use data from 1980-2010. We exclude financials, utilities and firms with SIC codes starting with 9. We also exclude firms with missing assets, equity, debt, and those with missing or negative PPE and cash balances. As in Covas and Den Haan (2011a), we also remove GM, GE, Chrysler, and Ford, since these
firms were the most affected by the accounting change in 1988 requiring firms to consolidate the balance sheets of their wholly owned subsidiaries.

**Computstat Data**

We first define liquidity accumulation, investment, and external finance as:

\[
\text{Investment} = i_k = \text{CAPEX} \quad \text{(Capital Expenditures)}
\]

\[
\text{Liquidity Accumulation} = i_l = \text{CHECH} \quad \text{(Cash and cash equivalents, change)}
\]

\[
\text{External Finance} = -e = -(CF_D + CF_E)
\]

For flows to debt and equity and operating cash flows we use the statement of cash flows:

*For statements of cash flows:*

\[
CF_E = - \text{Sale of common and pref. stock (SSTK)} + \text{Purchase of common and pref. stock (PRSTKC)} + \text{Cash dividends (DV)}
\]

\[
CF_D = - \text{Long-term debt issuance (DLTIS)} + \text{Long-term debt: reduction (DLTR)} + \text{Changes in current debt (DLCCH)} + \text{Interest paid (net) (XINT)}
\]

*For statements by source and use of funds:*

\[
CF_E = - \text{Sale of common and pref. stock (SSTK)} + \text{Purchase of common and pref. stock (PRSTKC)} + \text{Cash dividends (DV)}
\]

\[
CF_D = - \text{Long-term debt issuance (DLTIS)} + \text{Long-term debt: reduction (DLTR)} + \text{Changes in current debt (DLCCH)} + \text{Interest paid (net) (XINT)}
\]

*For working capital statements:*

\[
CF_E = - \text{Sale of common and pref. stock (SSTK)} + \text{Purchase of common and pref. stock (PRSTKC)} + \text{Cash dividends (DV)}
\]

\[
CF_D = - \text{Long-term debt issuance (DLTIS)} + \text{Long-term debt: reduction (DLTR)} + \text{Changes in current debt (DLCCH)} + \text{Interest paid (net) (XINT)}
\]

*For cash statements by activity:*

\[
CF_E = - \text{Sale of common and pref. stock (SSTK)} + \text{Purchase of common and pref. stock (PRSTKC)} + \text{Cash dividends (DV)}
\]

\[
CF_D = - \text{Long-term debt issuance (DLTIS)} + \text{Long-term debt: reduction (DLTR)} + \text{Changes in current debt (DLCCH)} + \text{Interest paid (net) (XINT)}
\]

**Flow of Funds Data**
We use annual data from the electronic ASCII flow of funds seasonally adjusted annual rates table F.102 available at http://www.federalreserve.gov/Releases/z1/Current/data.htm. Refer to the coded tables for definitions and relationships between entries. Codes appear in parentheses after variable names. Interest payments, not reported in table F.102, are from NIPA table 1.14 line 25 “Net interest and miscellaneous payments” for nonfinancial corporate business.

\[ CF_D = \text{Commercial paper (FA103169700) + Mortgages (FA103065003) - Credit market instruments (FA104104005) + NIPA interest} \]

\[ CF_E = \text{Net dividends (FA106120005) - Net new equity issues (FA103164003)} \]

\[ \text{Liquidity Accumulation} = i_t = \text{Net acquisition of financial assets - Commercial paper - Mortgages - Trade receivables - Other Assets} \]

\[ \text{Investment} = i_k = \text{Capital expenditures} \]

**Other Data**

The following series used can be found in the FRED database at the St Louis Fed website.

\[ Gdp = \text{Real gross domestic product} \]

\[ \text{Default Spread} = \text{Difference between Moody’s Seasoned Baa and Aaa yield. We use end of year values.} \]

\[ \text{Lending Standards} = \text{Net Percentage of Domestic Respondents Tightening Standards for Commercial and Industrial Loans Large and Medium Firms (DRTSCILM). We use end of year values.} \]

Finally, we obtain TFP data from John Fernald’s website at: [http://www.frbsf.org/economics/economists/staff.php?jfernald](http://www.frbsf.org/economics/economists/staff.php?jfernald) We construct the log level series from the series of annual changes provided, and detrend the series with two breaks as in Fernald (2007), which advocates breaks after 1974 and 1995. Shocks are then residuals from an AR(1) regression on the log level series.
IX. Figures, Tables. View in color.

Figure 1: We plot aggregate accumulation of liquid assets against aggregate external finance. Our sample excludes the largest 10% of firms. Plotted data are normalized by lagged assets, HP-filtered, and then scaled to have unit variance. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). The correlation between liquidity accumulation and external finance is 0.6. A univariate regression of liquidity accumulation on external finance yields an $R^2$ of 0.36 and a regression coefficient estimate of 0.19.
Figure 2: We plot the cross sectional correlation between liquidity accumulation and external finance and % of firms raising external finance in the top panel. These moments are highly correlated with the shock to the cost of external finance in our quantitative model. The bottom panel plots the negative of the default spread, the net % of banks tightening lending standards, and the sentiment index. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). All series are standardized to have mean zero and unit variance.
Figure 3: Two period model intuition for $\xi_H$, $\xi_M$, and $\xi_L$. $\theta z_i k^{\theta-1}$ represents the marginal product of capital, $\xi(-e)$ the marginal cost of external finance, and $r_l - 1$ the net return on liquid assets. $i_l$ and $i_k$ represent investment in liquid assets and physical capital, $-e$ represents external finance raised.

Figure 4: Two period model intuition for a panel of firms with heterogeneous levels of productivity $z_i$, and three levels of $\xi$, $\xi_H$, $\xi_M$, and $\xi_L$. $\theta z_i k^{\theta-1}$ represents the marginal product of capital, $\xi(-e)$ the marginal cost of external finance, and $r_l - 1$ the net return on liquid assets. $i_l$ and $i_k$ represent investment in liquid assets and physical capital, $-e$ represents external finance raised.
Figure 5: This figure plots firm policy functions. The x-axis is firm size. The left and right panels give the policy functions in low and high cost states. Productivity and liquidity are averaged out in order to construct the plots in two dimensions.
Figure 6: This figure plots firm policy functions for investment for the highest and lowest levels of productivity (top and bottom rows respectively), and for the low and high cost of external finance states (left and right columns respectively) in order to illustrate the difference in differences in investment across $\xi$ states.
Figure 7: This figure describes the average cost of external finance paid per dollar of external finance raised in the US time series estimated using cross sectional moments and the estimation procedure described in Section VI. The top graph uses the regression index method. The bottom graph uses the two state SMM procedure, and indicates a high cost for years in red.
Table I: This table displays the main aggregate issuance and savings waves facts. Except where noted, we use annual Compustat data from 1980-2010. We normalize aggregate series by the lag of total assets and hp-filter. Size bins are determined by total asset size. The main results in the paper use the [0,90]% sample. Flow of funds data are normalized by the trend in gross value added for the corporate sector. Narrow liquidity is the net acquisition of financial assets minus trade receivables minus miscellaneous assets. Broader liquidity also includes 1/3 of miscellaneous other assets as liquid assets. * indicates significance at a 5% level. $\rho_{t}^{x,a}(\frac{-e_{i,t}}{TA_{i,t}}, \frac{i_{i,t}}{TA_{i,t}})$

<table>
<thead>
<tr>
<th>Aggregate Issuance and Savings</th>
<th>$\rho \left( \frac{\sum(-e_{t})}{\sum TA_{t-1}}, \frac{\sum i_{t}}{\sum TA_{t-1}} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0,50] %</td>
<td>0.84*</td>
</tr>
<tr>
<td>[0,90] %</td>
<td><strong>0.60</strong></td>
</tr>
<tr>
<td>[0,100] %</td>
<td>0.12</td>
</tr>
<tr>
<td>Conditional on Raising funds: e &lt; 0</td>
<td>0.74*</td>
</tr>
<tr>
<td>No Dividends</td>
<td>0.68*</td>
</tr>
<tr>
<td>No Rating</td>
<td>0.56*</td>
</tr>
<tr>
<td>Flow of Funds: Narrow Liquidity</td>
<td>0.33</td>
</tr>
<tr>
<td>Flow of Funds: Broader Liquidity</td>
<td>0.38*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho \left( \rho_{t}^{x,a}(\cdot), \text{Aggregate State}_{t} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minus Baa-Aaa Spread</td>
</tr>
<tr>
<td>Minus Lending Standards</td>
</tr>
<tr>
<td>TFP Shock</td>
</tr>
</tbody>
</table>
Table II: We give our calibrated parameters below along with those in Riddick and Whited (RW) and the standard business cycle literature (RBC). The label e.c.f. denotes external cost of finance and i.a. denotes investment adjustment costs. The lower panel gives the implied average costs of issuance and investment firms pay with the given parameters. For example, the implied average cost of issuance gives the average cost paid for a firm raising external finance as a fraction of the amount of funds raised.

<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th>Symbol</th>
<th>Description</th>
<th>Model</th>
<th>RW</th>
<th>RBC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>τ</td>
<td>tax rate</td>
<td>0.1</td>
<td>0.20</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>δ</td>
<td>depreciation</td>
<td>0.08</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>curvature</td>
<td>0.65</td>
<td>0.75</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>ρ</td>
<td>persistence</td>
<td>0.66</td>
<td>0.66</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>σ</td>
<td>total vol of prod</td>
<td>0.121</td>
<td>0.121</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>σᵢ</td>
<td>idiosyncratic vol</td>
<td>0.11</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>σᵣ</td>
<td>aggregate vol</td>
<td>0.03</td>
<td>—</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>risk-free</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>Symbol</th>
<th>Description</th>
<th>Estimate</th>
<th>Std Err</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>invest adj cost</td>
<td>0.13</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>λ₁</td>
<td>e.c.f. fixed</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>λ₂</td>
<td>e.c.f. quad</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>σᵣ</td>
<td>vol cost of funds</td>
<td>2.08</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>γ</td>
<td>persistence</td>
<td>0.36</td>
<td>0.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Implied Average Costs Paid</th>
<th>Symbol</th>
<th>Description</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>E[φₑ(ₑ)]/ₑ</td>
<td></td>
<td>implied average issuance cost</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Table III: This table displays aggregate moments from the model using a simulated panel of firms. We compare these moments with those from annual Compustat data, 1980-2010. For correlations, we normalize each series by lagged assets and apply the hp-filter. All other series are normalized by current assets. $\rho_{\xi,t}^{x}$ is $\rho_t \left( \frac{-e}{TA_i}, \frac{i_l}{TA_i} \right)$. TFP are TFP level shocks. We use the Baa-Aaa default spread as an empirical proxy for $\xi$. * indicates significance at 5% level. We use notation from the model: $-e$ represents external finance (negative of payouts), $i_l$ liquidity accumulation, $i_k$ investment in physical capital, and $l$ liquid balances. We normalize each series by total assets except investment which is normalized by physical capital $k$.

<table>
<thead>
<tr>
<th>Estimation Moments</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>$E[l]$</td>
<td>0.11</td>
<td>0.19</td>
</tr>
<tr>
<td>$\sigma(i_l)$</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>$\sigma(i_k)$</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma(-e)$</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho(i_l, -e)$</td>
<td>0.60*</td>
<td>0.77</td>
</tr>
<tr>
<td>$\rho(i_l, i_k)$</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>$\rho(-e, i_k)$</td>
<td>0.46*</td>
<td>0.64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Additional Moments</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>$E[i_k]$</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>$\rho(i_{k,t}, i_{k,t-1})$</td>
<td>0.38*</td>
<td>0.34</td>
</tr>
<tr>
<td>$E[-e]$</td>
<td>-0.01</td>
<td>-0.10</td>
</tr>
<tr>
<td>$prob(-e &gt; 0)$</td>
<td>0.05</td>
<td>0.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments Conditional on $e&lt;0$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>$\rho(i_l, -e)$</td>
<td>0.74*</td>
<td>0.92</td>
</tr>
<tr>
<td>$\rho(i_l, i_k)$</td>
<td>0.37*</td>
<td>0.63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Business Cycle Correlations</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>$\rho(i_l, gdp)$</td>
<td>0.00</td>
<td>0.21</td>
</tr>
<tr>
<td>$\rho(-e, gdp)$</td>
<td>0.28*</td>
<td>0.69</td>
</tr>
<tr>
<td>$\rho(i_k, gdp)$</td>
<td>0.47*</td>
<td>0.62</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TFP vs. $\xi$ Moments</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>$\rho(xsrho_{i_l,e}, \xi)$</td>
<td>0.64*</td>
<td>0.86</td>
</tr>
<tr>
<td>$\rho(%raise, \xi)$</td>
<td>0.59*</td>
<td>0.84</td>
</tr>
<tr>
<td>$\rho(xsrho_{i_l,e}, TFP)$</td>
<td>0.48*</td>
<td>0.12</td>
</tr>
<tr>
<td>$\rho(%raise, TFP)$</td>
<td>0.25</td>
<td>0.44</td>
</tr>
</tbody>
</table>
**Table IV:** This table compares the model with stochastic costs to a model which turns off stochastic costs of external finance (restricted model). We compare the moments across models as well as the estimated parameter values. * indicates significance of parameter values. The bottom panel formally tests the restrictions and rejects the restricted model.

<table>
<thead>
<tr>
<th>Aggregate Moments</th>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Restricted Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E \left[ \frac{l}{l+k} \right]$</td>
<td>0.11</td>
<td>0.19</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(i_t, -e)$</td>
<td>0.60*</td>
<td>0.77</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(i_k, i_t)$</td>
<td>0.12</td>
<td>0.11</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(i_k, -e)$</td>
<td>0.46*</td>
<td>0.64</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>$\sigma(i_t)$</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$\sigma(i_k)$</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$\sigma(-e)$</td>
<td>0.03</td>
<td>0.05</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th></th>
<th>Model</th>
<th>Restricted Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.36*</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>2.08*</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.06*</td>
<td>0.05*</td>
<td></td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.005*</td>
<td>0.006*</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.13*</td>
<td>0.11*</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Formal Test of Restricted Model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$JT_{\text{rest}} - JT \sim \chi^2(2)$</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Test of over-identifying restrictions
**Table V:** This table displays moments for debt and equity separately using our Compustat sample. We normalize each aggregate series by lagged aggregate assets and apply the hp-filter. * indicates significance at 5% level.

<table>
<thead>
<tr>
<th>Aggregate Compustat Moments: Debt vs. Equity</th>
<th>Unconditional</th>
<th>Conditional on $e &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho (\text{liqacc, debt})$</td>
<td>0.16</td>
<td>0.33*</td>
</tr>
<tr>
<td>$\rho (\text{liqacc, equity})$</td>
<td>0.69*</td>
<td>0.77*</td>
</tr>
<tr>
<td>$\rho (\text{inv, debt})$</td>
<td>0.60*</td>
<td></td>
</tr>
<tr>
<td>$\rho (\text{inv, equity})$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table VI:** Firm Level Facts. The table gives firm level moments. We compute the relevant moment for the entire panel of firms and then take a median across firms. We use our simulated panel of data (Model column) and Compustat (Data column). We normalize the series by total book assets. * indicates significance at 5% level. We use notation from the model: $-e$ represents external finance (negative of payouts), $i_l$ liquidity accumulation, $i_k$ investment in physical capital, and $l$ liquid balances. We normalize each series by total assets except investment which is normalized by physical capital $k$.

<table>
<thead>
<tr>
<th>Firm Level Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[l]$</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>$\sigma(i_l)$</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>$E[i_k]$</td>
<td>0.06</td>
<td>0.10</td>
</tr>
<tr>
<td>$\sigma(i_k)$</td>
<td>0.07</td>
<td>0.19</td>
</tr>
<tr>
<td>$\rho(i_{k,t}, i_{k,t-1})$</td>
<td>0.29*</td>
<td>0.04</td>
</tr>
<tr>
<td>$E[-e]$</td>
<td>-0.02</td>
<td>-0.11</td>
</tr>
<tr>
<td>$\sigma(-e)$</td>
<td>0.17</td>
<td>0.22</td>
</tr>
<tr>
<td>$\rho(i_l, -e)$</td>
<td>0.18*</td>
<td>0.52</td>
</tr>
<tr>
<td>$\rho(i_l, i_k)$</td>
<td>-0.06*</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\rho(-e, i_k)$</td>
<td>0.20*</td>
<td>0.77</td>
</tr>
<tr>
<td>%raise</td>
<td>0.43</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Table VII: This table presents the diff in diff results to estimate the effect of a high cost of external finance on investment. * indicates statistical significance at 10%. Standard errors are clustered by firm.

Regression: \( \left( \frac{d^k}{dt^k} \right)_{n,t} = \alpha_t + \beta X_{n,t} + \delta_1 1_{(z_{n,t}>0)} + \delta_2 1_{(z_{n,t}>0), (\rho_{n,t}^{i,e}<0)} + \varepsilon_{n,t} \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Data</th>
<th>Data (smallest 50%)</th>
<th>Data (smallest 25%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital stock</td>
<td>-0.3</td>
<td>0.2*</td>
<td>0.4*</td>
<td>0.4</td>
</tr>
<tr>
<td>Liquid assets</td>
<td>0.2</td>
<td>-0.2*</td>
<td>-0.2*</td>
<td>-0.1*</td>
</tr>
<tr>
<td>Productivity</td>
<td>1.4</td>
<td>0.8*</td>
<td>0.5*</td>
<td>0.4*</td>
</tr>
<tr>
<td>Lagged investment</td>
<td>0.5*</td>
<td>0.3*</td>
<td>0.2*</td>
<td></td>
</tr>
<tr>
<td>High Prod</td>
<td>3.0</td>
<td>0.7*</td>
<td>0.9*</td>
<td></td>
</tr>
<tr>
<td>High Prod, High Cost</td>
<td>-0.8</td>
<td>-0.2*</td>
<td>-0.4*</td>
<td>-0.7*</td>
</tr>
<tr>
<td>Time fixed effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
</tbody>
</table>

Table VIII: This table estimates the cost of external finance in the model using regressions of cost measures on \( \rho_{i,t}^{e,x} \), the cross sectional correlation between liquidity accumulation and external finance, and, \%raise, the percentage of firms raising external finance. We measure the cost of external finance in two ways: first, as the average cost paid per dollar of external finance raised, \( E_N \left[ \frac{\phi_e}{e} \right] \), and second as the level of the stochastic cost series \( \xi \).

<table>
<thead>
<tr>
<th>y</th>
<th>Constant</th>
<th>( \rho_{i,t}^{e,x} )</th>
<th>%raise</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_N \left[ \frac{\phi_e}{e} \right] )</td>
<td>0.030</td>
<td>-0.018</td>
<td>66.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(52.27)</td>
<td>(-31.68)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E_N \left[ \frac{\phi_e}{e} \right] )</td>
<td>0.030</td>
<td>-0.017</td>
<td>64.3%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(51.19)</td>
<td>(-30.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E_N \left[ \frac{\phi_e}{e} \right] )</td>
<td>0.030</td>
<td>-0.011</td>
<td>-0.009</td>
<td>71.1%</td>
</tr>
<tr>
<td></td>
<td>(58.00)</td>
<td>(-10.78)</td>
<td>(-8.66)</td>
<td></td>
</tr>
<tr>
<td>( \xi )</td>
<td>-0.01</td>
<td>-0.86</td>
<td>73.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.16)</td>
<td>(-36.94)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi )</td>
<td>-0.01</td>
<td>-0.85</td>
<td>71.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.09)</td>
<td>(-35.35)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi )</td>
<td>-0.01</td>
<td>-0.49</td>
<td>-0.41</td>
<td>77.3%</td>
</tr>
<tr>
<td></td>
<td>(-1.34)</td>
<td>(-11.27)</td>
<td>(-9.40)</td>
<td></td>
</tr>
</tbody>
</table>
Table IX: This table gives correlations of our estimated cost of external finance estimated using the cross-section of firm policies, $\xi$, with other empirical proxies for the cost of external finance.

<table>
<thead>
<tr>
<th>Correlation with:</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default spread</td>
<td>0.64*</td>
</tr>
<tr>
<td>Index of lending standards</td>
<td>0.58*</td>
</tr>
<tr>
<td>Sentiment index</td>
<td>0.72*</td>
</tr>
<tr>
<td>Average of three</td>
<td>0.83*</td>
</tr>
</tbody>
</table>