Authority, Consensus and Governance*

Archishman Chakraborty†  Bilge Yilmaz‡

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Abstract

We investigate properties of shareholder-value maximizing corporate boards in situations where both the management and the board have expertise (or private information) relevant for the decision facing the corporation. We suppose that the board cannot directly monitor management in a way that eliminates distortions arising out of managerial agency. This creates a trade-off between improving information sharing and reducing distortions in decision-making, leading shareholders to often design a supervisory board that is imperfectly aligned with both shareholders and management. Indeed, even when management has complete control over decision-making and the board only has an advisory role, the optimal board may be designed to provide “bad” advice to management in order to limit the ex-post costs of managerial agency. We establish our results by investigating the relationship between shareholder value maximization and the possibility of generating ex-post consensus between management and the board. We show that the optimal advisory board must generate consensus and the value of authority within the organization is equal to the cost of creating consensus. We also show that the optimal board must be able to generate shareholder consensus and investigate how this affects the optimal board architecture, in particular the transparency of information flows within the board.

JEL classification: C7, D2, D7, D8, G3, L2.

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†Schulich School, York University, achakraborty@schulich.yorku.ca.
‡Wharton School, University of Pennsylvania, yilmaz@wharton.upenn.edu.
1 Introduction

Consider a corporation in a setting where effective communication between management and the board is important for making efficient decisions. In practice, such communication is often constrained by the fact that management has its own agency that predisposes it towards one of the decisions under consideration. For instance, management may favor going ahead with an investment motivated in part by the joy of empire-building, or be biased towards rejecting a raider’s offer due to private benefits of retaining control. If such conflicts are strong, decision making will be based on coarse information transmission between the management and the board of directors acting presumably in shareholder interest. From the normative perspective of maximizing shareholder value, what should an optimal board look like in such situations?

To answer this question we consider an environment where both the board as well as management have expertise (payoff-relevant private information) relevant for the decision facing the corporation. Furthermore, the board cannot effectively monitor management or directly control its agenda. It can, at best, approve or turn down management proposals, in light of its own expertise and experience as well as any information that management provides in its efforts to persuade the board. In making its decisions, the board must then take into account managerial discretion not only in the representation of information but also in the implementation of approved proposals.

In such situations shareholders face a fundamental trade-off. If they choose a board that shares the management’s interests then information transmission between the manager and a sympathetic board should be quite effective and the final decision of the board will be well informed. However, decision-making will be distorted away ex-post from shareholder value maximization. On the other hand, if shareholders choose a board that is less sympathetic to management, ex-post decision-making will be closer to shareholder interests, although this will be at the cost of poor information transmission between management and a skeptical board resulting in poorly informed decision-making. We show that the optimal board will often be imperfectly aligned with both shareholders and management, balancing the gain from improved information flows against the cost of distorted decision-making. In effect, shareholders will gain by replacing the agency problem between shareholders and management by two smaller agency problems, one between management and the board and a second one between the board and shareholders.

This trade-off and attendant benefit of an intermediate board depends on the fact that the final
decision requires the board’s approval. In such cases, we say that the board has a *supervisory* role. Our results on optimal expert supervisory boards that are conflicted with both shareholders and management are similar to results on delegation due to Dessein (2002) as well as related results due to Harris and Raviv (2005, 2008, 2010). However, our analysis goes beyond these papers in a number of respects.

First, we show that the benefits of an imperfectly aligned board extend even to the case where the management has total control over the firm and can implement its desired action with or without approval from the board. In such cases, the board only has an advisory role and the management may choose to solicit and/or ignore the board’s expert advice. If shareholders choose an advisory board that is conflicted relative to management, this can only hamper information flows between the board and management without affecting in any way the distortion in the final decision-making that is fully controlled by management. Nevertheless, shareholders may gain by choosing an *advisory* board that is imperfectly aligned with management. The conflict of interest between board and management allows shareholders to commit to credibly providing vague but valuable advice to management. When the manager is uncertain about his own agenda, vague advice from the board preserves managerial doubt about the benefits of insisting on his agenda. This may benefit shareholders in ex-ante expected terms by limiting the distortion arising out of managerial control over actual decision making, even after adjusting for the expected costs of relatively poor information transmission.

We compare also the relative merits of supervisory and advisory boards from the perspective of shareholders. This is the same as determining the value of allocating decision-making authority to the board or the management. When the conflict of interest between management and board is small, management is likely to agree with the final decision made by a supervisory board and the value of allocating authority is likely to be small. We pursue this intuition via developing a notion of agreement between the board and management that we call consensus. Consensus obtains when both sides agree with the final decision given their private information and the information revealed endogenously in equilibrium.

We use our notion of consensus to show that the value of authority is equal to the cost of creating consensus. In particular, the optimal advisory board from the perspective of shareholders is the optimal supervisory board but determined subject to the additional constraint that the board must obtain managerial consensus. The loss in shareholder value from imposing this additional
constraint of consensus is equal to the loss in value from letting management wrest control of final
decision-making from the board. It follows from this that shareholders can never find it strictly
optimal in give control over decision-making (let alone the board) to management. Furthermore,
even if management has de facto control over decision-making it always finds it in its own interest
to respect the opinion of shareholder value maximizing advisory boards.

A third important difference between our results and the delegation results in Dessein (2002) has
to do with commitment and the related notion of shareholder consensus. We say that shareholder
consensus obtains when shareholders necessarily agree with the final decisions made by the board
(or management). We show that the ability to generate shareholder consensus is a necessary
property of the optimal board at the optimal allocation of authority. It follows that shareholders
do not need to commit to cede control to the expert board. The optimal board need only be an
intermediary with final decision-making authority concentrated in a perfectly shareholder aligned
board member. This is true as long shareholders do not observe the details of deliberations between
board and management and often even when they do.

These results shed light both on features of one-tier boards that are commonly observed in
the U.S., Canada and the U.K. as well as those of two-tier boards that are more common in
continental Europe (see, e.g., Cadbury, 1995). One tier boards usually have both executive as
well as independent members and integrate the advisory function of a corporate board with its
supervisory role. In contrast, two-tier boards tend to separate these two functions. The lower tier
provides advice on decision management and typically has executive members. The upper tier is
concerned exclusively with a supervisory role and typically contains only independent directors (as
well as founding family members with controlling interests) who are presumably more aligned with
shareholders. Both these structures are consistent with our normative theory of shareholder value
maximizing boards. Since the optimal board generates shareholder consensus, it can be structured
as a two-tier board with a shareholder aligned upper tier and a (partially management aligned)
lower tier. Alternatively, we may replace the upper tier by the executive committee of a one-tier
board and the lower tier by an expert audit committee.

From the perspective of our model, the key difference between the two board architectures lies in
the transparency of information flows between the two tiers/sub-committees. When the board has
effective supervisory authority, we show that shareholders would prefer transparency, i.e., to commit
to freely sharing the board’s information between the different tiers and with management, vesting
decision-making authority in a perfectly shareholder aligned member. However, when management effectively has all authority, shareholders may prefer a more opaque board structure that often withholds information from management and provides only vague advice. At the same time, the board must be sufficiently aligned with management so that the latter always finds it optimal to voluntarily follows the board’s advice.

Beyond the context of designing corporate boards the present paper adds to our understanding of broader issues concerning the allocation of authority and communication rights within organizations. It belongs to the literature that takes an agency-based approach to understanding organizations based on a conflict in preferences between owners and managers (Alchian and Demsetz (1972), Jensen and Meckling (1976)). In part however, it also reconciles the conflict-based approach with the team-theoretic approach pioneered by Williamson (1985) that focuses more on costs of information transmission within hierarchies. Our results provide some general insight on the widespread practice of delegating collective decisions to specialized committees of possibly biased agents, both in cases where the committee has executive power and in cases where it performs merely an advisory role.

Finally, we show that our binary decision problem with two-sided private information has a close formal relationship, in terms of equilibria and payoffs, to the canonical model of Crawford and Sobel (1982) with a continuum of possible decisions but one-sided private information. Because of this, many results originally established in the Crawford-Sobel set-up, such as those of Dessein (2002), happily extend to our setting and we make heavy use of this ‘equivalence’ throughout the paper. Since our model has two-sided private information however we are also able to generate many novel results that have no direct parallel in the model where only one agent has private information relevant for the decision.

The rest of the paper is organized as follows. In Section 2 we set up our basic model. In Section 3.2 we establish our main results on communication, consensus and control. We investigate the implications of these results in more detail in Section 5 where we also discuss some extensions including those related to transparency, commitment and incentives. Section 6 contains a discussion of the related literature (not included in the present draft) while Section 7 concludes (not included in the present draft). The Appendix contains all proofs (not included in the present draft).
2  The Model

A principal or owner (the shareholders collectively) must undertake one of two actions: the status quo or the alternative. The status quo is safe and we normalize its value to shareholders equal to zero. The alternative has uncertain value \( x - y \) where \( x \in [0, 1] \) is the private information or expertise of management (the agent) and \( y \in [y_L, y_H] \) is the private information or expertise of the board (delegate). Let \( F_x \) and \( F_y \) denote the common knowledge priors associated with the random variables \( x \) and \( y \) with densities \( f_x \) and \( f_y \) and suppose that \( y_L < 1 \) and \( y_H > 0 \). We assume that \( x \) and \( y \) are statistically independent and, for tractability, assume throughout that \( F_x \) and \( F_y \) are uniform.

We will often refer to the alternative as a decision to ‘invest’ in a project with the status quo being the default option of not investing. In such a case managerial expertise could reflect specialized knowledge of the operational details of the project whereas the board’s information could reflect expertise on legal or regulatory issues. Alternatively, one may think of a situation where the firm is a potential takeover target and has to decide whether or not to accept a raider’s offer or adopt an anti-takeover measure. In such cases, management may have private information about the intrinsic value of the firm under current management whereas the board may have expertise in evaluating the legality of anti-takeover measures proposed by management or in estimating the value of keeping alive the interest of potential raiders in the market for corporate control.\(^1\)

The central conflict of interest in this paper is between management and shareholders. Management is biased in favor of the alternative and the value of the alternative to management is the sum of shareholder value \( x - y \) and a private benefit \( b_m \) whereas the value of the status quo is zero. The managerial bias parameter \( b_m > 0 \) reflects managerial agency in the form of private benefits of empire-building and expanded control.\(^2\)

\(^1\)Nothing depends on the status quo being safe and we may think of \( x - y \) as the net (possibly negative) benefit of the alternative relative to the status quo. We also permit the relative importance of the two sources of information to vary, e.g., by writing \( y = kz \), where \( z \) is a random variable capturing the board’s information and \( k \geq 0 \) a scalar parameter that measures the relative importance of the two sources of information for the decision at hand.

\(^2\)The importance of managerial agency in corporate governance and performance has been extensively analyzed (See, e.g., Jensen and Meckling, 1976). In this paper, we take this agency as given and suppose that the bias \( b_m \) is a necessary cost of having a manager with payoff-relevant expertise \( x \). For most of the paper, we also suppose that all private information is unverifiable and contracts are incomplete so that management is not perfectly aligned with shareholders even after an optimal provision of incentives. We also abstract away from reputational concerns. Apart
The central question of this paper concerns the composition of the optimal board from the perspective of shareholders. We model the board as having an (endogenous) bias \( b_d \in [0, b_m] \) that captures its “ideological” sympathy for management. Accordingly, the value of the alternative to the board is \( x - y + b_d \) while the value of the status quo is zero. In general, the board may consist of both shareholder aligned members (such as independent directors) or management aligned members (friends or associates of management). If the board takes its final decision via majority voting then we may think of \( b_d \) as the bias of the median board member.\(^3\) Our central question then boils down to asking what choice of \( b_d \) will maximize the ex-ante expected welfare of shareholders.\(^4\)

In determining the optimal choice of \( b_d \) from the perspective of shareholders, we contrast two different scenarios. In the first scenario, the board has some control over management and any decision made by the firm must obtain the board’s approval. We call this the case of a *supervisory board*. Notice that even a supervisory board can only approve or reject a proposal brought forward to it by management and that will be implemented ultimately by management. This implicitly captures a situation where most of the effective authority within the firm is in the hands of management and the board has a limited ability to monitor management ex-post and limit its agency. In particular, the board has no control over the details of the decision facing the corporation (such as the size or location of the investment) and management entirely controls this ‘agenda-setting’ aspect of the firm.

Our second scenario takes this situation of effective managerial authority to the extreme by assuming that management has total control over the firm and can take any decision it wishes to even in defiance of the board’s wishes. In such cases, the board does not have any effective supervisory authority over management and may only play an advisory role. Accordingly, we call this the case of an *advisory board*. We wish to find the shareholder-value maximizing choice of \( b_d \),

\(^3\) Alternatively, we may assume that the board maximizes a weighted sum of the welfare of its shareholder aligned members (with zero bias) and management aligned members (with bias \( b_m \)), with the weights being proportional to the size of each group. In this scenario, the ratio \( b_d/b_m \) measures the proportion of management aligned board members.

\(^4\) In reality, both management and shareholders have a say in board composition. The present draft presents an entirely normative theory that seeks to identify necessary properties of shareholder value maximizing corporate boards.
both in the case of a supervisory as well as an advisory board. We wish also to identify precisely the value of authority, i.e., the relative merits of optimal supervisory and advisory boards.

In what follows we will model both cases of a supervisory and an advisory board as a cheap talk game between management and the board. The assumption of cheap talk amounts to assuming that all private information is soft and cannot be verified by courts so that contracts written on these variables are not enforceable. Rather the decisions are taken by the agent with effective authority according to its own interests after taking into account the information voluntarily shared by the board and management.

3 Supervisory Boards and Optimal Delegation

Since a supervisory board makes the final decision, we model this case as a simple game where management first sends a cheap talk message to the board following which the board takes a final decision. In order to develop intuition we begin by considering the simple benchmark case of an uninformed supervisory board, i.e., the case where the board has no payoff-relevant private information. Subsequently, we turn to the case where the board also has private information $y$ relevant for all parties.

3.1 Uninformed supervisory boards: a benchmark

Let $\bar{y} \equiv E[y]$ and assume that no party has any information about $y$. The following lemma is immediate.

**Lemma 1** Assume an uninformed supervisory board. If communication is influential, the board implements the manager’s optimal decision rule.

In any influential equilibrium, the management recommends the alternative whenever it is in the management’s interest to choose the alternative, i.e., if and only if $x - \bar{y} + b_m > 0$. The board finds it in its interest to follow the manager’s advice (i.e., an influential equilibrium exists) whenever $1 + b_m > \bar{y} > b_m$ and further

$$E[x|x - \bar{y} + b_m > 0] - \bar{y} + b_d > 0$$

(1)

Our first result follows immediately from the lemma.
Proposition 1 With uninformed supervisory boards, a perfectly shareholder aligned supervisory board is always weakly optimal and sometimes strictly optimal.

When communication is influential under a perfectly shareholder-aligned supervisory board, shareholders find it optimal to follow the management’s advice (in its own interest). Changing the board’s preferences can only destroy informative communication in this case which is not in the interest of shareholders. On the other hand, if communication is not influential with a perfectly shareholder aligned board, then a shareholders do not find in their interest to follow management’s advice. Although decision-making is uninformed in this case, this is better than following the decision rule that is in management’s interest. It is therefore not in the interest of shareholders to design a board that is more closely aligned with management in order to improve management credibility and information flows between management and the board. In the next section we show that when the board also has private information, shareholders have stronger incentives to improve information flows between management and the board.

3.2 Expert supervisory boards

Suppose that the board has independent expertise captured by its private observed signal $y$ that is payoff-relevant for all parties.\footnote{We suppose that this information is observed by a monolithic board. The effect of privately informed board members in a diverse board will be considered in Section 5.2.} If $b_d = b_m$, then the management and board have perfectly aligned interests. Therefore they should be willing to perfectly share their information in order to implement the optimal decision rule given their common interests. This involves investing whenever $x - y + b_m > 0$.

We turn now to the case where $b_d \neq b_m$. In such cases, given any message $\mu$ from the management, the board will prefer to invest whenever its own signal $y$ is less than a threshold value $t = E[x|\mu] + b_d$. From the perspective of management, the expected payoff from sending message $\mu$ is then

$$\Pr[y < t] (x - E[y|y < t] + b_m) \equiv q(x + b_m) - \int_0^q F_y^{-1}(z)dz$$

where $q = F_y(t)$ is probability that the board chooses the alternative. Notice that the last expression is a supermodular function of $x$ and $q$ that is concave in $q$. This provides the key insight for our
next result in which we fully characterize the set cheap talk equilibria with an expert supervisory board.

**Proposition 2** Fix $b_d \neq b_m$ and assume an expert supervisory board. In any cheap talk equilibrium, management discloses the interval $[c_{i-1}, c_i)$ in which $x$ lies, $i = 1, \ldots, N$ where $c_0 = 0$, $c_N = 1$ and for $i = 1, \ldots, N - 1$,

$$c_i = E[y|t_i < y < t_{i+1}] - b_m$$

where, following management’s message that $x \in [c_{i-1}, c_i)$, $i = 1, \ldots, N - 1$, the board chooses the alternative iff $y < t_i$ where

$$t_i = E[x|c_{i-1} < x < c_i] + b_d$$

for $i = 1, \ldots, N$. Furthermore, there exists $N^*(b_d, b_m)$ that is an upper bound on the number of messages that can be sent by management in equilibrium.

Figure 1 illustrates Proposition 2. In the figure we take $b_m = \frac{3}{16}$, $b_d = 0$, $y_H = \frac{22}{16}$, $y_L = 0$. In such a case, the most informative equilibrium consists of the management sending two messages, a ‘low’ message $\mu'$ whenever $x < c_1 = \frac{1}{8}$ and a ‘high’ message $\mu$ when $x > \frac{1}{8}$. For the low message $\mu'$, the board chooses the alternative whenever $y < t_1 = \frac{1}{16}$ whereas for the high message the board chooses the alternative whenever $y < t_2 = \frac{9}{16}$. The resulting step function depicts the decision rule
that is implemented in equilibrium—below the step function the alternative is chosen by the board whereas above the step function the status quo is chosen. To provide a contrast, the figure also shows the decision rule that would be used if shareholders had all the information (the line \( y = x \)) and the rule that would be used if management had all the information (line \( y = x + b_m \)).

Figure 1 and, more generally, Proposition 2 shows that the equilibrium set of our cheap talk game with binary decisions and two-sided private information is similar to that of the cheap talk game with 1-sided private information and continuous actions considered in Crawford and Sobel (1982, henceforth CS). In fact, since we have assumed that \( x \) and \( y \) are uniformly distributed, when \( t_i \in [y_L, y_H] \), the equilibrium conditions (3) and (4) can be rewritten to obtain the difference equation

\[
l_{i+1} = l_i + 4B, \quad i = 1, ..., N
\]

where \( l_i = c_i - c_{i-1} \) is the length of the \( i \)th interval in the manager’s announcement strategy and \( B = b_m - b_d \) measures the net conflict of interest between the board and management. This is of course identical to the equilibrium condition in the leading uniform-quadratic example in CS. In our setting, the fact that the lengths increase for higher values of \( x \) reflects the fact that the manager’s incentive to overstate the case for investing limits his credibility and forces him to disclose coarser information exactly when \( x \) is high and the case for investing his strong. As (2) illustrates, the similarity with CS extends beyond the double uniform (uniform-quadratic case)—in general, the game with a supervisory board is like a CS game where \( x \) is the information of the sender and \( q \in [0, 1] \), the probability with which the board chooses the alternative, the action of the receiver.\(^6\)

The similarity with CS is however not perfect. For instance, when \( t_N > y_H \) (resp. \( t_1 < y_L \)), the actions \( q \) of the receiver hit the boundaries \( q = 1 \) (resp., \( q = 0 \)). In such cases, even under the double uniform assumption, (5) does not obtain. Relative to CS, this difference indicates that the conflict of interest between the sender and the receiver in general exhibits only a weak but not strict upward bias (see Gordon, 2007). As a result, there may be multiple equilibria with a given number of messages. However, one can show that the only the equilibrium with maximum number of possible messages \( N^*(b_m, b_d) \) satisfies the refinement proposed in Chen, Kartik and Sobel (2008) and accordingly, we will focus attention on this equilibrium throughout what follows.

\(^6\)It should be noted that the correspondence with CS is not only in terms of strategies but also in terms of payoffs.
A second key difference from CS is of course that in our setting both parties are privately informed. This raises the question whether the equilibrium characterized by Proposition 2 (where manager speaks first and then the board makes a decision) fully exhausts the communication possibilities between the two parties. Our next result addresses this question. To state it concisely, we use the short-hand \( mb^* \) to denote the sequence of moves where the manager speaks first and the board then makes a decision, where the asterisk denotes that the board also has decision-making authority. Similarly, let \( bmb^* \) denote the game where there is more extensive communication with the bard speaking first, followed by management followed by the board making a decision. In contrast, in the game \( b^*mb^* \) the board can also take a (unilateral) decision in the first round or choose to speak to management. If it does the latter, then management speaks to the board following which the board takes a decision. Finally, in game \( b^*m^* \), the board first takes a decision or speaks to management. In the latter case, it also “hands over” decision making authority to management and management makes the final decision.

**Proposition 3** Fix \( b_d, b_m \) and assume an expert supervisory board. The set of decision rules that can be implemented in equilibrium are identical across the games \( mb^* \), \( bmb^* \), \( b^*mb^* \) and \( b^*m^* \).

The result shows that extended (sequential) communication has no additional benefit in our setting. While the proof contains some important technical details, the intuition is relatively straightforward— the information that the two parties can bring to bear on the decision depends on their conflict of interest and each party can take this conflict (and the other party’s incentives) into account even with one round of communication by conditioning on the event when its own message makes a difference for the decision.\(^7\) In this sense, it is reminiscent of a ‘revelation principle’ type argument, although no party can commit ex-ante to a decision rule or mechanism.

What is the optimal choice of \( b_d \) from the ex-ante perspective of shareholders? The next result provides a partial characterization.

**Proposition 4** Suppose \( y_H > 1 + b_m \) and \( y_L < b_m \). With an expert supervisory board, the optimal

\(^7\)This is for sequential communication (polite conversation) only. It is an open question at the time of writing if the possibility of simultaneous messages (or longer sequential protocols) alters the equilibrium set. See Krishna and Morgan (2004) and Aumann and Hart (2003).
board $b^*_d$ satisfies:

$$
b^*_d = \begin{cases} 
  b_m & \text{if } b_m \leq \frac{1}{6} \\
  \in (0, b_m) & \text{if } \frac{1}{6} < b_m \leq \frac{1}{2\sqrt{2}} \\
  0 & \text{otherwise}
\end{cases}
$$

The parameter restriction in the Proposition ensures that all the thresholds $t_i$ in Proposition 2 are interior (i.e., belong to $[y_L, y_H]$). As a result the equations of CS apply (in particular, expression (5)). The result then follows from the analysis in the proof of Proposition 5 in Dessein (2002).\(^8\)

The result states that when the manager’s agency is sufficiently low ($b_m < \frac{1}{6}$), it does not pay the shareholders to impair information exchange between the board and management. On the other hand, when $b_m > \frac{1}{2\sqrt{2}}$, managerial agency sufficiently is value destructive for shareholders to forego eliciting information from management. The optimal choice of $b_d$ in this case ensures that the board makes its decision without learning anything about the manager’s information $x$. In the intermediate case the shareholders limit the distortion in decision-making by choosing a partially management aligned board at the cost of some information loss. Figure 2 depicts the optimal choice $b^*_d = \frac{5}{32}$ for the parameter values of Figure 1, in particular when $b_m = \frac{3}{16}$. As Figure 2 depicts, for these values of $b_d$ and $b_m$, the equilibrium consists of four messages.

**INSERT TRIANGLE INTUITION HERE**

For the interesting intermediate zone where $b^*_d \in (0, b_m)$ one can show that

$$b^*_d = \frac{N^*_2 - 1}{N^*_2 + 2} b_m$$

where $N_*$ is the number of messages sent in equilibrium by management to the optimum board. Furthermore, $N_*$ can be shown to equal to either the integer least upper bound or the integer greatest lower bound of $\sqrt{\frac{2}{b_m-1}}$. If we ignore these integer constraints, then one obtains an explicit formula for $b^*_d$ when $b_m \in \left(\frac{1}{6}, \frac{1}{2\sqrt{2}}\right)$:

$$b^*_d = \begin{cases} 
  \frac{1}{4} - \frac{1}{2} b_m & \text{if } \frac{1}{6} < b_m \leq \frac{1}{4} \\
  \frac{1}{2} b_m & \text{if } \frac{1}{4} < b_m \leq \frac{1}{2\sqrt{2}}
\end{cases}
$$

\(^8\)Similar results will obtain when $y_H < 1 + b_m$ or $y_L > b_m$ although in such cases we may have corner solutions and the CS equations will not immediately apply. We postpone an explicit calculation of expressions for $b^*_d$ in such cases for future drafts.
As the formula depicts, the optimal $b_d^*$ is non-monotonic in $b_m$. Figure 3 plots $b_d^*$ as a function of $b_m$ without ignoring integer constraints. Since shareholder value is decreasing in $b_m$ and shareholder alignment of the board can be measured by $b_d$, the figure shows that there is no monotonic (indeed, continuous) relationship between shareholder value and shareholder alignment of a supervisory board. As we argue in the next Section, one may also obtain similar relationships between $b_m$ and the bias $b_{d^{**}}$ of the optimal advisory board.

4 Advisory Boards, Consensus and Authority

In this section we begin by characterizing the equilibria of the game between the board and management when the management has all decision-making authority. We begin with the game $bm^*$ in which the board first sends a message to management following which management takes a decision. The next result is the analogue of Proposition 2 for this case of an advisory board.

**Proposition 5** Fix $b_d \neq b_m$ and assume an expert advisory board. In any cheap talk equilibrium, the board discloses the interval $[t_{i-1}', t_i')$ in which $y$ lies, $i = 1, ..., M$ where $t_0' = y_L$, $t_M' = y_H$ and for $i = 1, ..., M - 1$,

$$t_i' = E[x|c_{i-1}' < x < c_i'] + b_d$$

(6)
where, following the board’s message that $y \in [t_{i-1}', t_i')$, management chooses the alternative iff $x > c_i'$ where

$$c_i' = E[y | t_i' < y < t_{i+1}'] - b_m \quad (7)$$

for $i = 1, ..., M$. Furthermore, there exists $M^*(b_d, b_m)$ that is an upper bound on the number of messages that can be sent by management in equilibrium.

As with a supervisory board, we ask next if the equilibrium outcomes characterized by Proposition 5 are robust to extended communication when the board has only an advisory role. Letting $b_m^*$ denote the game where the board speaks to the manager who has decision making authority and using analogous notation to define the games $mbm^*$, $m^*bm^*$ and $m^*b^*$, we obtain the analog to Proposition 3 for the case of advisory boards.

**Proposition 6** Fix $b_d, b_m$ and assume an expert advisory board. The set of decision rules that can be implemented in equilibrium are identical across the games $b_m^*$, $mbm^*$, $m^*bm^*$ and $m^*b^*$.

Because of the similarity between our results on advisory and supervisory boards, a natural question is to ask when the same decision rule will be implemented regardless of who has decision making authority. To answer this question we now introduce our notion of consensus. We say that consensus obtains in our setting when the agent without decision-making authority agrees (i.e., does not wish to overturn) with the final decision made by the agent who has such authority, given his own information and the information revealed by the equilibrium play of the game. Thus, when
the board has decision-making authority, consensus obtains when the manager agrees with every possible board decision after every possible sequence of messages sent in equilibrium and never wishes to overturn the board’s decision after it is made even if he could do so. Similarly, when the board has only an advisory role and consensus obtains, the board does not wish to overturn the management’s final decision if it had the power to do so. We emphasize that consensus is a property that a particular equilibrium may or may not possess. The next result shows that consensus obtains whenever it obtains at the extremes, i.e., whenever, with a supervisory board, management does not wish to overturn the board’s decision not to invest, when it has the most favorable private information in favor of investment. If consensus obtains in an equilibrium, the same decision rule can be implemented even if one switches the allocation of decision-making authority from one party to the other.

Proposition 7  Fix $b_d, b_m$. When consensus obtains the allocation of decision-making authority is irrelevant for shareholder value.

1. With an expert supervisory board, consensus obtains iff $E[y|y > t_N] \geq 1 + b_m$ or $t_N = y_H$.

2. With an expert advisory board, consensus obtains iff $E[x|x > c'_M] + b_d \geq y_H$ or $c'_M = 1$.

In the proof of the Proposition we show that with a supervisory board, management with information $x$ agrees with the board’s decision to invest (not invest) only if a higher (lower) cutoff type $c_i$ does and further all interior cutoff types agree with every possible board decision. As a result, consensus obtains as long as management agrees with the board’s decision not to invest even when it has the most favorable information in favor of investment. In this sense, Proposition 7 identifies an interesting property of cheap talk equilibria—communication in any such equilibrium is an exercise in building consensus. More precisely, if consensus obtains in any equilibrium it obtains in the most informative equilibrium (for fixed $b_d, b_m$). This is because $t_N$ (or $c_M$) can be shown to be increasing in $N$ (resp., $M$). In this sense more informative communication facilitates consensus. However, since the two parties have conflicting interests, one can never obtain consensus if, for instance, more information than what is consistent with equilibrium is exogenously disclosed. Therefore, the most informative equilibrium maximizes the chances of consensus. If more or less
information is revealed, this may jeopardize the chances of consensus.\textsuperscript{9} The notion of consensus is important for characterizing the shareholder value maximizing advisory board as we show now.

Let $V_{b\text{-authority}}(b_d; b_m)$ be the ex-ante expected shareholder value when the supervisory board controls decision-making. Let $b_d^*$ be the optimal supervisory board, i.e., $b_d^*$ solves

$$\max_{b_d} V_{b\text{-authority}}(b_d; b_m)$$

Similarly, let $V_{m\text{-authority}}(b_d; b_m)$ be the ex-ante expected shareholder value when management controls decision-making and the board merely has an advisory role. Let $b_d^{**}$ be the optimal optimal advisory board, i.e., $b_d^{**}$ solves

$$\max_{b_d} V_{m\text{-authority}}(b_d; b_m)$$

The following result characterizes the optimal advisory board and as a corollary, the value of authority for shareholders.

**Proposition 8** The optimal advisory board $b_d^{**}$, which is the solution to the program in (9), is also the solution to the program in (8) but subject to the additional constraint of obtaining consensus.

Proposition 8 contains the central conceptual insight of this paper. Since the additional constraint of consensus can only lower the (optimized) shareholder value relative to the case with a supervisory board where such a constraint is not imposed, Proposition 8 shows that the value to shareholders of allocating decision making authority to the board is exactly equal to the cost of imposing the constraint of consensus. By obtaining consensus, shareholders in effect make management willing to hand back decision-making authority to the board.

Proposition 8 is interesting for reasons that go beyond determining the optimal allocation of authority. For instance, we may think of the case with an advisory board as a case where the final decision is determined by voting among the general body of shareholders and where management owns or controls the majority of voting shares. In such a case, Proposition 8 says that the expected welfare of minority shareholders is exactly equal to their expected payoff when they are in the majority but face the constraint of ensuring that the board has to obtain minority consensus with its decisions.

\textsuperscript{9}Even when consensus obtains, the two parties only agree with the decision but typically do not agree on the value of the decision.
Finally, Proposition 8 when combined with Proposition 7 immediately allows us to obtain the following result on the alignment of the optimal advisory board.

**Proposition 9** The shareholder value maximizing advisory board is weakly more management aligned than the shareholder value maximizing supervisory board: \( \hat{b}_d^* \geq \hat{b}_s^* \). When \( y_H \leq 1 + b_m \), \( \hat{b}_d^* = b_m \), while if \( y > 1 + b_m \), \( \hat{b}_d^* = b_d^* \in [0, b_m] \)

The first part of the result follows from observing that if consensus does not obtain under the optimal supervisory board then to obtain consensus it is necessary to raise \( b_d \). The second part follows from observing via Proposition 7 that the only possibility of obtaining consensus when \( y_H \leq 1 + b_m \) is by making the board share the management’s interests. The third part of Proposition 9 is interesting when combined with Proposition 4 for it shows that the shareholders may sometimes prefer to commit to withhold information from management via choosing a board with bias \( b_d^{**} < b_m \). Such a board provides vague or imprecise advice to management that preserves managerial doubt about insisting on his agenda when \( x \) and \( y \) are both large, leading management to voluntarily forego the investment. This may may benefit shareholders enough to overcome the value loss created by the conflict between the advisory board and management.

INSERT LEADING EXAMPLE, FIGURE AND INTUITION HERE

CONTRAST WITH UNINFORMED MANAGER BENCHMARK HERE

5 Extensions

The previous results on optimal advisory and supervisory boards, together with those on shareholder consensus, provide an argument in favor of ‘deep’ hierarchies in organizations that are mainly concerned with the efficacy of information transmission between stakeholders with conflicts of interest. In this section we shed further light on these results by briefly discussing some variations on the basic model. We begin by considering the possibility of obtaining shareholder consensus, i.e., ask if shareholders need to commit to respect the decisions of the board and to what extent this depends on the transparency of board–management deliberations. Subsequently, we consider the value to shareholders from committing to reveal the board’s information perfectly and discuss as well optimal mechanisms (without transfers). Finally, we discuss some alternative notions of agency as well as the possibility of optimal contracting.
5.1 Shareholder consensus and transparency

So far we have assumed that shareholders can commit to their ex-ante choice of a board and characterized the optimal board as well as the optimal allocation of authority. In this section we ask whether shareholders will agree with the final decision reached by the optimal board after deliberations with management. Our next result shows that the decisions of the optimal supervisory board will necessarily obtain ‘shareholder consensus.’

**Proposition 10** With the optimal board and at the optimal allocation of authority, shareholders will agree with the final decision, provided they do not observe the deliberations between the board and management.

The last result follows from the following two observations. First, shareholders will necessarily agree with a board decision to choose the status quo because both the management and the board are biased against the status quo relative to shareholders, and a recommendation from the board against the direction of its own bias will meet with shareholder approval. Furthermore, shareholders will also agree with the optimal board’s decision to choose the alternative, since the optimal board is weakly better in expected terms than a perfectly shareholder aligned board and since shareholders will always agree ex-post with the decision of a perfectly shareholder aligned board.

Proposition 10 shows that the optimal board may be designed as a two-tier structure with a perfectly shareholder aligned upper tier that has final decision-making authority. As long as the lower tier is also chosen optimally, and as long as the upper tier does not observe the details of the deliberations between the lower tier and management, shareholder aligned decision-making authority within the board is optimal for shareholders. In this sense, the basic message of Proposition 1 extends to the environment with expert boards that we have analyzed in this section.

A natural question is whether shareholder consensus obtains when shareholders observe the communication between (but not the signals of) board and management. It turns out that the answer to this question depends on details of the communication protocol between board and management, and the extent of managerial agency, even though changes in these protocols have no effect on decision-making (see Proposition 3). In particular, we can show that when \( b_m \) is not too small, then under the optimal supervisory board \( b_d^* \), shareholders will agree with the final decision of the board provided communication takes place according to the game \( bmb^* \) but not
if it takes place according to the game $mb^*$, even though the two games implement the same set of equilibrium decision rules. The intuition is that in game $bmb^*$, the less biased party relative to shareholders (the board) reveals more information publicly compared to the more biased party (management). This allows for shareholder consensus even though shareholder consensus will not obtain in the outcome-equivalent equilibrium of the game $mb^*$ where management discloses more information and the board only takes a final decision. While communication protocols have no decision-relevant effects, they may be important for third party consensus. The effect of shareholder activism (board membership) on shareholder value may therefore depend on the board architecture and the transparency and structure of deliberations between board members and management.

5.2 Optimal mechanisms & board architecture

What is the optimal mechanism in the class of games that we have studied? Translating well-known results (Holmstrom, 1984) to our context, we obtain the following result.

**Proposition 11** When the board has decision-making authority, shareholders would prefer to reveal $y$ to management. This can be implemented with a two-tier board where both tiers observe $y$, with the lower perfectly management aligned tier communicating first with management and then with the perfectly shareholder aligned upper tier that holds final decision making rights.

The board structure described above implements the optimal mechanism in the class of all mechanisms. In essence, it allows the management to take its own optimal action unless $y$ is higher than a cut-off value equal to $1 - b_m$, in which case the upper tier vetoes the investment. As such it has a close correspondence with the closed-loop system used in the U.S. Congress. In a sense it is also the same result as Proposition 1 with the modification that the shareholders can choose a different $b_d$ for every realization of $y$.

Two caveats present themselves however in evaluating Proposition 11. The first is interpretational. If the private information of the board is held by a particular board member (who is not perfectly aligned with other board members) then it will not be possible to share freely share this information across the two tiers as the result assumes. Indeed it is not difficult to see that if the information is held by a shareholder aligned upper tier member the situation is the same as a supervisory board of Section 3.2 with $b_d = 0$, whereas if the information is held by a management
aligned lower tier member then the situation is identical to the case with $b_d = b_m$. Instead, if the information arrives to all board members (from some expert outside auditor or consultant) then Proposition 11 is relevant. The second caveat to Proposition 11 arises out of noting that the result requires final decision making authority to be held by the board/shareholders. If instead management is entrenched and has hijacked authority, then from Proposition 9 and the discussion that follows, shareholders may prefer to withhold information from management.

5.3 Incentives and contracts

Throughout the paper we have completely ignored the possibility that the conflict of interest between management and the shareholders may be mitigated by providing management with incentives that align interests. Our results however are robust to at least limited forms of contracting.

Consider for example the case where $x - y$ corresponds to long run expected equity value. Suppose that managerial messages to the board are unverifiable and so are the decisions made. However, the shareholders can align management imperfectly with them by providing a long run (vested) equity share $\alpha \in (0, 1)$, keeping the residual fraction $1 - \alpha$ for themselves. In such a case, managerial payoffs from choosing the alternative is equal to $\alpha(x - y) + b_m$ whereas the payoff to shareholders is $(1 - \alpha)(x - y)$. Notice that for each choice of $\alpha$, the board bias $b_d$ typically matters for shareholder value. Indeed, our results above extend to this setting with the managerial bias recalibrated to equal $b_m/\alpha$ for each choice of $\alpha$. We leave for future drafts a full analysis of the interaction between the optimal choice of incentives $\alpha$ and governance $b_d$ as a function of the underlying agency $b_m$.

5.4 Alternative notions of bias

The notion of managerial bias $b_m$ can be interpreted in terms of private benefits of empire-building or control or even in terms of behavioral biases such as hubris or overconfidence. Similarly, the board’s bias $b_d$ has so far been interpreted in terms of “ideological sympathy” for management.

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10 Proposition 11 does show however, that shareholders will benefit if for instance, a management aligned board member privately “leaks” the board’s information to management, if decision-making authority is held by a shareholder aligned board.

11 Non-equity contracts such as executive options or golden parachutes may have added value in our setting.
This however is not the only possible interpretation of our set-up.

As one alternative, we may think of $b_m$ as cash or other resources that management directly “steals” from shareholders while implementing his favored decision after it is approved by the board. We may of course think of $b_d$ also in these terms. For instance, a board with bias $b_d$ may be one that obtains a “cut” of the manager’s share of the “loot” leaving only $b_m - b_d$ for management. Such a self-serving board may mitigate the agency problem not only by improving communication but also via directly forcing the manager to share his loot with the board and so reducing management’s incentives to push his biased agenda in the first place. On the other hand, if the board’s bias $b_d$ arises out of additional resources that the board also steals from shareholders, then shareholders have to trade off the gains from improved communication (as measured by $b_m - b_d$) against not only the distortion in decision-making but also the loss from increased looting by both the management and the board (as measured by $b_m + b_d$). We leave a full investigation of these issues for future drafts.

6 Related Literature

TO BE ADDED

7 Conclusion

TO BE ADDED

8 Appendix

Proof of Lemma 1.
Straightforward and omitted. ■

Proof of Proposition 1.
Straightforward and omitted. ■

Proof of Proposition 2.
To Be Added.

Proof of Proposition 3.
We start by comparing the games \( mb^* \) and \( bmb^* \). Notice that the decision rule implemented by any equilibrium of the short protocol \( mb^* \) can be replicated by an equilibria of the long protocol \( bmb^* \) with the board babbling in stage 1. We wish to prove the converse. To this end, we identify necessary properties of perfect Bayesian equilibria of the long protocol in what follows. Without loss of generality we suppose that all messages in \( \mathcal{M} \) are chosen by \( \sigma_d \) with strictly positive probability and for each \( \mu_d \), all messages in \( \mathcal{M} \) are chosen by \( \sigma_m \) with strictly positive probability. Also, the result is immediate when \( b_d = b_m \), so suppose \( b_d \neq b_m \).

Stage 3: decision making

Given messages \( \mu = (\mu_d, \mu_m) \), let \( x(\mu) = E[x|\mu_m, \mu_d] \). The board would like to choose the alternative \( y < x(\mu) + b_d \equiv t(\mu) \) (10)

Stage 2: manager’s message

Fix a message \( \mu_d \) in stage 1. The expected payoff to the management from sending a message \( \mu_m \) is

\[
G(t(\mu)|\mu_d)(x + b_m - E[y|y < t(\mu), \mu_d])
\]

where the expectation is arbitrarily defined if \( G(t(\mu)|\mu_d) = 0 \). The manager of type is indifferent between two messages \( \mu_m, \mu'_m \) with \( x(\mu_m, \mu_d) < x(\mu'_m, \mu_d) \) iff

\[
x + b_m = E[y|t(\mu_m, \mu_d) < y < t(\mu'_m, \mu_d), \mu_d]
\]

whenever the expectation is well-defined. It follows that almost every type must cannot be indifferent and that the equilibrium given each \( \mu_d \) is monotone partitional with at most a finite number of elements (using \( b_m \neq b_d \) and compactness of \( X,Y \)). Let \( \mathcal{C}(\mu_d) = \{c_i(\mu_d)\}_{i=0}^{k(\mu_d)} \) represent the partition (i.e., the cutoffs \( c_i(\mu_d) = E[y|t(\mu_i^{i}, \mu_d) < y < t(\mu_{i+1}, \mu_d), \mu_d] \) chosen by the sender after message \( \mu_d \), with \( k(\mu_d) < \infty \) the number of partition elements, \( [c_{i-1}(\mu_d), c_i(\mu_d)] \) the \( i \)th, \( x_i(\mu_d) = E[x|c_{i-1}(\mu_d) < x < c_i(\mu_d)] \).

Stage 1: board’s message

For any type \( y \), a message \( \mu_d \) matters for \( y \) if the final decision of \( y \) after sending message \( \mu_d \) depends on the message \( \mu_m \) that the management sends back. That is, \( k(\mu_d) > 1 \) and there exists \( i(y) \), with \( 1 \leq i(y) < k(\mu_d) \), with

\[
x_i(y)(\mu_d) + b_d < y < x_{i(y)+1}(\mu_d) + b_d
\]
For such a message $\mu_d$, type $y$ of the board chooses both the status quo and the alternative with strictly positive probability—the former if $x$ is announced to be in the $i$th interval for $i \leq i(y)$ and the latter otherwise.

No type $y$ may have a message that matters if, for instance, $k(\mu_d) = 1$ for all $\mu_d$. In such equilibria, management is babbling in stage 2 and the board learns no information from management. The decision rules implemented by these equilibria can be replicated by equilibria of the short protocol where the management babbles. Accordingly, in what follows we restrict attention to equilibria in which $k(\mu_d) > 1$ for some $\mu_d$ that is sent in equilibrium with strictly positive probability.

However, a particular type $y$ also may not have a message that matters if for instance $y < x_1(\mu_d) + b_d \equiv t_1(\mu_d)$ for all $\mu_d$. Such types choose the alternative in equilibrium regardless of the message they send and the message sent back by the management. Similarly, types $y > x_k(\mu_d)(\mu_d) + b_d \equiv t_k(\mu_d)$ for all $\mu_d$ choose the status quo in equilibrium regardless of the message they send and the message sent back by the management. Notice from (11) that the behavior of such types of the board does not affect the incentives of the manager, since the manager’s message matters only when the board has the complementary set of types $Y^*$ for which there is some message that matters.

If $Y^*$ is empty, then the board learns no information from the management and the management learns no information from the board. Clearly, such equilibria can be implemented by the short protocol and accordingly we focus on the case where the set $Y^*$ is non-empty, indeed has strictly positive probability. By Blackwell’s theorem (Blackwell, 1963), every type $y \in Y^*$ that has a message that matters strictly prefers every message that matters for it to every message that does not. Indeed, the expected payoff to $y$ from sending a message $\mu_d$ that matters is

$$[1 - F_m(c_{i(y)+1}(\mu_d))]\left[E[x | x > c_{i(y)+1}(\mu_d)] + b_d - y\right]$$ (13)

Fix $y$ and consider any two messages $\mu_d$ and $\mu_d'$ that matter and are in the support of $\sigma_d(\cdot | y)$ with $c_{i(y)+1}(\mu_d) = c_{i(y)+1}(\mu_d')$. Type $y$ must be indifferent between all such messages. Next assume without loss of generality that $c_{i(y)+1}(\mu_d) < c_{i(y)+1}(\mu_d')$. Type $y$ is indifferent between the two messages if and only if

$$y = E[x | c_{i(y)+1}(\mu_d) < x < c_{i(y)+1}(\mu_d')] + b_d$$ (14)

It follows that no type $y$ can be indifferent between three or more messages that matter where
the corresponding \( c_{i(y)+1} \) are all distinct. Furthermore, only a zero measure of \( y \) can be indifferent between two distinct \( c_{i(y)+1} \)'s. Finally, from (11) and (14), using the compactness of \( X, Y \) and \( b_d \neq b_m \), there is a finite number of distinct \( c^*(y) \)'s.

Consider the following new strategy profile in game \( bmb^* \). Each \( y \in Y^* \) reveals the \( c^*(y) \in (0,1) \) that it induces in the original equilibrium (ignoring the set of measure zero that induces multiple distinct \( c^* \)). Each \( y \in Y - Y^* \) that always chooses the alternative reveals that it belongs to this set (equivalently, \( c^*(y) = 0 \)) and similarly each \( y \in Y - Y^* \) that always chooses the status quo reveals that it belongs to this set (equivalently, \( c^*(y) = 1 \)). Given an announced cutoff \( c^* \), management reveals whether or not \( x > c^* \), i.e., recommends the alternative or the status quo. The board follows the management's advice if it is behaved according to the prescribed profile in stage 1, behaving sequentially rationally otherwise.

We show now that this new strategy profile is an equilibrium of the game \( bmb^* \). Clearly, stage 3 behavior of the board under the new profile is sequentially rational, from the definition of \( c^* \) (see (12) in particular). Consider now the stage 2 behavior of the management given an announced or inferred cutoff \( c^* \). In the original equilibrium, consider types \( y \in Y^* \) who induce the same cutoff \( c_i(\mu_d) = c^* \) by sending some message \( \mu_d \) and for some \( i, 1 \leq i < k(\mu_d) \). Notice using (11) that

\[
c^* + b_m = c_i(\mu_d) + b_m = E[y | t(\mu^i_m, \mu_d) < y < t(\mu^{i+1}_m, \mu_d), \mu_d]
\]

But in the conditioning event in the last expression above is exactly the set of \( y \in Y^* \) who announce the cutoff \( c^* \), using (12) and (10). It follows management management would choose exactly the cutoff \( c^* \) when it knows that the board has announced a (desired) cutoff \( c^* \). Finally, if in stage 1 a type \( y \in Y^* \) behaves as prescribed and reveals its own \( c^*(y) \), then it induces the decision rule to choose the alternative iff \( x > c^*(y) \), the same as in the original equilibrium. If it achieves a strictly higher payoff by revealing some other \( c^* \neq c^*(y) \) and then following a sequential rational decision rule, it could have achieved the same payoff by following the same decision rule but deviating to some other message \( \mu'_d \), a contradiction with the original equilibrium. The same argument shows also that all types \( y \notin Y^* \) also find it optimal to behave as specified. It is immediate that this implements the same decision rule as the original equilibrium almost everywhere.

We show now that this decision rule can also be implemented in an equilibrium of the game \( mb^* \). Consider the management strategy where the management discloses the interval \([c^*_{j-1}, c^*_j]\), \( j \geq 1 \), to which \( x \) belongs. If management is believed to following this strategy, then following the
message corresponding to the interval \([c_{j-1}^*, c_j^*]\), it is sequentially rational for the board to choose the alternative if \(y < t_j^* = E[x | c_{j-1}^* < x < c_j^*] + b_d\). But then it is an equilibrium for management to follow the prescribed strategy since in particular the indifferent types of the management satisfy \(c_j^* + b_m = E[y | t_{j-1}^* < y < t_{j+1}^*]\). This completes the proof of the equivalence of \(bmb^*\) with \(mb^*\). The equivalence of \(bmb^*\) with \(b^*mb^*\) as well as \(b^*m^*b\) follows from the construction of the alternative decision equivalent strategy profile in game \(bmb^*\) provided above. 

**Proof of Proposition 5.**

To Be Added.

**Proof of Proposition 6.**

Identical to the proof of Proposition 6 and therefore omitted.

**Proof of Proposition 4.**

See proof of Proposition 5 in Dessein.

**Proof of Proposition 7.**

To Be Added.

**Proof of Proposition 8.** (For the case \(y_L = 0\) and \(c_1 = 0\))

Suppose the manager has authority. Let \(V(b_d, b_m)\) be shareholder value at \(b_d, b_m\) and let \(\hat{V} = V(b_m, b_m)\). We can write

\[
V(b_d, b_m) = \hat{V} + \sum_{i=1}^{M} \left[ \int_{c_i}^{\max(y_L, \min(t_i-b_m, 1)]} \int_{x+b_m}^{t_i} (x-y) \frac{1}{y_H} dy dx - \int_{\max(y_L, t_i-b_m)}^{c_i} \int_{t_{i-1}}^{x+b_m} (x-y) \frac{1}{y_H} dy dx \right]
\]

Using the equilibrium relationship \(c_i = \frac{d_{i-1}+d_i}{2} - b_m\) for all \(i\), and adjusting for the first and last intervals, we have, in the case \(y_L < b_m\),

\[
V(b_d, b_m) = \hat{V} - \frac{1}{24y_H} \sum_{i=2}^{M} t_i^3 - \frac{1}{6y_H} (b_m-t_1)^2(t_1+2b_m) + 1_{y_H>1+b_m} \frac{1}{6y_H} (y_H - (1 + b_m))^2 (y_H + 2b_m - 1)
\]

It follows immediately that if \(y_H \leq 1 + b_m\), then setting \(b_d = b_m\) is optimal. Since consensus obtains in this case the result follows.

Henceforth we assume \(y_H > 1 + b_m\) and write

\[
V(b_d, b_m) = \hat{V} - L_1 - L_2 + G
\]
where

\[ L_1(b_d, b_m) = \frac{1}{24y_H} \sum_{i=2}^{M} l_i^3 \]  

(17)

\[ L_2(b_d, b_m) = \frac{1}{6y_H}(b_m - t_1)^2(t_1 + 2b_m) \]  

(18)

are the two losses from conflict of interest and

\[ G(b_m) = \frac{1}{6y_H} (y_H - (1 + b_m))^2(y_H + 2b_m - 1) \]  

(19)

Consider a \( M \) message equilibrium without consensus under \( m \)-authority and suppose that no \( M+1 \) message without consensus exists. It suffices to show that \( \frac{\partial V(b_d, b_m)}{\partial b_d} > 0 \) in such a case.

Using CS (expression 25 on pp 1442)

\[ \sum_{i=2}^{M} l_i^3 = \sum_{j=1}^{N} l_j^3 = \frac{(y_H - t_1)^3}{(M - 1)^2} + 4B^2(y_H - t_1)((M - 1)^2 - 1) \]

so that

\[ L_1 = \frac{1}{24y_H} \left[ \frac{(y_H - t_1)^3}{(M - 1)^2} + 4B^2(y_H - t_1)((M - 1)^2 - 1) \right] \]

\[ \frac{\partial L_1}{\partial b_d} = -\frac{1}{24y_H} \left[ \frac{3(y_H - t_1)^2}{(M - 1)^2} + 4M(M - 2)B^2 \frac{\partial t_1}{\partial b_d} + 8M(M - 2)B(y_H - t_1) \right] < 0 \]

since \( \frac{\partial t_1}{\partial b_d} = \frac{2M(M - 1)}{2M - 1} > 0 \) using the equilibrium formulas (This will prove the result when \( c_1 > 0 \)).

Furthermore, \( \frac{\partial G}{\partial b_d} = 0 \) and

\[ \frac{\partial L_2}{\partial b_d} = \frac{1}{2y_H} (t_1^2 - b_m^2) \frac{\partial t_1}{\partial b_d} \]  

(20)

Then,

\[ \frac{\partial L}{\partial b_d} = \frac{\partial L_1}{\partial b_d} + \frac{\partial L_2}{\partial b_d} \]

\[ = -\frac{M(M - 2)B(y_H - t_1)}{3y_H} + \left[ \frac{1}{2y_H} (t_1^2 - b_m^2) - \frac{1}{24y_H} \left( \frac{3(y_H - t_1)^2}{(M - 1)^2} + 4M(M - 2)B^2 \right) \right] \frac{\partial t_1}{\partial b_d} \]

Notice that this is increasing in \( t_1 \).

For there to be no \( M+1 \) partition equilibrium with \( c_1 = 0 \), it is necessary and sufficient that

\[ b_d < b_d^{M+1} = \left( \frac{2M^2b_m - y_H}{2M(M + 1)} \right) \]

But then \( \frac{\partial L}{\partial b_d} < 0 \) at \( b_d = b_d^{M+1} \) if and only if
Substituting into (21), in particular,

\[
- \frac{M(M-2)B(y_H - t_1)}{3y_H} + \left[ \frac{1}{2y_H} (t_1^2 - b_m^2) - \frac{1}{24y_H} \left( \frac{3(y_H - t_1)^2}{(M-1)^2} + 4M(M-2)B^2 \right) \right] \frac{\partial t_1}{\partial b_d} < 0
\]

Using \( \frac{\partial t_1}{\partial b_d} = \frac{2M(M-1)}{2M-1} \) and the formula for \( t_1 \), considerable algebraic manipulation yields the equivalent inequality

\[
y_H^2 (4M^3 - 3M + 7) + 4b_m y_H (2M^4 - 9M^2 - M^3 - M - 1) + 4b_m^2 M (3M + 4M^3 + 1) > 0
\]

All coefficients are positive in the case \( M > 2 \). When \( M = 2 \), the expression is

\[
33y_H^2 - 60y_H b_m + 312b_m^2 = 3 (11y_H^2 - 20y_H b_m + 104b_m^2)
\]

which is also positive since \( y_H > 1 + b_m \).

**Proof of Proposition 9.**

To Be Added.

**Proof of Proposition 10.**

To Be Added.

**Proof of Proposition 11.**

To Be Added.

**References**


INCOMPLETE BIBLIOGRAPHY