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THE CAPITAL ASSET PRICING MODEL:
SOME OPEN AND CLOSED ENDS

by

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Introduction

Portfolio analysis, in the modern sense, has barely been with us 20 years—its beginning is usually attributed to Markowitz's 1952 paper and to Tobin (1958). Over this brief span, it has thoroughly pervaded every aspect of finance, gained a strong foothold in economics, and is even beginning to make its presence felt in accounting. Portfolio theory has not only become a field of study in its own right but is an indispensable part of the foundations of corporation finance, the study of financial intermediation, and the modern analysis of money and capital markets.

At least four distinct models of portfolio choice have been "put on the market": the expected utility model, the mean-variance approach (in many versions), the long-run growth model, and the chance-constrained programming approach. Based on the market acceptance test, there is clearly one winner at this point. The mean-variance model of portfolio choice is so dominant that to many business school graduates portfolio theory is synonymous with mean-variance theory.

Why does the mean-variance (MV) model occupy such a dominant position? The usual explanation for the prominence, at any particular time, of a given theory in science is that a theory prevails until it is replaced by a better one. Thus, we might expect—and this will indeed be borne out—that the answer to our question lies as much in the problems associated with the other models as in the net strength of the MV approach.

The chance-constrained programming (CCP) model of portfolio choice [see, e.g., Naslund and Whinston (1962), Agnew, Agnew, Rasmussen, and Smith (1969)] came into being in conjunction with the evolution of chance-constrained programming as a branch of linear programming. Typically, the
objective is to maximize expected return subject to the return not falling below some specified rate with a minimum probability (such as .95). This feature, which reveals an intimate association with classical statistics, clearly offers some intuitive appeal. But CCP suffers from noncontractability except in the case of normal distributions, and never attracted much following.\(^1\) A fatal shortcoming in the view of many is the model's lack of consistency with the von Neumann-Morgenstern postulates [Borch (1968, pp. 41-42)].

The expected utility approach, sometimes referred to as the decision theoretic approach in situations involving risk, offers its converts a "completely" rational and general decision framework. Yet, most investors, as Roy (1952, p. 433) pointed out, are not grateful for the advice that they maximize expected utility (subject to environmental and other constraints). The model is widely viewed as rather nonoperational and many investors, after taking into account the work involved in employing it, are not sure they still want to be (completely) rational in the prescribed sense. However, the expected utility model is still gaining supporters and numerous individuals employ it as the primary criterion for judging the soundness of other portfolio models.

The long-run growth model,\(^2\) which calls for maximizing the geometric

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\(^1\)Some chance-constrained portfolio models, referred to as "safety-first" models [e.g., Roy (1952)], have, via Chebychev's Inequality, tended to become classified in the mean-variance category [see, e.g., Lintner (1965a) and Pyle and Turnovsky (1970)].

\(^2\)Generally credited to Williams (1936), Kelly (1956), Latané (1959), and Breiman (1960).
mean of principal plus return in each period, is consistent with logarithmic utility (only). Even though the optimal policy of this model almost surely leads to more capital (though generally not expected utility) under reinvestment in the long run than competing policies do, it is rather nonintuitive and somewhat cumbersome computationally. While still not widely understood, the long-run growth model appears to be gaining in absolute terms.

In contrast, the MV model is highly intuitive—the very concepts mean and variance have almost become synonyms for risk and return. It also possesses a near-perfect balance between richness and simplicity: the separation property permits certain differences in risk tolerance among investors, yet results in linear relationships between "risk" and "return" for optimal portfolios as well as individual securities in equilibrium. This, in turn, provides an ideal setting for empirical tests—linear relations based on the most commonly used statistical measures. As a result, a great deal of empirical evidence has been gathered, much of it supportive of the model. Add to this the goodwill generated by familiarity and understandability, and the social dominance of the MV model no longer seems a mystery.

It is not the intent of this paper to review the (mean-variance) capital asset-pricing model [Sharpe (1964) and Lintner (1965a, 1965b)] and the empirical tests, both supportive and nonsupportive, which have been performed in attempts to determine its explanatory power. An excellent summary of the literature on this subject has already been prepared by Jensen (1972). Rather, I shall ponder and/or attempt to explain some questions touching the larger context of capital asset pricing. In particular, I will wonder out loud why, in view of the fact that the
essential equilibrium structure of the (present) capital asset-pricing model fits a richer structure than the mean-variance framework, the MV concept has been granted such a divine position. Among other issues I will also attempt to give some insight into why the standard MV model was so slow in being extended to an intertemporal framework and why, when it was, this extension took on a very special form.

Is the Market Portfolio Efficient?

Let us turn first to the standard (single-period) capital asset pricing model when the limited liability of financial assets is recognized. If all investors are rational in the von Neumann-Morgenstern sense, the MV assumptions now imply that preferences must be quadratic, i.e., investor k's utility of wealth function is of the form

\begin{equation}
    u_k(w) = a_k w - w^2 - \left( b_k - w \right)^2
\end{equation}

where \( \sim \) means equivalent and the \( a_k = 2b_k \) are constants. Under homogeneous probability beliefs and unlimited borrowing and lending at the same rate, all investors hold the market portfolio (M) in conjunction with borrowing, lending, or neither, in equilibrium. As is well known, this implies that the means \( \mu_{p_2} \) and standard deviations \( \sigma_{p_2} \) of return of the optimal portfolios of all investors form a linear (upward-sloping) plot, usually called the capital market line (CML). Its equation may be written

\begin{equation}
    \mu_{p_2} = \mu_2 + \sigma \sigma_{p_2} \mu_2
\end{equation}

where the parameters \( \mu_2 \) and \( \sigma_2 \) represent the (equilibrium) interest
rate and the (equilibrium) market price of risk. (The subscript 2 refers to the fact that we have assumed quadratic utility.) In this case we also obtain a linear relationship between $\mu_{i2}$ and $\beta_{i2}$, the equilibrium means and betas of returns ($r_i$) of all securities (and portfolios), where

$$
(3) \quad \beta_{i2} = \frac{\text{Cov}(r_{i2}, r_{M2})}{\sigma_{M2}^2} ;
$$

the equation of the security market line may be written

$$
(4) \quad \mu_{i2} = r_2 + (\mu_{M2} - r_2)\beta_{i2}.
$$

Equations (2) and (4), of course, are the foundations of virtually all empirical tests of the capital asset-pricing model.

Observe that the sole basis for the linearity of the mean-standard deviation plot of optimal portfolios is the fact that in equilibrium all investors hold portfolios combining the risk-free asset with the market portfolio of risky assets. This, in turn, is due to the separation property of the preference class (1). But the separation property holds for a considerably wider class of risk-averse functions than (1), namely, [Hakansson (1969), Cass and Stiglitz (1970)].

$$
(5) \quad u_k(w) = \frac{1}{\gamma} (w + a_k)^{\gamma} \quad \gamma < 1 \text{ and fixed}
$$

$$
(6) \quad u_k(w) = -(a_k - w)^{\gamma} \quad a_k \text{ large, } \gamma > 1 \text{ and fixed}
$$

and
(7) \[ u_k(w) = -\exp(a_k w) \quad a_k < 0. \]

(In (5), \( \gamma = 0 \) represents \( \log(w + a_k) \).) Clearly, (1) is but one member of (6), with \( \gamma = 2 \). Thus, whenever investor preferences are described either by (5), (6) (with \( \gamma \) fixed), or (7), all investors, under our previous assumptions, choose the same mix of risky assets, which in equilibrium can only be that of the market portfolio. Thus, in mean-standard deviation space of return (or ending wealth), the plot of the optimal portfolios \( p \) is linear (and upward-sloping) and can be represented by the line

(8) \[ \mu_p = \gamma + \mu_m \sigma_p. \]

In general, we would, of course, expect the parameters \( \gamma \) (the equilibrium interest rate) and \( \mu_m \) (the market risk premium) to depend on the risk tolerance parameter \( \gamma \), as do the mean and standard deviation of the market portfolio, \( \mu_m \) and \( \sigma_m \). But perhaps more important, the optimal portfolios need not be efficient in a mean-variance sense unless \( \gamma = 2 \) as in (1). Figure 1 shows the capital market line \(^4\) when \( \gamma = 2 \) (line AML) and Fig. 2 when \( \gamma > 2 \) (or (7) holds) (line A'M'L'). As Fig. 2

\(^3\)The numbers \( \gamma, \mu_m, \sigma_m \) clearly also depend on the individual investor parameters \( a_k \) (in (1), (5), (6), and (7)).

\(^4\)Here we implicitly adopt the definition: CML is that function which relates \( \mu_p \) to \( \sigma_p \) for optimal portfolios in equilibrium. The "efficiency" of CML under assumption (1) is then viewed as a property of CML, not as a part of its definition.
indicates there will in general exist portfolios, under the equilibrium return structure (for any \( \gamma \neq 2 \) or (7)), which dominate the optimal portfolios in a mean-variance sense, yet no one will seek such portfolios simply because they are less desirable. In other words, when \( \gamma \neq 2 \) (or (7) holds), every investor will prefer some portfolio on line \( A'M'L' \) to every portfolio on the (MV)-efficient frontier \( A'B' \). And the market portfolio \( M' \) is clearly not (MV) efficient.

It is not difficult to construct examples for which the distance between \( A'M'L' \) and \( A'B' \) is substantial. However, when returns are "compact," Samuelson (1970) has shown that the optimal portfolios will be close to those derived from a quadratic function; thus, in this case we would expect \( A'M'L' \) to lie close to \( A'B' \). The same is true, under certain conditions, when the number of securities is large [Ross(1972)].

In the single-period limited liability case, our examination of the capital market line has revealed that only the a priori insistence on MV efficiency favors (1) over (7) and over the members of (5) and (6) for fixed \( \gamma \). We should also note that the class \( (w + a_k)^{1/2} \), or \( \log (w + a_k) \) (both belonging to (5)), say, is just as "rich" in preference patterns as (1) (it is actually "richer" since there are no constraints on the parameters \( a_k \) as in (1)). It is readily verified that all utility functions in (5) exhibit decreasing [Arrow-Pratt (1963, 1964)] absolute risk aversion, while the members of (6) possess increasing absolute risk aversion and the class (7) constant absolute risk aversion. In this case, the significance of these properties is that only with respect to the class (5) are risky assets normal goods. No one, to my knowledge, has ever argued, or purported to have given evidence, in favor of the proposition that risky assets such as securities are not normal goods. In sum, then, the contest (in the single-period limited liability
case) between the capital market line in Fig. 1 and that in Fig. 2 (A'M'L') is a contest as to which of two notions reigns supreme: MV efficiency or the normal goods nature of securities.

Upon further examination, however, there are at least two considerations which complicate the issue. First, if one chooses in favor of Fig. 2, which $\gamma$ in (5) is the "correct" one? This is, of course, an empirical matter, but on the basis of suggestions by Samuelson and Merton (1974) and evidence by Friend and Blume (1974), a somewhat negative $\gamma$ may have the best "fit." Second, only (1) of all the functions in (5), (6), and (7) generally yields a linear relationship between $\mu_{iY}$ and $\beta_{iY}$, the means and betas of securities in equilibrium.\(^5\) With this in mind, and remembering the important role that the security market line has come to play in the capital asset-pricing model, it is perhaps easier to understand why the MV-efficiency concept has not been challenged despite its lack of normal goods ammunition (in the single-period limited liability case).

When $\gamma \neq 2$, a plot of the points $(\mu_{iY}, \beta_{iY})$ produces a scatter which may contain negative or flat "relationships." Such relationships have also been obtained from real data [see, e.g., Douglas (1969), Miller and Scholes (1972), Black, Jensen, and Scholes (1972), Blume and Friend (1972)].

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\(^5\)The linearity of this relationship holds more generally for (1) than under our present assumptions (which include limited liability), namely, in the absence of a risk-less asset [Black (1972)], in the absence of riskless borrowing [Vasicek (1971)], and in the presence of differential borrowing and lending rates [Brennan (1971)]; it also holds in a continuous trading intertemporal model with arbitrary preferences under the geometric Brownian motion hypothesis [Merton (1971, 1973)]. In the latter case the linearity holds only instantaneously, not over a discrete interval [see, e.g., Jensen (1972, p. 386) for the discretized equation].
The open question, then, is whether the data already collected, which has only been related to the MV-based capital asset-pricing model, gives any support to an alternative capital asset-pricing model based on one of the (normal goods) classes (i.e., γ) in (5), which, as noted, yields a capital market line as in Fig. 2, a security market "scatter" in lieu of a "line," and for which the market portfolio is likely to be (MV) inefficient. 6

Intertemporal Mean-Variance Analysis

Few, of course, have based their defense for using the capital asset pricing model on the quadratic utility assumption. The majority rely, rather, on the normal distribution of return assumption when they wish to achieve consistency with the von Neumann-Morgenstern postulates. The general comfort most writers on the subject seem to feel in the presence of this assumption appears to stem from the near normality of empirical distributions of the log of price relatives [see, e.g., Fama (1965)], coupled with the closeness of the functions r and log (1+r) for r near 0 (there are, of course, exceptions). Those who are most disturbed about the normal return assumption seem to object on the following grounds:

1. It clashes with the undisputed empirical fact known as limited liability.

2. It seems difficult to reconcile with the opportunity (generally present) to borrow at a riskless rate, given that the probability of repayment then is less than 1.

3. The normal distribution cannot be integrated with "most"

6A beginning in this direction has been made by Kraus and Litzenberger (1972); see also Roll (1973) and Rubinstein (1973, 1974).
utility functions exhibiting decreasing absolute risk aversion; in particular, normal distributions cannot be integrated with any member of the class (5), the class of decreasing absolute risk aversion utility functions for which the separation property holds.  

4. The normal distribution does not reproduce itself multiplicatively. Thus, the assumption can be valid for at most one time-period length in a random-walk model (of returns).

5. In an n-period (Fisherian) consumption-investment model with $n \geq 2$, the nonnegativity of consumption can only be satisfied if the investor avoids all risk-bearing.

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7 As is well known, integrability is no problem with respect to (6) and (7).

8 With respect to this point, several comments are in order. While it is true, as Fama (1970) has shown, that risk aversion with respect to consumption streams implies risk aversion with respect to current investment under weak conditions, the "interpretations" which this result has been given are fallacious and misleading. First, it is correct that a consumer who lives only for two periods and faces symmetric stable return distributions (with finite means) will choose an (MV) efficient portfolio [Fama (1971)]—but only if his utility function is defined for negative consumption levels and he is willing to "tolerate" such consumption.
In sum, it is apparent that while the normality assumption can rescue the MV approach, and in particular the standard capital market and security market lines, it can only do so (1) in a single-period framework, (2) by assuming preferences, which, with the exception of a "small" class, possess nondcreasing absolute risk aversion, and (3) by disregarding some obvious empirical realities.

Having observed that the normal distribution assumption does not yield a viable mean-variance framework in the multiperiod case, the remaining possibility for justifying the MV model in a (discrete-time) sequential context rests on the assumption of quadratic utility. First, consider the case of simple reinvestment in which decisions are governed by the (quadratic) terminal utility-of-wealth function\(^9\)

\[ u_0(w) = -(a-w)^2 \quad \text{for large;} \]

this gives

levels (if not, he will choose a riskless portfolio). If the investor will not be around for the second period (i.e., his holdings are bequeathed) the assumption is more palatable—but then we do not really have a two-period model. One can, of course, argue that the probability of negative second-period consumption is negligible, or equal to \(\varepsilon\). But then, under the random-walk-of-returns assumption, the probability of avoiding negative consumption over \(t\) periods is "on the order of magnitude" of \((1 - \varepsilon)^t\).

\(^9\)Tobin's (1965) multi-period mean-variance analysis, which is based on non-normal distributions and disregards consumption, belongs in this category.
(9) \( u'_0(w) > 0 \quad w < a \) only,

(10) \( u''_0(w) < 0 \quad \text{all } w. \)

Let \( r_{n2} \) be the (known) interest rate with \( n \) periods to go and let the returns in any two periods be independent; then the induced utility of wealth functions with \( n \) periods remaining to the horizon (ignoring possible solvency requirements) are given by [Mossin (1968)]

(11) \( u_n(w) = -k_1 \cdots k_n(a_n - w)^2 \sim -(a_n - w)^2, \)

since \( k_1, \ldots, k_n \) are positive constants. But, while \( u''_n(w) < 0 \) for all \( w \),

(12) \( u'_n(w) > 0 \quad w < a_n \equiv \frac{a}{(1+r_{12}) \cdots (1+r_{n2})} \) only,

and if the interest rates are bounded away from zero,

(13) \( a_n \to 0 \quad \text{as } n \text{ increases.} \)

This is clearly an awkward situation: the more distant the horizon, the closer to 0 is the upper end of the interval over which our (currently relevant) utility function is increasing.

As a second illustration, consider a (Fisherian) multiperiod consumption-investment model in which the utility of consumption is additive and quadratic. The separation property will now hold, in the next to last period, only for large wealth levels, since the nonnegative
consumption constraint is always binding for low wealth levels, altering the optimal mix of risky assets in no particular pattern.\textsuperscript{10} As we move backwards in time, the minimum wealth level for which the separation property holds increases, systematically destroying the common portfolio basis on which the capital asset pricing model in any particular period rests.\textsuperscript{11}

While, as we have seen, the possibilities of extending the standard capital asset pricing model to a discrete-time intertemporal setting are barred, this is not the case if decisions are presumed to be made continuously by consumer/investors. As shown by Merton (1971), when asset returns are lognormally distributed in such a model, all investors will hold the same mix of risky assets regardless of their preferences.\textsuperscript{12} Thus, the preconditions of the (standard) capital asset-pricing model are met within the assumption of limited liability and without having to rely on quadratic utility. The most crucial assumptions embedded in the continuous-time version of the capital asset-pricing model are those of (1) "compact" returns, i.e., essentially that

\textsuperscript{10} The fact that this constraint is binding for "low" wealth levels is partly due to the inferior goods nature of securities in this model. In models in which investors are increasingly risk averse and securities are normal goods the nonnegativity constraint on consumption is often not binding [e.g., Hakansson (1970); see also Sibley (1974)]. The quadratic model is one example for which the separation property holds only locally [Hakansson (1969)].

\textsuperscript{11} Let \(-(a_k - c)^2\) be the single-period utility of consumption function component for individual \(k\), where \(c\) is the amount of consumption. Then the optimality of the market portfolio mix in equilibrium, under nonnegative consumption, is preserved only if current individual wealth levels \(w_k\) are such that the ratios \(w_k/a_k\) are small or approximately equal for all individuals. Small ratios imply that all individuals are nearly risk-neutral in the relevant decision region while near-equal (but not small) ratios clearly represent an exceedingly restrictive condition. Among other things, both cases virtually rule out secular mobility of individuals.

\textsuperscript{12} Mathematically, the crucial property at work is that portfolios of lognormal assets will also be lognormally distributed in the continuous-time model—but not in the discrete-time case—with parameters that are simple weighted averages of the individual asset parameters [see also, Ohlson (1972)].
for increasingly small time intervals the skewness of the return distribution becomes small compared to the first two moments.\footnote{For a more precise description of a compactness, see Samuelson (1970) and Ohlson (1973).} (2) stationary opportunities, and (3) absence of transaction costs. The second of these is clearly unrealistic but can be relaxed, giving rise to a richer capital asset pricing structure involving more than two "mutual funds" in which optimal portfolios need not be (MV) efficient [Merton (1973)]. As to the third assumption, it is clearly correct, as Merton points out (1973), that the absence of transaction costs (assumed in all of the models cited) is an argument in favor of continuous-time decision-making and that explicit cognizance of such costs almost surely would affect the optimal time between decisions. But it is not at all certain that recognition of realistic transaction costs implies that frequent portfolio revision is optimal.\footnote{For example, the fact that mutual funds as a group have performed worse than the "market" has in part been attributed to excessive portfolio turnover [see, e.g., Jensen (1969)].} In any case, the assumption of zero transaction costs is clearly easier to justify when the (prespecified) time between decisions is fairly long.

A "Nonstandard" Mean-Variance Model

The weakness of the mean of a distribution as a measure of "wealth" was amply demonstrated by Bernoulli (1954 transl.). In a multiperiod setting, the shortcomings of expected return or expected wealth are further magnified. For example, consider a reinvestment setting with stationary, temporally independent returns, including a positive interest rate and at least one positive risk premium. Denote wealth at the end of period \( n \)
by \( w_n \) and return on portfolio \( p \) in period \( n \) by \( r_n(p) \). Then we may have

\[
E[w_n] \to \infty,
\]

\[
\text{Mode}[w_n] < \$1 \quad n \text{ large},
\]

(14)

\[
\text{Median}[w_n] < \$1 \quad n \text{ large},
\]

\[
\Pr\{w_n < \$1\} \to 1.
\]

In fact, a sufficient condition for (14) to hold is

\[
E[r_n(p)] > \epsilon > 0
\]

(15)

\[
E[\ln(1 + r_n(p))] < -\epsilon \quad \text{all } n
\]

for some \( \epsilon \), a condition not nearly as difficult to satisfy as generally believed. For example, suppose that the (ex ante) return distribution of the market portfolio in each period (year) is constructed from the realized returns "on the market" for the period 1926-65 as reported by Fisher and Lorie (1968, Table 1A) and assume an interest rate of 8%. Then the market portfolio, purchased on a margin of 54.4% or less, satisfies (15) and hence (14); i.e., such portfolios, many of which clearly are incapable of making the investor insolvent in any one year, will gradually ruin him. 

\[\text{The choice of interest rate, for the purpose at hand, matters very little.}\]

\[\text{For an intuitive exposition of the forces at work, consider the following simplified example. Suppose } r_n(p) \text{ is either } -60\% \text{ or } 100\% \text{ with}\]

\[\text{...}\]
An expectation which alerts the investor to the last part of (15), when it holds, is that of average compound return, $C_n - 1$, given by

$$C_n = \left( \prod_{j=1}^{n} (1 + r_j(p)) \right)^{1/n}. \tag{16}$$

The reason for this is that the last part of (15) holds (under weak regularity conditions) if and only if

$$E[C_n] - 1 \to a \text{ negative number}. \tag{17}$$

As noted in Hakansson (1971, Sec. IV), maximization of $E[C_n]$ induces the decreasingly risk averse (myopic decision rule) utility of wealth function $w^{-1/n}$ in each period.

Now consider the mean and variance of $C_n - 1$, average compound return over (the first) $n$ periods (ACRN). It should be noted that the portfolio sequences obtained by invoking the standard efficiency notion are not (exactly) consistent with those obtained by a utility of wealth function except in certain cases (such as when there are two assets, one of which is riskless, and returns are stationary). But the class of utility equal probability in each period. Then expected return is 20% in each period, causing expected capital to grow at 20% per period. Thus, after 10 periods our expected capital is more than six times what we started with since $E[w_{10}] \approx 6.19 w_0$. But $\text{Mode}(w_{10}) = \text{Median}(w_{10}) \approx 0.33 w_0$ and $\Pr(w_{10} < w_0) \approx 0.62$, i.e., there is roughly a 62% chance that after 10 periods of investment we will have less capital than we started out with.
functions

\[ \frac{1}{\gamma} u^\gamma \quad : \gamma \leq 1/n, \]

with the same \( \gamma \) applied in each period, appears to yield good approximations to the exactly efficient sequences. Besides offering myopic policies, this class (as well as the exactly efficient (ACRN) portfolio sequences) avoids long-run "ruin" if \( n \) is sufficiently large (just how large depends on the return structure—for the empirical example based on the Fisher-Lorie data, \( n \) need only be greater than or equal to 2) [Hakansson and Miller (1973)]. But perhaps the strongest evidence in favor of the investment policies implied by the class (18) is the fact that the (first-period) optimal investment policies for a very large class of (fairly risk-averse) terminal utility functions converges to the set which is optimal for (18) as the horizon becomes more distant [Hakansson (1974)].

Unfortunately, the optimal mix of risky assets for one \( \gamma \) in (18) is not obtainable as a linear combination of the optimal mixes for two different \( \gamma \). As a consequence, the ACRN model does not yield a "simple" capital asset pricing model with a capital market line and a security market line.\(^{17}\) The preference structure is too "rich" for the market portfolio, combined with the riskless asset, to be optimal for everyone. The central lesson of the second half of this paper then is that, from all indications, the possibility of a meaningful extension of the standard capital asset pricing model to a discrete-time multiperiod, setting is closed; the assumptions which make the extension to an intertemporal setting possible in continuous time are not relaxable.

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\(^{17}\) The same situation arises when nonmarketable assets [Mayers (1972)] or nonhomogeneous beliefs are introduced into the standard capital asset pricing model.
References


10. David Cass and Joseph Stiglitz, "The Structure of Investor Preferences and Asset Returns, and Separability in Portfolio Selection: A Contri-


